Cosmic Inflation & Gravity Waves

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Workshop on Cosmic Inflation
Thessaloniki, Greece

Inflationary Cosmology [Starobinsky, Mukhanov, Chibisov, Guth, Linde, Hawking, $\cdots$]

Successful Primordial Inflation should:

- Explain flatness, isotropy;
- Provide origin of $\frac{\delta T}{T}$;
- Offer testable predictions for n_s, r (gravity waves)$, dn_s/d$ ln$k$;
- Recover Hot Big Bang Cosmology;
- Explain the observed baryon asymmetry;
- Offer plausible CDM candidate;

Slow-roll Inflation

- Inflation is driven by some potential $V(\phi)$:
- Slow-roll parameters:

$$\epsilon = \frac{m_p^2}{2}\left(\frac{V'}{V}\right)^2, \ \eta = m_p^2\left(\frac{V''}{V}\right).$$

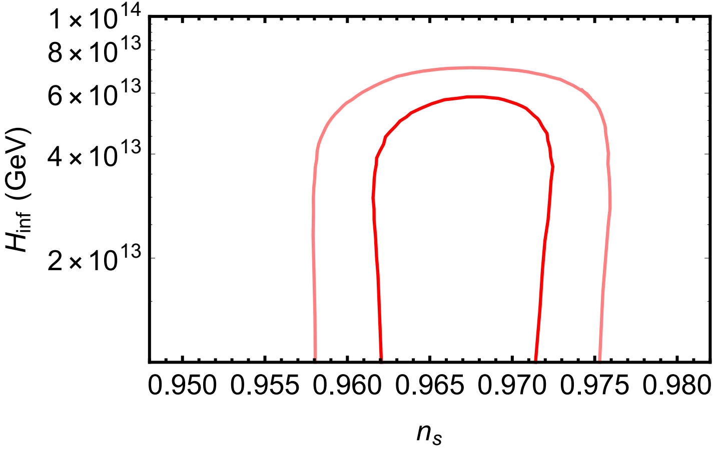
- The spectral index n_s and the tensor to scalar ratio r are given by

$$n_s - 1 \equiv \frac{d\ln\Delta_{\mathcal{R}}^2}{d\ln k}, \ r \equiv \frac{\Delta_h^2}{\Delta_{\mathcal{R}}^2},$$

  where Δ_h^2 and $\Delta_{\mathcal{R}}^2$ are the spectra of primordial gravity waves and curvature perturbation respectively.
- Assuming slow-roll approximation (i.e. $(\epsilon, |\eta|) \ll 1$), the spectral index n_s and the tensor to scalar ratio r are given by

$$n_s \simeq 1 - 6\epsilon + 2\eta, \ r \simeq 16\epsilon.$$

Constraint on Inflation Planck (2018), BK (2015)



SUSY Higgs (Hybrid) Inflation

[Dvali, Shafi, Schaefer; Copeland, Liddle, Lyth, Stewart, Wands '94]
[Lazarides, Schaefer, Shafi '97][Senoguz, Shafi '04; Linde, Riotto '97]
[Buchmüller, Domcke and Schmitz]

- Attractive scenario in which inflation can be associated with symmetry breaking $G \longrightarrow H$
- Simplest inflation model is based on

$$W = \kappa\, S\, (\Phi\, \overline{\Phi} - M^2)$$

$S =$ gauge singlet superfield, $(\Phi\, , \overline{\Phi})$ belong to suitable representation of G

- Need $\Phi\, , \overline{\Phi}$ pair in order to preserve SUSY while breaking $G \longrightarrow H$ at scale $M \gg$ TeV, SUSY breaking scale.
- R-symmetry

$$\Phi\, \overline{\Phi} \to \Phi\, \overline{\Phi}, \;\; S \to e^{i\alpha}\, S, \;\; W \to e^{i\alpha}\, W$$

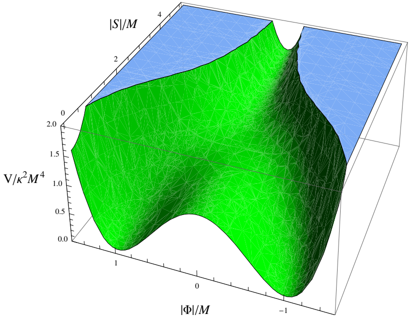
SUSY Higgs (Hybrid) Inflation

- Tree Level Potential

$$V_F = \kappa^2 \left(M^2 - |\Phi^2|\right)^2 + 2\kappa^2 |S|^2 |\Phi|^2$$

- SUSY vacua

$$|\langle\overline{\Phi}\rangle| = |\langle\Phi\rangle| = M,\ \langle S\rangle = 0$$



SUSY Higgs (Hybrid) Inflation

Take into account radiative corrections (because during inflation $V \neq 0$ and SUSY is broken by $F_S = -\kappa\, M^2$)

- Mass splitting in $\Phi - \overline{\Phi}$

$$m_\pm^2 = \kappa^2\, S^2 \pm \kappa^2\, M^2, \quad m_F^2 = \kappa^2\, S^2$$

- One-loop radiative corrections

$$\Delta V_{\mathsf{1loop}} = \frac{1}{64\pi^2}\mathsf{Str}[\mathcal{M}^4(S)(\ln \frac{\mathcal{M}^2(S)}{Q^2} - \frac{3}{2})]$$

- In the inflationary valley ($\Phi = 0$)

$$V \simeq \kappa^2\, M^4 \left(1 + \frac{\kappa^2 \mathcal{N}}{8\pi^2} F(x)\right)$$

where $x = |S|/M$ and

$$F(x) = \frac{1}{4}\left(\left(x^4+1\right)\ln\frac{\left(x^4-1\right)}{x^4} + 2x^2 \ln\frac{x^2+1}{x^2-1} + 2\ln\frac{\kappa^2 M^2 x^2}{Q^2} - 3\right)$$

SUSY Higgs (Hybrid) Inflation

Tree level + radiative corrections + minimal Kähler potential yield:

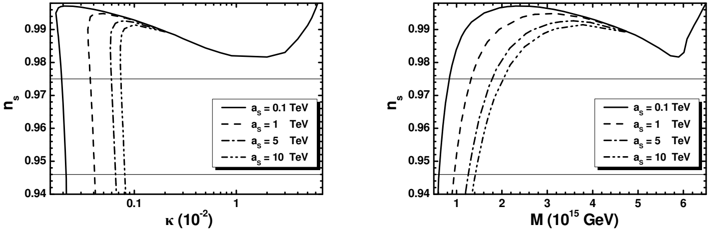
$$n_s = 1 - \frac{1}{N} \approx 0.98.$$

$\delta T/T$ proportional to M^2/M_p^2, where M denotes the gauge symmetry breaking scale. Thus we expect $M \sim M_{GUT}$ for this simple model. In practice, $M \approx (1-5) \times 10^{15}$ GeV

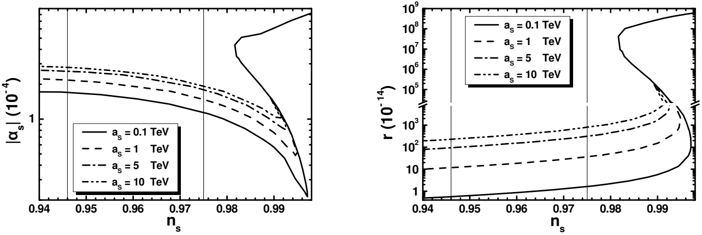
Since observations suggest that n_s lie close to 0.97, there are at least two ways to realize this slightly lower value:

- include soft SUSY breaking terms, especially a linear term in S;
- employ non-minimal Kähler potential.

[Pallis, Shafi, 2013; Rehman, Shafi, Wickman, 2010]



(a) (b)

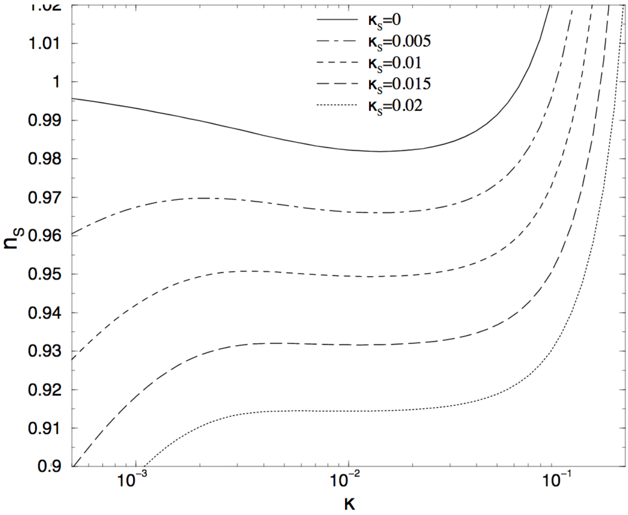


(a) (b)

SUSY Higgs (Hybrid) Inflation

- $K \supset \kappa_s (S^\dagger S)^2$

[M. Bastero-Gil, S. F. King and Q. Shafi, 2006]



Susy Hybrid Inflation

- Some examples of gauge groups G such that

$$G \xrightarrow{\langle\Phi\rangle\neq 0} H \supseteq SM \equiv SU(3)_c \times SU(2)_L \times U(1)_Y$$

where

$G = SM \times U(1)_{B-L}$, (cosmic strings)

$G = SU(5)$, $(\Phi = \overline{\Phi} = 24)$, (monopoles)

$G = SU(5) \times U(1)$, $(\Phi = 10)$, (Flipped $SU(5)$)

$G = SU(4)_c \times SU(2)_L \times SU(2)_R$, $(\Phi = (\overline{4}, 1, 2))$, (monopoles)

$G = SO(10)$, $(\Phi = 16)$ (monopoles)

(Non-minimal) Sugra Hybrid Inflation

[M. Bastero-Gil, S. F. King, Q. Shafi 2006; M. Rehman, V. N. Senoguz, Q. Shafi 2006]

- The superpotential is given by

$$W = \kappa S \left[\Phi \overline{\Phi} - M^2 \right]$$

- The Kähler potential can be expanded as

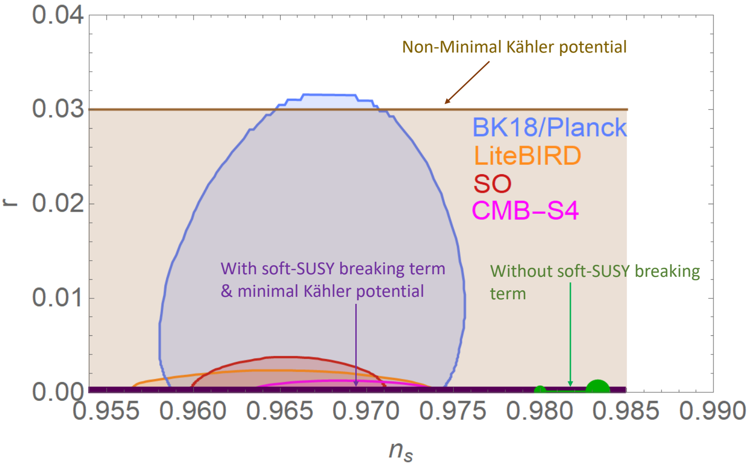
$$\begin{aligned} K &= |S|^2 + |\Phi|^2 + \left|\overline{\Phi}\right|^2 \\ &+ \kappa_S \frac{|S|^4}{4\, m_P^2} + \kappa_\Phi \frac{|\Phi|^4}{4\, m_P^2} + \kappa_{\overline{\Phi}} \frac{|\overline{\Phi}|^4}{4\, m_P^2} \\ &+ \kappa_{S\Phi} \frac{|S|^2 |\Phi|^2}{m_P^2} + \kappa_{S\overline{\Phi}} \frac{|S|^2 |\overline{\Phi}|^2}{m_P^2} + \kappa_{\Phi\overline{\Phi}} \frac{|\Phi|^2 |\overline{\Phi}|^2}{m_P^2} \\ &+ \kappa_{SS} \frac{|S|^6}{6\, m_P^4} + \cdots \end{aligned}$$

- Now including all other corrections potential takes the following form

$$V \simeq \kappa^2 M^4 \left(1 - \kappa_S \left(\frac{S}{m_P} \right)^2 + \frac{\gamma_S}{2} \left(\frac{S}{m_P} \right)^4 \right) + V_{1-loop} + V_{soft}$$

where, $\gamma_S = 1 - \frac{7\kappa_S}{2} - 2\kappa_S^2 - 3\kappa_{SS}$.

Results



Non-Minimal Kähler potential
BK18/Planck
LiteBIRD
SO
CMB−S4
With soft-SUSY breaking term
& minimal Kähler potential
Without soft-SUSY breaking
term
0.04
0.03
0.02
0.01
0.00
r
0.955
0.960
0.965
0.970
0.975
0.980
0.985
0.990
n_s

μ-HYBRID INFLATION

N. Okada and Q. Shafi, 2017

$$W = \kappa S(\Phi\bar{\Phi} - M^2) + \lambda S H_u H_d$$

- The S field gets a destabilizing tadpole term $\simeq 2\kappa m_{3/2} M^2 S + h.c.$, and taking account of the term $\simeq 2\kappa^2 M^2 |S|^2$, the resulting vev of S is $\simeq m_{3/2}/\kappa$.

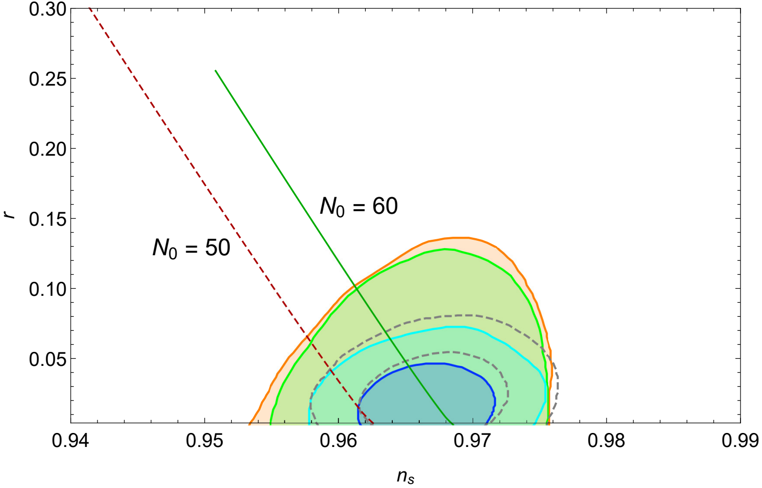
- The vev of S will generate a μ term with

$$\mu = \lambda\langle S\rangle = m_{3/2}(\lambda/\kappa) \simeq (10^2 - 10^3) GeV.$$

$$\Gamma_S(S \to \tilde{H}_u \tilde{H}_d) = \frac{\lambda^2}{8\pi} m_S,$$

ϕ^4 Inflation with non-minimal coupling to gravity

[Okada, Rehman, Shafi, 2010]



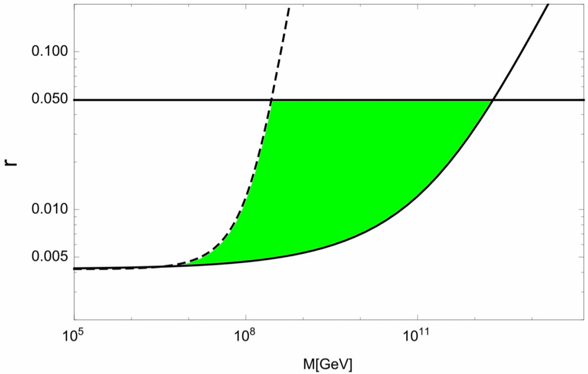
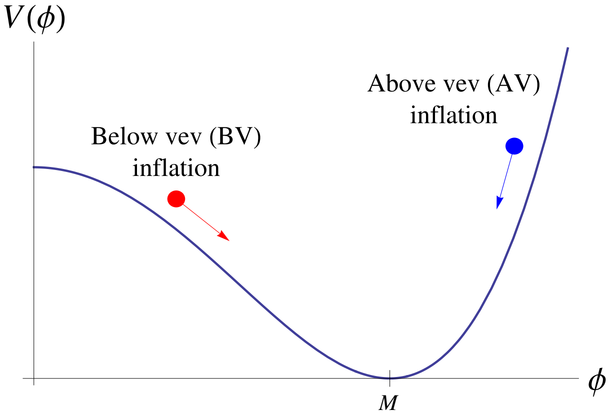


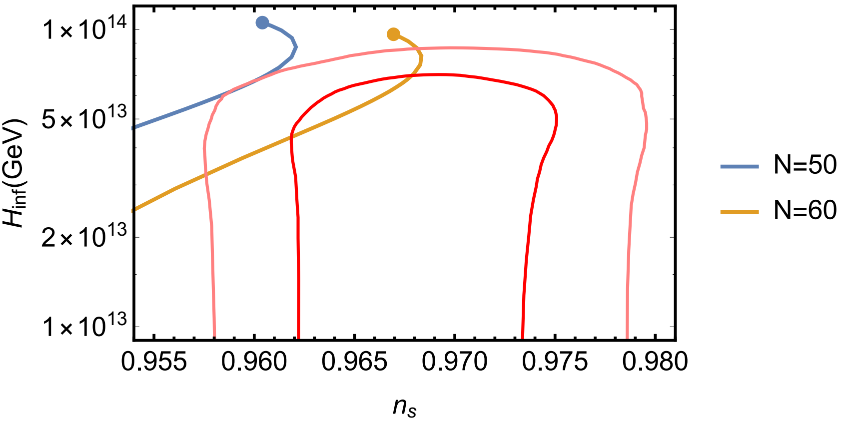
FIG. : Allowed parameter regions in (M, r)-plane. The solid curve corresponds to the solid diagonal line (top), while the dashed curve corresponds to the (left) dashed diagonal line in Figure 2. The horizontal solid line depicts the upper bound from the Planck measurements, $r \leq 0.0496$ for $N_0 = 50$. The shaded region satisfies all the constraints.

Inflation with a CW Higgs Potential



Note: This is for minimal coupling to gravity

Inflation with a CW Higgs Potential



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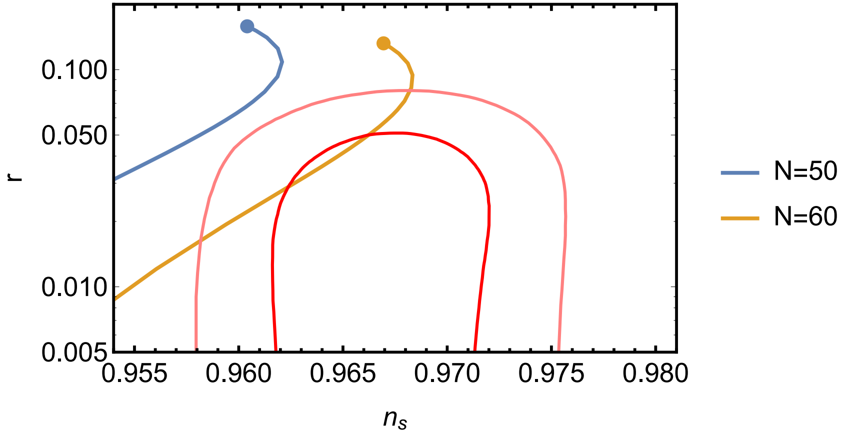
Inflation with a CW Higgs Potential



Note: This is for minimal coupling to gravity

Evolution of Intermediate-mass Monopoles

Number density at production, $\xi = \min\left(H^{-1}, m_{\rm eff}^{-1}\right)$

Dilution during Inflation

Dilution from Inflaton oscillation

Monopole yield after reheating :

$$\boxed{Y_M \simeq \frac{\frac{\xi^{-3}}{10}\ \exp(-3N_M)\ \left(\frac{\tau}{t_r}\right)^2}{\frac{2\pi^2}{45} g_* T_r^3}}$$

Entropy density after reheating

- MACRO bound: $Y_M \lesssim 10^{-27}$.

Ambrosio et al. [MACRO Collaboration], EPJC 25, 511 (2002)

- Adopted threshold for observability: $Y_M \gtrsim 10^{-35}$.

Intermediate Mass Monopoles and MACRO

$\frac{V_0^{1/4}}{10^{16}\text{GeV}}$	ϕ_+/m_{Pl}	ϕ_-/m_{Pl}	H_+ (10^{13} GeV)	H_- (10^{13} GeV)	N_+	N_-	$\log_{10}\left(\frac{M_{I+}}{\text{GeV}}\right)$	$\log_{10}\left(\frac{M_{I-}}{\text{GeV}}\right)$
1.51	14.41	13.07	3.40	3.91	9.8	16.2	13.30	13.40
1.59	16.04	14.67	3.54	4.10	9.9	16.2	13.30	13.41
1.66	17.91	16.51	3.67	4.28	9.9	16.2	13.31	13.41
1.74	20.05	18.62	3.78	4.45	9.9	16.2	13.31	13.41
1.82	22.51	21.04	3.88	4.59	9.9	16.2	13.31	13.41

Table: Values of the various parameters (indicated by a subscript $+$) corresponding to the MACRO bound ($Y_M < 10^{-27}$) on the flux of monopoles formed at the scale M_I and their values (indicated by a subscript $-$) corresponding to the adopted observability threshold ($Y_M > 10^{-35}$) for the monopole flux.

Chakrabortty, Lazarides, RM, Shafi JHEP 02 (2021) 114

ϕ^4 Inflation with non-minimal coupling to gravity

- Consider the following action in the Jordan frame:

$$S_J = \int d^4x \sqrt{-g} \left[\left(\frac{m_P^2 + \xi\phi^2}{2} \right) \mathcal{R} - \frac{1}{2} (\partial\phi)^2 - \frac{\lambda}{4}\, \phi^4 \right].$$

- In the Einstein frame the potential turns out to be:

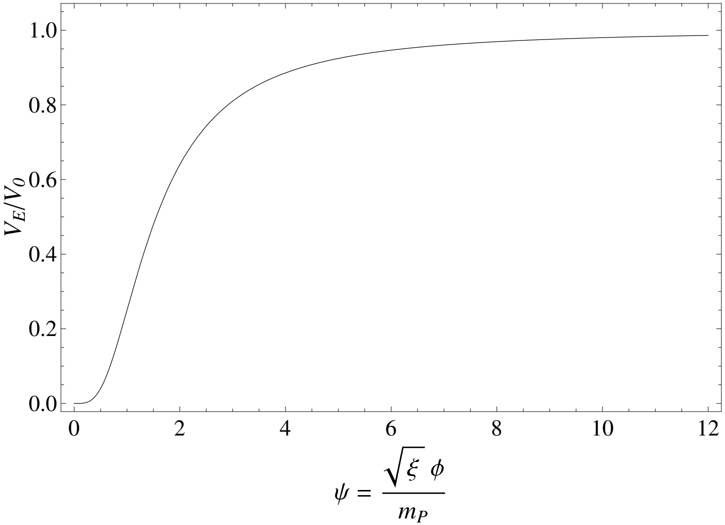
$$V_E(\phi) = \frac{\frac{1}{4}\lambda\phi^4}{\left(1+\frac{\xi\,\phi^2}{m_P^2}\right)^2} = V_0 \frac{\psi^4}{(1+\psi^2)^2},$$

where $V_0 = \left(\frac{\lambda\, m_P^4}{4\,\xi^2} \right)$ and $\psi \equiv \frac{\sqrt{\xi}\,\phi}{m_P}$.

- The kinetic energy of the scalar field is made canonical with respect to a new field σ as

$$\left(\frac{d\sigma}{d\phi} \right)^{-2} = \frac{\left(1+\frac{\xi\phi^2}{m_P^2}\right)^2}{1+(6\xi+1)\frac{\xi\phi^2}{m_P^2}}.$$

ϕ^4 Inflation with non-minimal coupling to gravity



ϕ^4 Inflation with non-minimal coupling to gravity

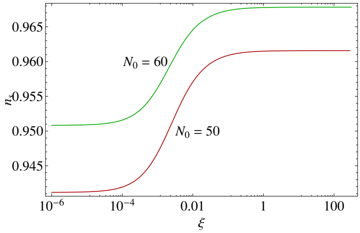
[Okada, Rehman, Shafi, 2010]

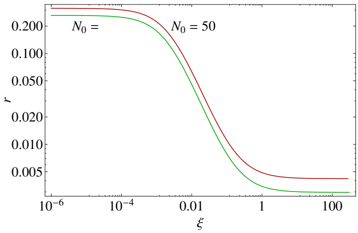
- CMB observables in the large ξ limit:

$$n_s \simeq 1 - \frac{2}{N_0} \sim 0.967, \quad r \simeq \frac{12}{N_0^2} \sim 0.003, \quad \text{for } N_0 = 60$$

with

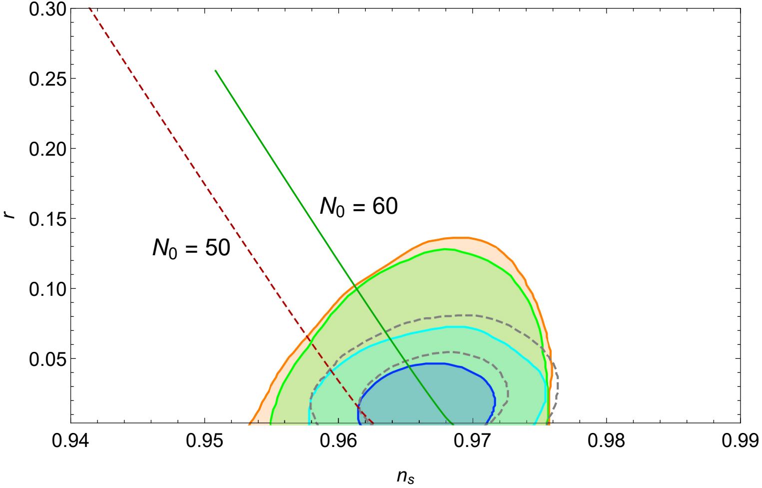
$$A_s \simeq \frac{\lambda}{\xi^2} \frac{N_0^2}{72\pi^2} \;\Rightarrow\; \xi \simeq \left(\frac{N_0}{\sqrt{72}\,\pi\, A_s}\right)\sqrt{\lambda} \sim 10^4 \text{ for } \lambda \sim 1 \text{ [SM Higgs Inflation?]}$$





ϕ^4 Inflation with non-minimal coupling to gravity

[Okada, Rehman, Shafi, 2010]



$SO(10) \times U(1)_{PQ}$ [redacted]

Holman, Lazarides, Shafi; Lazarides, Maji, Shafi

- $$\begin{aligned} & SO(10) \times U(1)_{\rm PQ} \xrightarrow[M_U]{\langle 210(0) \rangle} \\ & SU(2)_L \otimes SU(2)_R \otimes SU(4)_C \times U(1)_{\rm PQ} \xrightarrow[M_I]{\langle (1,1,15) \in 210(0) \rangle} \\ & SU(2)_L \otimes SU(2)_R \otimes SU(3)_C \otimes U(1)_{B-L} \times U(1)_{\rm PQ} \xrightarrow[M_{II}]{\left\langle (1,3,1,-2) \in (1,3,10) \in \overline{126}(-2) \right\rangle} \\ & SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes \mathbb{Z}_2 \times U(1)'_{\rm PQ} \xrightarrow[f_a]{\langle (1,3,1)+(1,1,15) \in 45(4) \rangle} \\ & SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes \mathbb{Z}_2 \xrightarrow[m_W]{\left\langle (1,2,\pm\frac{1}{2}) \in 10(-2) \right\rangle} SU(3)_C \otimes U(1)_Q \otimes \mathbb{Z}_2. \end{aligned}$$
- $\psi_{16}^{(i)} (i = 1,2,3) \underset{U(1)_{PQ}}{\rightarrow} e^{i\theta} \psi_{16}^{(i)}$
- Residual discrete PQ symmetry is $Z_{12} \Rightarrow$ domain wall problem ($U(1)_{PQ}$ broken after inflation)
- Introduce two $SO(10)$ fermion 10-plets $\psi_{10}^{(\alpha)} \rightarrow e^{-2i\theta} \psi_{10}^{(\alpha)} \quad (\alpha = 1,2)$
- Residual discrete symmetry is now Z_4, which coincides with the center Z_4 of $SO(10)$ (Spin(10)) (for the full theory) (Cosmic Strings and Dark Matter)

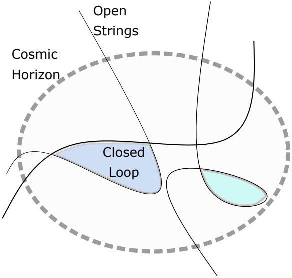
- $\phi_{210} \to \phi_{210}, \quad \phi_{126} \to e^{2i\theta}\phi_{126}, \quad \phi_{45} \to e^{4i\theta}\phi_{45}, \quad \phi_{10} \to e^{-2i\theta}\phi_{10}$. These transformation properties ensure that the action of the residual PQ symmetry on these fields is identical to that of the center of $SO(10)$.
- Yukawa couplings: $\psi_{16}\psi_{16}\phi_{10}, \psi_{16}\psi_{16}\phi_{126}, \psi_{10}\psi_{10}\phi_{45}$
- Higgs couplings include $\phi_{210}\phi_{126^\dagger}\phi_{126^\dagger}\phi_{45}, \phi_{210}\phi_{126^\dagger}\phi_{10}\phi_{45}, \phi_{210}\phi_{126}\phi_{10}$
- These couplings guarantee that $U(1)_{PQ}$ is the only global symmetry present.

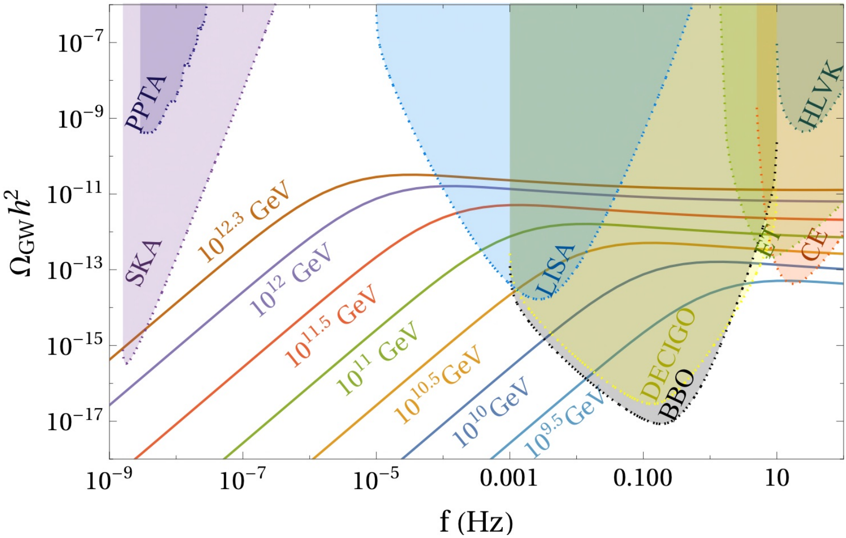
- First breaking produces GUT monopoles that are inflated away.
- Second breaking makes intermediate scale cosmic strings which can appear after inflation $\Rightarrow$ astrophysical test of GUTs.
- Dark Matter:
  In addition to axions there could exist WIMP-like DM in this class of models because of the fermion 10-plets.

Cosmic Strings from SO(10)

Cosmic Strings arise during symmetry breaking of $G \to H$ if $\pi_1(G/H)$ is non-trivial. Consider

$SO(10) \xrightarrow{M_{GUT}} SU(4) \times SU(2)_L \times SU(2)_R \xrightarrow{M_I} SM \times Z_2$ Mass per unit length of string is $\mu \sim M_I^2$, with $M_I \ll M_P$. The strength of string gravity is determined by the dimensionless parameter $G\mu \ll 1$.





PPTA
SKA
LISA
DECIGO
BBO
ET
CE
HLVK
$10^{12.3}$ GeV
10^{12} GeV
$10^{11.5}$ GeV
10^{11} GeV
$10^{10.5}$GeV
10^{10}GeV
$10^{9.5}$GeV
$\Omega_{GW} h^2$
f (Hz)
10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}
10^{-17}
10^{-9}
10^{-7}
10^{-5}
0.001
0.100
10

Quasi-stable strings & gravitational waves

Consider the symmetry breaking: $G \to H \times U(1) \to H$

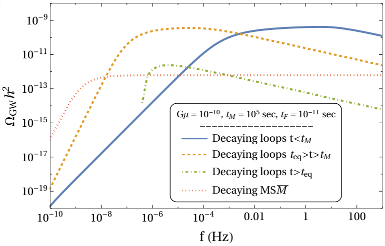
- The first step yields monopoles (& antimonopoles), which then connected to one another by strings from $U(1)$ breaking.
- If there is adequate hierarchy between the two breakings the strings are quantum mechanically stable.

Early Universe:

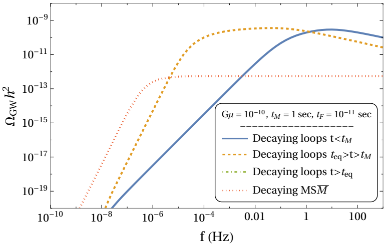
- Suppose monopoles undergo a period of inflation, but are not inflated away.
- Cosmic strings also may experience a period of inflation. As long as the monopoles are absent, we obtain gravitational radiation from the strings in the usual way.
- However, once the monopoles reenter the horizon we obtain strings with ends attached to monopoles antimonopoles. We expect that the long wavelength portion of the gravitational spectrum is modified.
- Models incorporating this scenario can be realized in $SO(10)$ breaking, for instance.

Gravitational Waves from Quasi-stable Strings

The gravitational wave spectra from loops decaying before and after t_M (the horizon reentrance time of the monopoles) during radiation dominance, from loops decaying after the equidensity time t_{eq}, and from the decaying $MS\bar{M}$ structures.



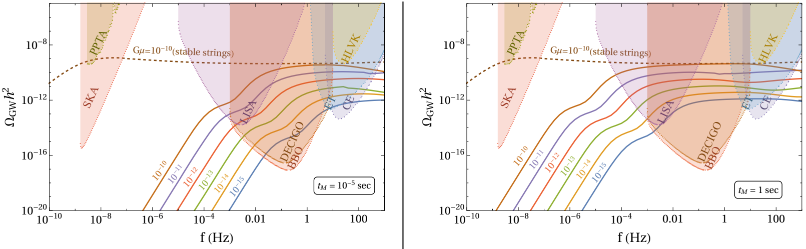
$m_M = 4.45 \times 10^{15} GeV$



$m_M = 4.17 \times 10^{15} GeV.$

Gravitational Waves from Quasi-stable Strings

The total gravitational wave spectra from quasi-stable cosmic strings with varying G_μ values as indicated and for different horizon reentrance times of the monopole-antimonopole pairs.



Gravitational Waves from Quasi-stable Strings

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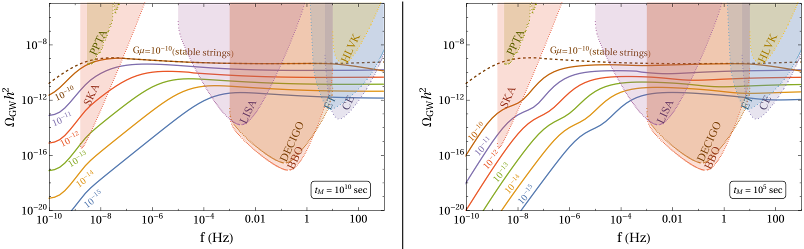


Thank You

Stochastic Gravity Waves from Strings

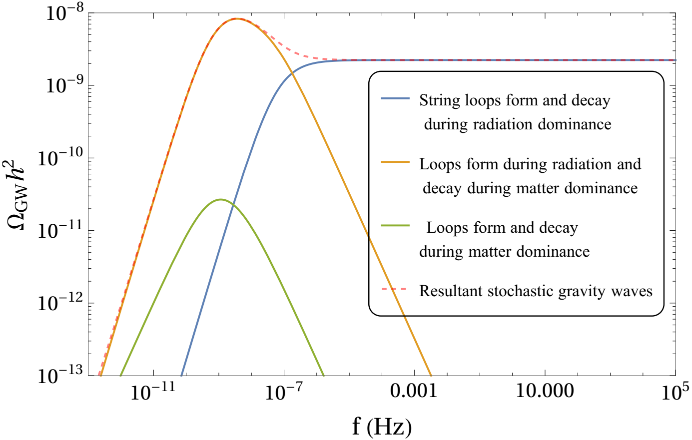
- Unresolved GWs bursts from string loops at different cosmic era produces the stochastic background.

- Loops that are formed and decay during radiation produce a plateau in the spectrum in the high frequency regime.

- Loops that are produced during radiation dominance but decay during matter dominance generate a sharply peaked spectrum at lower frequencies.

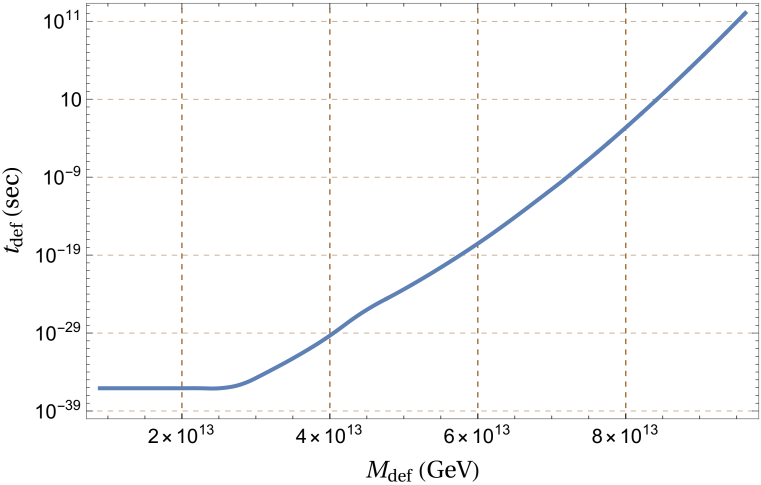
- Loops that are produced and decay during matter domination also generate a sharply peaked spectrum which, however, is overshadowed by the previous case.

Stochastic Gravity Wave Background: Analytic Approximation



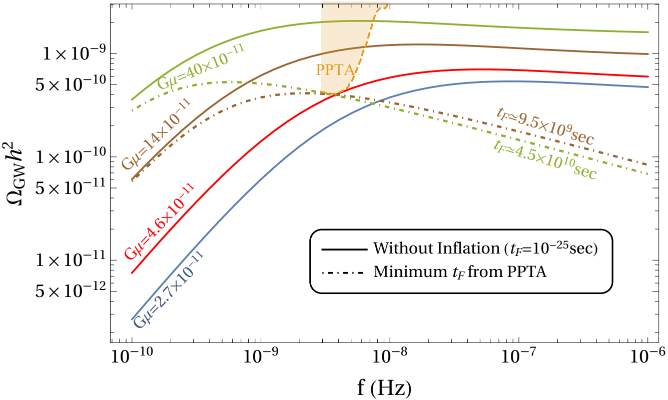
Sousa, Avelino, Guedes, arXiv:2002.01079

Gravitational Waves from Quasi-stable Strings



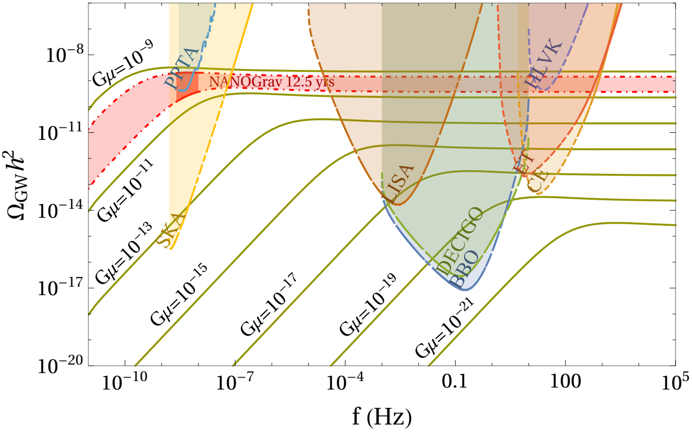
Horizon reentry time t_{def} of the topological defects (monopoles or strings) versus the symmetry breaking scale M_{def} (M_I or M_{II}).

Inflation, GWs and PPTA bound



- Partially inflated strings re-enter horizon at a time t_F in post-inflationary universe and can decay via GWs emission.
- Modified GWs spectra from ‘diluted’ strings can satisfy the PPTA bound.

GWs without Inflation and Observational Prospects



- Strongest constraint has come from PPTA: $G\mu \lesssim 10^{-11}$.
- Provisional GWs signal in NANOGrav: $G\mu \sim 10^{-10}$.