Formulating E- & T-Model Inflation in Supergravity

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Based on:

- C.P., *J. Cosmol. Astropart. Phys.* **05**, *043 (2021)* [arXiv:2103.05534].
- C.P., *Eur. Phys. J. C* **82**, *no. 5, 444 (2022)* [arXiv:2204.01047].

Outline

Hellenic Foundation for Research & Innovation

HEP 2022: Recent Developments in High Energy Physics and Cosmology
15-18 June 2022, Thessaloniki, Greece
Workshop: Cosmic Inflation: From Observations to Particle Models

Observational Status of Chaotic Inflation (CI)

- Motivation: The Power-Law Potentials, Employed in Modes of CI, of the form

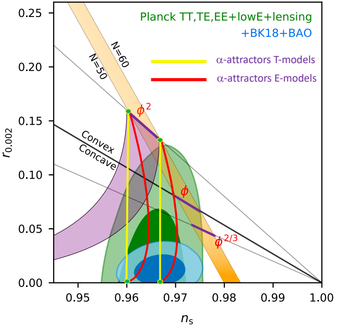
$$V_{\rm I} = \lambda^2\phi^n \quad \text{or} \quad V_{\rm I} = \lambda^2(\phi^2 - M^2)^{n/2} \quad \text{For} \quad M \ll m_{\rm P} = 1. \quad (:\!\mathbf{I})$$

are Very Common in Physics and so It is Easy the **Identification** of the **Inflaton** ϕ With a Field Already Present in the Theory; E.g., Within Higgs Inflation (**HI**) the Inflaton Could Play, At The End Of Inflation, The Role Of A Higgs Field.

- **However**, For $n = 2, 4$ The Theoretically Derived Values For **Spectral Index** $n_{\rm s}$ and/or **Tensor-to-Scalar Ratio** r Are Not Consistent With the Observational Ones.

- The Combined Bicep2/*Keck Array* and *Planck* Results Require, for Fitted $A_{\rm s}$ and $N_\star$ – see Below –,

$$n_{\rm s} = 0.965 \pm 0.009 \quad \text{and} \quad r \lesssim 0.032 \quad \text{at } 95\% \text{ c.l.}$$



- On the Contrary, **Observationally Friendly** Are Models of CI Collectively Named **α-Attractors**.
- These can be Classified into **E-Model Inflation** (**EMI**) (or α-Starobinsky model) and **T-Model Inflation** (**TMI**) And Are Based on a Specific Relation Established Between **the Initial**, ϕ, and the **Canonically Normalized** Inflaton $\widehat{\phi}$. I.e.

$$V_\alpha = \begin{cases} V_{\rm E}\left(1 - {\rm Exp}\left(-\sqrt{2/N}\widehat{\phi}\right)\right) & \text{for EMI,} \\ V_{\rm T}\left(\tanh\left(\widehat{\phi}/\sqrt{2N}\right)\right) & \text{for TMI,} \end{cases}$$

where $N > 0$ and $V_{\rm E,T} = V_{\rm I}(\phi)$ – See Eq. (I).

- Such Relations Between ϕ and $\widehat{\phi}$ Can be Achieved in the The Presence Of A **Pole** In The Inflaton **Kinetic Term**.

Introducing A Kinetic Pole In the Inflaton Sector

• To Analyze Systematically the **non-Minimal Kinetic Mixing** in the inflaton Sector We Consider the Lagrangian Of The Homogenous Inflaton Field $\phi = \phi(t)$

$$\mathcal{L} = \sqrt{-\mathfrak{g}}\left(\frac{N_p}{2f_p^2}\dot{\phi}^2 - V_{\rm I}(\phi)\right) \quad \text{with} \quad f_p = 1-\phi^p,\ p=1,2 \quad \text{and} \quad N_p > 0.$$

Where we set $m_{\rm P} = 1$ and $\mathfrak{g}$ is the Determinant Of The Background Metric $g^{\mu\nu}$.

• If we Introduce the **Canonically Normalized Field**, $\widehat{\phi}$, Defined As Follows:

$$\frac{d\widehat{\phi}}{d\phi} = J = \frac{\sqrt{N_p}}{f_p} \quad \Rightarrow \quad \phi = \begin{cases} 1 - e^{-\widehat{\phi}/\sqrt{N_1}} & \text{for}\ \ p=1, \\ \tanh\left(\dfrac{\widehat{\phi}}{\sqrt{N_2}}\right) & \text{for}\ \ p=2\,, \end{cases}$$

$\mathcal{L}$ in terms of $\widehat{\phi}$ Takes the Form

$$\mathcal{L} = \sqrt{-\mathfrak{g}}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\widehat{\phi}\partial_\nu\widehat{\phi} - V_{\rm I}\left(\widehat{\phi}\right)\right) \quad \text{With} \quad V_{\rm I}(\widehat{\phi}) = V_{\rm E/T}\left(\widehat{\phi}(\phi)\right)\cdot$$

• We can Show that for a Suitable Choice of f_p Including A **Pole**[1] the Potential $V_{\rm I}(\widehat{\phi})$ Develops A **Plateau**, and so it Becomes Suitable to Drive Observationally Acceptable CI.

• The Analysis of **E- and/or T-Model Inflation** (**ETM**) Can Be Performed Exclusively in terms of $V_{\rm I}$ and $\widehat{\phi}$ Using The Standard Slow-Roll Approximation.

[1] B.J. Broy et al. (2015); T. Terada (2016); T. Kobayashi et al. (2017).

Inflationary Observables and Requirements

- The **Number of e-foldings**, $N_\star$, that the Scale $k_\star = 0.05/\mathrm{Mpc}$ Underwent During CI has to be Sufficient to Resolve the Horizon and Flatness Problems of Standard Big Bang:

$$\widehat{N}_\star = \int_{\widehat{\phi}_\mathrm{f}}^{\widehat{\phi}_\star} d\widehat{\phi}\, \frac{V_\mathrm{I}}{V_{\mathrm{I},\widehat{\phi}}} = \int_{\phi_\mathrm{f}}^{\phi_\star} d\phi\, J^2 \frac{V_\mathrm{I}}{V_{\mathrm{I},\phi}} \simeq 44-56 \ \text{Depending on}\ w_\mathrm{rh} \simeq (-0.24-0.58), \ \text{Where}$$

  - The **Barotropic Index** w_rh Depends on the Degree of the Polynomial in V_I;
  - $\phi_\star$ $[\widehat{\phi}_\star]$ is The Value of ϕ $[\widehat{\phi}]$ When $k_\star$ Crosses Outside The Inflationary Horizon;
  - ϕ_f $[\widehat{\phi}_\mathrm{f}]$ is the Value of ϕ $[\widehat{\phi}]$ at the end of HI Which Can Be Found From The Condition:

$$\max\{\widehat{\epsilon}(\phi_\mathrm{f}), |\widehat{\eta}(\phi_\mathrm{f})|\} = 1, \quad \text{With}\ \ \widehat{\epsilon} = \frac{1}{2}\left(\frac{V_{\mathrm{I},\widehat{\phi}}}{V_\mathrm{I}}\right)^2 = \frac{1}{2J^2}\left(\frac{V_{\mathrm{I},\phi}}{V_\mathrm{I}}\right)^2 \ \ \text{and}\ \ \widehat{\eta} = \frac{V_{\mathrm{I},\widehat{\phi}\widehat{\phi}}}{V} = \frac{1}{J^2}\left(\frac{V_{\mathrm{I},\phi\phi}}{V_\mathrm{I}} - \frac{V_{\mathrm{I},\phi}}{V_\mathrm{I}}\frac{J_{,\phi}}{J}\right).$$

- The **Amplitude A_s of the Power Spectrum** of the Curvature Perturbations is To Be Consistent with ***Planck*** Data:

$$A_\mathrm{s}^{1/2} = \frac{1}{2\sqrt{3}\,\pi}\,\frac{V_\mathrm{I}(\widehat{\phi}_\star)^{3/2}}{|V_{\mathrm{I},\widehat{\phi}}(\widehat{\phi}_\star)|} = \frac{|J(\phi_\star)|}{2\sqrt{3}\,\pi}\,\frac{V_\mathrm{I}(\phi_\star)^{3/2}}{|V_{\mathrm{I},\phi}(\phi_\star)|} = 4.588\cdot 10^{-5}$$

- The **Remaining Observables** are found as:

$$n_\mathrm{s} = 1 - 6\widehat{\epsilon}_\star + 2\widehat{\eta}_\star, \quad \alpha_\mathrm{s} = 2\left(4\widehat{\eta}^2_\star - (n_\mathrm{s}-1)^2\right)/3 - 2\widehat{\xi}_\star \ \ \text{and}\ \ r = 16\widehat{\epsilon}_\star,$$

  Where $\widehat{\xi} = V_{\mathrm{I},\widehat{\phi}} V_{\mathrm{I},\widehat{\phi}\widehat{\phi}\widehat{\phi}}/V_\mathrm{I}^2 = V_{\mathrm{I},\phi}\,\widehat{\eta}_{,\phi}/V_\mathrm{I}\,J^2 + 2\widehat{\eta}\widehat{\epsilon}$ And The Variables With Subscript $\star$ Are Evaluated at $\phi = \phi_\star$.

- We Have To Check The Hierarchy Between **The Ultraviolet Cut-off** $\Lambda_\mathrm{UV} \sim m_\mathrm{P}$, of the Effective Theory And The Inflationary Scale. In Particular, The Validity Of The Effective Theory Implies:

$$\text{(a)}\ \ V_\mathrm{I}(\phi_*)^{1/4} \leq \Lambda_\mathrm{UV} \ \ \text{for}\ \ \text{(b)}\ \ \phi \leq \Lambda_\mathrm{UV}$$

E-Model Inflation (Pole of Order One)

- The **Simplest** Choice It would be The **Pole** in in the Kinetic Part of $\mathcal{L}$ to be of Order One. I.e.,:

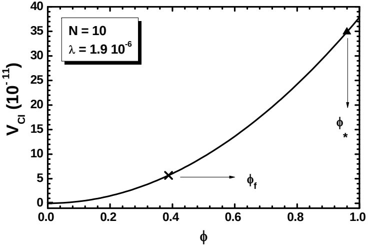
$$f_1 = 1 - \phi \ \text{ and } \ V_{\rm I} = V_{\rm E} = \lambda^2\phi^n/n \ \text{ With } \ N_1 > 0\,.$$

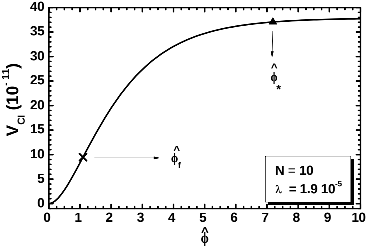
- **Canonically Normalizing** ϕ, we Obtain

$$\widehat{\phi} = -\sqrt{N_1}\,\ln(1-\phi) \ \text{ or } \ \phi = 1 - e^{-\sqrt{N_1}\widehat{\phi}}$$

- $V_{\rm I}$ in Terms of $\widehat{\phi}$ Experiences A **Stretching** For $\widehat{\phi} > 1$ Which Results To A Plateau, i.e., $V_{\rm I} = \lambda^2(1 - e^{-\sqrt{N_1/2}\widehat{\phi}})^n/n$

– E.g., For $n = 2$ we Obtain the Well-Known **Starobinsky Model** and the Plots Below.





Here, $\epsilon \simeq n{f_1}^2/2N_1\phi^2$ and $\eta \simeq nf_1(nf_1-1)/N_1\phi^2$. Therefore, $N_\star \simeq N_1\phi_\star^2/nf_{1\star} \Rightarrow \phi_\star = \sqrt{nN_\star}/(nN_\star+N_1) \sim 1 \gg \phi_{\rm f}$.

- The **Constraint on $A_{\rm s}$** Yields $A_{\rm s}^{1/2} \simeq \lambda N_\star/2\sqrt{3nN}\pi = 4.588\cdot 10^{-5} \Rightarrow \lambda \simeq 2\sqrt{3nNA_{\rm s}}\pi/N_\star \Rightarrow \underline{\lambda \sim 10^{-6}}$ for $N_\star \simeq 55$
- The **Other Observables** Are $n_{\rm s} \simeq 1 - 2/N_\star \simeq 0.965$, $\alpha_{\rm s} \simeq -2/N_\star^2 = 9.5\cdot 10^{-4}$ and $r \simeq 8N_1/N_\star^2 \leq 0.07 \Rightarrow \underline{N_1 \lesssim 19}$.

T-Model Inflation (Pole of Order Two)

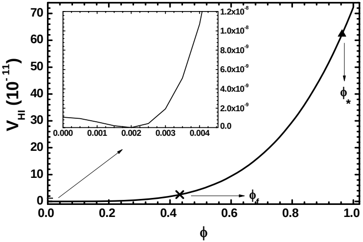
- If We Introduce a **Pole of Order Two** in the Kinetic Part of $\mathcal{L}^2$ We Obtain:

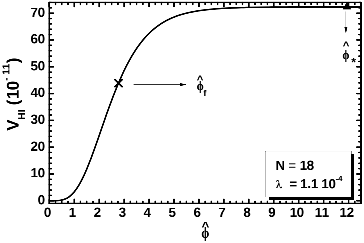
$$f_2 = 1-\phi^2 \ \text{ and } \ V_{\rm T} = \lambda^2\left(\phi^2-M^2\right)^2/16 \ \text{ With } \ M \ll 1 \ \& \ N_2>0\,.$$

- **Canonically Normalizing** ϕ, we Obtain $\phi \sim \tanh\widehat\phi$ and hence the Name **T-Model (TMI)** HI

$$\widehat\phi = \sqrt{N_2}\,\ln\left((1+\phi)/(1-\phi)\right) \ \text{ or } \ \phi=\tanh\left(\widehat\phi/\sqrt{N_2}\right)$$

- $V_{\rm I}$ in Terms of $\widehat\phi$ Experiences a **Stretching** For $\widehat\phi>1$ Which Results To A Plateau, i.e., $V_{\rm I}=\lambda^2\tanh^4(\widehat\phi/\sqrt{2N_2})/16$.





Here, $\epsilon \simeq 16{f_2}^2/N_2\phi^2$ and $\eta\simeq 8f_2(3-5\phi^2)/N_2\phi^2$. Therefore, $N_\star \simeq N_2\phi_\star^2/4f_{2\star} \Rightarrow \phi_\star = \sqrt{4N_\star}/\sqrt{4N_\star+N_2}\sim 1 \gg \phi_{\rm f}$.

- The **Constraint on** $A_{\rm s}$ Yields $A_{\rm s}^{1/2}\simeq \sqrt{2}\lambda N_\star/\sqrt{3N_2}\pi = 4.588\cdot 10^{-5} \Rightarrow \lambda\simeq 4\sqrt{6N_2A_{\rm s}}\pi/N_\star \Rightarrow \underline{\lambda\sim 10^{-5}}$ for $N_\star\simeq 55$
- <u>The **Other Observables** Are $n_{\rm s}\simeq 1-2/N_\star\simeq 0.965$,</u> $\alpha_{\rm s}\simeq -2/N_\star^2 = 9.5\cdot 10^{-4}$ and $r\simeq 2N_2/N_\star^2\leq 0.032 \Rightarrow N_2\lesssim 55$.

[2] R. Kallosh and A. Linde (2013); J. Ellis, D.V. Nanopoulos and K.A. Olive (2013).

SUGRA Scalar Potential

- How We Can Formulate **Pole-Inflation** Within SUGRA?
- The General **Lagrangian For The Scalar Fields z^α** Plus Gravity In Four Dimensional, $\mathcal{N}=1$ SUGRA is:

$$\mathcal{L}=\sqrt{-\mathfrak{g}}\left(-\frac{1}{2}\mathcal{R}+K_{\alpha\bar\beta}g^{\mu\nu}D_\mu z^\alpha D_\nu z^{*\bar\beta}-V\right)\ \text{Where}\ V=V_{\rm F}+V_{\rm D}\ \text{With}\ \begin{cases}V_{\rm D}=g^2{\rm D}_a^2/2\\ V_{\rm F}=e^K\left(K^{\alpha\bar\beta}{\rm F}_\alpha{\rm F}^*_{\bar\beta}-3|W|^2\right)\end{cases}$$

Also $K_{\alpha\bar\beta}=\dfrac{\partial^2K}{\partial z^\alpha\partial z^{*\bar\beta}}>0$ and $K^{\bar\beta\alpha}K_{\alpha\bar\gamma}=\delta^{\bar\beta}_{\bar\gamma}$; $D_\mu z^\alpha=\partial_\mu z^\alpha+igA^a_\mu T^a_{\alpha\beta}z^\beta$, ${\rm F}_\alpha=W_{,z^\alpha}+K_{,z^\alpha}W$ and ${\rm D}_a=z_\alpha\,(T_a)^\alpha_\beta\,K_{,z^\beta}$

A^a_μ is The Vector Gauge Fields, g is the Gauge Coupling and T_a are the Generators of the Gauge Transformations Of z^α.
- The Kinetic Mixing is Controlled by The **Kähler Potential** K Which Affects **Also** V. This Consists a **Complication** With Respect the non-SUSY case And We Show Below How We Arrange it in **two** Ways. V Depends on **Superpotential** W Too.
- We Concentrate on **CI Driven by $V_{\rm F}$** – As we show Below We Can Easily Assure $V_{\rm D}=0$ During CI.

Introduction of the Stabilizer Field

- EMI Can be Systematically Formulated in SUGRA If We Introduce A Gauge Singlet Superfield $z^1=S$ called **Stabilizer or Goldstino**. Its Introduction is Necessary For the Following Reasons:
  - It Generates the **non-SUSY Potential** From the term $|W_{,S}|^2$ for $S=0$. E.g., For $W=\lambda S\,\Phi^{\mathbf{n/2}}$ We Obtain
  $$\langle V_{\rm F}\rangle_{\rm I}=\langle e^KK^{SS^*}|W_{,S}|^2\rangle_{\rm I}\in V_{\rm non-SUSY}=\lambda^2\phi^n\quad\text{with}\quad\phi={\rm Re}(\Phi)\ \text{the (initial) inflaton.}$$
  - It Assures the **Boundedness of $V_{\rm F}$:** If We set $S=0$ During Inflation, the Terms $K_{,z^\alpha}W$, $\alpha\neq1$, and $-3|W|^2$ Vanish. The 2nd one May Render $V_{\rm F}$ Unbounded From Below.
  - It can be **Stabilized** at $S=0$ Without Invoking Higher Order Terms, if we Select [3]:
  $$K_2=N_S\ln\left(1+|S|^2/N_S\right)\ \Rightarrow\ K_2^{SS^*}=1\ \text{With}\ 0<N_S<6\ \text{Which Parameterizes the Compact Manifold}\ SU(2)/U(1).$$

[3] C.P. and N. Toumbas (2016).

E-Model Inflation (EMI)

- We Select another Gauge Singlet Superfield $z^1 = \Phi$ (the Inflaton) and the Most General W Consistent With the **R Symmetry** Under Which $R(S) = R(W)$, $W = S\left(\lambda_1\Phi + \lambda_2\Phi^2 - M^2\right)$.
- We Obtain A Pole Of order 1 in the Kinetic Terms, If we Adopt

$$K_{1\rm s} = -N\ln\left(1-(\Phi+\Phi^*)/2\right) \quad \text{or} \quad \widetilde K_{1\rm s} = -N\ln\frac{(1-\Phi/2-\Phi^*/2)}{(1-\Phi)^{1/2}(1-\Phi^*)^{1/2}}, \quad \text{with } {\rm Re}(\Phi)<1 \text{ and } N>0.$$

- Keeping in Mind that the Pole has to be Eliminated from $V_{\rm I}$ for $K = K_{1\rm s}$, We analyze the Following Models:
  - δ E-Model **(δEM)** With $K = K_{21\rm s} = K_2 + K_{1\rm s}$ with $N = 2, M \ll 1$ and $\lambda_2 \simeq \lambda_1(1+\delta_{21})$ with $\delta_{21} = \mathcal{O}(10^{-5})$;
  - E-Model 2 & 4 **(EM2 & EM4)** With $K = \widetilde K_{21\rm s} = K_2 + \widetilde K_{1\rm s}$ with Free N, λ_1, λ_2 and $M \ll 1$ Since $\langle e^K\rangle_{\rm I} = 1$.

T-Model Inflation (TMI)

- We Use 2 Extra (Gauge non-Singlet) Superfields $z^2 = \Phi$, $z^3 = \bar\Phi$, **Charged** Under a Local Symmetry, e.g. $U(1)_{B-L}$.
- **Superpotential** $W = S\left(\lambda_2\bar\Phi\Phi/2 - M^2/4 + \lambda_4(\bar\Phi\Phi)^2\right)$
- W Is Uniquely Determined Using $U(1)_{B-L}$ and an R Symmetry and Leads to a **Grand Unified Theory (GUT)** Phase Transition

  At The SUSY Vacuum $\langle S\rangle = 0, |\langle\Phi\rangle| = |\langle\bar\Phi\rangle| \sim M/\sqrt{2}$

Charge Assignments

Superfields:	S	Φ	$\bar\Phi$
$U(1)_R$	1	0	0
$U(1)_{B-L}$	0	1	-1

- We Obtain A **Pole Of order 2** in the Kinetic Terms, If we Adopt

$$K_{(11)^2} = -\frac{N}{2}\ln\left(1-2|\Phi|^2\right)\left(1-2|\bar\Phi|^2\right) \quad \text{or} \quad \widetilde K_{(11)^2} = -\frac{N}{2}\ln\frac{\left(1-2|\Phi|^2\right)\left(1-2|\bar\Phi|^2\right)}{(1-2\bar\Phi\Phi)^{1/2}(1-2\bar\Phi^*\Phi^*)^{1/2}},$$

- Keeping in Mind that the Pole has to be Eliminated from $V_{\rm I}$ for $K = K_{(11)^2}$, We Analyze The Following Models:
  - δ T-Model **(δTM)** With $K = K_{2(11)^2} = K_2 + K_{(11)^2}$ with $N = 2$ and $\lambda_4 = \lambda_2(1+\delta_{42})$ in W with $\delta_{42} = \mathcal{O}(10^{-5})$;
  - T-Model 4 & 8 **(TM4 & TM8)** With $K = \widetilde K_{2(11)^2} = K_2 + \widetilde K_{(11)^2}$ with Free N, λ_2, λ_4 and M Since $\langle e^K\rangle_{\rm I} = 1$.

The Kähler Manifold Corresponding to K_{1s} and $\widetilde{K}_{1s}$

- The **Riemannian Metric And The Scalar Curvature** of K_{1s} and $\widetilde{K}_{1s}$ is calculated by

$$ds^2_{1s} = K_{\Phi\Phi^*}\, d\Phi d\Phi^* = \frac{N}{4}\frac{d\Phi d\Phi^*}{(1-(\Phi+\Phi^*)/2)^2} \quad \text{and} \quad \mathcal{R}_{1s} = -K^{\Phi\Phi^*}\partial_\Phi\partial_{\Phi^*}\ln\left(K_{\Phi\Phi^*}\right) = -\frac{2}{N}.$$

- ds^2_{1s} Remains **Invariant** under the Transformations

$$\frac{\Phi}{2} \to \frac{a\Phi/2+b}{c\Phi/2+d} \quad \text{Represented By} \quad \mathcal{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = a\begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}, \quad \text{With} \quad |a|^2 = 1. \quad (T_1)$$

- The Matrix $\mathcal{M}$ Is a Conjugate Anti-Symplectic Matrix, I.e.,

$$\mathcal{M}^\dagger E \mathcal{M} = -E \quad \text{with} \quad E = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad \text{It is Written As} \quad \mathcal{M} = \mathcal{S}\sigma_3 \quad \text{With} \quad \mathcal{S} \in U(1,1).$$

The Kähler Manifold Corresponding to $K_{(11)^2}$ and $\widetilde{K}_{(11)^2}$

- The **Riemannian Metric And The Scalar Curvature**, of $K_{(11)^2}$ and $\widetilde{K}_{(11)^2}$ is Calculated by

$$ds^2_{(11)^2} = K_{\alpha\bar\beta}dz^\alpha dz^{*\bar\beta} = \frac{N|d\Phi|^2}{(1-2|\Phi|^2)^2} + \frac{N|d\bar\Phi|^2}{\left(1-2|\bar\Phi|^2\right)^2} \quad \text{and} \quad \mathcal{R}_{(11)^2} = -K^{\alpha\bar\beta}\partial_\alpha\partial_{\bar\beta}\ln\left(\det M_{\bar\Phi\Phi}\right) = -\frac{8}{N}, \quad z^{\alpha,\beta} = \Phi, \bar\Phi$$

- $ds^2_{(11)^2}$ Remains **Invariant** under the Transformations – We Assign the Charges $(B-L)(\alpha_1, b_1, \alpha_2, b_2) = (0, 1, 0, -1)$:

$$\sqrt{2}\Phi \to \frac{\alpha_1\sqrt{2}\Phi + b_1}{b_1^*\sqrt{2}\Phi + \alpha_1} \quad \text{and} \quad \sqrt{2}\bar\Phi \to \frac{\alpha_2\sqrt{2}\bar\Phi + b_2}{b_2^*\sqrt{2}\bar\Phi + \alpha_2}, \quad \text{Where} \quad \alpha_i^2 - |b_i|^2 = 1 \quad \text{With} \quad i = 1, 2$$

- The Corresponding Matrices $\mathcal{U}_i$ Are Elements Of The Coset Spaces $(SU(1,1)/U(1))_\Phi$ and $(SU(1,1)/U(1))_{\bar\Phi}$ Since

$$\mathcal{U}_i = \begin{pmatrix} \alpha_i & b_i \\ b_i^* & \alpha_i \end{pmatrix} \quad \text{with} \quad \alpha_i \in \mathbb{R}_+, \quad b_i \in \mathbb{C} \quad \text{and} \quad \alpha_i^2 - |b_i| = 1.$$

E-Model Inflation (Gauge Singlet Inflaton)

- Expanding Φ and S as Follows: $\Phi = \phi e^{i\theta}$ and $S = (s_1 + is_2)/\sqrt{2}$ We Can Introduce The **Canonically Normalized Fields**

$$d\widehat{\phi}/d\phi = J \simeq \sqrt{N/2}/f_1 \;\Rightarrow\; \phi = 1 - \exp\left(-\sqrt{2/N}\widehat{\phi}\right),\;\; \widehat{\theta} \simeq J\phi\theta \;\text{ and }\; \widehat{s}_i = s_i \;\text{ with }\; i = 1,2 \;\;(\text{Recall }\; f_1 = 1 - \phi)$$

Where We observe that We Established the Correct **Non-Minimal Kinetic Mixing**.

- Along The **Inflationary Path** $\langle S\rangle_{\rm I} = \langle\theta\rangle_{\rm I} = 0$, the only Surviving term of $V_{\rm F}$ is (where $r_{ij} = -\lambda_i/\lambda_j$ with $i, j = 1, 2$)

$$V_{\rm I} = \langle e^K K^{SS^*} |W_{,S}|^2\rangle_{\rm I} = \begin{cases} \left(\phi - r_{21}\phi^2 - M_1^2\right)^2/f_1^N & \text{for } \delta\text{EM}, \\ \left(\phi - r_{21}\phi^2 - M_1^2\right)^2 & \text{for EM2}, \\ \left(\phi^2 - r_{12}\phi - M_2^2\right)^2 & \text{for EM4}, \end{cases} \quad \text{where } \lambda = \begin{cases} \lambda_1 \text{ and } M_1 = M/\sqrt{\lambda_1} & \text{for } \delta\text{EM and EM2}, \\ \lambda_2 \text{ and } M_2 = M/\sqrt{\lambda_2} & \text{for EM4}. \end{cases}$$

Scalar **Mass-Squared Spectrum** for $K = K_{21s}$ and $\widetilde{K}_{21s}$ Along The Inflationary Trajectory

Fields	Eigen-states	Masses Squared		
			$K = K_{21s}$	$K = \widetilde{K}_{21s}$
1 real scalar	$\widehat{\theta}$	$\widehat{m}^2_{\theta}$	$6H^2_{\rm I}$	
2 real scalars	$\widehat{s}_1,\ \widehat{s}_2$	$\widehat{m}^2_{s}$	$6H^2_{\rm I}/N_S$	
2 Weyl spinors	$(\widehat{\psi}_\Phi \pm \widehat{\psi}_S)/\sqrt{2}$	$\widehat{m}^2_{\psi\pm}$	$6n(1-\phi)^2H^2_{\rm I}/N\phi^2$	

We Observe the Following:

- All mass^2 > 0. Especially $m^2_{\widehat{S}} > 0 \;\Leftrightarrow\; N_S < 6$;
- All mass^2 $> H^2_{\rm I}$ and So Any Inflationary Perturbations Of The Fields Other Than The Inflaton Are Safely Eliminated.
- The One-Loop Radiative Corrections Have No Significant Effect.

T-Model Inflation (Gauge Non-Singlet Inflaton)

- If We Use The **Parametrizations**: $\Phi = \phi e^{i\theta}\cos\theta_\Phi$ and $\bar\Phi = \phi e^{i\bar\theta}\sin\theta_\Phi$ with $0 \leq \theta_\Phi \leq \pi/2$ and $S = (s + i\bar{s})/\sqrt{2}$

We Select as **Inflationary Path** The **D-Flat Direction** Is $\langle\theta\rangle_{\rm I} = \langle\bar\theta\rangle_{\rm I} = 0,\ \langle\theta_\Phi\rangle_{\rm I} = \pi/4$ and $\langle S\rangle_{\rm I} = 0$ **(: P)**

- The only Surviving term of $V_{\rm F}$ Along the Path in Eq. (P) is (With $r_{ij} = -\lambda_i/\lambda_j$ with $i, j = 2, 4$)

$$V_{\rm I} = \langle e^K K^{SS^*}|W_{,S}|^2\rangle_{\rm I} = \frac{\lambda^2}{16}\begin{cases}\left(\phi^2 - r_{42}\phi^4 - M_2^2\right)^2/f_2^N & \text{for } \delta\text{TM},\\ \left(\phi^2 - r_{42}\phi^4 - M_2^2\right)^2 & \text{for TM4},\\ \left(\phi^4 - r_{24}\phi^2 - M_4^2\right)^2 & \text{for TM8},\end{cases} \quad \text{where } \lambda = \begin{cases}\lambda_2 \text{ and } M_2 = M/\sqrt{\lambda_2} & \text{for } \delta\text{TM and TM4},\\ \lambda_4 \text{ and } M_4 = M/\sqrt{\lambda_4} & \text{for TM8}.\end{cases}$$

- To Obtain TMI, We Have to Establish the Correct **Non-Minimal Kinetic Mixing**.
- To This End We Compute The **Kähler Metric $K_{\alpha\bar\beta}$** Along the Path in Eq. (P) Which Takes The Form

$$\left(\langle K_{\alpha\bar\beta}\rangle_{\rm I}\right) = \left(\langle M_{\Phi\bar\Phi}\rangle_{\rm I}, \langle K_{SS^*}\rangle_{\rm I}\right) \quad \text{with } \langle M_{\Phi\bar\Phi}\rangle_{\rm I} = \kappa\,\mathrm{diag}(1,1), \kappa = N/f_2^2 \text{ and } K_{SS^*} = 1.$$

- The EF **Canonically Normalized Fields**, Which Are Denoted By Hat, Can Be Obtained As Follows:

$$d\widehat\phi/d\phi = J = \sqrt{2N}/f_2 \;\Rightarrow\; \phi = \tanh\frac{\widehat\phi}{2\sqrt{N}},\quad \widehat\theta_\pm = \sqrt{\kappa}\phi\theta_\pm \text{ and } \widehat\theta_\Phi = \sqrt{2\kappa}\phi\,(\theta_\Phi - \pi/4),\quad \left(\widehat{s},\widehat{\bar{s}}\right) = (s,\bar{s})\cdot$$

- We Have, Also, to Check the **Stability** of the Trajectory in Eq. (P) w.r.t the Fluctuations Of The Various Fields, i.e.

$$\left.\frac{\partial V}{\partial\widehat{z}^\alpha}\right|_{\text{Eq. (P)}} = 0 \text{ and } \widehat{m}^2_{z^\alpha} > 0 \text{ Where } \widehat{m}^2_{z^\alpha} = \text{Egv}\left[\widehat{M}^2_{\alpha\beta}\right] \text{ With } \widehat{M}^2_{\alpha\beta} = \left.\frac{\partial^2 V}{\partial\widehat{z}^\alpha\partial\widehat{z}^\beta}\right|_{\text{Eq. (P)}} \text{ and } z^\alpha = \theta_-, \theta_+, \theta_\Phi, s, \bar{s}.$$

Here Egv are the Eigenvalues of the Matrix $\widehat{M}^2_{\alpha\beta}$.

Stability of The Inflationary Direction

Scalar **Mass-Squared Spectrum** for $K = K_{2(11)^2}$ and $\widetilde{K}_{2(11)^2}$ Along The Inflationary Trajectory

Fields	Eigen-states	Masses Squared		
			$K = K_{221}$	$K = \widetilde{K}_{221}$
2 real scalars	$\widehat{\theta}_+$	$m^2_{\widehat{\theta}+}$	$3H_{\rm I}^2$	
	$\widehat{\theta}_\Phi$	$\widehat{m}^2_{\theta_\Phi}$	$M^2_{BL} + 6H_{\rm I}^2(1 + 4/N - 2/N\phi^2 - 2\phi^2/N)$	
1 complex scalar	$s, \bar{s}$	$\widehat{m}^2_s$	$6H_{\rm I}^2(1/N_S - 8(1-\phi^2)/N + N\phi^2/2$ $+2(1-2\phi^2) + 8\phi^2/N)$	$6H_{\rm I}^2(1/N_S - 4/N$ $+2/N\phi^2 + 2\phi^2/N)$
1 gauge boson	A_{BL}	M^2_{BL}	$2Ng^2\phi^2/f_2^2$	
4 Weyl spinors	$\widehat{\psi}_\pm$	$\widehat{m}^2_{\psi\pm}$	$12 f_2^2 H_{\rm I}^2/N^2\phi^2$	
	$\lambda_{BL}, \widehat{\psi}_{\Phi-}$	M^2_{BL}	$2Ng^2\phi^2/f_2^2$	

- We can Obtain $\forall\, \alpha,\ \widehat{m}^2_{\chi^\alpha} > 0$. Especially $\widehat{m}^2_s > 0 \ \Leftrightarrow\ N_S < 6$.
- We can Obtain $\forall\, \alpha,\ \widehat{m}^2_{\chi^\alpha} > H_{\rm I}^2$ and So Any Inflationary Perturbations Of The Fields Other Than ϕ Are Safely Eliminated;
- $M_{BL} \neq 0$ Signals the Fact that That $U(1)_{B-L}$ Is Broken and so, **no Topological Defects** are Produced.
- We Determine M **Demanding** That The Unification Scale $M_{\rm GUT} \simeq 2/2.433 \times 10^{-2}$ is Identified with M_{BL} at the Vacuum, i.e.,

$$\langle M_{BL}\rangle = \sqrt{2N} gM/\langle f_2\rangle = M_{\rm GUT} \ \Rightarrow\ M \simeq M_{\rm GUT}/g\sqrt{2N} \ \text{with}\ \ g \simeq 0.7 \ \text{(GUT Gauge Coupling)}.$$

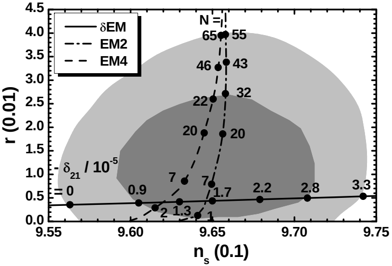
- The One-Loop **Radiative Corrections** à la Coleman-Weinberg to $V_{\rm I}$ Can Be Kept Under Control.

Testing Against The Inflationary Data

- Enforcing $N_\star \simeq 44 - 56$ and $\sqrt{A_s} = 4.588 \cdot 10^{-5}$, we Obtain the Allowed Curves for Our Models In the $n_s - r_{0.002}$ Plane
- In **Both Models** $\phi_\star \sim 1$ and the Relevant **Tuning** can be Qualified by Computing $\Delta_\star = (1 - \phi_\star)/1$.

E-Model Inflation

- The Free **Parameters** For δEM, EM2, EM4 are (δ_{21}, M_1), (N, r_{21}, M_1) and (N, r_{12}, M_2).
- We Fix $M_1 = 0.001$ for δEM, $M_1 = 0.01$ and $r_{21} = 0.001$ for EM2 and $M_2 = 0.01$ and $r_{12} = 0.001$ for EM4.



Model:	δEM	EM2	EM4
δ_{21} / r_{21} / r_{12}	$-1.7 \cdot 10^{-5}$	0.001	0.001
N	2	10	10
$\phi_\star/0.1$	9.9	9.53	9.84
$\Delta_\star(\%)$	1	4.7	2
$\phi_f/0.1$	6.66	3.7	5.6
w_{rh}	-0.24	-0.08	0.26
$N_\star$	44.4	51.5	55.5
$\lambda/10^{-5}$	1.2	2.1	1.9
$n_s/0.1$	9.65	9.64	9.65
$r/10^{-2}$	0.44	1.3	1.1

- For δ**EM** the Whole Observationally Favored Range **Can Be Covered** For δ_{21}'s close to 10^{-5} and r Remaining Below 0.01. For $n_s = 0.967$ We find $\delta_{21} = -1.7 \cdot 10^{-5}$ And $r = 0.003$.
- For **EM2 & EM4** we see that $r \lesssim 0.04$ Increases With N and $\Delta_\star$ Yielding **Upper Bounds**

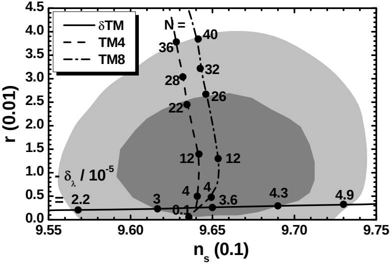
$$\text{I.e.,}\quad 0.96 \lesssim n_s \lesssim 0.965, \quad 0.5 \lesssim N \lesssim 65, \quad 0.24 \gtrsim \Delta_\star/10^{-2} \gtrsim 65 \quad \text{and} \quad 0.00076 \lesssim r \lesssim 0.04.$$

T-Model Inflation

- The **Free Parameters** For

$$\delta\text{TM},\ \text{TM4},\ \text{TM8}\ \text{ are }\ (\delta_{42}, M_2),\ (N, r_{42}, M_2)\ \text{ and }\ (N, r_{24}, M_4).$$

- M is Determined Requiring $\langle M_{BL}\rangle = M_{\rm GUT} \simeq 2/2.433\times10^{-2} \ \Rightarrow\ M_2, M_4 \simeq 0.001.$



Model:	δTM	TM4	TM8
δ_{42} / r_{42} / r_{24}	$-3.6\cdot10^{-5}$	0.01	10^{-6}
N	2	12	12
$\phi_\star/0.1$	9.9555	9.75	9.877
$\Delta_\star(\%)$	0.445	2.5	1.23
$\phi_{\rm f}/0.1$	5.9	3.9	6.5
$w_{\rm rh}$	0.33	0.266	0.58
$N_\star$	55.2	56.4	58
$\lambda/10^{-5}$	3.6	8.6	8.5
$n_{\rm s}/0.1$	9.65	9.64	9.65
$r/10^{-2}$	0.26	1.4	1.3

- For **δTM** We Obtain Results Similar to those for **δEM**. For δ_{21}'s close to 10^{-5} and r Remaining Below 0.01. For **$n_{\rm s} = 0.965$** We find $\delta_{21} = -3.6\cdot10^{-5}$ And $r = 0.0026$.
- For **TM4& TM8** $n_{\rm s}$ is Concentrated Close to Its Central Value And $r \lesssim 0.04$ Increases With $N \lesssim 40$ and $\Delta_\star$

$$0.963 \lesssim n_{\rm s} \lesssim 0.965,\quad 0.1 \lesssim N \lesssim 40,\quad 0.45 \gtrsim \Delta_\star/10^{-2} \gtrsim 13.6\ \text{ and }\ 0.0025 \lesssim r \lesssim 0.039\,.$$

- In the case of **EMI**, We obtain **Less Tuning** Regarding $\Delta_\star$.

Conclusions

- We Proposed New **Implementations** of E- and T-Model **Inflation** Within SUGRA.
- In our Approach, We Realized **EMI and TMI** With a Gauge Singlet and a non-Singlet Inflatons Respectively.
- For Both Models We Employ W's Consistent with an **R Symmetry** and We Can Single out two **Sub-Classes**:
  - One (with Representatives δEM & δTM) Where K has **one** Logarithmic Term and the Pole Appears not only in the Inflationary Kinetic term but also in $V_{\rm I}$.
    Selecting **Specific Curvature $\mathcal{R}_K$** and **Mildly Tuning** two W terms We can Almost Eliminate the Pole from $V_{\rm I}$.
    All $n_{\rm s}$ Values are Possible and **r is Rather Low**, $r \leq 5 \cdot 10^{-3}$.
  - One (with Representatives EM2 & EM4, TM4 & TM8) Where K has **Three** Logarithmic Terms and the Pole Appears Only in the Inflationary Kinetic term.
    In this Class Of Models, **$n_{\rm s}$ is Close to its Central Value**, 0.965, and **r Increases** with $\mathcal{R}_K \sim -1/N$.

Thank You!