

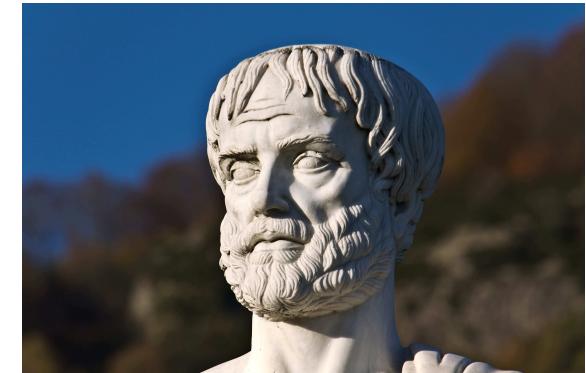
Tri-Resonant Leptogenesis and Charged LFV

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Upcoming article:

P. Candia da Silva, D. Karamitros, T. McKelvey, A.P., arXiv:2206.08352

Outline:

- Matter–AntiMatter Asymmetry and Leptogenesis
- Resonant and Tri-Resonant Leptogenesis
- Flavour Covariant Transport Equations
- Charged Lepton Flavour and Number Violation
- Numerical Estimates
- Conclusions

• Matter–AntiMatter Asymmetry and Leptogenesis

[N. Aghanim *et al.* [PLANCK Collaboration], Astron. Astrophys. 641 (2020) A6;
B.D. Fields, K.A. Olive, T.H. Yeh, C. Young, JCAP03 (2020) 010.]

$$\eta_B^{\text{CMB}} = \frac{n_B}{n_\gamma} = (6.104 \pm 0.058) \times 10^{-10}, \quad \eta_B^{\text{BBN+D}} = (6.148 \pm 0.161) \times 10^{-10}.$$

Sakharov's conditions for generating the BAU

(from an *initially B-symmetric Universe*): [A.D. Sakharov, JETP Lett. 5 (1967) 24.]

- B-violating interactions
- C and CP violation (assuming CPT invariance)
- Out-of-equilibrium dynamics

Popular Scenarios for Baryogenesis:

- **Baryogenesis through the decay of a heavy particle**

Out-of-equilibrium, *B-violating* decay of a heavy GUT particle,
e.g. in $SO(10)$.

[M. Yoshimura, PRL41 (1978) 281; S. Dimopoulos and L. Susskind, PRD18 (1978) 4500; . . .
K. S. Babu and R. N. Mohapatra, PRL109 (2012) 091803.]

- **Baryogenesis at the electroweak phase transition**

BAU generated by *(B + L)-violating* sphaleron interactions
at $T \sim T_c \approx 140$ GeV, through a 1st order phase transition.

[V.A. Kuzmin, V.A. Rubakov, M.E. Shaposhnikov, PLB155 (1985) 36;
MSSM: M. Carena, M. Quirós, C. Wagner '96; K. Rummukainen, M. Laine '98; . . .]

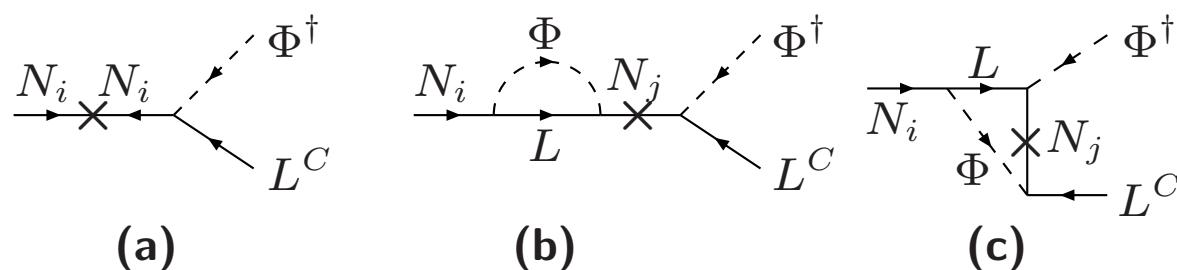
MSSM parameter space **squeezed** by **EDMs** and direct **LHC** searches:

[e.g., T. Cohen, D. E. Morrissey and A. Pierce, PRD86 (2012) 013009; . . .]

⇒ Baryogenesis through Leptogenesis

Out-of-equilibrium *L-violating* decays of heavy Majorana neutrinos produce a *net lepton asymmetry*, converted into the **BAU** through *(B + L)-violating sphaleron interactions*.

[M. Fukugita, T. Yanagida, PLB174 (1986) 45.]



$$\partial_\mu j_B^\mu = \partial_\mu j_L^\mu = i \frac{N_F}{8\pi} \left(-W_{\mu\nu}\tilde{W}^{\mu\nu} + B_{\mu\nu}\tilde{B}^{\mu\nu} \right).$$

In-equilibrium sphaleron rates:

$$120 \text{ GeV} \lesssim T \lesssim 10^{12} \text{ GeV}$$

– Models of Leptogenesis

- Hierarchical Leptogenesis

[M. Fukugita, T. Yanagida, PLB174 (1986) 45.]

Lower mass bound on $m_{N_1} \gtrsim 10^9$ GeV

[S. Davidson, A. Ibarra, PLB535 (2002) 25;

W. Buchmüller, P. Di Bari, M. Plümacher, Annals Phys. **315** (2005) 305.]

- Resonant Leptogenesis

[A.P. and T. Underwood, NPB692 (2004) 303,
and references therein]

- Dirac Leptogenesis

[K. Dick, M. Lindner, M. Ratz, D. Wright, PRL84 (2000) 4039.]

- Other scenarios: Non-thermal leptogenesis, Affleck–Dine, spontaneous leptogenesis, CPT-Violating Leptogenesis . . .

[For a review, see, M. Dine and A. Kusenko, Rev. Mod. Phys. **76** (2004) 1;
N. Mavromatos and S. Sarkar, Eur. Phys. J. C **73** (2013) 2359.]

- **The Flavourdynamics of Leptogenesis**

BAU can be generated from and protected in a single lepton flavour:

$$\frac{1}{3}B - L_{e,\mu,\tau}.$$

[e.g. J.A. Harvey, M.S. Turner, PRD42 (1990) 3344;
H. Dreiner, G.G. Ross, NPB410 (1993) 188;
J.M. Cline, K. Kainulainen, K.A. Olive, PRD49 (1994) 6394.]

Two sources of flavour effects:

- Heavy-neutrino Yukawa couplings $h_{l\alpha}^\nu$

[A.P., PRL95 (2005) 081602 [hep-ph/0408103];
T. Endoh, T. Morozumi and Z. h. Xiong, PTP111 (2004) 123;
A.P., T.E.J. Underwood, PRD72 (2005) 113001; P. Di Bari, NPB727 (2005) 318.]

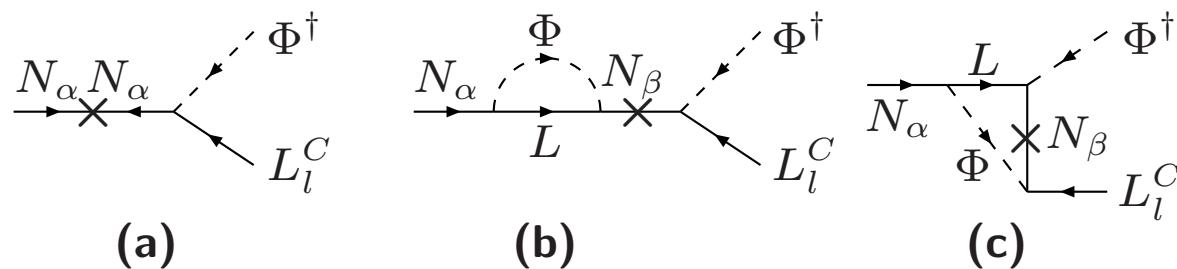
Modify BAU by many orders of magnitude, e.g. $> 10^6$, in RL models.

- Charged-lepton Yukawa couplings $h_{e,\mu,\tau}$

[E. Nardi, Y. Nir, J. Racker, E. Roulet, JHEP0601 (2006) 068;
A. Abada, S. Davidson, F. X. Josse-Michaux, M. Losada, A. Riotto, JCAP0604 (2006) 004.]

Modify BAU by up to 1-order of magnitude at $T \sim m_N \sim 10^9$ GeV.

- **Resonant and Tri-Resonant Leptogenesis**



Importance of self-energy effects (when $|m_{N_1} - m_{N_2}| \ll m_{N_{1,2}}$)

[J. Liu, G. Segré, PRD48 (1993) 4609;
 M. Flanz, E. Paschos, U. Sarkar, PLB345 (1995) 248;
 L. Covi, E. Roulet, F. Vissani, PLB384 (1996) 169.]

Importance of the heavy-neutrino width effects: Γ_{N_α}

[A.P., PRD56 (1997) 5431; NPB504 (1997) 61;
 A.P. and T. Underwood, NPB692 (2004) 303;
 (inspired by A.P., ZPC47 (1990) 95)]

Variants of Resonant Leptogenesis:

- **Soft RL**

[Y. Grossman, T. Kashti, Y. Nir, E. Roulet, PRL91 (2003) 251801;
G. D'Ambrosio, G. F. Giudice, M. Raidal, PLB575 (2003) 75.]

- **Radiative RL**

[R. Gonzalez Felipe, F. R. Joaquim and B. M. Nobre, PRD70 (2004) 085009;
G. C. Branco, A. J. Buras, S. Jager, S. Uhlig, A. Weiler, JHEP0709 (2007) 004;
G. C. Branco, R. Gonzalez Felipe, M. N. Rebelo, H. Serodio, PRD79 (2009) 093008.]

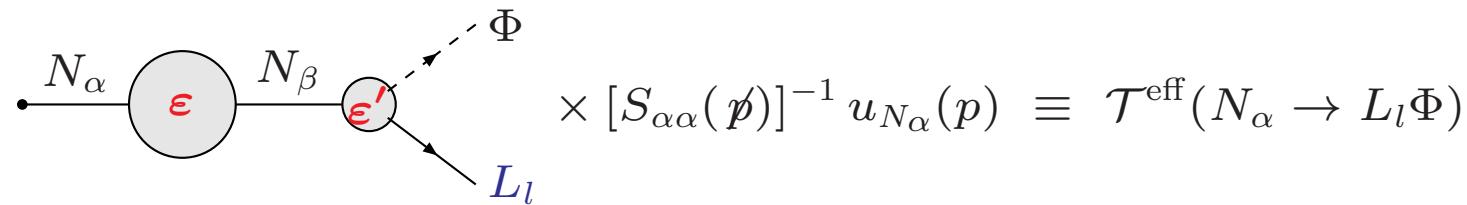
- **Coherent RL** (via sterile neutrino oscillations)

[E. K. Akhmedov, V. A. Rubakov, A. Y. Smirnov, PRL81 (1998) 1359;
T. Asaka, M. Shaposhnikov, PLB620 (2005) 17.]

- The Field-Theory of Resonant Leptogenesis:

[A.P., PRD56 (1997) 5431; NPB504 (1997) 61.]

LSZ-type formalism for mixing and decay of heavy Majorana neutrinos



2- N Mixing Model:

$$S_{\alpha\beta}(\not{p}) = \begin{pmatrix} \not{p} - m_{N_1} + \Sigma_{11}(\not{p}) & \Sigma_{12}(\not{p}) \\ \Sigma_{21}(\not{p}) & \not{p} - m_{N_2} + \Sigma_{22}(\not{p}) \end{pmatrix}^{-1}$$

[For 3- N mixing, see, A.P., T. Underwood, NPB692 (2004) 303;
P. Candia da Silva, D. Karamitros, T. McKelvey, A.P., arXiv:2206.08352.]

Effective (Resummed) Neutrino Yukawa Couplings:

$$\mathcal{T}^{\text{eff}}(N_\alpha \rightarrow L_l \Phi) = \mathbf{h}_{l\alpha} \bar{u}_l P_R u_{N_\alpha}$$

For 2- N mixing:

$$\begin{aligned}\mathbf{h}_{l\alpha} &= h_{l\alpha}^\nu + iB_{l\alpha} - \frac{i h_{l\beta}^\nu m_{N_\alpha} (m_{N_\alpha} A_{\alpha\beta} + m_{N_\beta} A_{\beta\alpha})}{m_{N_\alpha}^2 - m_{N_\beta}^2 + 2i A_{\beta\beta} m_{N_\alpha}^2} \\ \mathbf{h}_{l\alpha}^c &= h_{l\alpha}^{\nu*} + iB_{l\alpha}^* - \frac{i h_{l\beta}^{\nu*} m_{N_\alpha} (m_{N_\alpha} A_{\alpha\beta}^* + m_{N_\beta} A_{\beta\alpha}^*)}{m_{N_\alpha}^2 - m_{N_\beta}^2 + 2i A_{\beta\beta} m_{N_\alpha}^2}\end{aligned}$$

Lepton Flavour Asymmetries

[A.P., T. Underwood, PRD72 (2005) 113001.]

$$\delta_{\alpha l} \equiv \frac{\Gamma_{\alpha l} - \Gamma_{\alpha l}^C}{\sum_{l=e,\mu,\tau} (\Gamma_{\alpha l} + \Gamma_{\alpha l}^C)} = \frac{|\mathbf{h}_{l\alpha}|^2 - |\mathbf{h}_{l\alpha}^c|^2}{(\mathbf{h}^\dagger \mathbf{h})_{\alpha\alpha} + (\mathbf{h}^c \mathbf{h}^c)_{\alpha\alpha}}$$

ε' -type CP violation :

$$\varepsilon'_{N_\alpha} = \frac{\text{Im} (h^{\nu\dagger} h^\nu)_{\alpha\beta}^2}{(h^{\nu\dagger} h^\nu)_{\alpha\alpha} (h^{\nu\dagger} h^\nu)_{\beta\beta}} \left(\frac{\Gamma_{N_\beta}}{m_{N_\beta}} \right) f \left(\frac{m_{N_\beta}^2}{m_{N_\alpha}^2} \right),$$

where

$$\Gamma_{N_\beta} = \frac{(h^{\nu\dagger} h^\nu)_{\beta\beta}}{8\pi} m_{N_\beta}$$

ε -type CP violation :

$$\varepsilon_{N_\alpha} = \frac{\text{Im} (h^{\nu\dagger} h^\nu)_{\alpha\beta}^2}{(h^{\nu\dagger} h^\nu)_{\alpha\alpha} (h^{\nu\dagger} h^\nu)_{\beta\beta}} \frac{(m_{N_\alpha}^2 - m_{N_\beta}^2) m_{N_\alpha} \Gamma_{N_\beta}}{(m_{N_\alpha}^2 - m_{N_\beta}^2)^2 + m_{N_\alpha}^2 \Gamma_{N_\beta}^2}$$

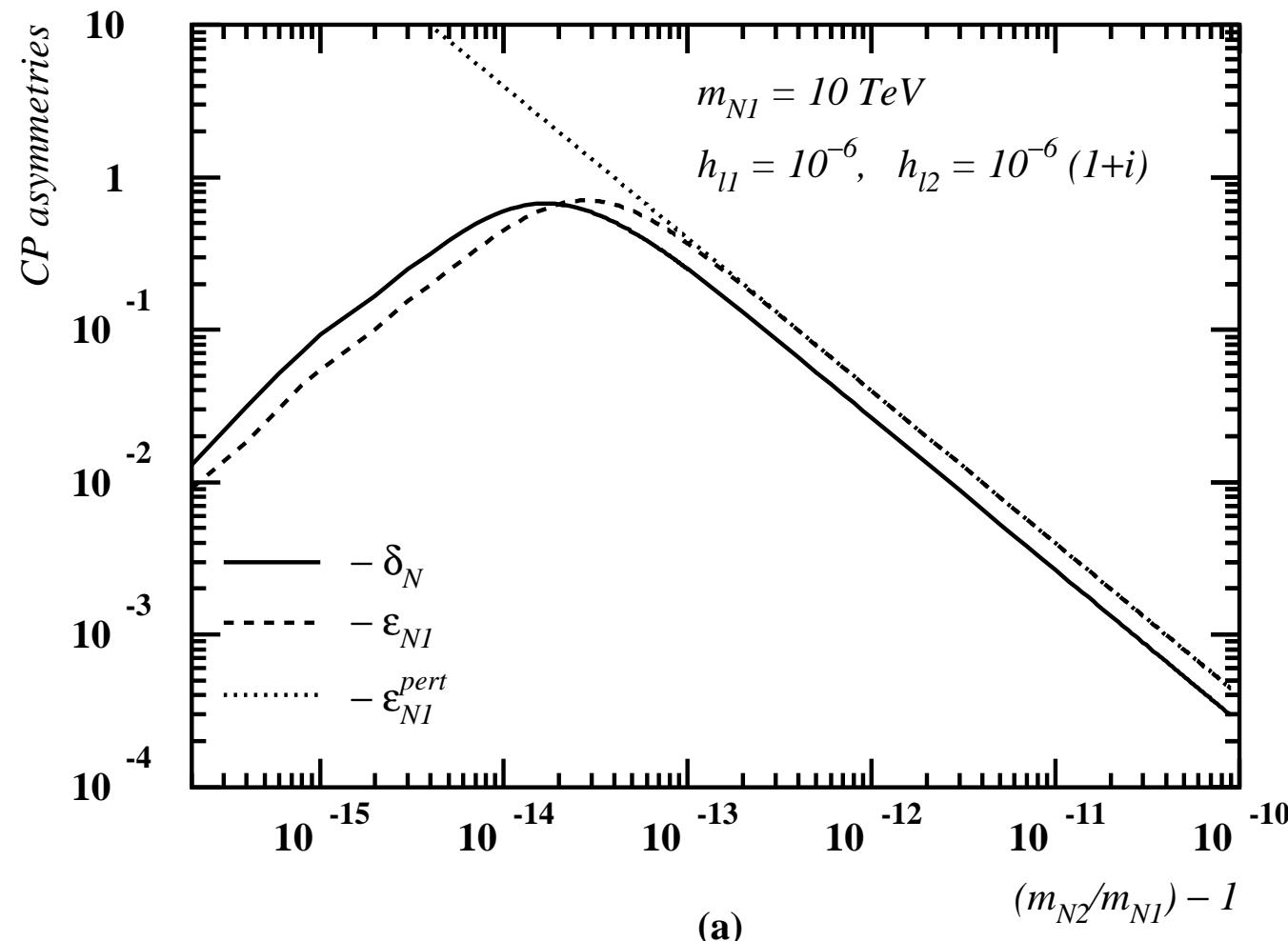
Note: $\varepsilon_{N_{1,2}}$ have the same sign.

Resonant conditions for $O(1)$ leptonic asymmetries:

[A.P., PRD56 (1997) 5431.]

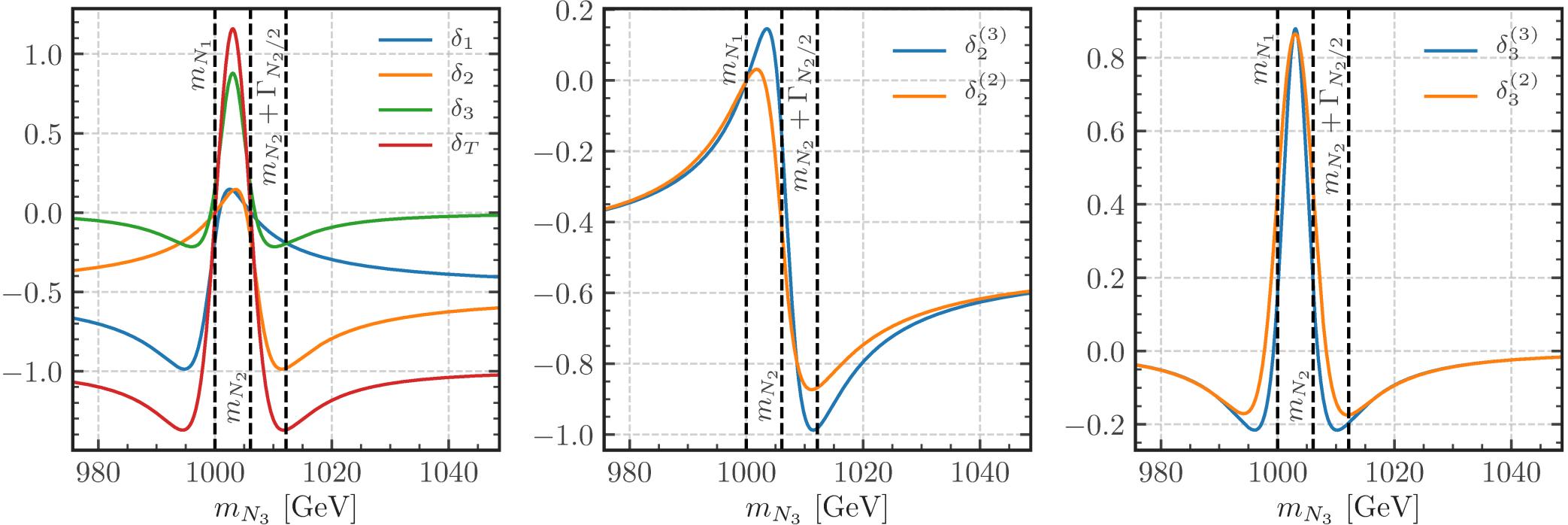
$$\Rightarrow m_{N_2} - m_{N_1} \sim \frac{1}{2} \Gamma_{N_{1,2}}$$

$$\Rightarrow \frac{\text{Im} (h^{\nu\dagger} h^\nu)_{\alpha\beta}^2}{(h^{\nu\dagger} h^\nu)_{\alpha\alpha} (h^{\nu\dagger} h^\nu)_{\beta\beta}} \sim 1$$



– Tri-Resonant leptonic asymmetries

[P. Candia da Silva, D. Karamitros, T. McKelvey, A.P., arXiv:2206.08352.]



Mass and Yukawa parameters (with approximate Z_6 symmetry):

$$m_{N_2} = m_{N_1} + \frac{\Gamma_{N_1}}{2}, \quad m_{N_3} = m_{N_2} + \frac{\Gamma_{N_2}}{2}, \quad \mathbf{h}_0^\nu = \begin{pmatrix} a & a\omega & a\omega^2 \\ b & b\omega & b\omega^2 \\ c & c\omega & c\omega^2 \end{pmatrix},$$

with $\omega = \exp(i\pi/3)$, and $a, b, c \lesssim 10^{-3}$ for $m_\nu \lesssim 0.05$ eV.

- Flavour Covariant Transport Equations

[E. W. Kolb and S. Wolfram, NPB172 (1980) 224.]

- Flavour Diagonal Boltzmann Equations

$$\frac{dn_a}{dt} + 3Hn_a = \sum_{aX' \leftrightarrow Y} \left(-\frac{n_a n_{X'}}{n_a^{\text{eq}} n_{X'}^{\text{eq}}} \gamma(aX' \rightarrow Y) + \frac{n_Y}{n_Y^{\text{eq}}} \gamma(Y \rightarrow aX') \right),$$

where n_a is the **number density**:

$$\begin{aligned} n_a(T) &= g_a \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \exp \left[-\left(\sqrt{\mathbf{p}^2 + m_a^2} - \mu_a(T) \right)/T \right] \\ &= \frac{g_a m_a^2 T e^{\mu_a(T)/T}}{2\pi^2} K_2 \left(\frac{m_a}{T} \right) \end{aligned}$$

and $\gamma(X \rightarrow Y)$ is the **collision term**:

$$\gamma(X \rightarrow Y) = \int d\pi_X d\pi_Y (2\pi)^4 \delta^{(4)}(p_X - p_Y) e^{-p_X^0/T} |\mathcal{M}(X \rightarrow Y)|^2.$$

– Flavour Diagonal BEs for Leptogenesis

[A.P., T.E. Underwood, **NPB692** (2004) 303; **PRD72** (2005) 113001.]

Define first

$$\eta^X \equiv n_X/n_\gamma , \quad z \equiv m_{N_1}/T , \quad H \equiv H(T = m_N) \approx 17 m_N^2/M_{\text{Planck}}$$

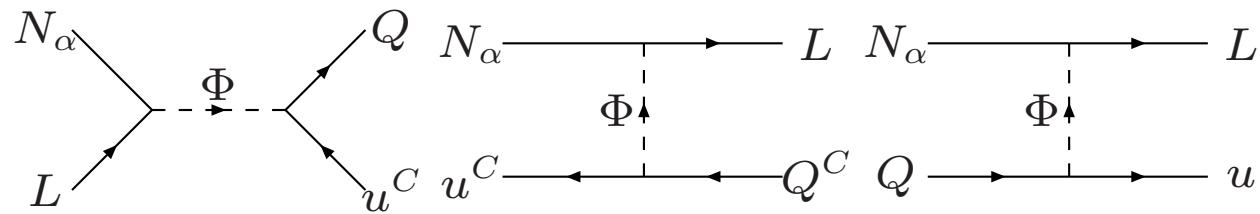
and the short-hands:

$$\begin{aligned} \gamma_Y^X &\equiv \gamma(X \rightarrow Y) + \gamma(\bar{X} \rightarrow \bar{Y}) \stackrel{\text{CPT}}{=} \gamma_X^Y , \\ \delta\gamma_Y^X &\equiv \gamma(X \rightarrow Y) - \gamma(\bar{X} \rightarrow \bar{Y}) \stackrel{\text{CPT}}{=} -\delta\gamma_X^Y . \end{aligned}$$

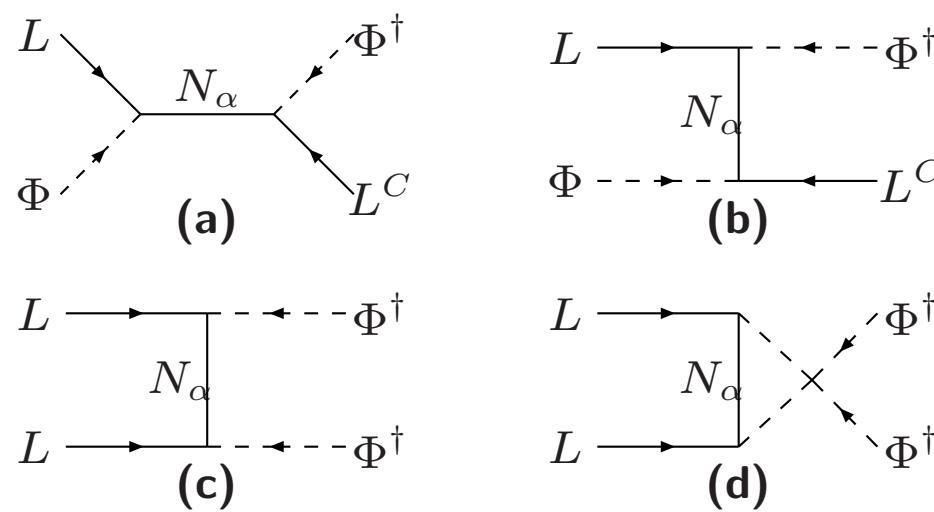
Write down the **Boltzmann equations**:

$$\begin{aligned} \frac{H n_\gamma}{z} \frac{d\eta_\alpha^N}{dz} &= \left(1 - \frac{\eta_\alpha^N}{\eta_{\text{eq}}^N}\right) \sum_k \gamma_{L_k \Phi}^{N_\alpha} + \dots \\ \frac{H n_\gamma}{z} \frac{d\delta\eta_l^L}{dz} &= \sum_\alpha \left(\frac{\eta_\alpha^N}{\eta_{\text{eq}}^N} - 1 \right) \delta\gamma_{L_l \Phi}^{N_\alpha} - \frac{2}{3} \delta\eta_l^L \sum_k \left(\gamma_{L_k^c \Phi^c}^{L_l \Phi} + \gamma_{L_k \Phi}^{L_l \Phi} \right) \\ &\quad - \frac{2}{3} \sum_k \delta\eta_k^L \left(\gamma_{L_l^c \Phi^c}^{L_k \Phi} - \gamma_{L_l \Phi}^{L_k \Phi} \right) + \dots \end{aligned}$$

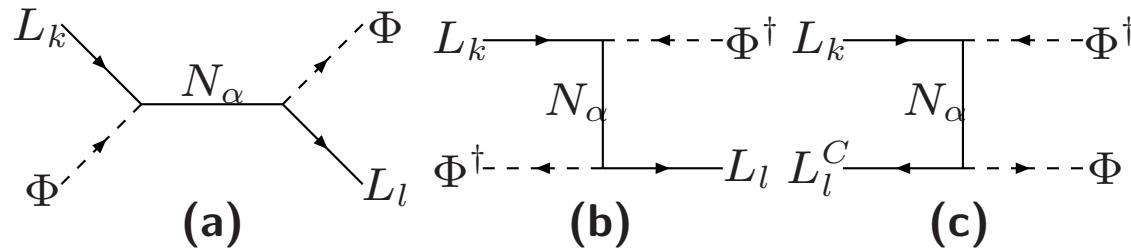
$\Delta L = 1$ scatterings involving L , N_α and quarks



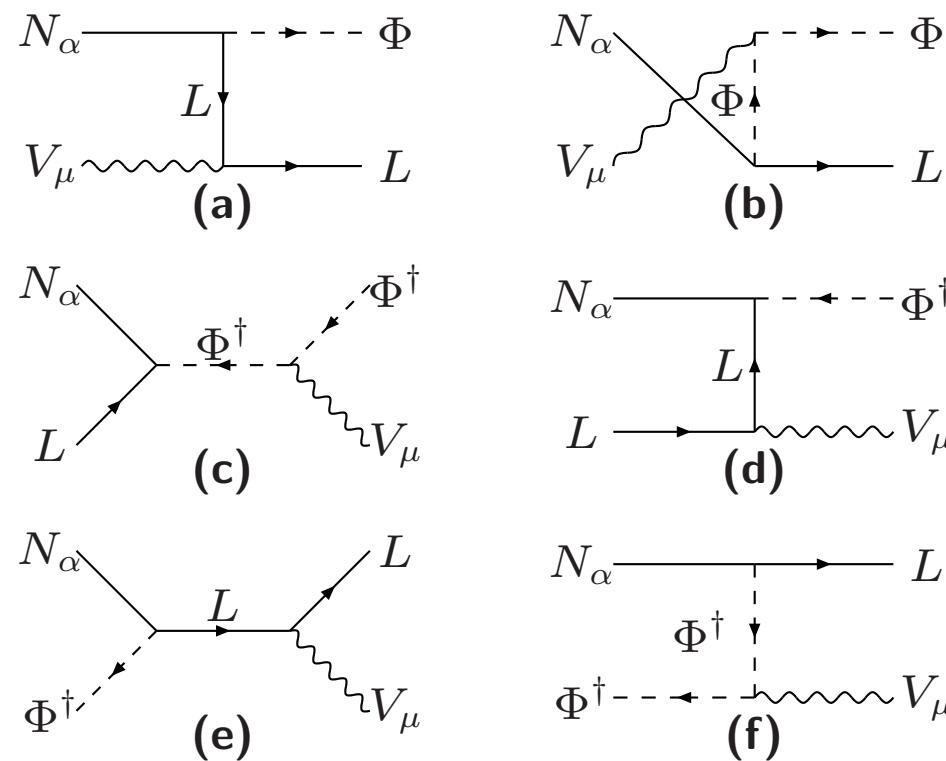
$\Delta L = 2$ scatterings involving L , Φ and N_α



$\Delta L = 0$ scatterings involving L , Φ and N_α



Gauge-mediated $\Delta L = 1$ scatterings



- Order-of-magnitude estimate of the BAU

Flavour-dependent decay width of heavy Majorana neutrino N_α :

$$\Gamma_{N_\alpha \rightarrow l} \equiv \Gamma(N_\alpha \rightarrow L_l \Phi) = (h^{\nu\dagger})_{\alpha l} h_{l\alpha}^\nu \frac{m_{N_\alpha}}{8\pi}$$

Define the effective wash-out K -factors:

$$K_l^{\text{eff}} \equiv \frac{\sum_{N_\alpha} \Gamma_{N_\alpha \rightarrow l}}{H}$$

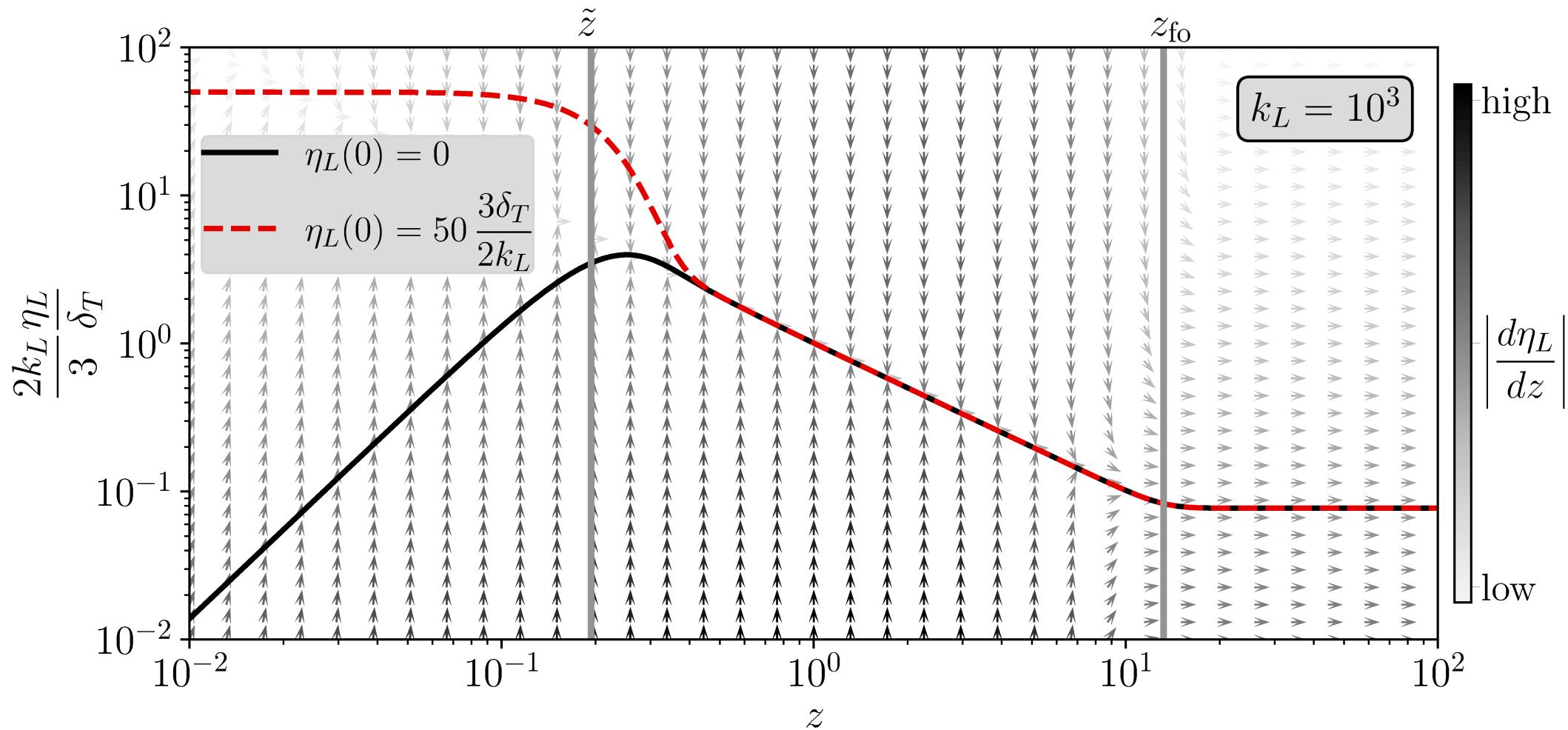
Estimate of the BAU (strong wash-out regime):

[F. Deppisch, A.P., PRD83 (2011) 076007.]

$$\eta_B^{\text{mix}} \sim -3 \cdot 10^{-2} \sum_{l=e,\mu,\tau} \frac{\delta_l^{\text{mix}}}{K_l^{\text{eff}} \min \left[m_N/T_c, 1.25 \ln(25 K_l^{\text{eff}}) \right]} .$$

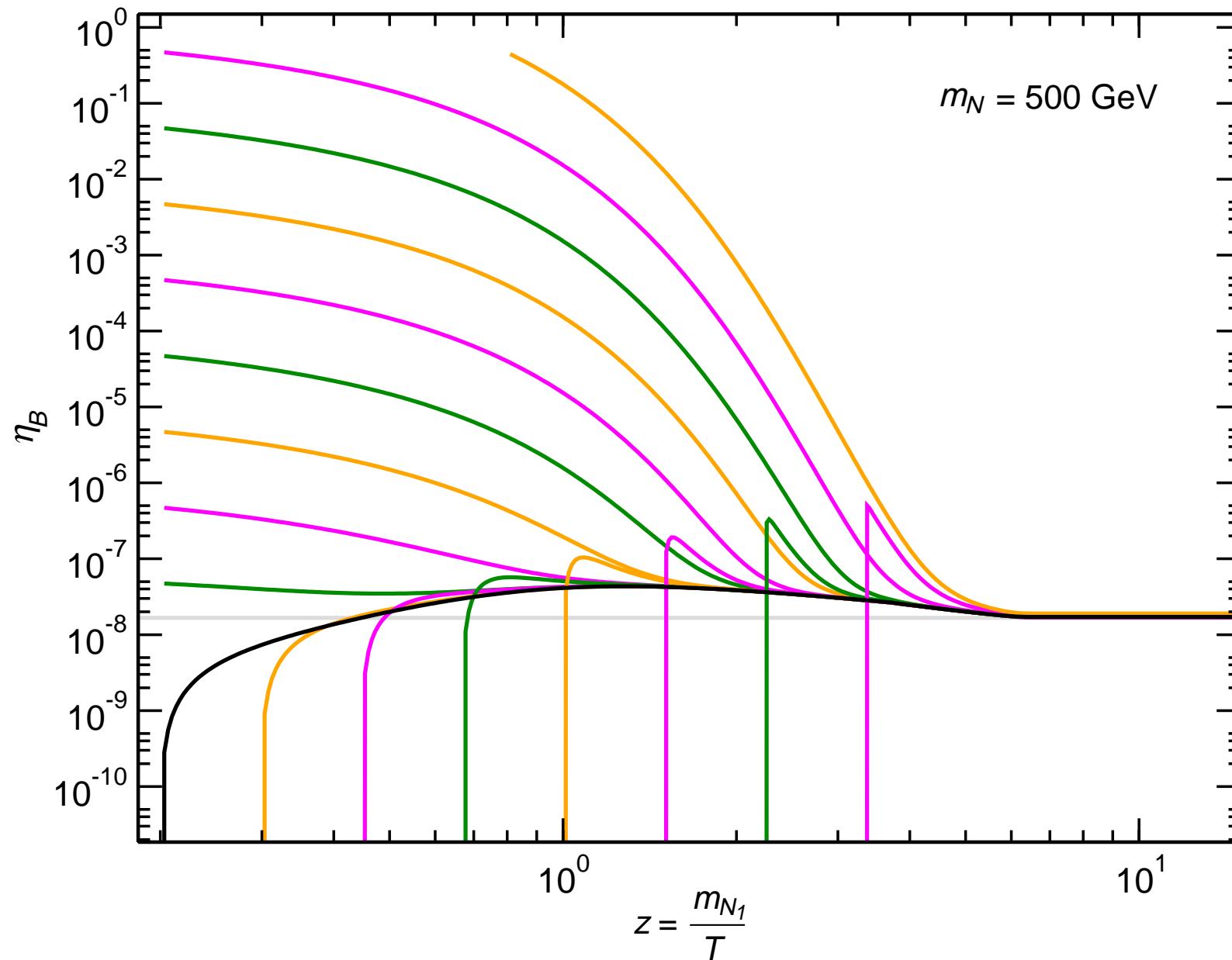
Independence of BAU on Initial Conditions

[P. Candia da Silva, D. Karamitros, T. McKelvey, A.P., arXiv:2206.08352.]



Resonant τ -Genesis

[A.P., T. Underwood, PRD72 (2005) 113001.]



– Flavour Covariant Rate Equations (Markovian approximation)

[P.S.B. Dev, P. Millington, A.P., D. Teresi, NPB886 (2014) 569.]

$$\frac{H n_\gamma}{z} \frac{d[\underline{\eta}^N]_\alpha^\beta}{dz} = -i \frac{n_\gamma}{2} \left[\mathcal{E}_N, \delta\eta^N \right]_\alpha^\beta + [\text{Re}(\gamma_{L\Phi}^N)]_\alpha^\beta - \frac{1}{2\eta_{\text{eq}}^N} \left\{ \underline{\eta}^N, \text{Re}(\gamma_{L\Phi}^N) \right\}_\alpha^\beta$$

$$\begin{aligned} \frac{H n_\gamma}{z} \frac{d[\delta\eta^N]_\alpha^\beta}{dz} &= -2i n_\gamma \left[\mathcal{E}_N, \underline{\eta}^N \right]_\alpha^\beta + 2i [\text{Im}(\delta\gamma_{L\Phi}^N)]_\alpha^\beta \\ &\quad - \frac{i}{\eta_{\text{eq}}^N} \left\{ \underline{\eta}^N, \text{Im}(\delta\gamma_{L\Phi}^N) \right\}_\alpha^\beta - \frac{1}{2\eta_{\text{eq}}^N} \left\{ \delta\eta^N, \text{Re}(\gamma_{L\Phi}^N) \right\}_\alpha^\beta \end{aligned}$$

$$\begin{aligned} \frac{H n_\gamma}{z} \frac{d[\delta\eta^L]_l^m}{dz} &= -[\delta\gamma_{L\Phi}^N]_l^m + \frac{[\underline{\eta}^N]_\beta^\alpha}{\tilde{\eta}_{\text{eq}}^N} [\delta\gamma_{L\Phi}^N]_l^m{}_\alpha^\beta + \frac{[\delta\eta^N]_\beta^\alpha}{2\tilde{\eta}_{\text{eq}}^N} [\gamma_{L\Phi}^N]_l^m{}_\alpha^\beta \\ &\quad - \frac{1}{3} \left\{ \delta\eta^L, \gamma_{L\tilde{c}\Phi\tilde{c}}^{L\Phi} + \gamma_{L\Phi}^{L\Phi} \right\}_l^m - \frac{2}{3} [\delta\eta^L]_k^n ([\gamma_{L\tilde{c}\Phi\tilde{c}}^{L\Phi}]_n^k{}_l^m - [\gamma_{L\Phi}^{L\Phi}]_n^k{}_l^m) \\ &\quad - \frac{2}{3} \left\{ \delta\eta^L, \gamma_{\text{dec}} \right\}_l^m + [\delta\gamma_{\text{dec}}^{\text{back}}]_l^m \end{aligned}$$

- **Unified Description of 3 Physically Distinct Phenomena:**

[P.S.B. Dev, P. Millington, A.P., D. Teresi, NPB886 (2014) 569.]

- **Resonant Mixing between Heavy Neutrinos,**

through: $\mathbf{h}_{l\alpha}$ and $\mathbf{h}_{l\alpha}^c$ in $[\gamma_{L\Phi}^N]_l{}^m{}_\alpha{}^\beta$ and $[\delta\gamma_{L\Phi}^N]_l{}^m{}_\alpha{}^\beta$.

- **Coherent Oscillations between Heavy Neutrinos** ($\Delta m_N \ll m_N$),

from $[\mathcal{E}_N, \underline{\eta}^N]$ and the rank-4 tensor term $\frac{[\delta\eta^N]_\beta{}^\alpha}{2\tilde{\eta}_{\text{eq}}^N} [\gamma_{L\Phi}^N]_l{}^m{}_\alpha{}^\beta$, yielding:

$$\delta\eta_{\text{osc}}^L \sim \frac{3}{2Kz} \frac{\text{Im}[(h^\dagger h)_{12}^2]}{(h^\dagger h)_{11}(h^\dagger h)_{22}} \frac{2(m_{N_1}^2 - m_{N_2}^2)m_N\Gamma_N}{(m_{N_1}^2 - m_{N_2}^2)^2 + \left(\frac{2m_N\Gamma_N \text{Im}[h^\dagger h]_{12}}{|[h^\dagger h]_{12}|}\right)^2},$$

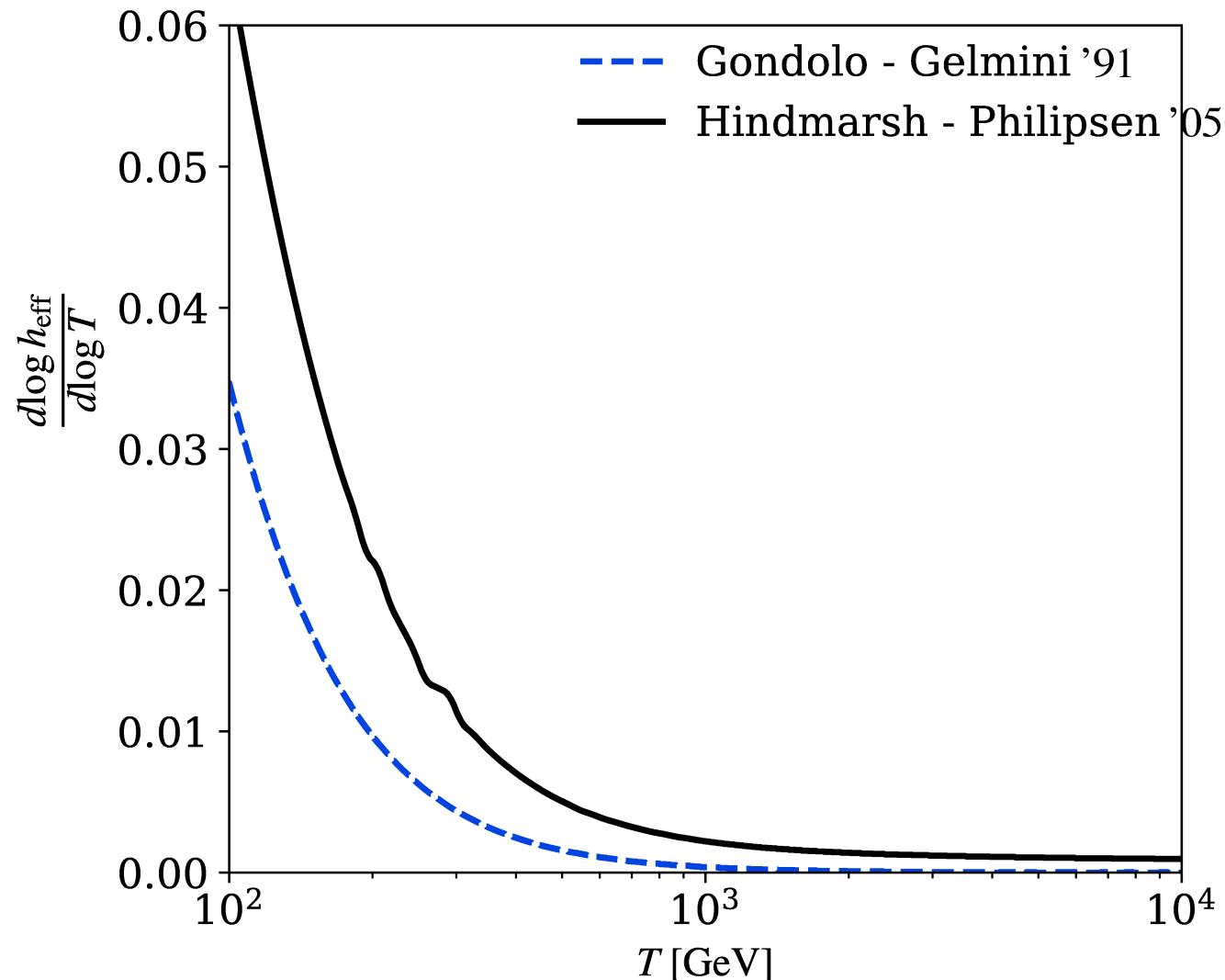
with $\Gamma_N = \frac{1}{2}\left(\Gamma_{N_1}^{(0)} + \Gamma_{N_2}^{(0)}\right)$. [Note: Different from the ARS mechanism.]

- **Decoherence Effects due to Charged Lepton Yukawa Couplings,**

from $-\frac{2}{3} \left\{ \delta\eta^L, \gamma_{\text{dec}} \right\}_l{}^m + [\delta\gamma_{\text{dec}}^{\text{back}}]_l{}^m$

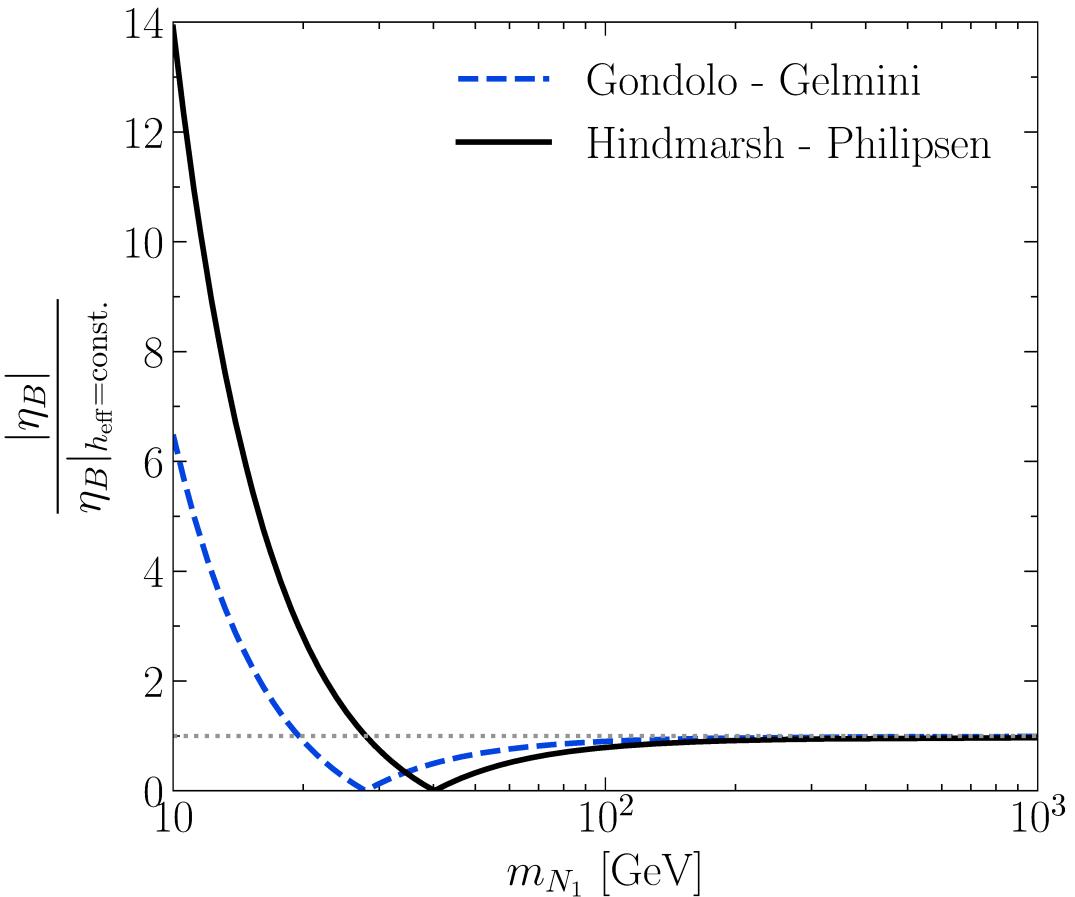
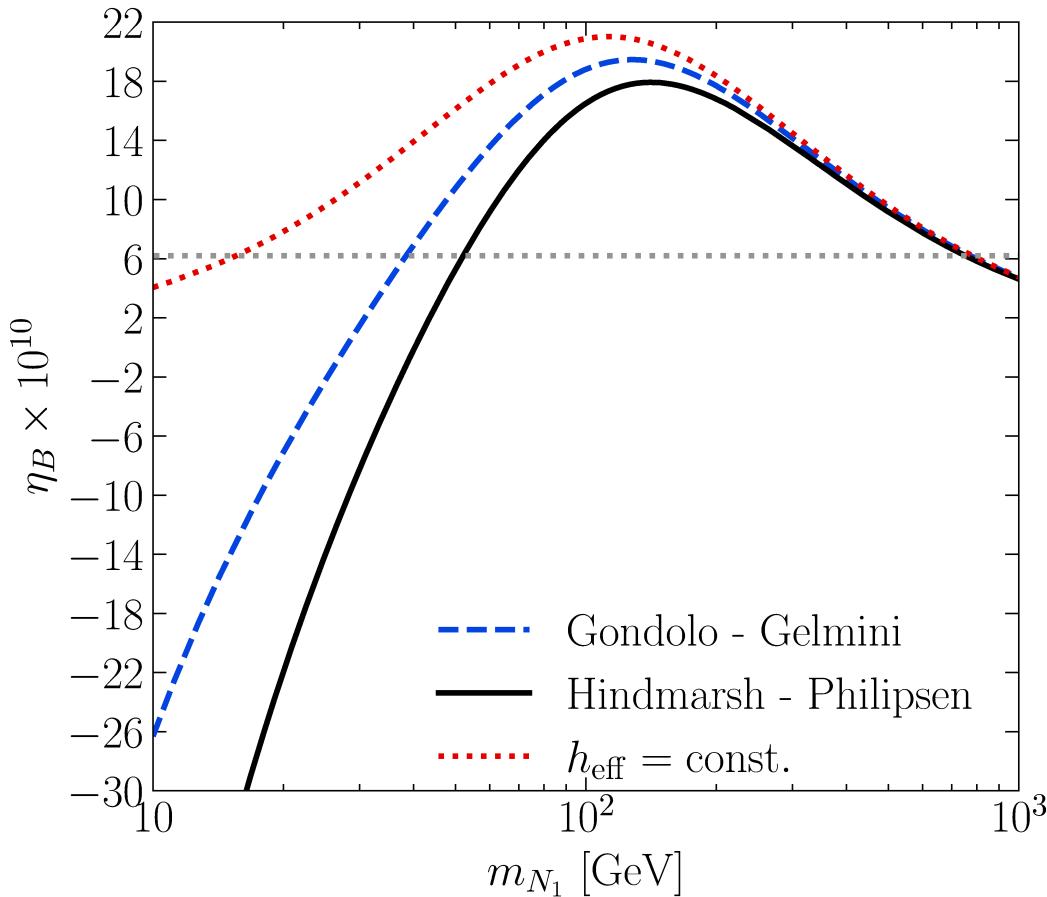
- The effect of varying relativistic dofs on Transport Equations

$$\rho(T) = \frac{\pi^2}{30} g_{\text{eff}}(T) T^4, \quad s(T) = \frac{\pi^2}{45} h_{\text{eff}}(T) T^3$$



Modification of the BAU predictions due to varying $h_{\text{eff}}(T)$

[P. Candia da Silva, D. Karamitros, T. McKelvey, A.P., arXiv:2206.08352.]



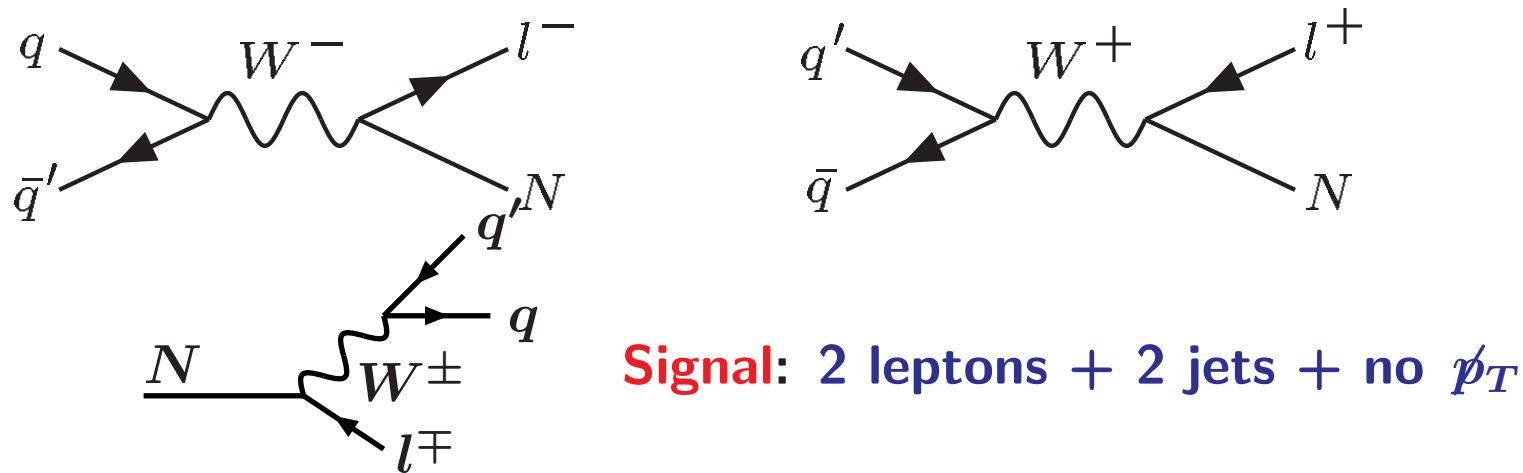
Isentropic FRW Universe ($ds/dt = -3Hs$):

$$\frac{d}{dt} = \frac{ds}{dt} \frac{dT}{ds} \frac{d}{dT} = H \delta_h^{-1} \frac{d}{dz}, \quad \text{with } \delta_h = 1 - \frac{1}{3} \frac{d \ln h_{\text{eff}}}{d \ln z}.$$

- **Charged Lepton Flavour and Number Violation**

- Heavy Majorana Neutrinos at the LHC

[A.P., ZPC55 (1992) 275; A. Datta, M. Guchait, A.P., PRD50 (1994) 3195;
 J. Kersten, A. Y. Smirnov, PRD76 (2007) 073005;
 F. del Aguila, J. A. Aguilar-Saavedra, R. Pittau, JHEP0710 (2007) 047.]



- **LNV signatures:** $pp \rightarrow e^+e^+, e^+\mu^+, e^-e^-, e^-\mu^-, e^-\tau^- \dots$

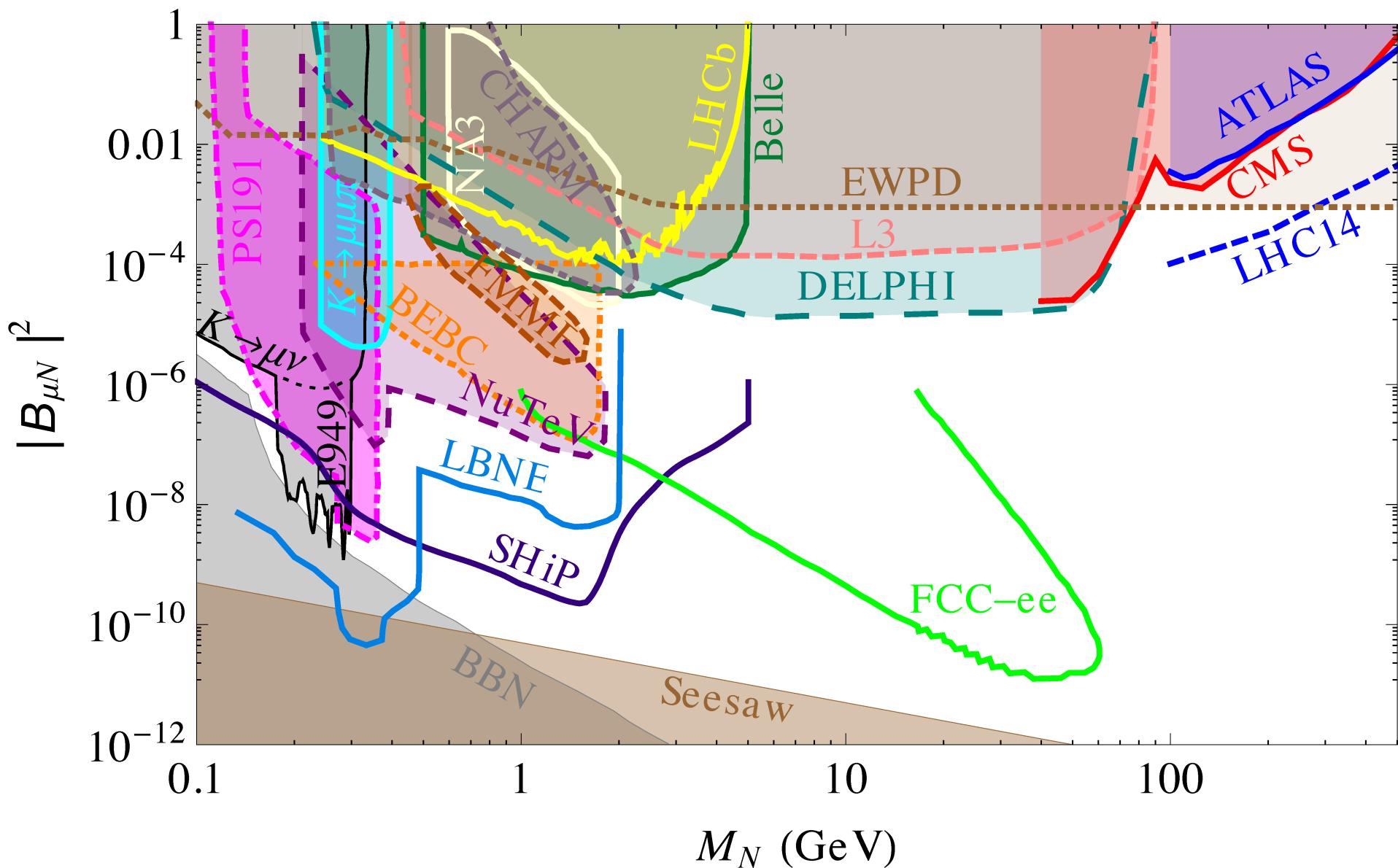
- **LFV signatures:** $pp \rightarrow e^+\mu^-, e^-\mu^+, e^-\tau^+ \dots$

- **CP Asymmetries**

[S. Bray, J.S. Lee, A.P., NPB786 (2007) 95.]

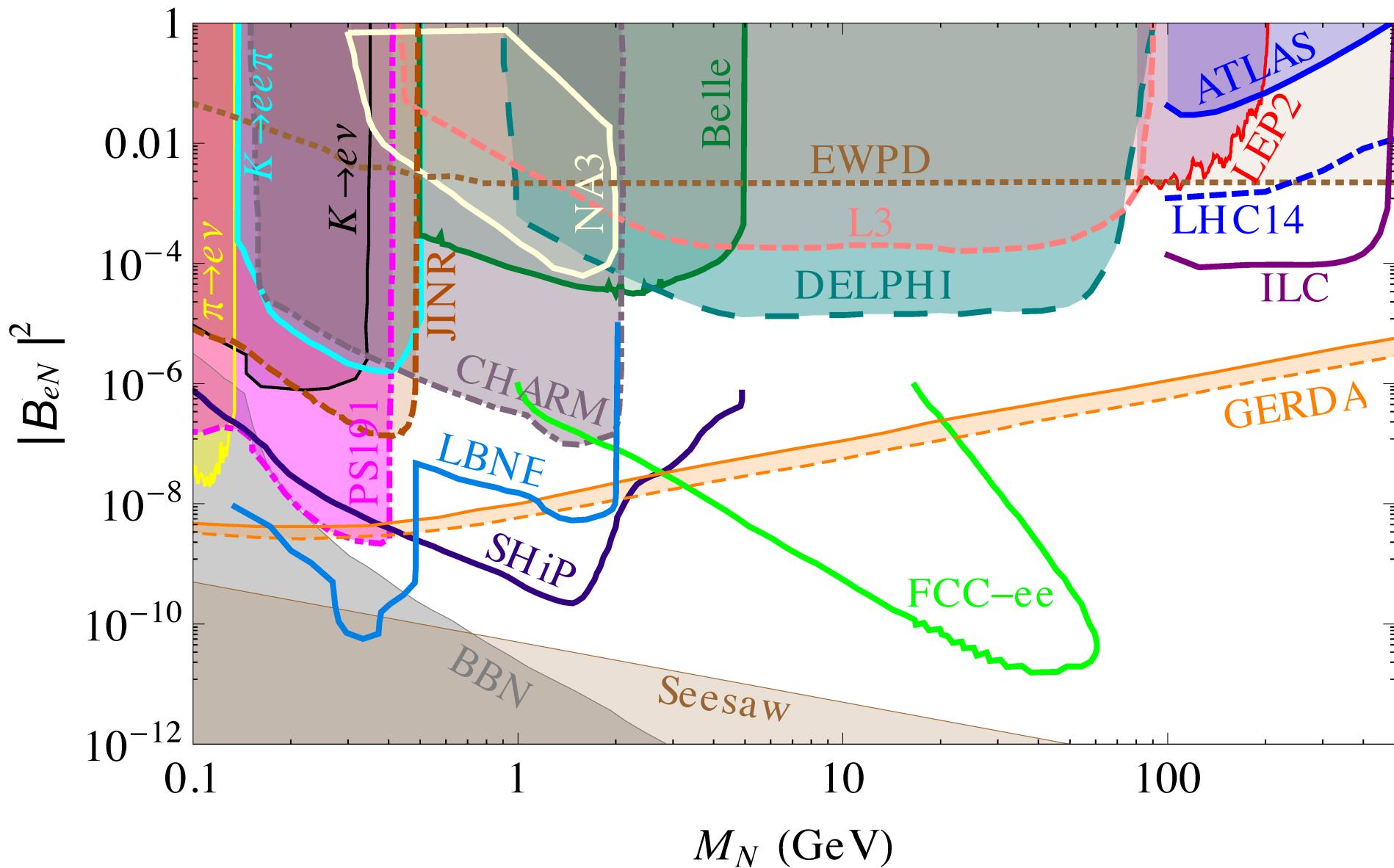
LHC and Other Constraints: μN -Sector

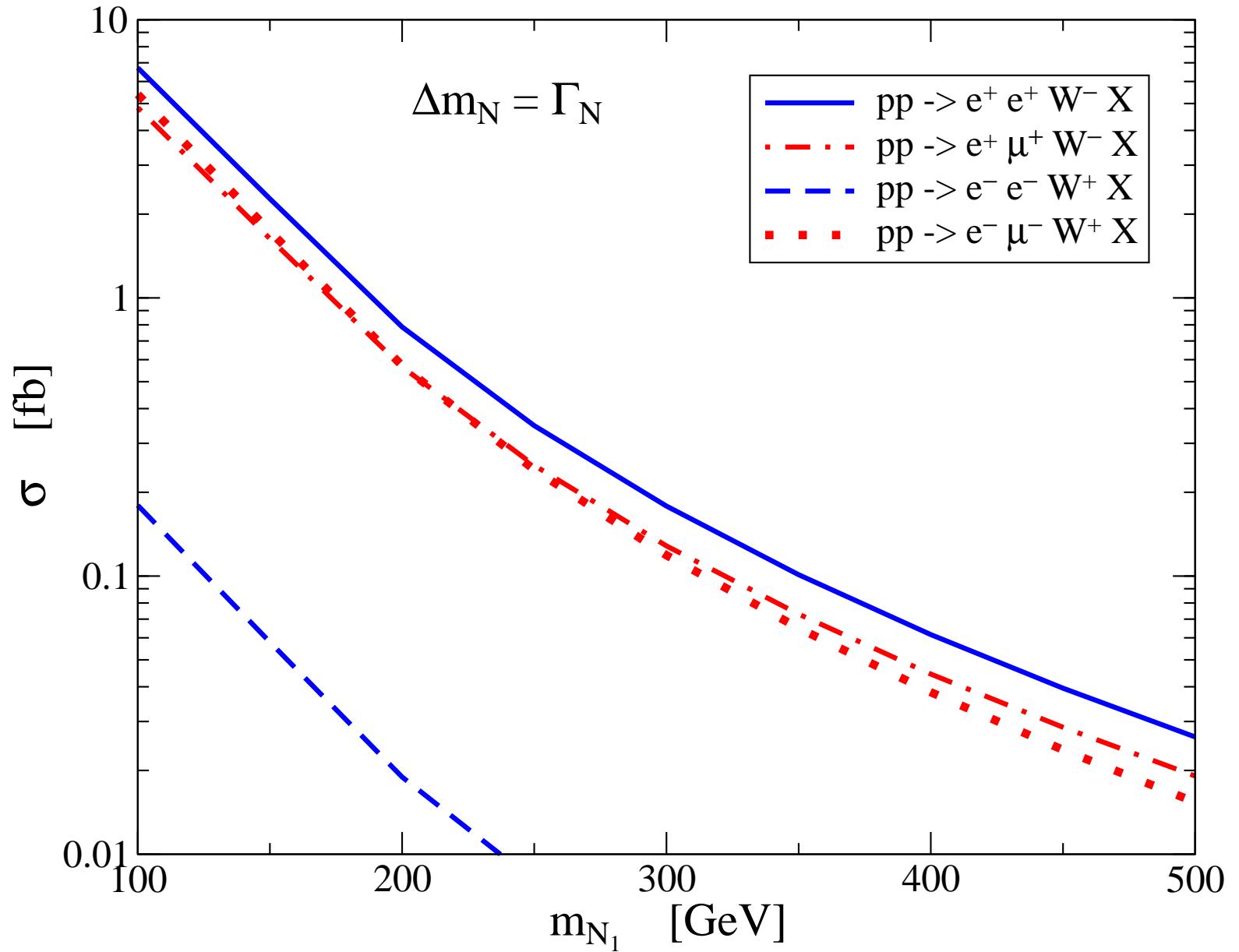
[F.F. Deppisch, P.S.B. Dev, A.P., NJP17 (2015) 7.]



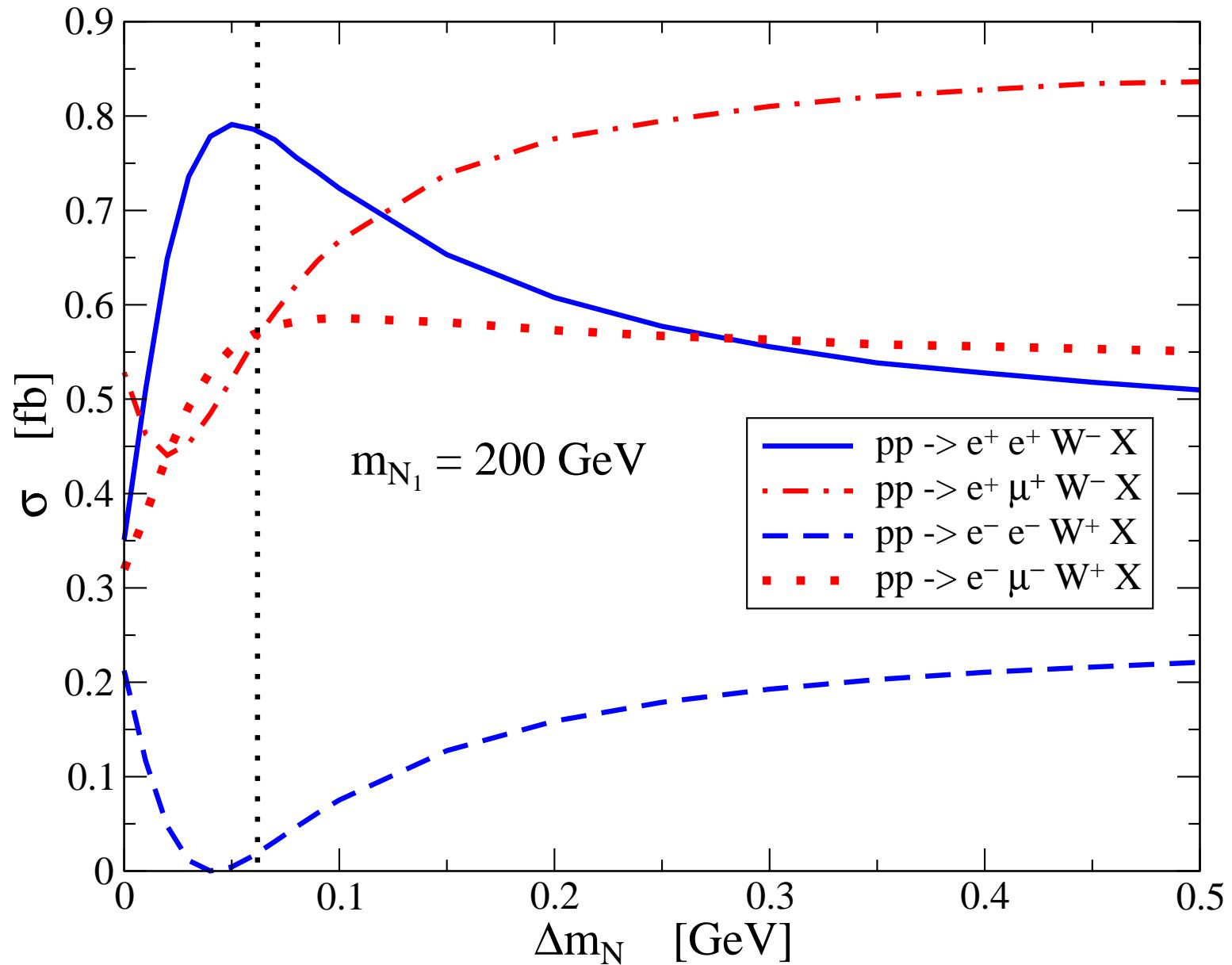
LHC and Other Constraints: eN -Sector

[F.F. Deppisch, P.S.B. Dev, A.P., NJP17 (2015) 7.]





[S. Bray, J.S. Lee, A.P., NPB786 (2007) 95.]



[S. Bray, J.S. Lee, A.P., NPB786 (2007) 95.]

- Lepton Number Violation:

$$A_{\text{CP}}(\text{LN}V1) = \frac{\sigma(pp \rightarrow e^+e^+W^-X) - K\sigma(pp \rightarrow e^-e^-W^+X)}{\sigma(pp \rightarrow e^+e^+W^-X) + K\sigma(pp \rightarrow e^-e^-W^+X)},$$

$$A_{\text{CP}}(\text{LN}V2) = \frac{\sigma(pp \rightarrow e^+\mu^+W^-X) - K\sigma(pp \rightarrow e^-\mu^-W^+X)}{\sigma(pp \rightarrow e^+\mu^+W^-X) + K\sigma(pp \rightarrow e^-\mu^-W^+X)},$$

$$R_{\text{CP}}(\text{LN}V) = \frac{\frac{\sigma(pp \rightarrow e^+e^+W^-X)}{\sigma(pp \rightarrow e^+\mu^+W^-X)} - \frac{\sigma(pp \rightarrow e^-e^-W^+X)}{\sigma(pp \rightarrow e^-\mu^-W^+X)}}{\frac{\sigma(pp \rightarrow e^+e^+W^-X)}{\sigma(pp \rightarrow e^+\mu^+W^-X)} + \frac{\sigma(pp \rightarrow e^-e^-W^+X)}{\sigma(pp \rightarrow e^-\mu^-W^+X)}}.$$

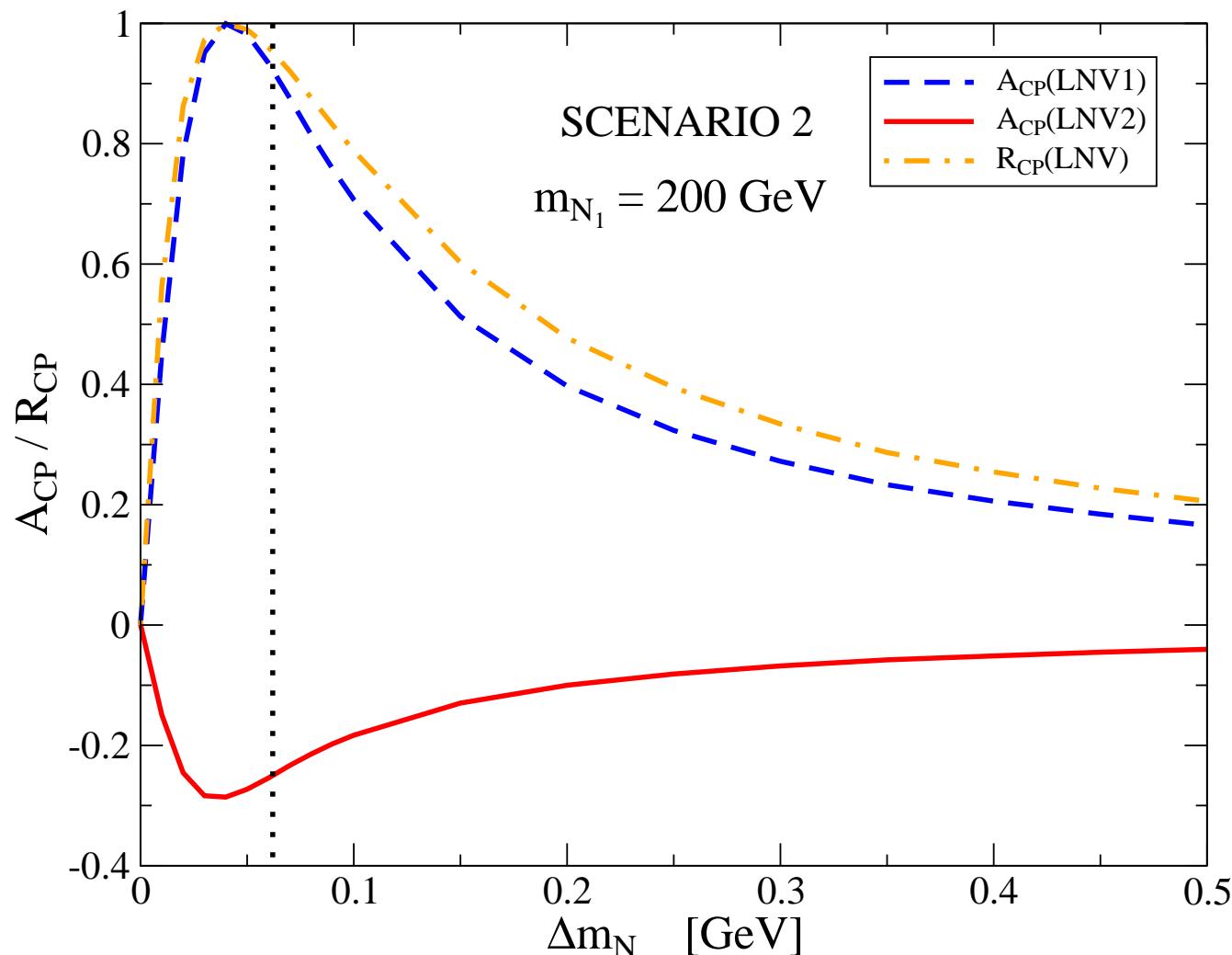
- Lepton Flavour Violation:

$$A_{\text{CP}}(\text{LNC}) = \frac{\sigma(pp \rightarrow e^+\mu^-W^\pm X) - \sigma(pp \rightarrow e^-\mu^+W^\pm X)}{\sigma(pp \rightarrow e^+\mu^-W^\pm X) + \sigma(pp \rightarrow e^-\mu^+W^\pm X)},$$

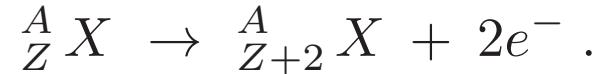
$$R_{\text{CP}}(\text{LNC}) = \frac{\frac{\sigma(pp \rightarrow e^+\mu^-W^\pm X)}{\sigma(pp \rightarrow e^-\mu^+W^\pm X)} - \frac{\sigma(pp \rightarrow e^-\mu^+W^\pm X)}{\sigma(pp \rightarrow e^+\mu^-W^\pm X)}}{\frac{\sigma(pp \rightarrow e^+\mu^-W^\pm X)}{\sigma(pp \rightarrow e^-\mu^+W^\pm X)} + \frac{\sigma(pp \rightarrow e^-\mu^+W^\pm X)}{\sigma(pp \rightarrow e^+\mu^-W^\pm X)}}.$$

Resonant CP Violation via Heavy Neutrino Mixing

[AP, NPB504 (1997) 61;
S. Bray, J.S. Lee, AP, NPB786 (2007) 95.]



- Charged LNV and LFV at the Intensity Frontier
- $0\nu\beta\beta$ Decay



Half-life for $0\nu\beta\beta$ decay:

$$[T_{1/2}^{0\nu\beta\beta}]^{-1} = \frac{|\langle m \rangle|^2}{m_e^2} |\mathcal{M}_{0\nu\beta\beta}|^2 G_{01} .$$

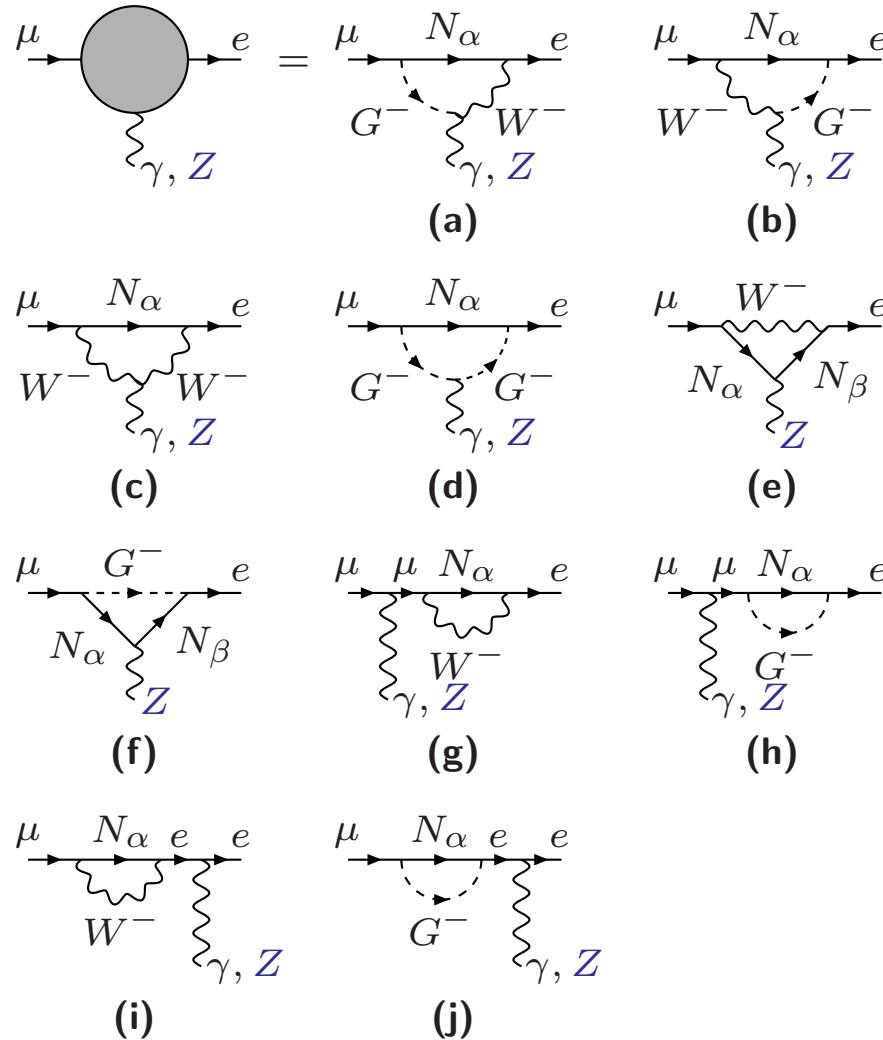
Weak limits on RL models:

$$|\langle m_{0\nu\beta\beta} \rangle| \simeq |(\mathbf{m}^\nu)_{ee}| \lesssim 0.015 \text{ eV} .$$

Future $0\nu\beta\beta$ experiments will be sensitive to $|\langle m \rangle| \sim 0.01\text{--}0.05$ eV, such as SuperNEMO . . .

• $\mu \rightarrow e\gamma$

[T.P. Cheng, L.F. Li, PRL45 (1980) 1908;
 J.G. Körner, A.P., K. Schilcher, PLB300 (1993) 381;
 J. Bernabéu, J.G. Körner, A.P., K. Schilcher, PRL71 (1993) 2695.]



$$\Omega_{ll'} \equiv \sum_\alpha B_{l\alpha} B_{l'\alpha}^*,$$

with $B_{l\alpha} \simeq (\mathcal{M}_D)_{l\alpha} m_{N_\alpha}^{-1}$

$$B(\mu \rightarrow e\gamma) \approx 7 \cdot 10^{-4} |\Omega_{e\mu}|^2,$$

for $m_N \gtrsim 100$ GeV.

Compare with

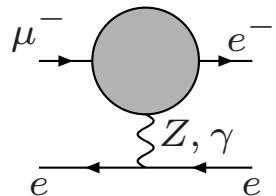
$$B^{\text{exp}}(\mu \rightarrow e\gamma) < 4.2 \times 10^{-13}$$

MEG sensitivity:

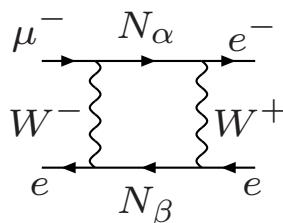
$$B(\mu \rightarrow e\gamma) \sim 10^{-13} - 10^{-14}$$

- $\mu \rightarrow eee$

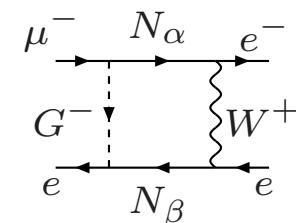
[A. Ilakovac, A.P., NPB437 (1995) 491.]



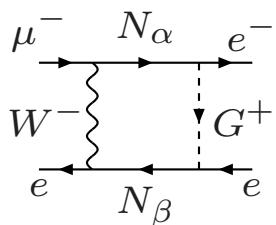
(a)



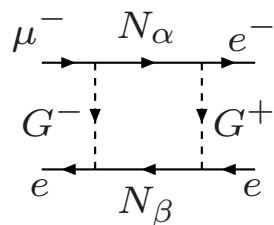
(b)



(c)



(d)



(e)

$$+ (e \leftrightarrow e^-)$$

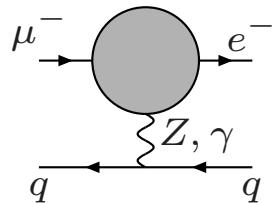
$$m_N = 250 \text{ GeV}: \quad \mathcal{B}(\mu \rightarrow eee) \approx 1.4 \cdot 10^{-2} \times \mathcal{B}(\mu \rightarrow e\gamma) \sim 1.4 \times 10^{-14}.$$

Current experimental limit: $\mathcal{B}^{\text{exp}}(\mu \rightarrow eee) < 10^{-12}$ (SINDRUM)

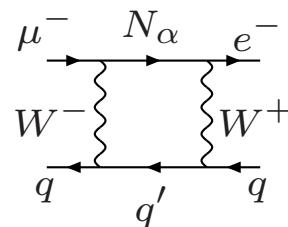
[Mu3E, A. Blondel et al, arXiv:1301.6113 [hep-ex].]

Future proposed sensitivity: $\mathcal{B}^{\text{exp}}(\mu \rightarrow eee) \sim 10^{-16}$

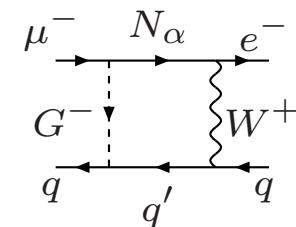
- Coherent $\mu \rightarrow e$ Conversion in Nuclei ($^{48}_{22}\text{Ti}$, $^{197}_{79}\text{Au}$)



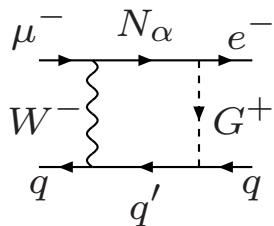
(a)



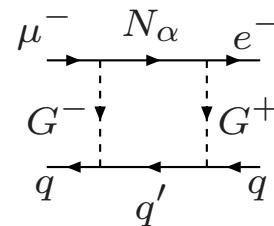
(b)



(c)



(d)



(e)

[A. Ilakovac, A.P., PRD80 (2009) 091902;

R. Alonso, M. Dhen, M. B. Gavela, T. Hambye, arXiv:1209.2679.]

$$m_N = 250 \text{ GeV}: \quad B_{\text{Ti}}(\mu \rightarrow e) \approx 5 \times B(\mu \rightarrow e\gamma).$$

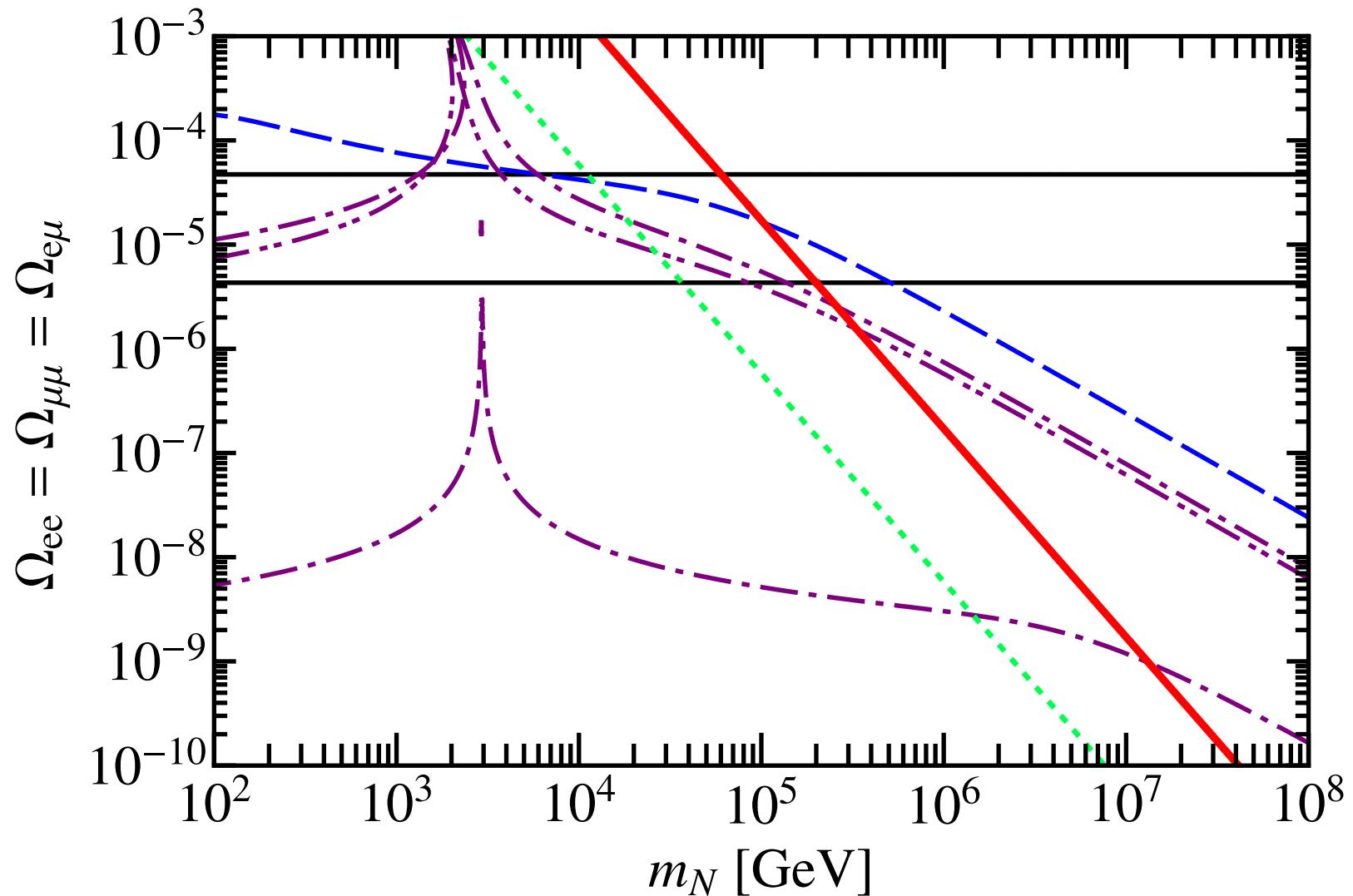
$$B_{\text{Ti}}^{\text{exp}}(\mu \rightarrow e) < 4.3 \times 10^{-12}, \quad B_{\text{Au}}^{\text{exp}}(\mu \rightarrow e) < 7 \times 10^{-13}.$$

$$\text{COMET/PRISM sensitivity: } B_{\text{Ti}}^{\text{exp}}(\mu \rightarrow e) \sim 10^{-13} - 10^{-18}.$$

[M. Aoki et al, cLFV at FERMILAB, arXiv:2203.08278.]

- $\mu \rightarrow e\gamma$, $\mu \rightarrow eee$ and $\mu \rightarrow e$ conversion

[A. Ilakovac, A.P., PRD80 (2009) 091902.]

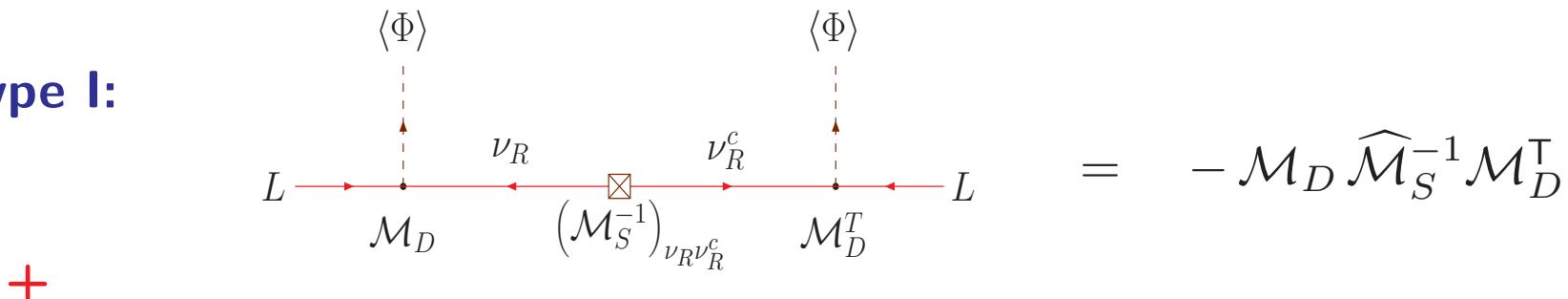


• Numerical Estimates

– The Type I+II Radiative Seesaw Model

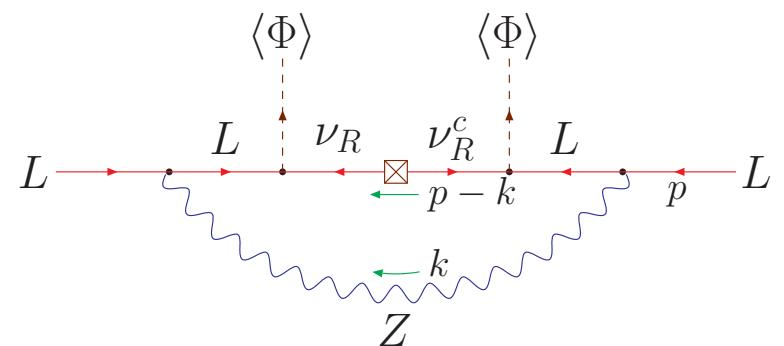
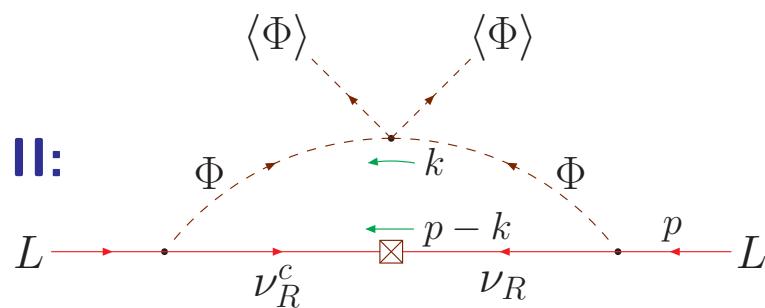
[A.P., ZPC55 (1992) 275;
P.S.B. Dev, A.P., PRD86 (2012) 113001.]

Type I:



+

Type II:

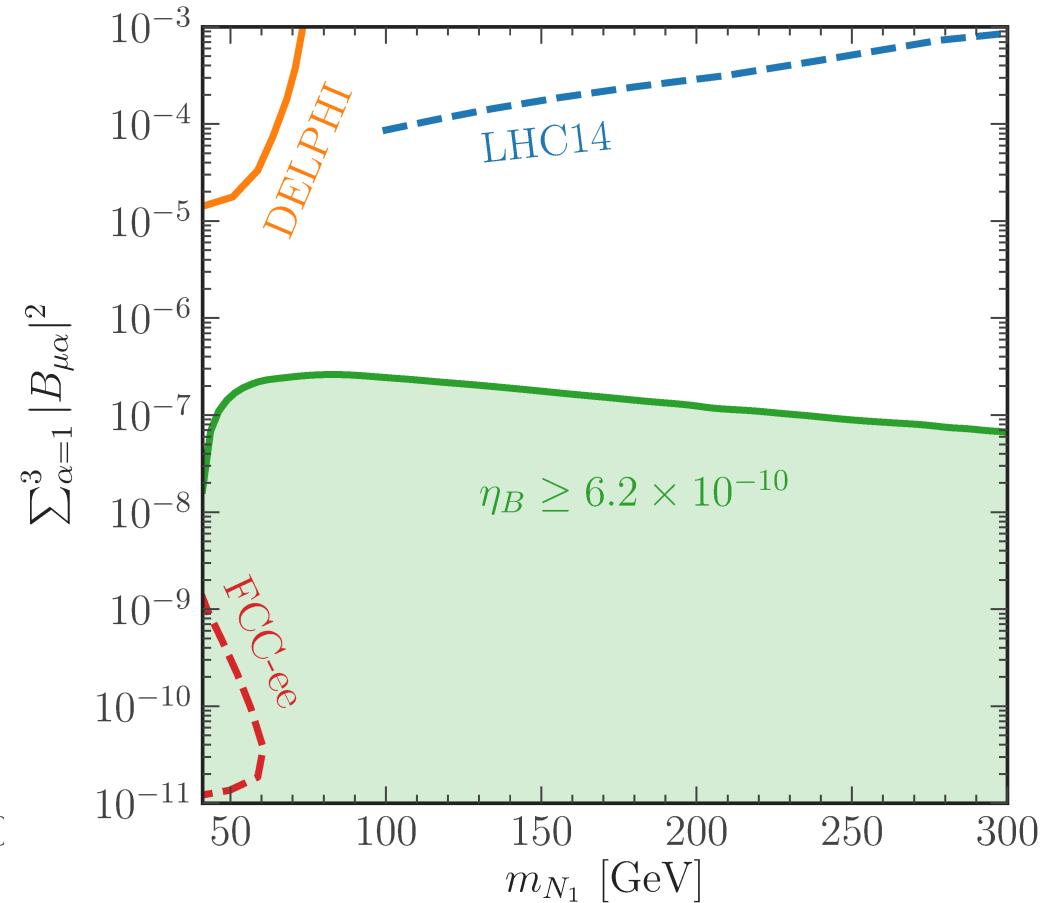
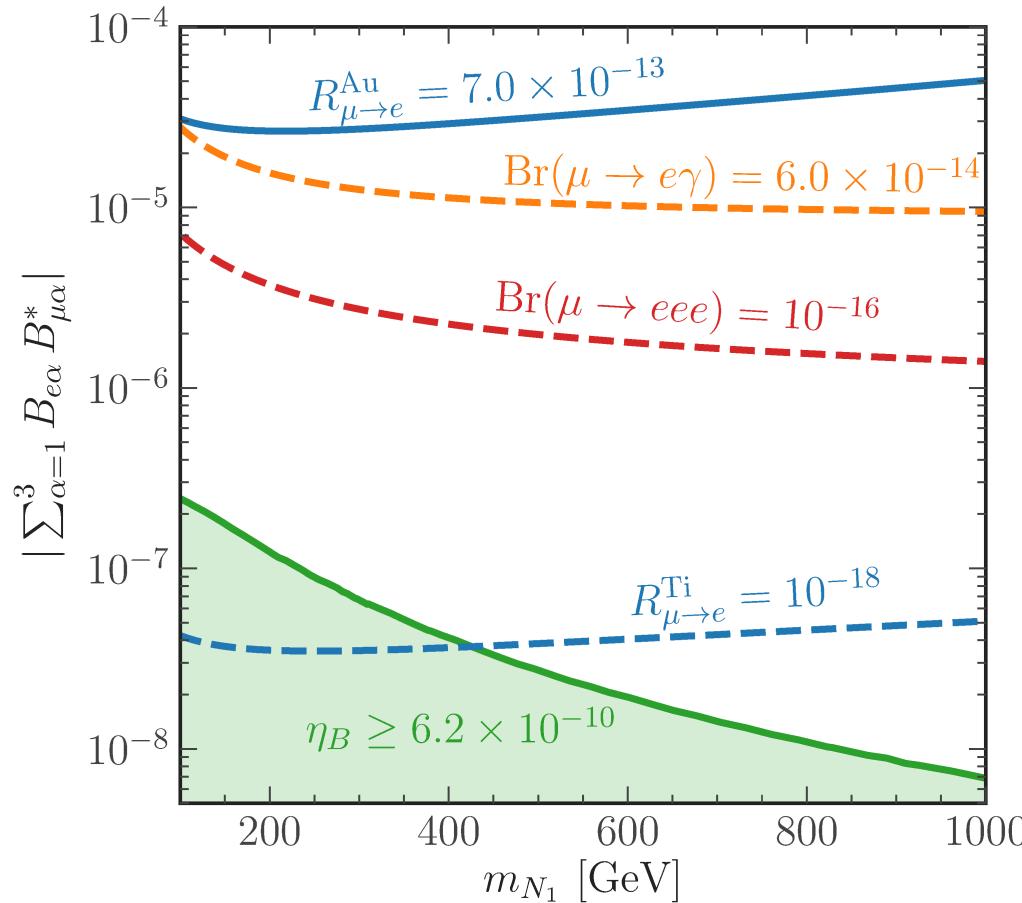


Light-neutrino mass matrix:

$$m_\nu = -\mathcal{M}_D \widehat{\mathcal{M}}_S^{-1} \mathcal{M}_D^\top + \mathbf{M}_{LL}^{1\text{-loop}}$$

Allowed parameter space from cLFV processes and Colliders

[P. Candia da Silva, D. Karamitros, T. McKelvey, A.P., arXiv:2206.08352.]



$B_{l\alpha} \sim (\mathcal{M}_D)_{l\alpha} m_{N_\alpha}^{-1}$: light-to-heavy neutrino mixings

Hypothesis: Democratic flavour models based on $\sim Z_3$ or $\sim Z_6$ symmetries.

• Conclusions

- Matter–AntiMatter Asymmetry through Resonant Leptogenesis

$$\implies \delta\eta_{\text{tot}}^L \sim \delta\eta_{\text{mix}}^L + \delta\eta_{\text{osc}}^L + \delta\eta_{\text{dec}}^L$$

independent of the *initial B-number state* of the early Universe.

Complete treatment may enhance BAU by 1-order of magnitude.

- Tri– or Multi–Resonant Leptogenesis offer new possibilities in building models with observable cLFV and LNV.
- Varying relativistic dofs modify significantly the Transport Equations and so the BAU predictions for $m_N \lesssim 100$ GeV.
- $B(\mu \rightarrow e\gamma) \sim 10^{-13} + \text{successful leptogenesis}$ [A.P. '04]
 $\implies \geq 3$ RHNs + Non-trivial Flavour Effects.
- Electroweak-Scale Heavy Majorana Neutrinos can give rise to observable signatures of LNV and CPV at the LHC.

BACK-UP SLIDES

- Flavour Covariant Transport Equations

[E. W. Kolb and S. Wolfram, NPB172 (1980) 224.]

- Flavour Diagonal Boltzmann Equations

$$\frac{dn_a}{dt} + 3Hn_a = \sum_{aX' \leftrightarrow Y} \left(-\frac{n_a n_{X'}}{n_a^{\text{eq}} n_{X'}^{\text{eq}}} \gamma(aX' \rightarrow Y) + \frac{n_Y}{n_Y^{\text{eq}}} \gamma(Y \rightarrow aX') \right),$$

where n_a is the **number density**:

$$\begin{aligned} n_a(T) &= g_a \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \exp \left[-\left(\sqrt{\mathbf{p}^2 + m_a^2} - \mu_a(T) \right)/T \right] \\ &= \frac{g_a m_a^2 T e^{\mu_a(T)/T}}{2\pi^2} K_2 \left(\frac{m_a}{T} \right) \end{aligned}$$

and $\gamma(X \rightarrow Y)$ is the **collision term**:

$$\gamma(X \rightarrow Y) = \int d\pi_X d\pi_Y (2\pi)^4 \delta^{(4)}(p_X - p_Y) e^{-p_X^0/T} |\mathcal{M}(X \rightarrow Y)|^2.$$

– Flavour Diagonal BEs for Leptogenesis

[A.P., T.E. Underwood, **NPB692** (2004) 303; **PRD72** (2005) 113001.]

Define first

$$\eta^X \equiv n_X/n_\gamma , \quad z \equiv m_{N_1}/T , \quad H \equiv H(T = m_N) \approx 17 m_N^2/M_{\text{Planck}}$$

and the short-hands:

$$\begin{aligned} \gamma_Y^X &\equiv \gamma(X \rightarrow Y) + \gamma(\bar{X} \rightarrow \bar{Y}) \stackrel{\text{CPT}}{=} \gamma_X^Y , \\ \delta\gamma_Y^X &\equiv \gamma(X \rightarrow Y) - \gamma(\bar{X} \rightarrow \bar{Y}) \stackrel{\text{CPT}}{=} -\delta\gamma_X^Y . \end{aligned}$$

Write down the **Boltzmann equations**:

$$\begin{aligned} \frac{H n_\gamma}{z} \frac{d\eta_\alpha^N}{dz} &= \left(1 - \frac{\eta_\alpha^N}{\eta_{\text{eq}}^N}\right) \sum_k \gamma_{L_k \Phi}^{N_\alpha} + \dots \\ \frac{H n_\gamma}{z} \frac{d\delta\eta_l^L}{dz} &= \sum_\alpha \left(\frac{\eta_\alpha^N}{\eta_{\text{eq}}^N} - 1 \right) \delta\gamma_{L_l \Phi}^{N_\alpha} - \frac{2}{3} \delta\eta_l^L \sum_k \left(\gamma_{L_k^c \Phi^c}^{L_l \Phi} + \gamma_{L_k \Phi}^{L_l \Phi} \right) \\ &\quad - \frac{2}{3} \sum_k \delta\eta_k^L \left(\gamma_{L_l^c \Phi^c}^{L_k \Phi} - \gamma_{L_l \Phi}^{L_k \Phi} \right) + \dots \end{aligned}$$

- Order-of-magnitude estimate of the BAU

Flavour-dependent decay width of heavy Majorana neutrino N_α :

$$\Gamma_{N_\alpha \rightarrow l} \equiv \Gamma(N_\alpha \rightarrow L_l \Phi) = (h^{\nu\dagger})_{\alpha l} h_{l\alpha}^\nu \frac{m_{N_\alpha}}{8\pi}$$

Define the effective wash-out K -factors:

$$K_l^{\text{eff}} \equiv \frac{\sum_{N_\alpha} \Gamma_{N_\alpha \rightarrow l}}{H}$$

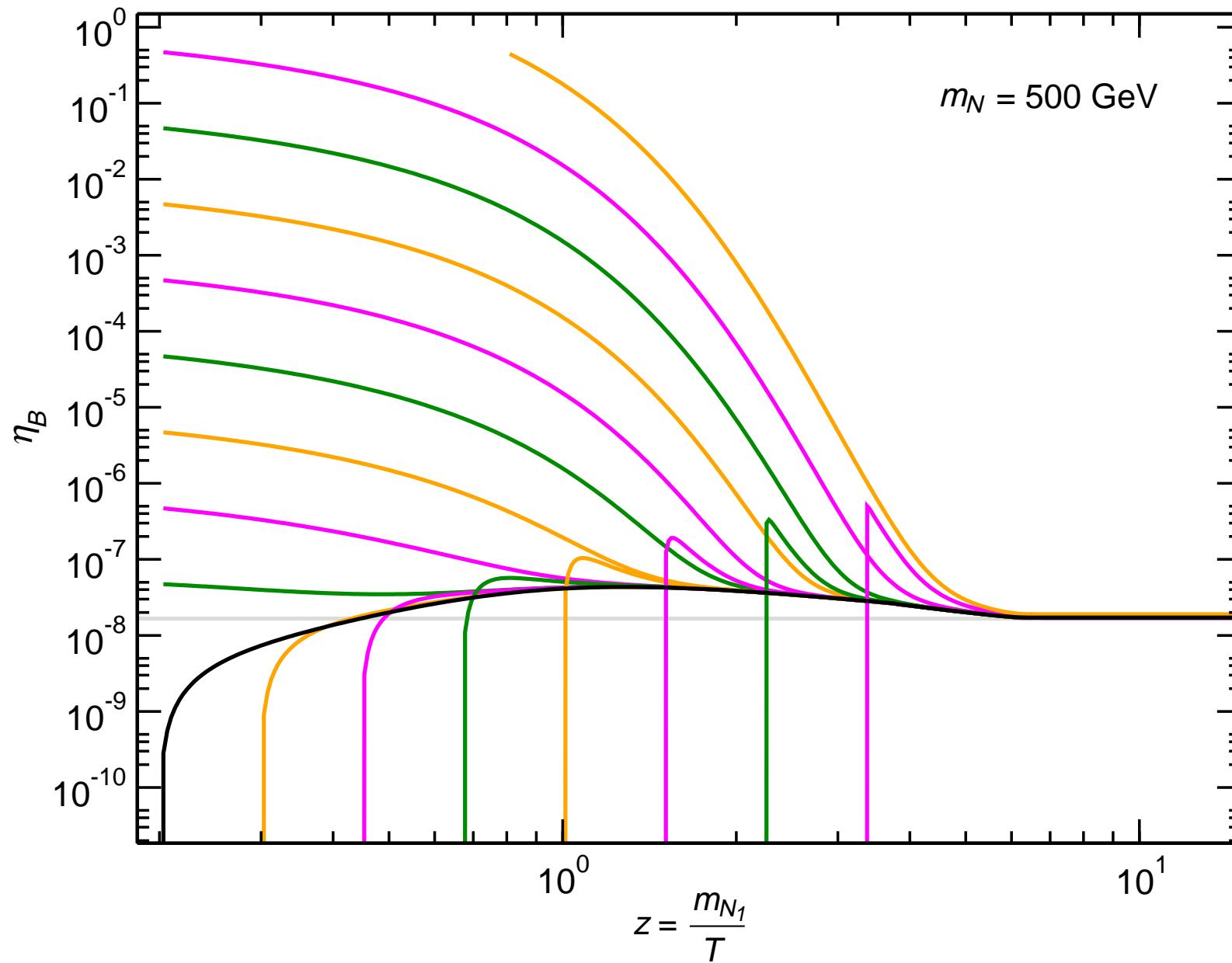
Estimate of the BAU (strong wash-out regime):

[F. Deppisch, A.P., PRD83 (2011) 076007.]

$$\eta_B^{\text{mix}} \sim -3 \cdot 10^{-2} \sum_{l=e,\mu,\tau} \frac{\delta_l^{\text{mix}}}{K_l^{\text{eff}} \min \left[m_N/T_c, 1.25 \ln(25 K_l^{\text{eff}}) \right]} .$$

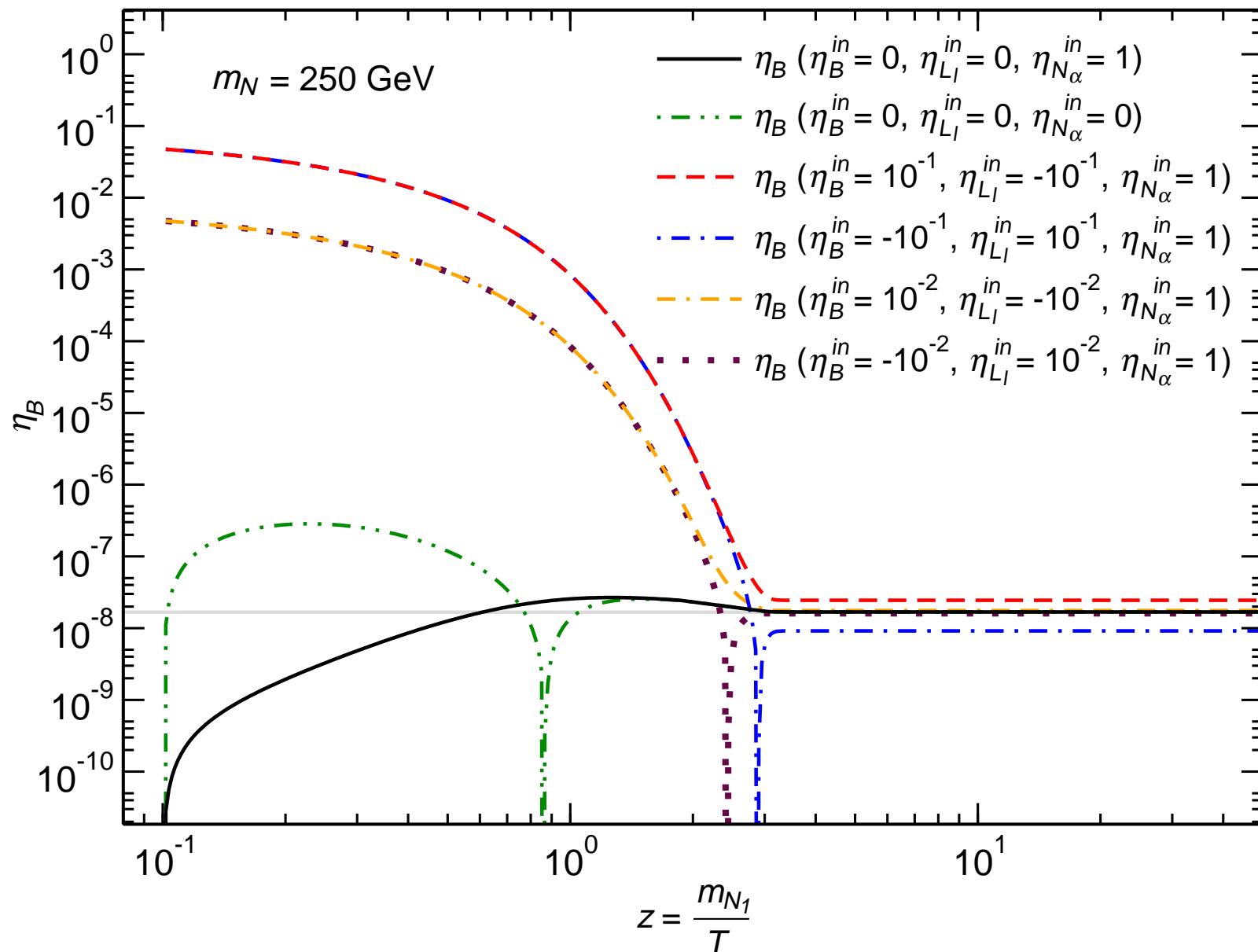
Resonant τ -Genesis

[A.P., T. Underwood, PRD72 (2005) 113001.]



Resonant τ -Genesis

[A.P., T. Underwood, PRD72 (2005) 113001.]



– Flavour Covariant Quantization

[P.S.B. Dev, P. Millington, A.P., D. Teresi, NPB886 (2014) 569.]

$U(\mathcal{N}_L) \otimes U(\mathcal{N}_N)$ flavour-invariant Lagrangian:

$$-\mathcal{L}_N = h_l^\alpha \bar{L}^l \tilde{\Phi} N_{R,\alpha} + \bar{N}_{R,\alpha}^C [M_N]^{\alpha\beta} N_{R,\beta} + \text{H.c.}$$

Under $U(\mathcal{N}_L) \otimes U(\mathcal{N}_N)$ flavour transformations:

$$\begin{aligned} L_l &\rightarrow L'_l = V_l^m L_m, & L^l &\equiv (L_l)^\dagger \rightarrow L'^l = V_m^l L^m, \\ N_{R,\alpha} &\rightarrow N'_{R,\alpha} = U_\alpha^\beta N_{R,\beta}, & N_R^\alpha &\equiv (N_{R,\alpha})^\dagger \rightarrow N_R'^\alpha = U_\beta^\alpha N_R^\beta, \end{aligned}$$

\mathcal{L}_N is invariant provided:

$$h_l^\alpha \rightarrow h'_l{}^\alpha = V_l^m U_\beta^\alpha h_m^\beta, \quad [M_N]^{\alpha\beta} \rightarrow [M'_N]^{\alpha\beta} = U_\gamma^\alpha U_\delta^\beta [M_N]^{\gamma\delta}.$$

Quantization rules:

$$\begin{aligned} \{b_l(\mathbf{p}, s), b^m(\mathbf{p}', s')\} &= \{d^{\dagger,m}(\mathbf{p}, s), d_l^\dagger(\mathbf{p}', s')\} = (2\pi)^3 \delta^{(3)}(\mathbf{p} - \mathbf{p}') \delta_{ss'} \delta_l^m \\ \{a_\alpha(\mathbf{k}, r), a^\beta(\mathbf{k}', r')\} &= (2\pi)^3 \delta^{(3)}(\mathbf{p} - \mathbf{p}') \delta_{rr'} \delta_\alpha^\beta \end{aligned}$$

– Flavour Covariant Number Densities and Collision Rates

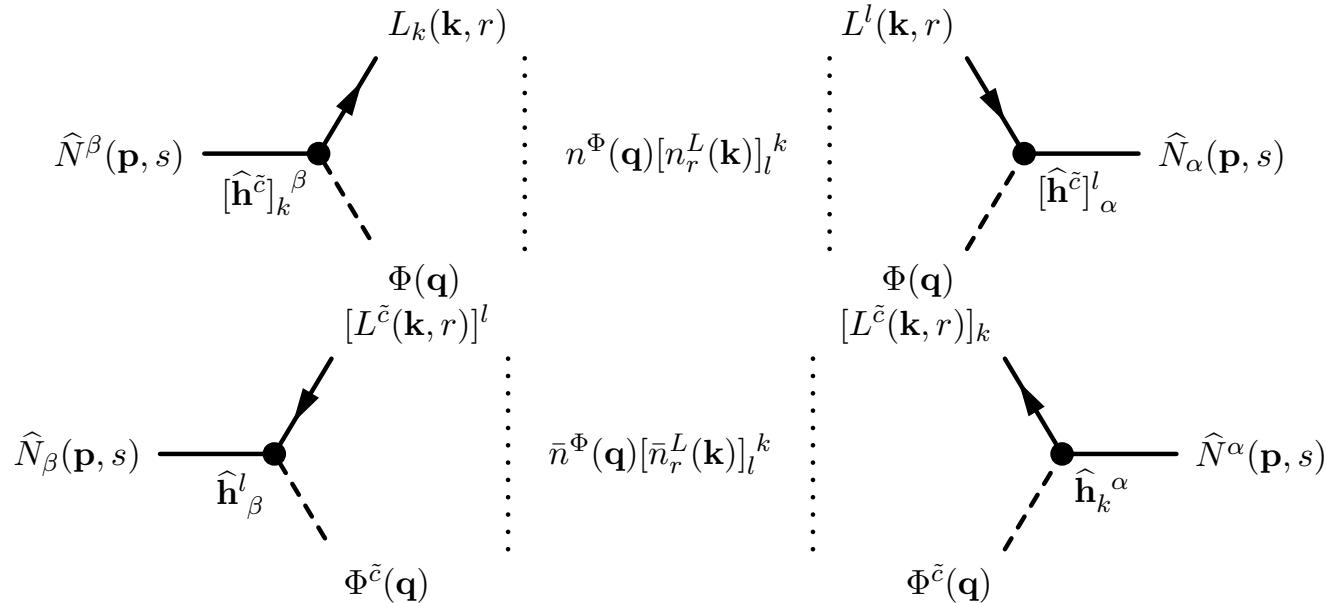
Flavour-covariant number densities:

$$\begin{aligned}[n_{s_1 s_2}^L(\mathbf{p})]_l{}^m &\equiv \frac{1}{\mathcal{V}} \langle b^m(\mathbf{p}, s_2) b_l(\mathbf{p}, s_1) \rangle , \\ [\bar{n}_{s_1 s_2}^L(\mathbf{p})]_l{}^m &\equiv \frac{1}{\mathcal{V}} \langle d_l^\dagger(\mathbf{p}, s_1) d_l^{\dagger, m}(\mathbf{p}, s_2) \rangle , \\ [n_{r_1 r_2}^N(\mathbf{k})]_\alpha{}^\beta &\equiv \frac{1}{\mathcal{V}} \langle a^\beta(\mathbf{k}, r_2) a_\alpha(\mathbf{k}, r_1) \rangle .\end{aligned}$$

Flavour-covariant generalization of the collision rates:

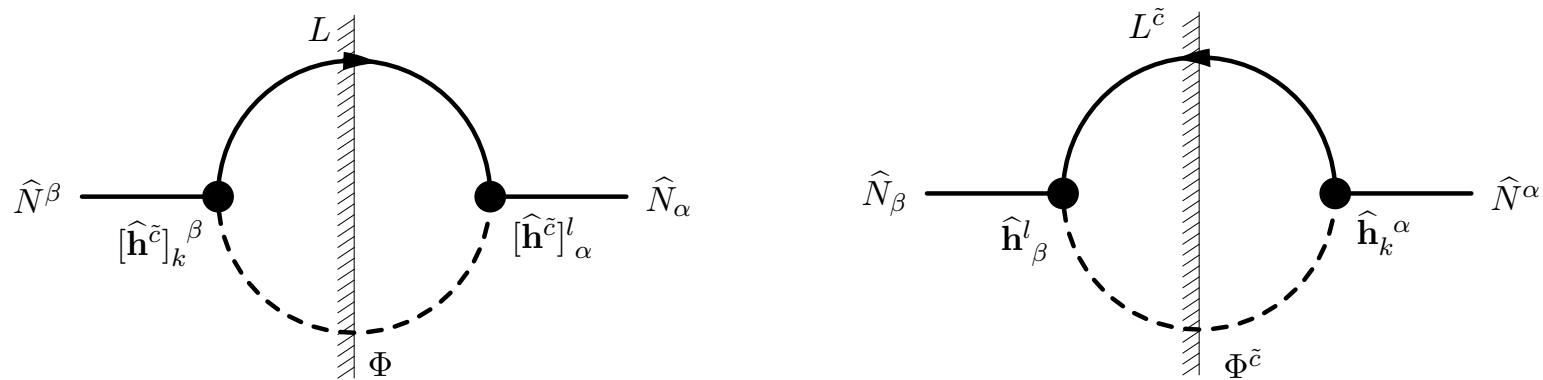
$$\begin{aligned}[\gamma(N \rightarrow L\Phi)]_l{}^m{}_\alpha{}^\beta &= \frac{m_N^4}{\pi^2 z} \frac{K_1(z)}{16\pi} \mathbf{h}_\alpha{}^m \mathbf{h}_l{}^\beta , \\ [\gamma(N \rightarrow L^{\tilde{c}}\Phi^{\tilde{c}})]_l{}^m{}_\alpha{}^\beta &= \frac{m_N^4}{\pi^2 z} \frac{K_1(z)}{16\pi} [\mathbf{h}^{\tilde{c}}]_\alpha{}^m [\mathbf{h}^{\tilde{c}}]_l{}^\beta , \\ [\gamma_{L\Phi}^N]_l{}^m{}_\alpha{}^\beta &\equiv [\gamma(N \rightarrow L\Phi)]_l{}^m{}_\alpha{}^\beta + [\gamma(N \rightarrow L^{\tilde{c}}\Phi^{\tilde{c}})]_l{}^m{}_\alpha{}^\beta , \\ [\delta\gamma_{L\Phi}^N]_l{}^m{}_\alpha{}^\beta &\equiv [\gamma(N \rightarrow L\Phi)]_l{}^m{}_\alpha{}^\beta - [\gamma(N \rightarrow L^{\tilde{c}}\Phi^{\tilde{c}})]_l{}^m{}_\alpha{}^\beta .\end{aligned}$$

– Feynman Diagrammatic Representation



– Closed Time Path (CTP) Representation

[e.g., P. Millington and A.P., PRD88 (2013) 085009.]



– Flavour Covariant Rate Equations (Markovian approximation)

[P.S.B. Dev, P. Millington, A.P., D. Teresi, NPB886 (2014) 569.]

$$\frac{H n_\gamma}{z} \frac{d[\underline{\eta}^N]_\alpha^\beta}{dz} = -i \frac{n_\gamma}{2} \left[\mathcal{E}_N, \delta\eta^N \right]_\alpha^\beta + [\text{Re}(\gamma_{L\Phi}^N)]_\alpha^\beta - \frac{1}{2\eta_{\text{eq}}^N} \left\{ \underline{\eta}^N, \text{Re}(\gamma_{L\Phi}^N) \right\}_\alpha^\beta$$

$$\begin{aligned} \frac{H n_\gamma}{z} \frac{d[\delta\eta^N]_\alpha^\beta}{dz} &= -2i n_\gamma \left[\mathcal{E}_N, \underline{\eta}^N \right]_\alpha^\beta + 2i [\text{Im}(\delta\gamma_{L\Phi}^N)]_\alpha^\beta \\ &\quad - \frac{i}{\eta_{\text{eq}}^N} \left\{ \underline{\eta}^N, \text{Im}(\delta\gamma_{L\Phi}^N) \right\}_\alpha^\beta - \frac{1}{2\eta_{\text{eq}}^N} \left\{ \delta\eta^N, \text{Re}(\gamma_{L\Phi}^N) \right\}_\alpha^\beta \end{aligned}$$

$$\begin{aligned} \frac{H n_\gamma}{z} \frac{d[\delta\eta^L]_l^m}{dz} &= -[\delta\gamma_{L\Phi}^N]_l^m + \frac{[\underline{\eta}^N]_\beta^\alpha}{\tilde{\eta}_{\text{eq}}^N} [\delta\gamma_{L\Phi}^N]_l^m{}_\alpha^\beta + \frac{[\delta\eta^N]_\beta^\alpha}{2\tilde{\eta}_{\text{eq}}^N} [\gamma_{L\Phi}^N]_l^m{}_\alpha^\beta \\ &\quad - \frac{1}{3} \left\{ \delta\eta^L, \gamma_{L\tilde{c}\Phi\tilde{c}}^{L\Phi} + \gamma_{L\Phi}^{L\Phi} \right\}_l^m - \frac{2}{3} [\delta\eta^L]_k^n ([\gamma_{L\tilde{c}\Phi\tilde{c}}^{L\Phi}]_n^k{}_l^m - [\gamma_{L\Phi}^{L\Phi}]_n^k{}_l^m) \\ &\quad - \frac{2}{3} \left\{ \delta\eta^L, \gamma_{\text{dec}} \right\}_l^m + [\delta\gamma_{\text{dec}}^{\text{back}}]_l^m \end{aligned}$$

- **Unified Description of 3 Physically Distinct Phenomena:**

- **Resonant Mixing between Heavy Neutrinos,**

through: $\mathbf{h}_{l\alpha}$ and $\mathbf{h}_{l\alpha}^c$ in $[\gamma_{L\Phi}^N]_l{}^m{}_\alpha{}^\beta$ and $[\delta\gamma_{L\Phi}^N]_l{}^m{}_\alpha{}^\beta$.

- **Coherent Oscillations between Heavy Neutrinos** ($\Delta m_N \ll m_N$),

from $[\mathcal{E}_N, \underline{\eta}^N]$ and the rank-4 tensor term $\frac{[\delta\eta^N]_\beta{}^\alpha}{2\tilde{\eta}_{\text{eq}}^N} [\gamma_{L\Phi}^N]_l{}^m{}_\alpha{}^\beta$, yielding:

$$\delta\eta_{\text{osc}}^L \sim \frac{3}{2Kz} \frac{\text{Im}[(h^\dagger h)_{12}^2]}{(h^\dagger h)_{11}(h^\dagger h)_{22}} \frac{2(m_{N_1}^2 - m_{N_2}^2)m_N\Gamma_N}{(m_{N_1}^2 - m_{N_2}^2)^2 + \left(\frac{2m_N\Gamma_N \text{Im}[h^\dagger h]_{12}}{|[h^\dagger h]_{12}|}\right)^2},$$

with $\Gamma_N = \frac{1}{2}\left(\Gamma_{N_1}^{(0)} + \Gamma_{N_2}^{(0)}\right)$. [NB: Different from the ARS mechanism.]

- **Decoherence Effects due to Charged Lepton Yukawa Couplings,**

from $-\frac{2}{3} \left\{ \delta\eta^L, \gamma_{\text{dec}} \right\}_l{}^m + [\delta\gamma_{\text{dec}}^{\text{back}}]_l{}^m$

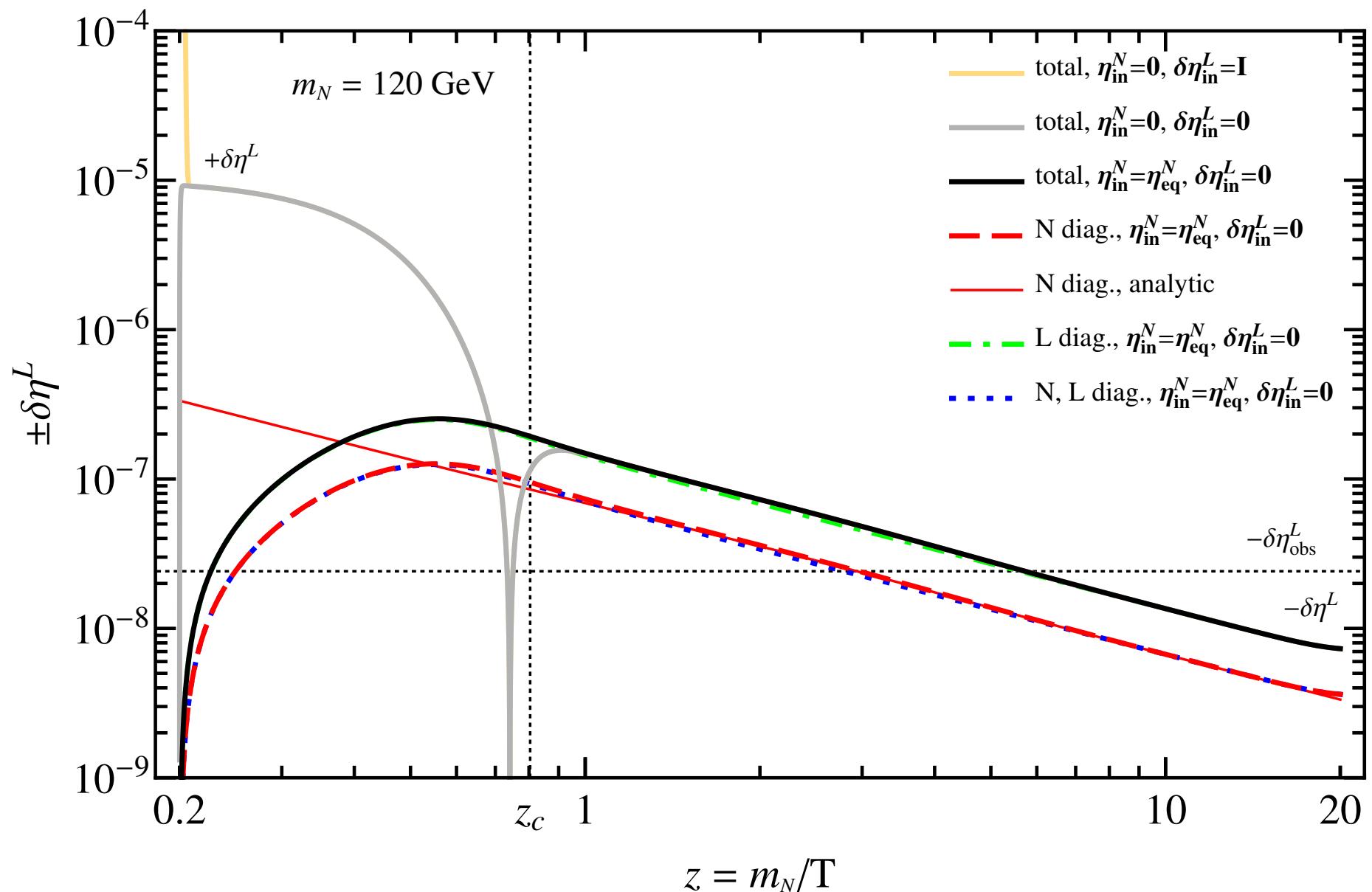
– Minimal Resonant Leptogenesis Model

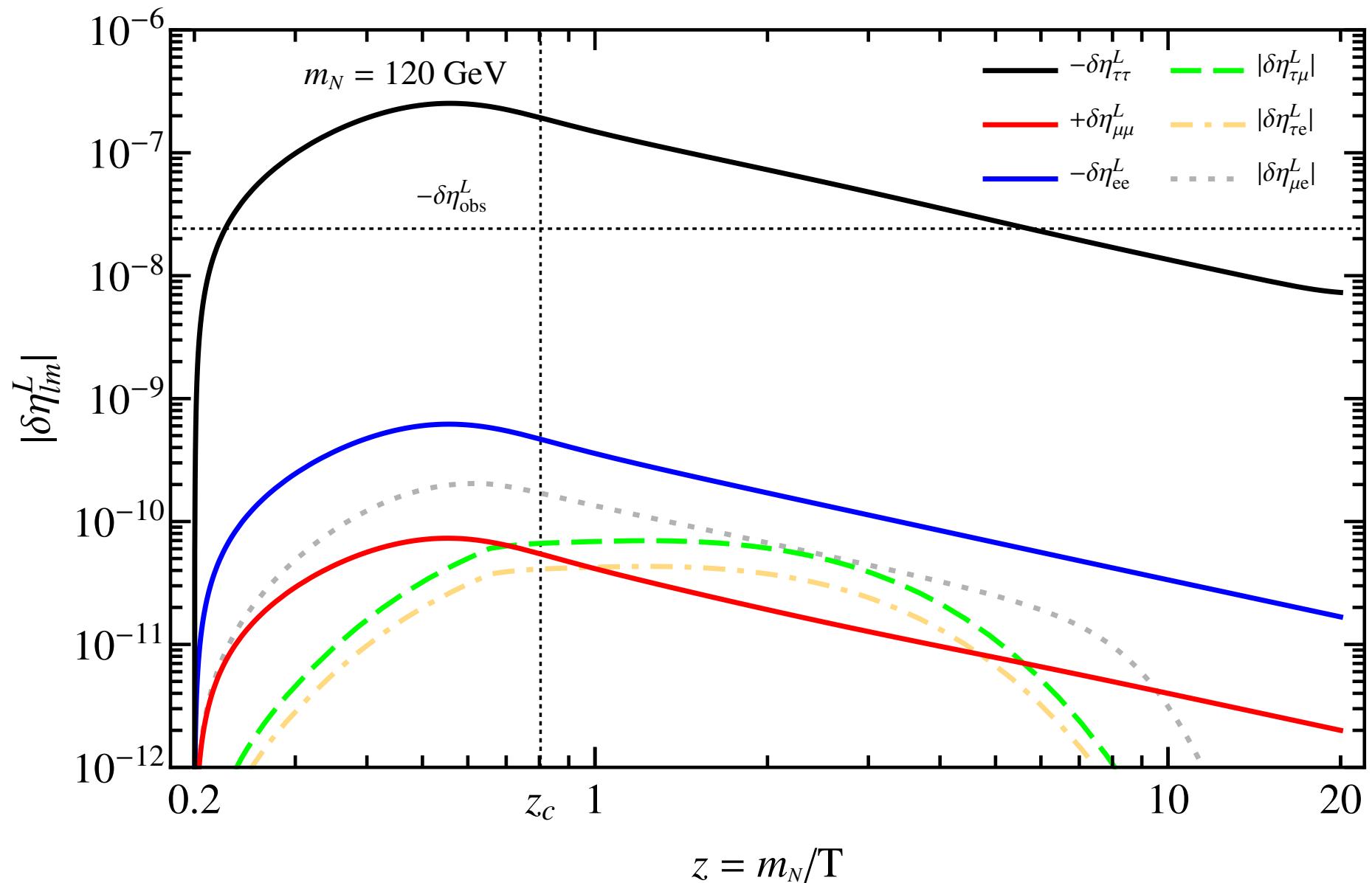
[F. Deppisch, A.P., PRD83 (2011) 076007.]

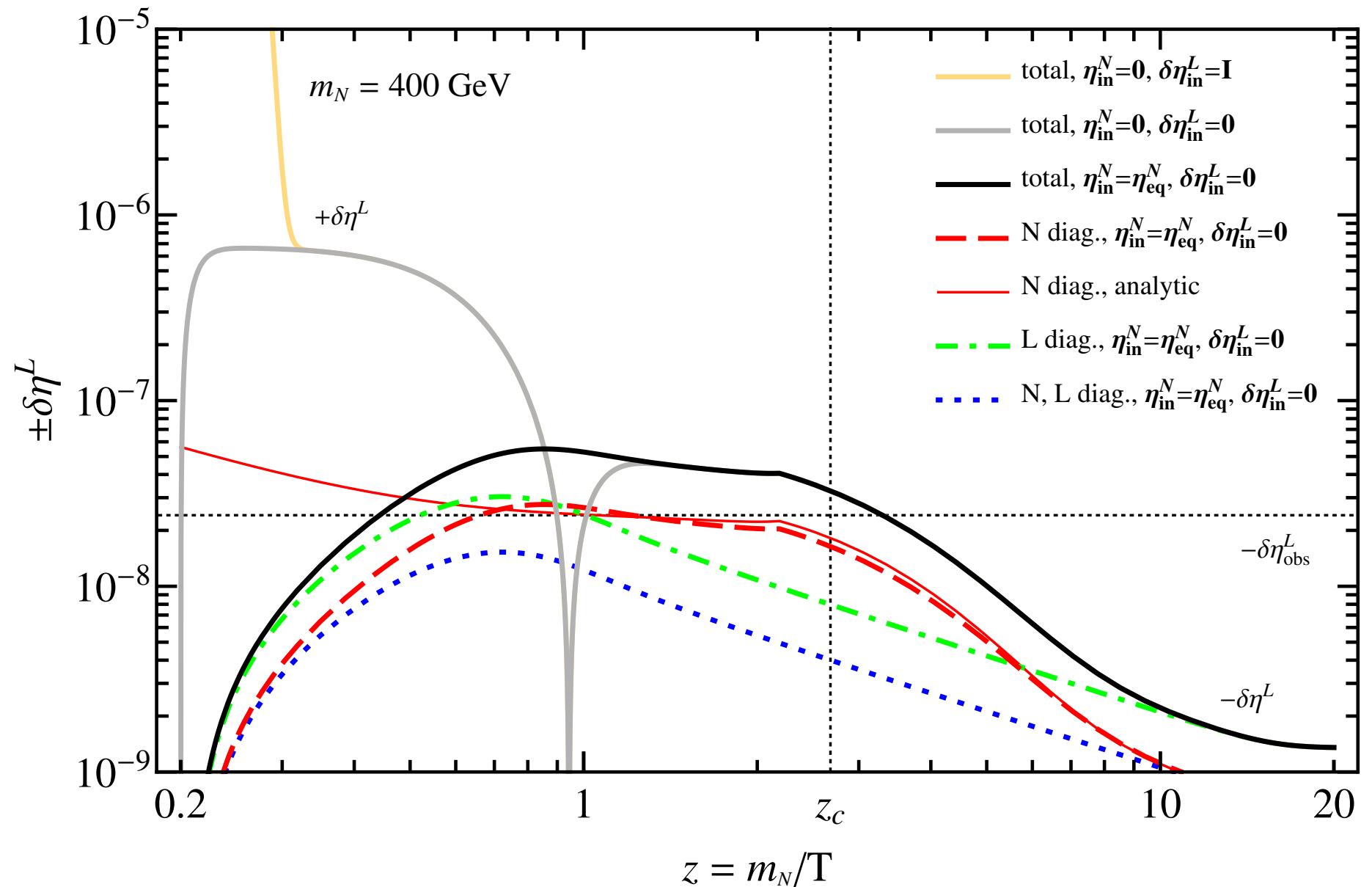
$$\mathbf{m}_M = m_N \mathbf{1}_3 \quad \text{and} \quad \mathbf{m}_D = \frac{v_{\text{SM}}}{\sqrt{2}} \begin{pmatrix} \epsilon_e & a e^{-i\pi/4} & a e^{i\pi/4} \\ \epsilon_\mu & b e^{-i\pi/4} & b e^{i\pi/4} \\ \epsilon_\tau & \kappa_1 e^{-i(\pi/4 - \gamma_1)} & \kappa_2 e^{i(\pi/4 - \gamma_2)} \end{pmatrix}$$

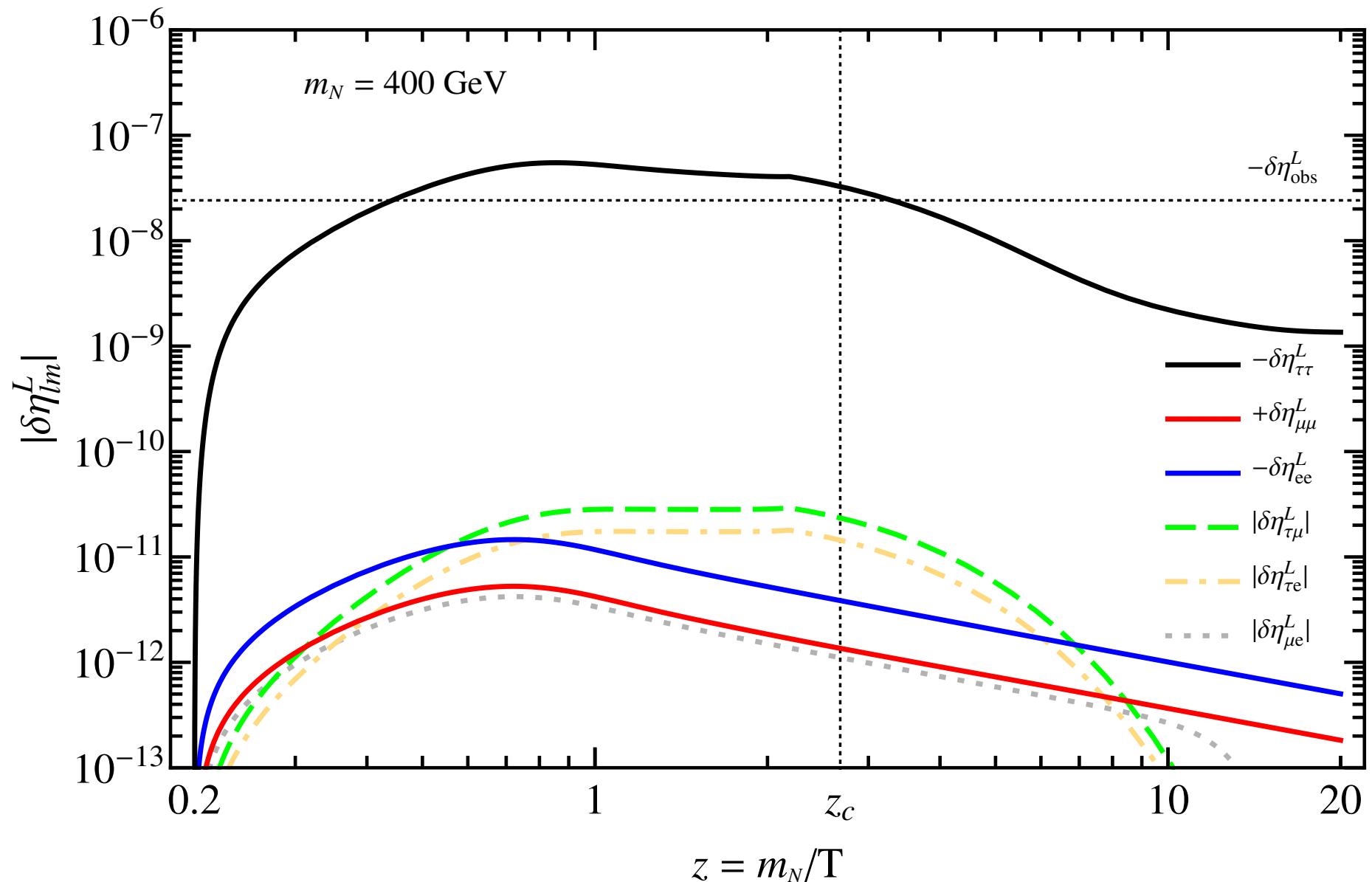
Parameters	BP1	BP2	BP3
m_N (GeV)	120	400	5000
γ_1	$\pi/4$	$\pi/3$	$3\pi/8$
γ_2	0	0	$\pi/2$
κ_1	4×10^{-5}	2.4×10^{-5}	2×10^{-4}
κ_2	2×10^{-4}	6×10^{-5}	2×10^{-5}
a	$(7.41 - 5.54 i) \times 10^{-4}$	$(4.93 - 2.32 i) \times 10^{-3}$	$(4.67 + 4.33 i) \times 10^{-3}$
b	$(1.19 - 0.89 i) \times 10^{-3}$	$(8.04 - 3.79 i) \times 10^{-3}$	$(7.53 + 6.97 i) \times 10^{-3}$
ϵ_e	3.31×10^{-8}	5.73×10^{-8}	2.14×10^{-7}
ϵ_μ	2.33×10^{-7}	4.3×10^{-7}	1.5×10^{-6}
ϵ_τ	3.5×10^{-7}	6.39×10^{-7}	2.26×10^{-6}

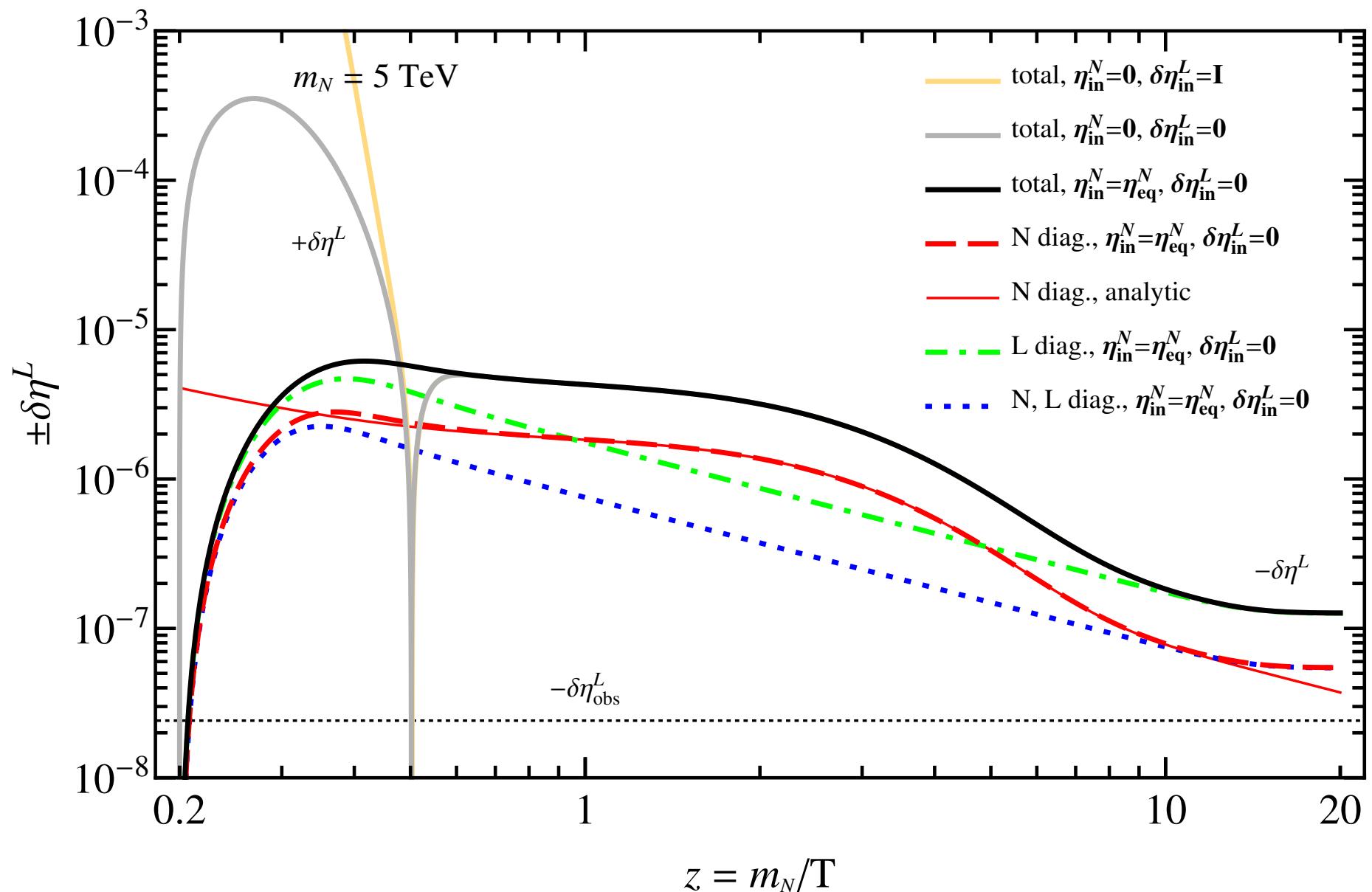
➡ **normal hierarchy for light neutrinos:** $\Delta m_{\text{sol}}^2 = 7.54 \times 10^{-5} \text{ eV}^2$, $\Delta m_{\text{atm}}^2 = 2.44 \times 10^{-3} \text{ eV}^2$, $\sin^2 \theta_{12} = 0.308$, $\sin^2 \theta_{23} = 0.425$, $\sin^2 \theta_{13} = 0.0234$.

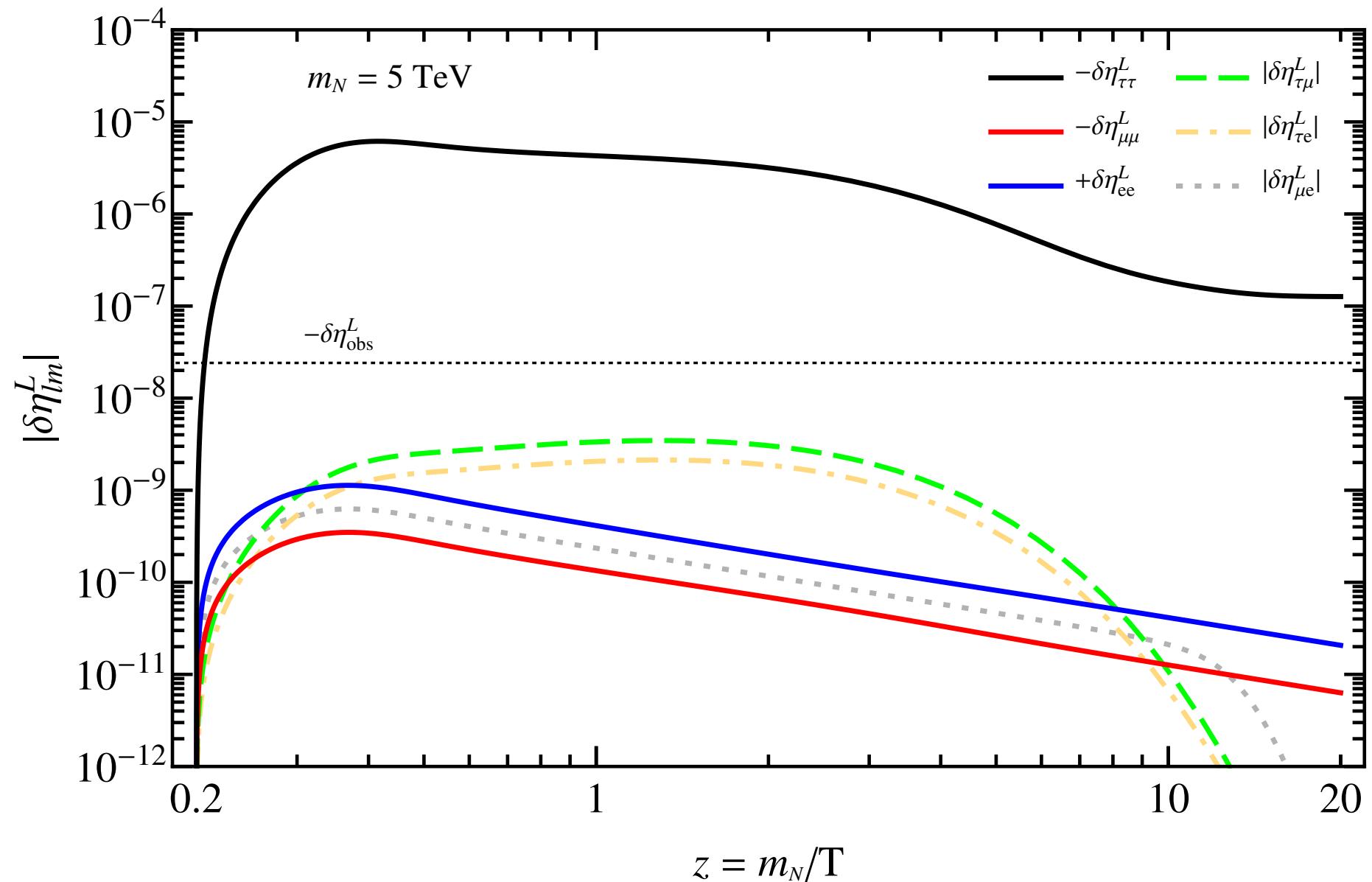


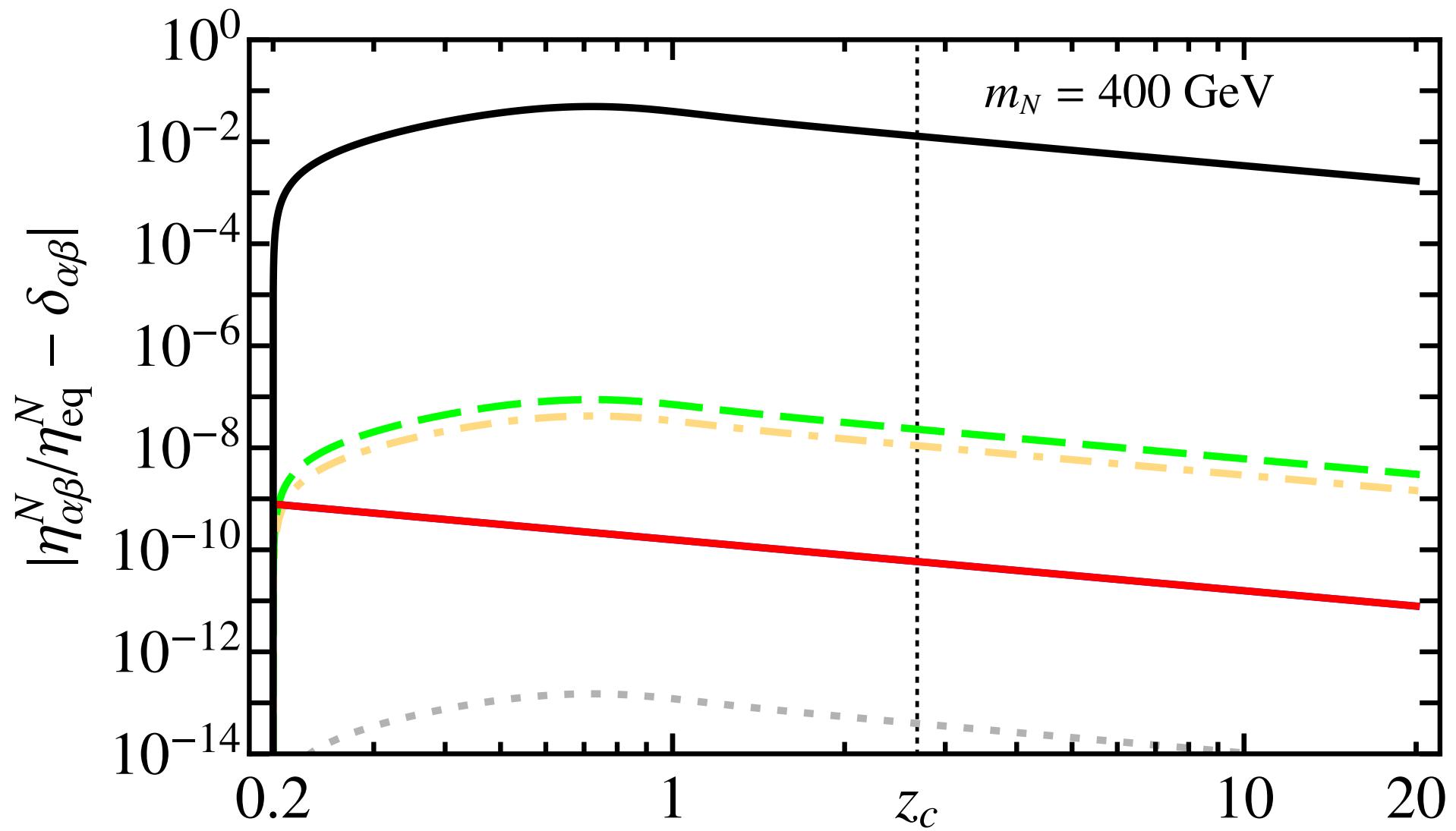












– Phenomenological Implications

[P.S.B. Dev, P. Millington, A.P., D. Teresi, arXiv:1404.1003.]

Low-energy observables	BP1 m_N 120 GeV	BP2 m_N 400 GeV	BP3 m_N 5 TeV	Experimental Limit
$\text{BR}(\mu \rightarrow e\gamma)$	4.5×10^{-15}	1.9×10^{-13}	2.3×10^{-17}	$< 5.7 \times 10^{-13}$
$\text{BR}(\tau \rightarrow \mu\gamma)$	1.2×10^{-17}	1.6×10^{-18}	8.1×10^{-22}	$< 4.4 \times 10^{-8}$
$\text{BR}(\tau \rightarrow e\gamma)$	4.6×10^{-18}	5.9×10^{-19}	3.1×10^{-22}	$< 3.3 \times 10^{-8}$
$\text{BR}(\mu \rightarrow 3e)$	1.5×10^{-16}	9.3×10^{-15}	4.9×10^{-18}	$< 1.0 \times 10^{-12}$
$R_{\mu \rightarrow e}^{\text{Ti}}$	2.4×10^{-14}	2.9×10^{-13}	2.3×10^{-20}	$< 6.1 \times 10^{-13}$
$R_{\mu \rightarrow e}^{\text{Au}}$	3.1×10^{-14}	3.2×10^{-13}	5.0×10^{-18}	$< 7.0 \times 10^{-13}$
$R_{\mu \rightarrow e}^{\text{Pb}}$	2.3×10^{-14}	2.2×10^{-13}	4.3×10^{-18}	$< 4.6 \times 10^{-11}$
$\langle m \rangle (\text{eV})$	3.8×10^{-3}	3.8×10^{-3}	3.8×10^{-3}	$< 0.11 - 0.25$

- **Comparison with Other Methods:** ε_{N_1} [F. Deppisch, A.P., PRD83 (2011) 076007.]

Consider a simple Inverse Seesaw-like Model ($1L + 2\nu_R$):

[R. N. Mohapatra, PRL56 (1986) 561;
 R. N. Mohapatra, J. W. F. Valle, PRD34 (1986) 1642;
 P. S. B. Dev and A.P., PRD86 (2012) 113001.]

$$M_\nu = \begin{pmatrix} 0 & vy & 0 \\ vy & \mu_1 & M \\ 0 & M & \mu_2 e^{i\alpha} \end{pmatrix}$$

Heavy neutrino masses:

$$M_{1,2} \approx M \mp \frac{\mu}{2}, \quad \text{with } \mu = |\mu_1 + \mu_2 e^{i\alpha}|$$

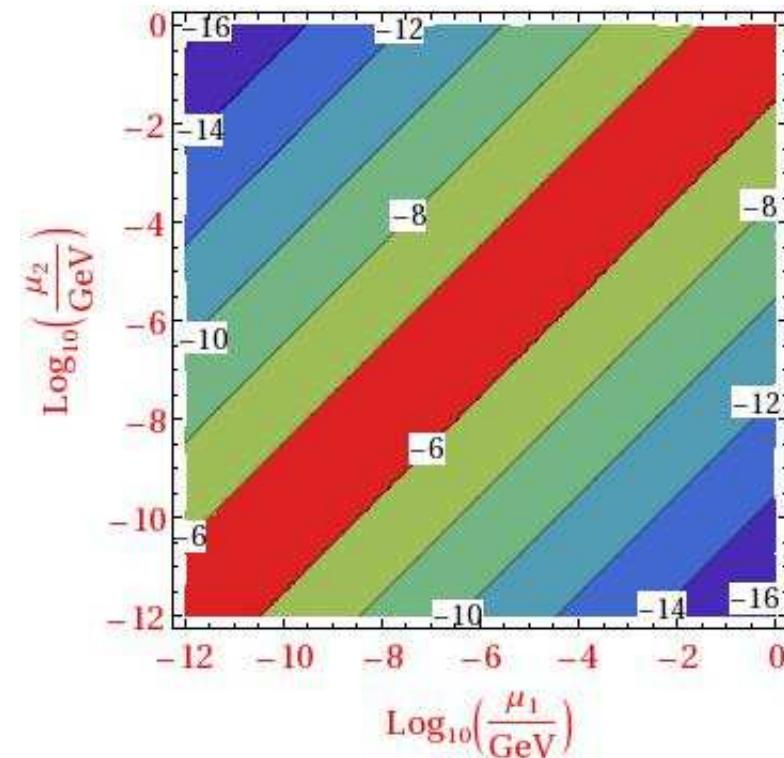
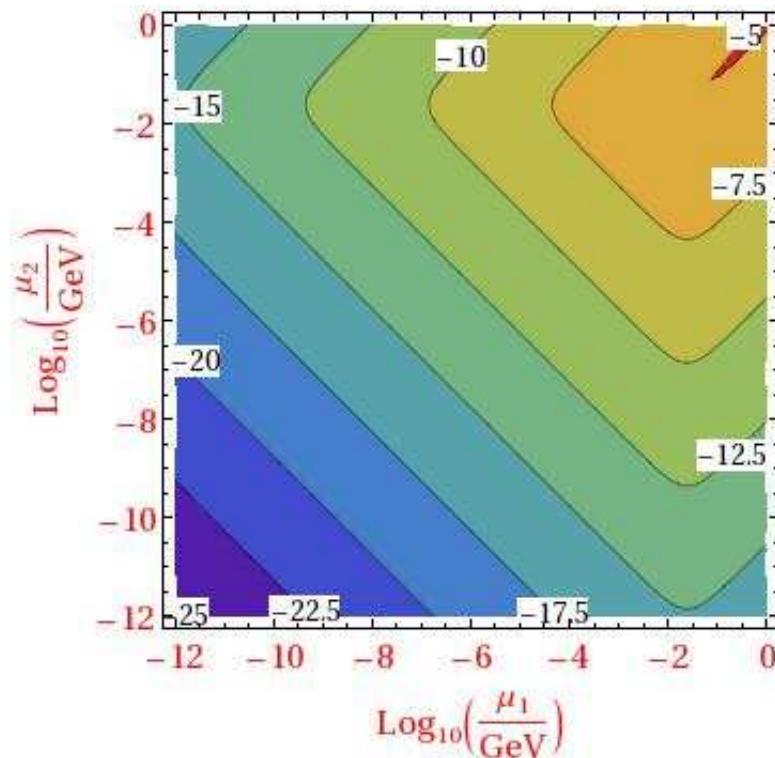
Lepton asymmetry ε_{N_1} :

$$\varepsilon_{N_1} = \frac{\text{Im} (h^{\nu\dagger} h^\nu)_{12}^2}{(h^{\nu\dagger} h^\nu)_{11} (h^{\nu\dagger} h^\nu)_{22}} f_{\text{reg}}$$

$$\implies \varepsilon_{N_1} \rightarrow 0, \text{ when } \mu_{1,2} \rightarrow 0.$$

L-conserving limits of ε_{N_1}

[P.S.B. Dev, P. Millington, A.P., D. Teresi, arXiv:1404.1003.]



Singular regulator:

[W. Buchmüller and M. Plümacher, PLB431 (1998) 354.]

$$f_{\text{reg}}^{\text{BP}} = \frac{\left| m_{N_1}^2 - m_{N_2}^2 \right| m_{N_1} \Gamma_{N_2}}{\left(m_{N_1}^2 - m_{N_2}^2 \right)^2 + \left(m_{N_1} \Gamma_{N_1} - m_{N_2} \Gamma_{N_2} \right)^2}$$

Thermal QFT Effects

G. F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia,
Towards a complete theory of thermal leptogenesis in the SM and MSSM,
Nucl. Phys. B685 (2004) 89.

A. De Simone and A. Riotto,
Quantum Boltzmann equations and leptogenesis, JCAP 0708 (2007) 002.

M. Garny, A. Hohenegger, A. Kartavtsev and M. Lindner,
Systematic approach to leptogenesis in nonequilibrium QFT: self-energy
contribution to the CP-violating parameter, Phys. Rev. D81 (2010) 085027.

M. Beneke, B. Garbrecht, C. Fidler, M. Herranen and P. Schwaller,
Flavoured leptogenesis in the CTP formalism, Nucl. Phys. B843 (2011) 177.

A. Anisimov, W. Buchmüller, M. Drewes and S. Mendizabal,
Quantum leptogenesis I, Annals Phys. 326 (2011) 1998.

Perturbative Non-Equilibrium Thermal Field Theory,
P. Millington and A.P., Phys. Rev. D88 (2013) 085009 (102 pages).