LISA: An opportunity to trace every bit of spacetime around a BH
Searching for deformed BHs

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EMRIs: *Extreme Mass Ratio Inspirals*

- One of the sources of GWs that are detectable by LISA are small (solar) BH orbiting around a massive one ($\sim 10^6 M_\odot$).
- The corresponding GW frequency for the ISCO orbit is

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What if the central object is not exactly a BH?

- A rotating black hole (Kerr BH) is the outcome of a physical process that has radiated away all the extra physical irregularities that are apparent in its progenitor (according to GR).

- A Kerr BH is an empty space, suitably curved according to vaccum Einstein's equations, in such a way that the only inprint it carries is the mass and the ang. momentum of the final collapsed object.

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Is it possible to discern a “normal” BH from a deviated one?

- The basic difference between a true BH and a deviated one is that the geodesic in the first case is described by an integrable system. This is not true for a deformed BH, though.
- A Kerr geodesic is fully characterized by 3 integrals of motion, E, L_z, Q.
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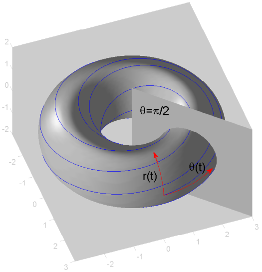
- The phase orbit for a perturbed integrable system, according to KAM theorem, still moves on tori (as for the integrable one) almost for all initial conditions.



- This is not the case though for these special tori of the integrable case, where the ratio of the corresponding fundamental frequencies is a rational number (especially a ratio of small integers).
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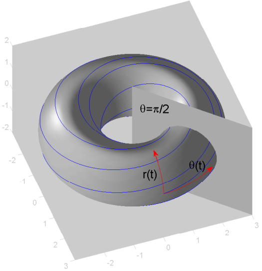
What to look for?

- By intersecting the winding phase orbit by a plane (Poincaré section) one obtains a series of points that form either a single curve (a KAM invariant curve),



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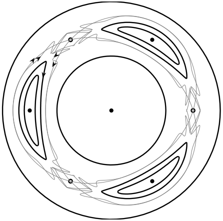

While an orbit evolves due to radiation reaction

- As the orbit evolves adiabatically, when it hits a resonance, it will enter a Birkhoff island
- ...and the ratio of the corrsponding frequencies will get locked to a constant number.
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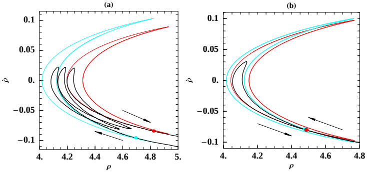

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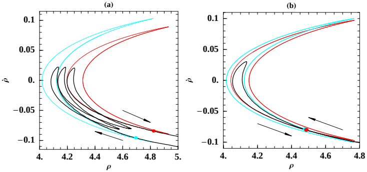

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Is this detectable?

- Using[1] a special metric to investigate the footprint of a non-integrable case we did induce a GW radiation, suitably adjusted, to make the orbit trace a resonance.
- The ratio of the two observable frequencies stayed almost fixed for a finite time interval.
- This is the best scenario though, since this time interval depends crucially on the initial conditions and in most cases it was quite short.

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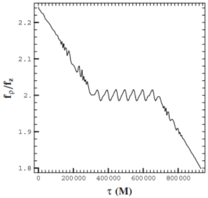

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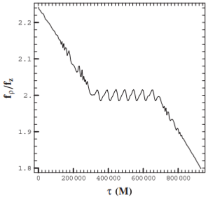

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Glitches in LISA's signals

- Kokkotas group has recently[2] studied the evolution of 2 types of EMRIs based on approximate solutions of Einstein's vacuum equation.
- (a) The first one is a deformed Kerr with integrable geodesics (they have a Carter constant)
- (b) The second one is a deviating Kerr (with no integrable geodesics).
- Both were evolved near a strong resonance by means of a quadrupole-like approximation scheme (representing GW radiation).

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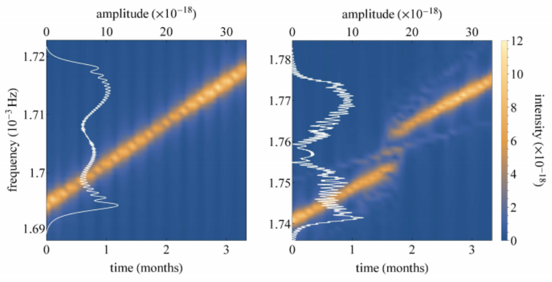

- The amplitude of the non-integrable system (right) shows a glitch, while the power spectrum is much less smoother than the corresponding integrable one (left). The period of the glitch is of the order of a few days (for a $m_2 = 2 \times 10^6 M_\odot$ – $m_1 = 2M_\odot$ system).

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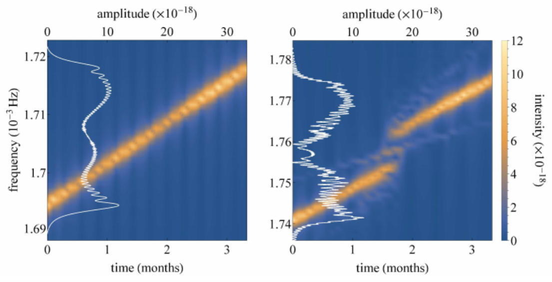
- 

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A Newtonian model

- In order to investigate these two aspect of adiabatic evolution we used a Newtonian model instead of a Kerr or a perturbed Kerr.
- This newtonian model[3] is a version of the Euler's problem of 2-fixed centers at an imaginary distance *ia* apart with an additional small mass μ at the center.
- For $\mu = 0$ this is an integrable analogue of a Kerr BH with mass M and spin a.
- By choosing a $\mu << M$, one could transform the problem into a slightly non-integrable one.

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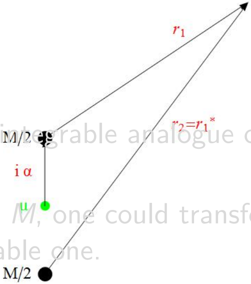

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Comparing the time-passage

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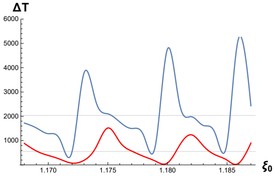

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