$\mathcal{R}^2$ Quantum Corrected Scalar Field Inflation

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Units and Conventions

$$c = 1$$

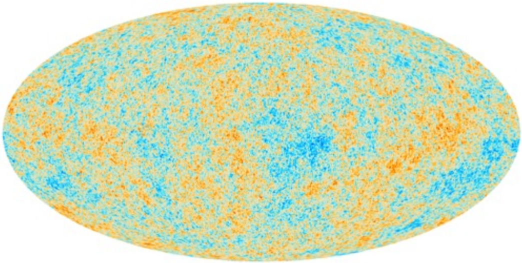
$$\hbar = 1$$

$$M_p = 1/\kappa = (8\pi G)^{-1/2}$$

$$(-, +, +, +)$$

Contents

CMB



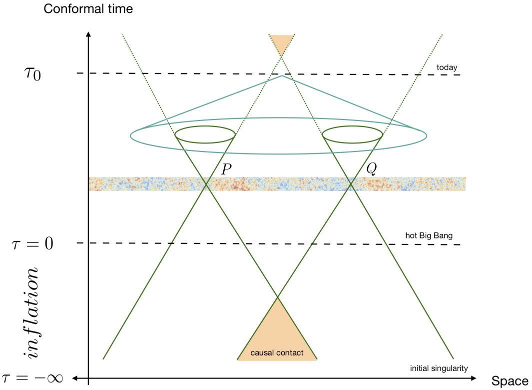
- Temperature today $\simeq 2.73$ K.
- Tiny anisotropies in the CMB temperature hint in fluctuations in the early universe energy density.
- What is the cause of these fluctuations? What insight in the primordial universe can the CMB provide?

Inflation

- **Cosmic Inflation** : a period of accelerating expansion in the early universe.
- It is theorised to have taken place 10^{-34} s after the Big Bang.
- Inflation follows the quantum epoch of the newborn universe, an era governed by high energy physics.
- It may hold a key role to the solution of some Big Bang puzzles such as: **the horizon problem**(the homogeneity problem), **the flatness problem**, the non-existence of magnetic monopoles.

Horizon Problem

During the inflationary era the comoving Hubble radius, $(H\alpha)^{-1}$ decreases, thus regions that now do not seem to have ever been in causal contact, in fact were before inflation. This causality established the homogeneity we observe.



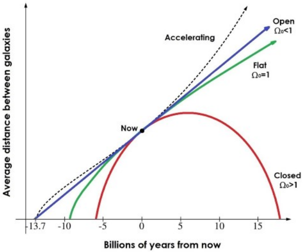
Horizon Problem

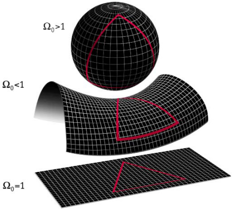
- The inflationary expansion amplifies microscopic quantum fluctuations in the primordial energy density, making them larger than the physical horizon at that time. They stay frozen outside the horizon.
- They re-enter the horizon at a later time, causing the perturbations depicted in the CMB and leading to the large scale structure we observe today via gravitational attraction.
- The study of the perturbations after they re-enter the horizon, using CMB data, can shed light in the physical conditions of the primordial universe.

Flatness Problem

$$1 - \Omega(\alpha) = \frac{-k}{(\alpha\mathcal{H})^2}\ , \ k = \begin{cases} 0 & \text{flat universe} \\ 1 & \text{closed universe} \\ -1 & \text{open universe} \end{cases} \tag{1}$$

where $\Omega(\alpha) = \frac{\rho}{\rho_{crit}}$, $\rho_{crit} = 3\mathcal{H}^2$.





The comoving Hubble radius decreases during inflation, driving the universe towards flatness. $\Omega = 1$ is an attractor during inflation.

Modeling Inflation

- The components of the universe are treated as fluids, thus the comoving Hubble horizon for a universe dominated by a fluid with equation of state w, is $(\alpha\mathcal{H})^{-1} \sim \alpha^{\frac{1}{2}(1+3w)}$. Therefore, the condition for inflation is:

$$\frac{d(\alpha\mathcal{H})^{-1}}{dt} < 0 \Rightarrow \frac{d^2\alpha}{dt^2} > 0 \Rightarrow \rho + 3p < 0 \tag{2}$$

- Using a scalar field is the simplest way to achieve a negative pressure and describe the inflationary era.
  The most general four dimensional scalar field Lagrangian containing at most two derivatives is,

$$\mathcal{S}_\varphi = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2}Z(\varphi)g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \mathcal{V}(\varphi) + h(\varphi)\mathcal{R}\right) \tag{3}$$

Modeling Inflation

$$\mathcal{S}_\varphi = \int \mathrm{d}^4 x \sqrt{-g} \left(\frac{1}{2} Z(\varphi) g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + \mathcal{V}(\varphi) + h(\varphi) \mathcal{R} \right)$$

When the scalar fields are evaluated at their vacuum configuration, the scalar field must be either minimally coupled of conformally coupled. We focus on the first possibility, so $Z(\varphi) = -1$ and $h(\varphi) = 1$.

The study of inflation can extend to other actions, such as using: non-minimally coupled fields, non-canonical kinetic terms, more than one fields, modified gravity, combinations.

Slow-Roll Approximation

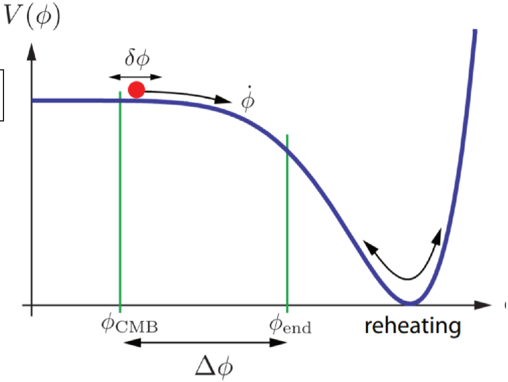
In the slow-roll regime:

$$\boxed{|\dot{\mathcal{H}}| \ll \mathcal{H}^2 \ , \ |\ddot{\mathcal{H}}| \ll |\mathcal{H}\dot{\mathcal{H}}|}$$

$$\boxed{\dot{\varphi}^2 \ll \mathcal{V}(\varphi)}$$

In order for inflation to last a sufficient time:

$$|\ddot{\varphi}| \ll |3\mathcal{H}\dot{\varphi}|, |\mathcal{V}_\varphi|$$



Canonical Scalar Field In The Presence Of $f(R)$ Gravity

- In our study we consider a $f(\mathcal{R})$ theory for gravity, thus our action has the form:

$$\mathcal{S} = \int d^4x\sqrt{-g}\left(\frac{f(\mathcal{R})}{2\kappa^2} - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \mathcal{V}(\varphi)\right) \tag{4}$$

- The background metric is a flat Friedmann-Robertson-Walker (FRW) metric:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)\sum_{i=1}^{3}\mathrm{d}x_i^2, \tag{5}$$

Canonical Scalar Field In The Presence Of $f(R)$ Gravity

$$\mathcal{S} = \int d^4x\sqrt{-g}\left(\frac{f(\mathcal{R})}{2\kappa^2} - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \mathcal{V}(\varphi)\right)$$

Using that $\mathcal{H} = \frac{\dot{a}}{a}$ and that for the flat FRW metric:

$$\mathcal{R} = 12\mathcal{H}^2 + 6\dot{\mathcal{H}}\,,\;\; \dot{\mathcal{R}} = 24\mathcal{H}\dot{\mathcal{H}} + 6\ddot{\mathcal{H}}, \tag{6}$$

and varying the action (13) with respect to the metric and the scalar φ, one can obtain the field equations:

$$3f_\mathcal{R}\mathcal{H}^2 = \frac{Rf_\mathcal{R} - f}{2} - 3\mathcal{H}\dot{F}_\mathcal{R} + \kappa^2(\frac{1}{2}\dot{\varphi}^2 + \mathcal{V}(\varphi))\;, \tag{7}$$

$$-2f_\mathcal{R}\dot{\mathcal{H}} = \kappa^2\dot{\varphi}^2 + \ddot{f}_\mathcal{R} - \mathcal{H}\dot{f}_\mathcal{R}\;, \tag{8}$$

$$\ddot{\varphi} + 3\mathcal{H}\dot{\varphi} + \mathcal{V}' = 0\;, \tag{9}$$

Canonical Scalar Field In The Presence Of $\mathcal{R}^2$ Gravity

We study a minimally coupled canonical scalar field inflation in the presence of a $\mathcal{R}^2$ quantum corrected modified gravity.
We use the following $f(\mathcal{R})$ gravity:

$$\boxed{f(\mathcal{R}) = \mathcal{R} + \frac{\mathcal{R}^2}{36M^2}} \tag{10}$$

The **motivation** for using combined scalar field and higher order gravity Lagrangians is that both scalar fields (string moduli) and higher order gravity terms originate from the underlying **string theory**. Those higher order terms appear as relics of the preceding quantum epoch of the universe in the inflationary Langrangian.

$$\mathcal{S}_{eff} = \int \mathrm{d}^4x\sqrt{-g}\Big(\Lambda_1 + \Lambda_2\mathcal{R} + \Lambda_3\mathcal{R}^2 + \Lambda_4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \Lambda_5\mathcal{R}_{\mu\nu\alpha\beta}\mathcal{R}^{\mu\nu\alpha\beta}$$
$$+ \Lambda_6\Box\mathcal{R} + \Lambda_7\mathcal{R}\Box\mathcal{R} + \Lambda_8\mathcal{R}_{\mu\nu}\Box\mathcal{R}^{\mu\nu} + \Lambda_9\mathcal{R}^3 + \mathcal{O}(\partial^8) + \ldots\Big)$$

Canonical Scalar Field In The Presence Of $\mathcal{R}^2$ Gravity

In this case, the Friedmann and Raychaudhuri equations become:

$$3\mathcal{H}^2 + \frac{3}{M^2}\mathcal{H}^2\dot{\mathcal{H}} = \frac{\dot{\mathcal{H}}^2}{2} - \frac{\ddot{\mathcal{H}}\mathcal{H}}{M^2} + \kappa^2\left(\frac{1}{2}\dot{\varphi}^2 + \mathcal{V}(\varphi)\right) \tag{11}$$

$$-2\dot{\mathcal{H}} - \frac{2}{M^2}\dot{\mathcal{H}}^2 = -\frac{\ddot{\mathcal{H}}\mathcal{H}}{M^2} + \kappa^2\dot{\varphi}^2 \tag{12}$$

We are using the slow-roll approximation which demands that

$$\dot{\mathcal{H}} \ll \mathcal{H}^2 \ , \ \ddot{\mathcal{H}} \ll \mathcal{H}\dot{\mathcal{H}}, \tag{13}$$

As in the single scalar field scenario, we require the scalar field to satisfy:

$$\frac{1}{2}\dot{\varphi}^2 \ll \mathcal{V}(\varphi) \ , \ \ddot{\varphi} \ll \mathcal{H}\dot{\varphi} \tag{14}$$

Canonical Scalar Field In The Presence Of $\mathcal{R}^2$ Gravity

Assuming that the following approximations hold true:

$$\boxed{\frac{\dot{\mathcal{H}}^2}{M^2} \ll \mathcal{H}^2, \ \ \frac{\dot{\mathcal{H}}^2}{M^2} \ll \mathcal{V}(\varphi)} \tag{15}$$

$$\boxed{\frac{2\kappa^2\dot{\varphi}^2}{M^2} \ll 1} \tag{16}$$

The field equations take the final form:

$$\mathcal{H}^2 \simeq \frac{\kappa^2\mathcal{V}(\varphi)}{3} + \mathcal{O}(\frac{\kappa^2\dot{\varphi}^2}{2}\mathcal{H}^2), \tag{17}$$

$$\dot{\mathcal{H}} \simeq -\frac{\kappa^2\dot{\varphi}^2}{2} - \frac{\kappa^4\dot{\varphi}^4}{4M^2} \tag{18}$$

$$\dot{\varphi} \simeq -\frac{\mathcal{V}'}{3\mathcal{H}} \tag{19}$$

Canonical Scalar Field In The Presence Of $\mathcal{R}^2$ Gravity

The slow-roll indices for a general $f(\mathcal{R}, \varphi)$ theory are defined as:

$$\epsilon_1 = -\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}, \quad \epsilon_2 = \frac{\ddot{\varphi}}{\mathcal{H}\dot{\varphi}}, \quad \epsilon_3 = \frac{\dot{f}_{\mathcal{R}}}{2\mathcal{H} f_{\mathcal{R}}}, \quad \epsilon_4 = \frac{\dot{E}}{2\mathcal{H}E} \tag{20}$$

where,

$$E = f_{\mathcal{R}} + \frac{3\dot{f}_{\mathcal{R}}^2}{3\kappa^2\dot{\varphi}^2} \tag{21}$$

In our case, that is $f(\mathcal{R}) = \mathcal{R} + \frac{\mathcal{R}^2}{36M^2}$, they become:

$$\epsilon_1 = \frac{1}{2\kappa^2}\left(\left(\frac{\mathcal{V}'}{V}\right)^2 + \frac{1}{6M^2}\left(\frac{\mathcal{V}'}{V}\right)^2 \frac{\mathcal{V}'^2}{V}\right), \quad \epsilon_2 = -\frac{\mathcal{V}''}{\kappa^2 V} + \epsilon_1, \quad \epsilon_3 = \frac{\epsilon_1}{-1 - \frac{3M^2}{2\mathcal{H}^2} + \frac{\epsilon_1}{2}},$$

$$E = 1 + \frac{2\mathcal{R}}{36M^2} + \frac{8}{3\kappa^2 M^4}\frac{\mathcal{H}^2\dot{\mathcal{H}}^2}{\dot{\varphi}^2}, \quad \dot{E} = \frac{4\mathcal{H}\dot{\mathcal{H}}}{3M^2} + \frac{16}{3\kappa^2 M^4 \dot{\varphi}^4}\left(\mathcal{H}\dot{\mathcal{H}}^3\dot{\varphi}^2 - \mathcal{H}^2\dot{\mathcal{H}}^2\dot{\varphi}\ddot{\varphi}\right)$$

Canonical Scalar Field In The Presence Of $\mathcal{R}^2$ Gravity

Our main focus lies in the observable quantities, the spectral index of the primordial curvature perturbations, the ratio of tensor over the scalar power spectrum and the tensor spectral index, given by:

$$n_{\mathcal{S}} = 1 - \frac{4\epsilon_1 + 2\epsilon_2 - 2\epsilon_3 + 2\epsilon_4}{1-\epsilon_1}, \;\; r = 16(\epsilon_1 + \epsilon_3), \;\; n_{\mathcal{T}} \sim -2(\epsilon_1 + \epsilon_3)$$

As well as in the amplitude of scalar perturbations $\mathcal{P}_\zeta(k)$, which is defined as follows,

$$\mathcal{P}_\zeta(k) = \frac{k_*^3}{2\pi^2} P_\zeta(k_*)\,,$$

a generalized $f(\mathcal{R},\varphi)$ gravity in the slow-roll approximation it becomes:

$$\mathcal{P}_\zeta(k) = \left(\frac{\mathcal{H}}{2\pi z}\left(1 - \epsilon_1 + (-2\epsilon_1 - \epsilon_2 + \epsilon_3 - \epsilon_4)\left(\ln\frac{1}{1-\epsilon_1} - 2 + \ln 2 + 0.57\right)\right)\right)^2$$

where $z = \frac{(\dot{\varphi}k)\sqrt{\frac{E(\varphi)}{f_{\mathcal{R}}/\kappa^2}}}{\mathcal{H}^2(\epsilon_3+1)}$ and for the first horizon crossing $k_* = a\mathcal{H}$.

Canonical Scalar Field In The Presence Of $\mathcal{R}^2$ Gravity

$$n_{\mathcal{S}} = 1 - \frac{4\epsilon_1 + 2\epsilon_2 - 2\epsilon_3 + 2\epsilon_4}{1 - \epsilon_1}, \quad r = 16(\epsilon_1 + \epsilon_3), \quad n_{\mathcal{T}} \sim -2(\epsilon_1 + \epsilon_3)$$

$$\mathcal{P}_\zeta(k) = \left(\frac{\mathcal{H}}{2\pi z} \left(1 - \epsilon_1 + (-2\epsilon_1 - \epsilon_2 + \epsilon_3 - \epsilon_4) \left(\ln \frac{1}{1 - \epsilon_1} - 2 + \ln 2 + 0.57 \right) \right) \right)^2$$

According to the Planck 2018 data [1807.06211], an inflationary phenomenology could be viable if it is compatible with the following constraints:

$$\boxed{n_{\mathcal{S}} = 0.9649 \pm 0.0042 \ , \ r < 0.064 \ , \ \mathcal{P}_\zeta(k) = 2.196^{+0.051}_{-0.06}}$$

The e-foldings number is computed by:

$$N = \int_{t_i}^{t_f} \mathcal{H} dt = \int_{\varphi_i}^{\varphi_f} \frac{\mathcal{H}}{\dot{\varphi}} d\varphi = \int_{\varphi_f}^{\varphi_i} \kappa^2 \frac{V}{\mathcal{V}'} d\varphi \qquad (22)$$

Inflation is believed to have lasted for $N \sim 50 - 70$ e-foldings.

Course of action

- We chose an inflationary potential to study
- We obtain φ_f by solving the equation $\epsilon_1(\varphi_f) = \mathcal{O}(1)$
- We solve $N = \int_{\varphi_f}^{\varphi_i} \kappa^2 \frac{V}{\mathcal{V}'} d\varphi$ analytically with respect to φ_i
- We can calculate the slow-roll indices for $\varphi = \varphi_i$ and, therefore, the scalar and tensor spectral indices and the tensor-to-scalar ratio as functions of the model's free parameters and the *e*-foldings number and test their compatibility with the Planck 2018 data.

We followed these steps for a simple power-law potential, the chaotic inflation potential, to study whether a viable inflationary phenomenology can occur.

Chaotic Inflation

We apply the framework developed in the case of a simple power-law scalar field potential:

$$\boxed{\mathcal{V}(\varphi) = \frac{\mathcal{V}_0}{\kappa^2}(\kappa\varphi)^2}$$

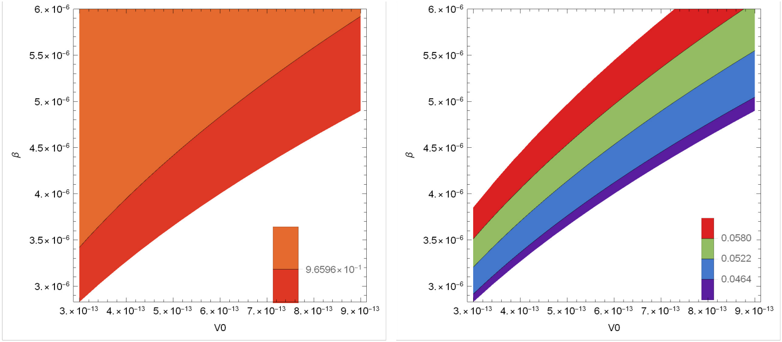
For this potential the slow-roll indices take the form:

$$\epsilon_1 = \frac{\frac{4}{\varphi^2} + \frac{8\mathcal{V}_0}{3M^2\kappa^2\varphi^2}}{2\kappa^2}, \quad \epsilon_2 = \frac{4\mathcal{V}_0}{3\kappa^4 M^2\varphi^2}, \quad \epsilon_3 = \frac{4\mathcal{V}_0\left(3\kappa^2 M^2 + 2\mathcal{V}_0\right)}{-27\kappa^4 M^4 - 6\kappa^2 M^2 \mathcal{V}_0\left(\kappa^2\varphi^2 - 1\right) + 4\mathcal{V}_0^2}$$

By solving the equation $\epsilon_1 \simeq \mathcal{O}(1)$ we obtain easily the final value of the scalar field at the end of inflation, which is, $\varphi_f = \frac{\sqrt{\frac{2}{3}}\sqrt{2\mathcal{V}_0 + 3M^2\kappa^2}}{M\kappa^2}$.

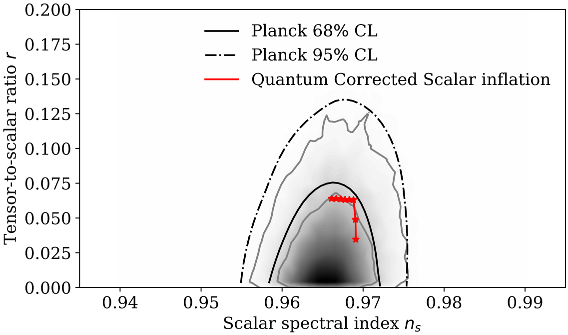
Using $N = \int_{\varphi_f}^{\varphi_i} \kappa^2 \frac{V}{\mathcal{V}'} d\varphi$, we find $\varphi_i = \frac{\sqrt{\frac{2}{3}}\sqrt{2\mathcal{V}_0 + 3M^2\kappa^2 + 6M^2 N\kappa^2}}{M\kappa^2}$.

Spectral Index and Tensor to Scalar Ratio



Contour plots of the spectral index of primordial curvature perturbations (left plot) and tensor-to-scalar ratio (right plot) at the first horizon crossing that correspond to values $n_{\mathcal{S}} = [0.9607, 0.9691]$ and $r < 0.064$ respectively and also satisfy our approximations, for $N = 60$, having set $M = \beta/\kappa$ and $\kappa = 1$.

Planck Likelihood Curves



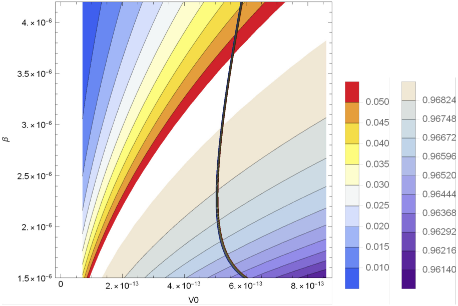
Confrontation of the $\mathcal{R}^2$ corrected chaotic inflation gravity model (red curve) with the 2018 Planck constraints for $N = [60, 67]$ and a selection of values for the free parameters that result in values of the observational indices in accordance with the latest Planck constraints, respecting the approximations following the theory.

Power Spectrum

There is an upper limit for the e-foldings number at $N = 67$, since for greater values there are no values of the free parameters that satisfy our approximations and the constraints on $n_{\mathcal{S}}$ and $\mathcal{P}_\zeta(k)$ at the same time!

$$\frac{\dot{\mathcal{H}}^2}{M^2} \ll \mathcal{H}^2, \quad \frac{\dot{\mathcal{H}}^2}{M^2} \ll \mathcal{V}(\varphi)$$

$$\boxed{\frac{2\kappa^2\dot{\varphi}^2}{M^2} \ll 1}$$



Spectral index of primordial curvature perturbations (lower contours), the factor $\frac{\kappa^2\dot{\varphi}^2}{M^2}$ (upper contours) and the amplitude of the scalar curvature perturbations (dark line) at the first horizon crossing, for $N = 68$.

Example

Our analysis of the free parameters' space showed that the amplitude of the scalar perturbations basically constrains severely the final model and it is compatible with the Planck constraints when the values for the free parameters are of order $\mathcal{V}_0 \sim \mathcal{O}(10^{-13})$ and $\beta \sim \mathcal{O}(10^{-6})$.

For example, if we set $\mathcal{V}_0 = 9.37 \times 10^{-13}$, $\beta = 6.8 \times 10^{-6}$ for $N = 60$. we get,

$$n_{\mathcal{S}} = 0.96611, \;\; r = 0.063968, \;\; n_{\mathcal{T}} = -0.00799592\,,$$

$$\mathcal{P}_\zeta(k) = 2.19216 \times 10^{-9}$$

We get red-tilted inflation and compatibility with the Planck constraints

$$\boxed{n_{\mathcal{S}} = 0.9649 \pm 0.0042 \;,\;\; r < 0.064 \;,\;\; \mathcal{P}_\zeta(k_*) = 2.196^{+0.051}_{-0.06}}$$

Summary and Conclusions

- We studied a generalized $f(\mathcal{R},\varphi)$ gravity framework, using a canonical minimally coupled scalar field in the presence of string theory originating higher order curvature quantum correction $\mathcal{R}^2$ term, in the string frame (Jordan frame) [Oikonomou:2022bqb]

- We derived the field equations and presented the final form of the slow-roll indices for this theory.

- For a test model we chose the chaotic inflation model, which is a non-viable single scalar field model when it is considered by itself.

Summary and Conclusions

- The $\mathcal{R}^2$-corrected chaotic inflation model yields a viable inflationary phenomenology, considering the spectral index of the primordial scalar curvature perturbations, the tensor-to-scalar ratio, the tensor spectral index and the amplitude of the scalar perturbations.
- The multiplicative parameter of the potential,$\mathcal{V}_0$, is constrained differently in comparison to the simple chaotic inflation model, due to the difference of the form of the amplitude of the scalar perturbations in the two cases.
- An extension of this study could be considering combinations of quantum corrections in the simple scalar field theory. The case of an $\mathcal{R}^2$-corrected Einstein-Gauss-Bonnet theory was studied in [2011.08680], and it was shown that a viable phenomenology can be obtained for these models, for both minimal and non-minimally coupled scalar field models. What has not been studied yet is considering $\mathcal{R}^3$ corrections.

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Thank you for your time!