

Thessaloniki

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de Sitter Vacua, Moduli Stabilisation and Hybrid Inflation  
in String Theory

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*based on works with:*

*I. Antoniadis, Y. Chen 1803.08941, 1909.10525*

*I. Antoniadis, O. Lacombe 2007.10362, 2109.03243*

*Pramod Shukla 2203.03362*

## A few facts about **Cosmology** and **de Sitter Vacua**

▲ *Major Observational Discovery* ~ 2 decades ago:

∃ ongoing **Accelerating Expansion of the Universe**

▲ *Standard Interpretation:*

Universe dominated by **Dark Energy** permeating all of space

▲ *in G.R. framework:*

Einstein's equs with a positive cosmological constant:

$$\Lambda \approx 10^{-120} \text{ (in 4 - d Planck units)}$$

▲ or equivalently,

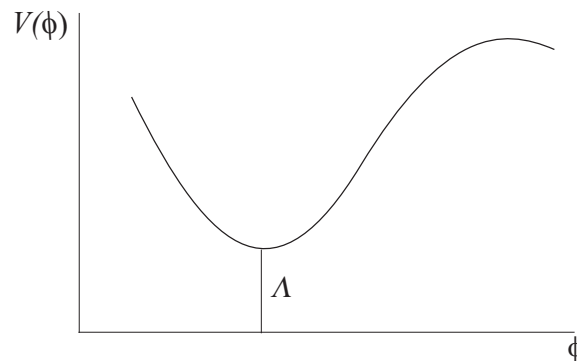
**Positive Vacuum Energy**

▲ *Simple Effective Field Theory description:*

with a scalar field,  $\phi$  acquiring

Potential Energy  $V(\phi)$

with **positive** vacuum energy:



▲ **de Sitter vacua** ▲

*...with some additional requirements:  $\phi \rightarrow$  inflaton*

suitable for **slow roll inflation**

A possible Interpretation within:

▲ **String Derived Effective Field Theories (EFT)** ▲

*Basic problems that must be solved*

▲ **Compactifications** characterised by **large numbers** of massless scalar fields (**moduli**)

▲ Find CY compactification inducing an EFT with  $V_{\min} > 0$  and **positive masses-squared** for all moduli fields  $\Rightarrow$

$\Rightarrow$  *Moduli Stabilisation*  $\Leftarrow$

▲ Look for possible **Inflaton** candidates among *moduli*

▲ at **String Theory** level: ▲

▲ # **CY** of **Compactifications** and # **fluxes**  $\Rightarrow$   
**Enormous** number of **String Vacua**



## String Landscape

▲ Long standing Question ▲

▲ Are there any **de Sitter vacua** in the **Landscape**?

... Even if the answer is **Yes**... we know that they are...

$\Rightarrow$  *Certainly Scarce*  $\Leftarrow$

*Focus of present Talk*

Context: type *IIB* theory:

- ▲ A solution to the **Moduli** Stabilisation problem
  
- ▲ Find a *de Sitter* vacuum in **String Theory**  
... based only on **perturbative** correction
  
- ▲ If **yes**:
  
- ▲ Examine cosmological implications such as **inflation**.

★ **Type II-B/F-theory**

★ **Moduli Space** (*notation*)

▲ Graviton, **dilaton** and Kalb-Ramond (**KR**)-field

$$g_{\mu\nu}, \phi, B_{\mu\nu} \rightarrow B_2$$

▲ **Scalar**, 2- and 4-index fields (*p-form potentials*)

$$C_0, C_{\mu\nu}, C_{\kappa\lambda\mu\nu} \rightarrow C_p, p = 0, 2, 4$$

1. ▲  $C_0, \phi \rightarrow$  combined to **axion-dilaton** *modulus*:

$$S = C_0 + i e^{-\phi} \equiv C_0 + \frac{i}{g_s}$$

2.  $z_a$  : **Complex Structure (CS)** moduli (*shape*)

3.  $T_i$  : **Kähler** moduli (*size*)

## Type II-B effective Supergravity

*Basic ‘ingredients’:*

*Superpotential  $\mathcal{W}$  and Kähler potential  $\mathcal{K}$*



▲ The Superpotential  $\mathcal{W}$  ▲

▲ Field strengths:

$$F_p := dC_{p-1}, \quad H_3 := dB_2, \quad \Rightarrow \mathbf{G}_3 := F_3 - SH_3$$

▲ Holomorphic  $(3,0)$ -form:  $\Omega(z_a)$

**Flux-induced superpotential** (*G.V.W. hep-th/9906070*):

$$\mathcal{W}_0 = \int \mathbf{G}_3 \wedge \Omega(z_a)$$

▲  $\mathcal{W}$ -Flatness conditions:

$$\mathcal{D}_{z_a} \mathcal{W} = 0, \quad \mathcal{D}_S \mathcal{W} = 0 :$$

$\Rightarrow z_a$  and  $S$  stabilised  $\Leftarrow$

**but!**

▲ Kähler moduli  $\notin \mathcal{W}_0 \Rightarrow$  remain unfixed! ▲

▲ The Kähler potential ▲

$$\mathcal{K}_0 = - \sum_{i=1}^3 \ln(-i(T_i - \bar{T}_i)) - \ln(-i(S - \bar{S})) - \ln(i \int \Omega \wedge \bar{\Omega}) .$$

▲ The classical scalar potential ▲

$$V = e^{\mathcal{K}} \left( \sum_{I,J} \mathcal{D}_I \mathcal{W}_0 \mathcal{K}^{I\bar{J}} \mathcal{D}_{\bar{J}} \mathcal{W}_0 - 3|\mathcal{W}_0|^2 \right) \equiv 0$$

due to *flatness* conditions and the *no-scale* structure



Kähler moduli completely **undetermined!**



Engineering the appropriate geometric set up and compute:

**Kähler** moduli-dependent **QUANTUM** corrections

*The Kähler potential  $\mathcal{K}$*   
*and*  
*PERTURBATIVE*  
String Loop Corrections

*Two types of expansions in String Theory:*

- i) Inverse string tension  $\propto \alpha'$ .*
- ii) String coupling  $g_s$*

▲  $\alpha'^3$  Corrections:

$$S \propto \int d^{10} \sqrt{-g} e^{-2\phi} (R + 4(\partial\phi)^2 + \alpha'^3 c_1 J_0)$$

Compactification  $\rightarrow$  redefinition of 4-d dilaton:

$$e^{-2\phi_4} = e^{-2\phi_{10}} (\mathcal{V} + \xi)$$

$$= e^{-\frac{1}{2}\phi_{10}} (\hat{\mathcal{V}} + \hat{\xi}) \quad (\text{Einstein frame})$$

$$\hat{\mathcal{V}} = \frac{1}{3!} \kappa_{ijk} \hat{t}^i \hat{t}^j \hat{t}^k = 6d \text{ volume}, \quad t^k = \text{Im} T^k$$

$$t^k = -\text{Im}(T^k) = \hat{t}^k \left( \frac{S - \bar{S}}{2i} \right)^{-1/2} \equiv \hat{t}^k g_s^{1/2}$$

$$\xi = -\frac{\zeta(3)}{4(2\pi)^3} \chi \left( \frac{S - \bar{S}}{2i} \right)^{-3/2} \equiv \hat{\xi} g_s^{3/2}$$

Hence,  $\hat{\xi}$  is incorporated into Kähler potential through the shift

$$\hat{\mathcal{V}} \rightarrow \hat{\mathcal{V}} + \hat{\xi}$$



## D7-branes and Logarithmic corrections

*Two ingredients needed for log-corrections:*



*A) Intersecting D7-brane configuration:*

D7s	Compact Dimensions									
	0	1	2	3	4	5	6	7	8	9
$D7_a$		*	*	*	*	*	*	*		
$D7_b$		*	*	*	*	*			*	*
$D7_c$		*	*	*			*	*	*	*

**B) Higher derivative couplings in curvature**

*(generated by multigraviton scattering)*

*(see hep-th/9704145; 9707013; 9707018)*

*Leading correction term in type II-B action:*

*proportional to the fourth power of curvature:*

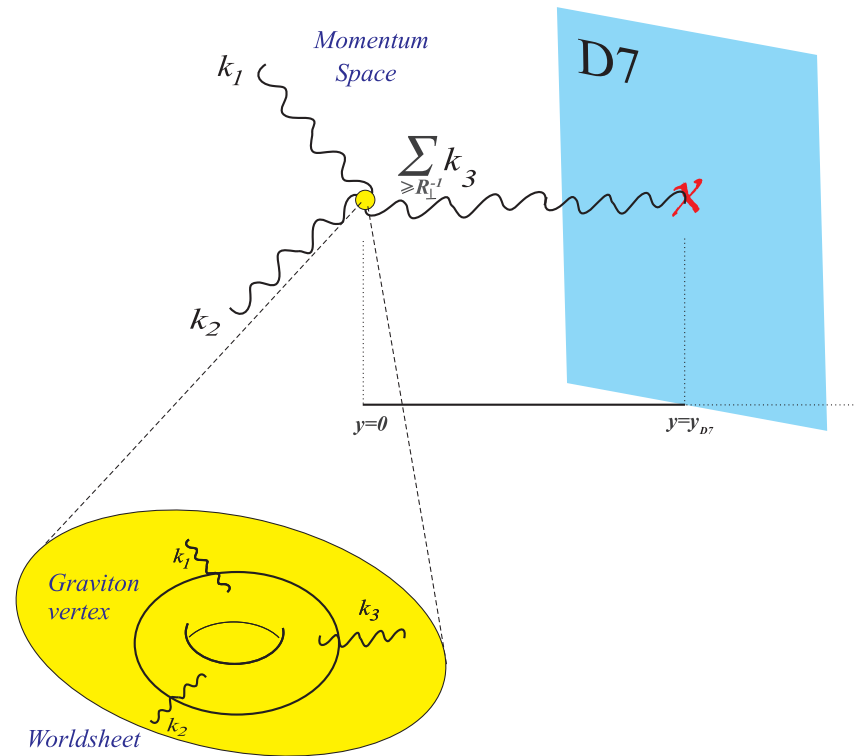
$$\boxed{\propto R^4}$$

After reduction on  $\mathcal{M}_4 \times \mathcal{X}_6$ , (with  $\mathcal{M}_4$  4-d Minkowski)  $R^4$  induces a **novel Einstein-Hilbert** term  $\mathcal{R}_{(4)} \propto$  by the Euler characteristic  $\chi$ :

$$\propto \underbrace{\chi \int_{\mathcal{M}_4} (\zeta(2) - \zeta(3)e^{-2\phi}) \mathcal{R}_{(4)}}_{\text{induced } \mathcal{EH} \text{ term}},$$

▲▲ *this  $\mathcal{EH}$  term possible in 4-dimensions only!*

▲▲ New  $\mathcal{EH}$ -term localised at points with  $\chi \neq 0$ ▲▲  
KK-exchange between graviton vertex and a  $D7$ -brane



## Corrections

$$\propto \zeta(2) \chi \int_{M_4} \left( 1 + \sum_{i=1,2,3} e^{2\phi} \mathcal{T}_i \log(R_{\perp}^i) \mathcal{R}_{(4)} \right),$$

▲  $\mathcal{T}_i$  : D7-brane tension ( $= g_s T_0$ )

▲  $R_{\perp}^i$  : D7-transverse 2-dimension

Kähler potential :

$$\mathcal{K} = -\log(-i(S - \bar{S})) - 2 \log(\mathcal{V} + \xi/2 + \eta \log \mathcal{V}) + K_{cs}$$

$$\eta = -\frac{1}{2} g_s T_0 \xi \quad ; \quad \xi = -\frac{\chi}{4} \times \begin{cases} \frac{\pi^2}{3} g_s^2 & \text{for orbifolds} \\ \zeta(3) & \text{for smooth CY} \end{cases} \quad (1)$$



Kähler moduli

*STABILISATION*

within a concrete Global Model:

(*GKL & Pramod Shukla* **2203.03362** )

Kreuzer-Skarke (*KS*) in *hep-th/0002240* introduced toric methods to  
construct Calabi-Yau manifolds in terms of  
*Reflexive Polyhedra*

*...exploring the **KS** dataset ...⇒*

## Explicit $CY_3$ Manifold

$$h^{1,1} = 3, \quad h^{2,1} = 115, \quad \chi = -224$$

$$\hat{\xi} = -\frac{\chi \zeta[3]}{2(2\pi)^3 g_s^{3/2}} = \frac{14 \zeta[3]}{\pi^3 g_s^{3/2}}, \quad \frac{\hat{\xi}}{\hat{\eta}} = -\frac{\zeta[3]}{\zeta[2]} \frac{1}{g_s^2}$$

Assuming a basis of smooth divisors  $D_1, D_2, D_3$ , the case under consideration gives intersection polynomial with **only one non-zero** intersection:

$$I_3 = 2D_1D_2D_3$$

with Kähler form  $J = 2 \sum_{k=1}^3 t^k D_k$ , and 6d-volume:

$$\mathcal{V} = 2 t^1 t^2 t^3 = \frac{1}{\sqrt{2}} \sqrt{\tau_1 \tau_2 \tau_3}$$

( $t^i \rightarrow 2$ -cycle,  $\tau_i \rightarrow 4$ -cycle moduli, subject to  $\tau_i = 2 t^j t^k$ )

▲ Kähler potential including  $\alpha'$  and loop corrections:

$$\mathcal{K}(T_i, S, z_a) = -\log\{-i(S - \bar{S})\} - 2 \ln \mathcal{U} + K_{cs}(z_a) \quad (2)$$

$$\mathcal{U}(T_i, S) = \mathcal{V} + \frac{\hat{\xi}}{2} + \mathcal{U}_1 \quad (3)$$

with  $\mathcal{U}_1(S, T^i)$  incorporating any loop corrections.

Computation of  $V_F$  requires inverse Kähler metric (basis  $S, T^i, z^a$ )

$$K^{A\bar{B}} = \begin{pmatrix} \tilde{\mathcal{P}}_1 & k_\alpha \tilde{\mathcal{P}}_2 & \mathcal{O} \\ k_\alpha \tilde{\mathcal{P}}_2 & k_\alpha k_\beta \tilde{\mathcal{P}}_3 - k_{\alpha\beta} \tilde{\mathcal{P}}_4 & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & K_{cs}^{i\bar{j}} \end{pmatrix} \quad (4)$$

$\Rightarrow$  block-diagonal for  $K_{cs}^{i\bar{j}}$  but  $S$  and  $T_i$  ( $\mathcal{V}$ ) mix :  $\tilde{\mathcal{P}}_I = \tilde{\mathcal{P}}_I(\mathcal{V}, S)$ .

Master formula for F-term potential (*generic*  $\mathcal{U}_1$ )



$$V_{\alpha'+\log} = e^{\kappa} \left( \frac{3\nu}{2\mathcal{U}^2} \left( 1 + \frac{\partial \mathcal{U}_1}{\partial \nu} \right)^2 \frac{4\nu^2 + \nu \hat{\xi} + 4\hat{\xi}^2}{\nu - \hat{\xi}} - 3 \right) |W_0|^2$$

For  $\alpha'$  and  $\log$  corrections  $\mathcal{U}_1 = -\hat{\eta} + \hat{\eta} \log \nu$ :

$$V_{\alpha'+\log} = 12g_s e^{K_{cs}} |W_0|^2 \underbrace{\hat{\xi} \frac{\nu^2 + 7\hat{\xi}\nu + \hat{\xi}^2}{(\nu - \hat{\xi})(2\nu + \hat{\xi})^4}}_{\alpha'^3\text{-corrections}} - \frac{3\kappa}{2} |W_0|^2 \underbrace{\frac{2\hat{\eta} - \hat{\eta} \log \nu}{2\nu^3}}_{\text{logarithmic}} + \dots$$

## Large Volume Limit

$$V_F \approx C \frac{\hat{\xi} - 4\hat{\eta} + 2\hat{\eta} \log(\mathcal{V})}{\mathcal{V}^3}$$

### Properties

- ▲ Minimum exists for  $\hat{\eta} < 0$ .
- ▲ Stabilisation at **large volume**:

$$\mathcal{V}_{\min} = e^{\frac{7}{3} + \frac{\hat{\xi}}{2|\hat{\eta}|}} \sim e^{\frac{1}{g_s^2}}$$

- ▲ For F-term potential, **AdS**-minimum

$$(V_F)_{\min} \propto \frac{\eta}{\mathcal{V}^3} < 0$$

▲ **Uplift** to **dS** occurs through **D-terms** (see *Lüst et al hep-th/0609211*) associated with universal  $U(1)$ 's of D7-stacks:

$$V_{\mathcal{D}} = \frac{g_{D7_i}^2}{2} \left( Q_i \partial_{T_i} K + \sum_j q_j |\Phi_j|^2 \right)^2, \quad \frac{1}{g_{D7_i}^2} = \text{Re} T_i + \dots$$

Minimising the total potential:

$$V_{\text{eff}} = V_F + V_{\mathcal{D}}$$

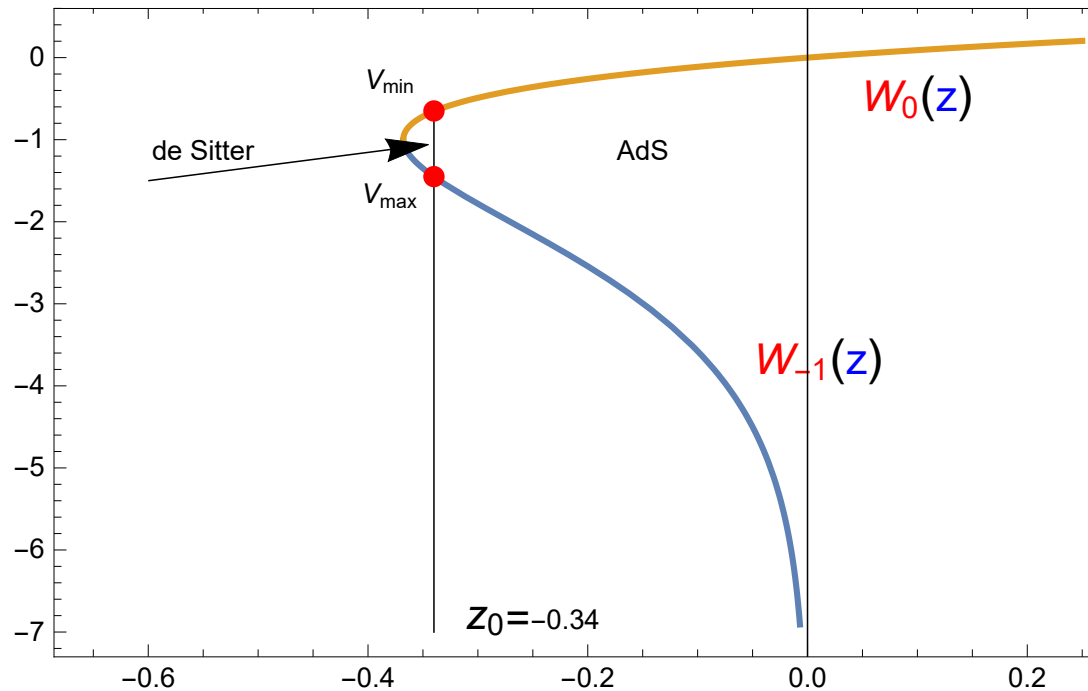
⇒ a minimum and a maximum defined by the **double-valued Lambert  $W$ -function** ( $we^w = z$ ):

$$\mathcal{V}_{\min} = \frac{\hat{n}}{d} \mathbf{W}_{0/-1} \left( \frac{d}{\hat{n}} e^{\frac{7}{3} - \frac{\hat{\xi}}{2\hat{n}}} \right)$$

▲ de Sitter vacua ▲

minimum  $V_{\text{eff}} = V_F + V_D$  at  $\mathcal{V}_0$  must be positive:

$$V_{\text{eff}}^{\text{min}} = \frac{c}{\mathcal{V}_0^3} + \frac{d}{\mathcal{V}_0^2} > 0]$$



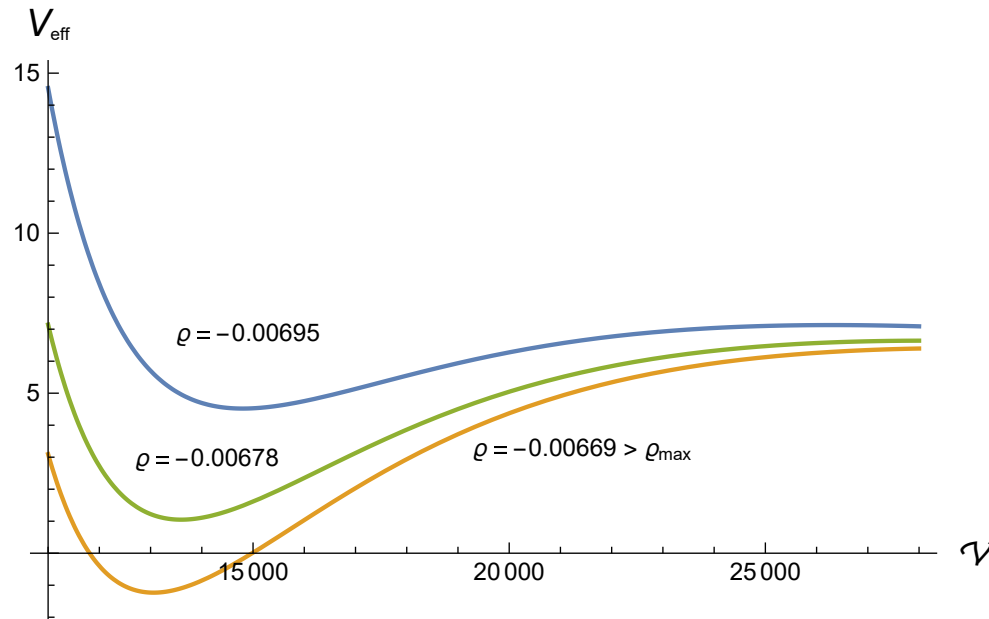
*This dramatically constrains acceptable string vacua (fluxes etc)*

## de Sitter

Plot of  $V_{\text{eff}}$  vs  $\mathcal{V}$  for fixed  $\varrho \propto \frac{d}{\hat{\eta}}$ .

The lower curve corresponds to AdS vacuum.

At large volume, the potential vanishes asymptotically





## Inflation

*Hybrid scenario with open string states at D7-brane intersections playing the role of waterfall fields*

*(Antoniadis, Lacombe, GKL, 2109.03243, JHEP):*

$$V_{total} = V_F + V_D + V_\chi$$

$$V_\chi = \sim m^2(\mathcal{V})\chi^2 + \lambda(\mathcal{V})\chi^4$$

For  $m^2 > 0$  minimum in the  $\chi$ -field direction is at the origin

$$m^2 > 0 \rightarrow \langle \chi \rangle = 0$$

When the mass of  $\chi$  becomes tachyonic, a phase transition occurs and the new vacuum is obtained at a non-vanishing  $\langle \chi \rangle$ :

$$m^2 < 0 \rightarrow \langle \chi \rangle \neq 0$$

A possible  $D7$  configuration with magnetic fluxes and Wilson lines with only one potential tachyonic state playing the role of **waterfall field**

	(45)	(67)	(89)
$D7_1$	$\cdot$	$\otimes$	$\times_{A_1}$
$D7_2$	$\times$	$\cdot \pm x_2$	$\otimes$
$D7_3$	$\otimes$	$\times_{A_3}$	$\cdot$

$$\alpha' m_{22}^2 \approx -\frac{A}{\nu^{1/3}} + B\nu^{1/3}$$

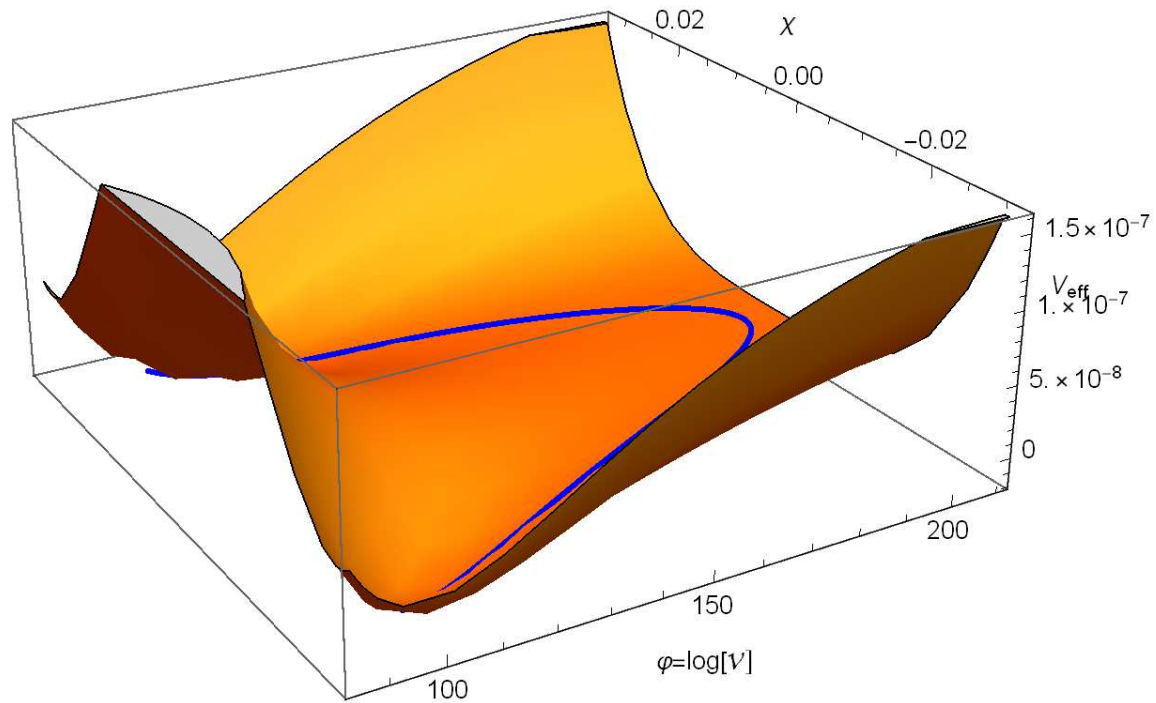
Tachyon for

$$\nu < \left(\frac{A}{B}\right)^{\frac{3}{2}}$$

Inflaton:  $\phi \propto \log \mathcal{V}$

▲ Blue curve: waterfall field  $\chi$  trajectory

*Hybrid scenario with open string states at D7-brane intersections playing the role of waterfall field:*



★ Conclusions ★

★ *IIB/F-theory*:

- Stabilisation of Kähler Moduli possible with  
Perturbative corrections only:

$$\mathcal{K} = -2 \ln \left( \mathcal{V} + \hat{\xi}/2 + \hat{\eta} \ln \mathcal{V} \right) + \dots$$

Origin of log-corrections:

Induced Einstein-Hilbert terms from  $R^4$ -couplings in 10-d theory.

This  $\mathcal{EH}$ -term  $\exists$  in  $4d$  only!



★ *induced*  $\mathcal{EH}$ -term ... indispensable element for a:  
 $4d$  de Sitter Universe

★ Hybrid Inflation with inflaton  $\phi \sim \log \mathcal{V}$  and  
waterfall fields open string states attached on  $D7$ 's.

*APPENDIX*

The analysis of the divisor topologies using *cohomCalc* shows that divisors are of *K3* and *SD* types and can be represented by the following Hodge diamonds:

$$\begin{array}{cccc}
 & & 1 & \\
 & 0 & & 0 \\
 K3 \equiv & 1 & 20 & 1 \text{ ,} \\
 & 0 & & 0 \\
 & & 1 & 
 \end{array}
 ,
 \quad
 \begin{array}{cccc}
 & & 1 & \\
 & 0 & & 0 \\
 SD \equiv & 27 & 184 & 27 \text{ .} \\
 & 0 & & 0 \\
 & & 1 & 
 \end{array}$$

## Cancellation of all D7-charges

Introduce  $N_a$  D7-branes wrapped around divisors  $D_a$  and orientifold images  $D'_a$  (0811.2936)

$$\sum_k N_k ([D_k + D'_k]) = 8[O7]$$

D7-branes and O7-planes also give rise to D3-tadpoles which receive contributions also from background 3-form fluxes

Assuming simple case:

D7-tadpoles are cancelled by placing 4  $D7 + D7'$ -branes on top of O7-plane:

$$N_{D3} + \frac{1}{2}N_{\text{flux}} + N_{\text{gauge}} = \frac{1}{4} (O3 + \chi(O7))$$



## Example

Specific brane setting involving 2 stacks of  $D7$ -branes wrapping the divisors  $D_1, D_6$  in the basis,

$$8[O7] = 4([D_1 + D'_1]) + 4([D_6 + D'_6])$$

$D3$  tadpole condition

$$N_{D3} + \frac{1}{2}N_{flux} + N_{gauge} = 12$$

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