Rescaled Einstein-Hilbert gravity: Inflation and the Swampland Criteria

Based on the work of V. K. Oikonomou, I. Giannakoudi, A. Gitsis and K. Revis

Introduction

- Modified gravity (f(R)) contains higher order curvature terms, appealing to describe an inflationary effective Lagrangian.
- The standard inflationary scenario consists of a canonical scalar field φ with equation of motion $\ddot{\phi} + 3H\dot{\phi} + V' = 0$ (FRW Metric).
- The rescaling ($0 \leq \alpha \leq 1$) may give rise to the compliance of the inflationary theory with the Swampland criteria.

Equations of motion

- The general f(R) gravity is of the form $f(R) = R - \gamma\lambda\Lambda - \lambda R \exp\left(-\frac{\gamma\Lambda}{R}\right) - \frac{\Lambda\left(\frac{R}{m_s^2}\right)^\delta}{\zeta}$

yielding the eom $3H^2 f_R = \frac{Rf_R - f}{2} - 3H\dot{f}_R + \kappa^2\left(\frac{1}{2}\dot{\phi}^2 + V\right)\,, \quad -2\dot{H}f_R = \kappa^2\dot{\phi}^2 + \ddot{f}_R - H\dot{f}_R$

- In the large curvarure limit, we can approximate the f(R) gravity action as

$$\mathcal{S} = \int d^4x\sqrt{-g}\left(\frac{1}{2\kappa^2}\left(\alpha R + \frac{\gamma^3\lambda\Lambda^3}{6R^2} - \frac{\gamma^2\lambda\Lambda^2}{2R} - \frac{\Lambda}{\zeta}\left(\frac{R}{m_s^2}\right)^\delta + \mathcal{O}(1/R^3) + \ldots\right) - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right)$$

yielding at leading order the eom $3H^2\alpha \simeq \kappa^2\left(\frac{1}{2}\dot{\phi}^2 + V\right)\,, \quad -2\dot{H}\alpha \simeq \kappa^2\dot{\phi}^2$ $(\alpha = 1 - \lambda)$

- From the 2 sets of eom, it appears at leading order as if we have rescaled the E-H action (αR instead of R).

Slow-roll inflation

(Flat FRW metric assumed)

- The conditions for slow-roll inflation are $\frac{|\dot{H}|}{H^2} \ll 1$ and $|\ddot{\phi}| \ll 3H|\dot{\phi}|$

- The spectral index is given by $n_s = 1 + 2\alpha\eta - 6\alpha\epsilon$

  and the tensor-to-scalar ratio by $r = 16\alpha\epsilon$

  where $\epsilon = \frac{1}{2\kappa^2}\frac{V'^2}{V^2}$ and $\eta = \frac{1}{\kappa^2}\frac{V''}{V}$

- The constraints from the observational data of Planck in 2018 are

$$n_s = 0.9649 \pm 0.0042 \ , \ r < 0.056$$

Swampland criteria

- Conjectures ensuring the concordance of the theory with quantum gravity (expressed in reduced Planck units).
- The distance conjecture: limits the maximum traversable range of a scalar field: $\Delta\phi \leq f \sim \mathcal{O}(1)$
- The De Sitter conjecture: sets a limit to the gradient of the scalar potential:

$$\frac{V'}{V} \geq g \sim \mathcal{O}(1)$$

or, alternatively,

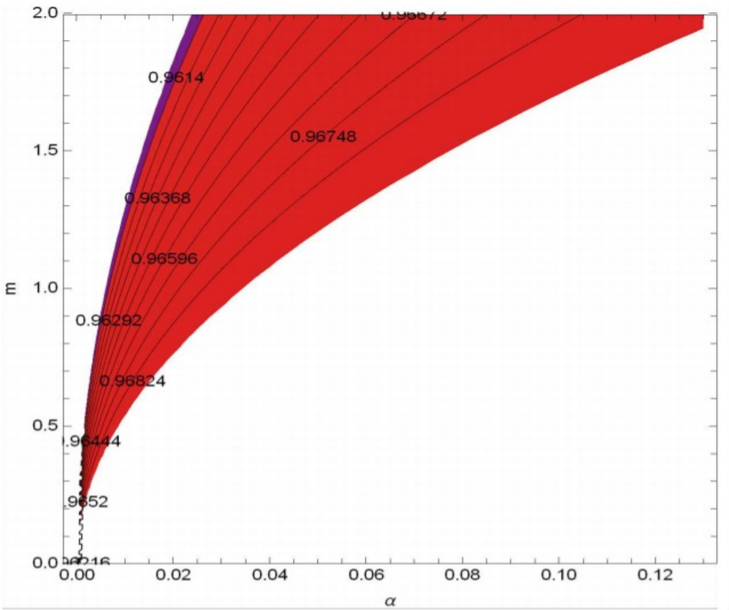
$$\frac{V''}{V} \leq -h \sim \mathcal{O}(1)$$

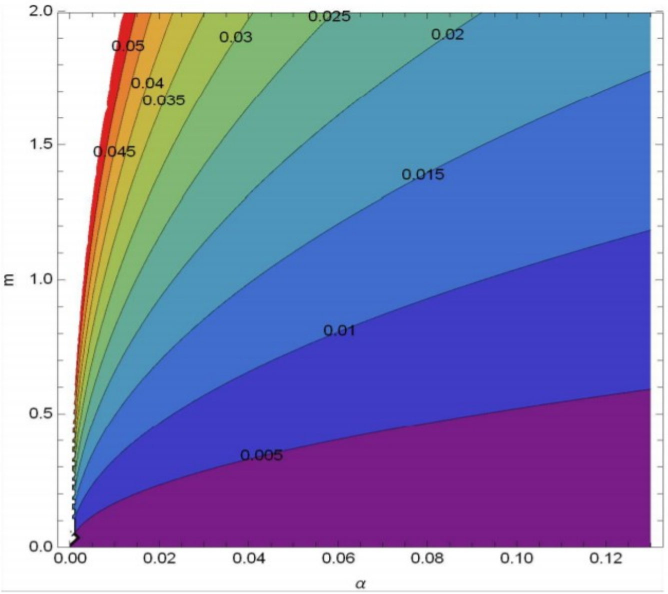
- We are looking for values of α that satisfy both the inflationary constraints and the Swampland criteria.

D – Brane (p = 2)

$$V(\phi) = \Lambda^4 \left(1 - \left(\frac{m}{\kappa\phi}\right)^2\right)$$

$$m = [10^{-6}, 10^{0.3}]$$





The constraints for α are

$$0.00598m^2 \leq \alpha \leq 0.03438m^2$$

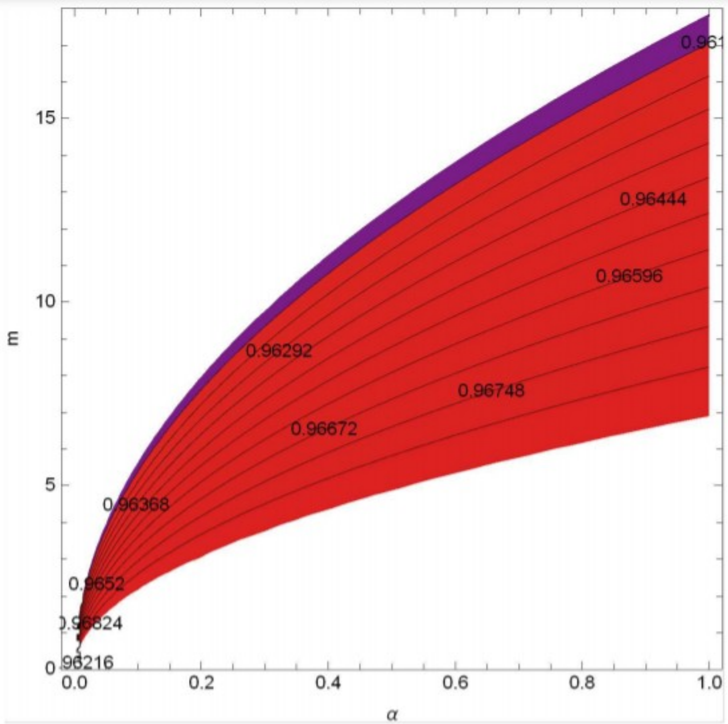
$$n_s \simeq \frac{3m^{2/3}}{8\sqrt[3]{2}\sqrt[3]{\alpha}N^2} - \frac{3\sqrt{\alpha m^2 N}}{4\sqrt{2}\alpha N^2} - \frac{3}{2N} + 1$$

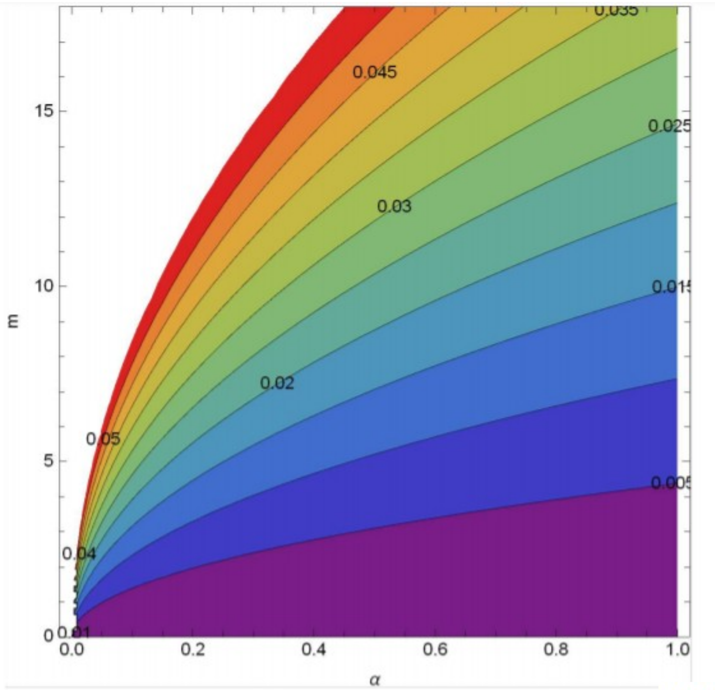
$$r \simeq \frac{\sqrt{2}\sqrt{\alpha m^2 N}}{\alpha N^2}$$

D – Brane (p = 4)

$$V(\phi) = \Lambda^4 \left(1 - \left(\frac{m}{\kappa\phi}\right)^4\right)$$

$$m = [10^{-6}, 10^{0.3}]$$





$$n_s \simeq \frac{5m^{4/5}}{18\sqrt[5]{2}\alpha^{2/5}N^2} - \frac{\sqrt[3]{\alpha m^4 N}}{2\ 3^{2/3}\alpha N^2} - \frac{5}{3N} + 1,$$

$$r \simeq \frac{4\sqrt[3]{\alpha m^4 N}}{3\ 3^{2/3}\alpha N^2}.$$

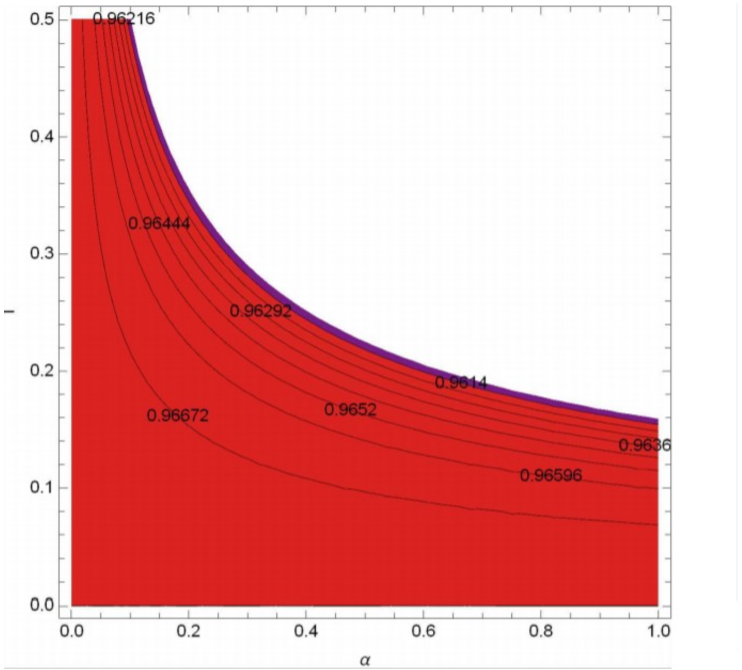
The constraints for α are

$$0.00313782m^2 \leq \alpha \leq 0.0209707m^2$$

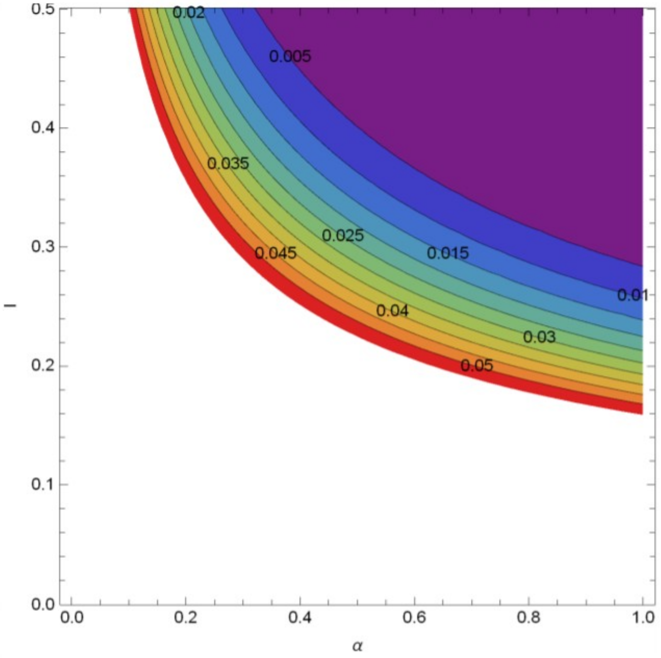
Natural inflation

$$V(\phi) = \Lambda^4(1 - \cos(l\kappa\phi))$$

$$l = [10^{0.3}, 10^{2.5}]$$



$$n_s = -\frac{2\alpha l^2 + (\alpha^2 l^4 + \alpha l^2 - 2)\, e^{\alpha l^2 N} + 2}{(\alpha l^2 + 2)\, e^{\alpha l^2 N} - 2}$$



$$r = \frac{16\alpha l^2}{(\alpha l^2 + 2)\, e^{\alpha l^2 N} - 2}$$

The constraints for α are

$$\frac{0.02525}{l^2} < \alpha < \frac{0.02553}{l^2}$$

- In this case, we have a very narrow range.

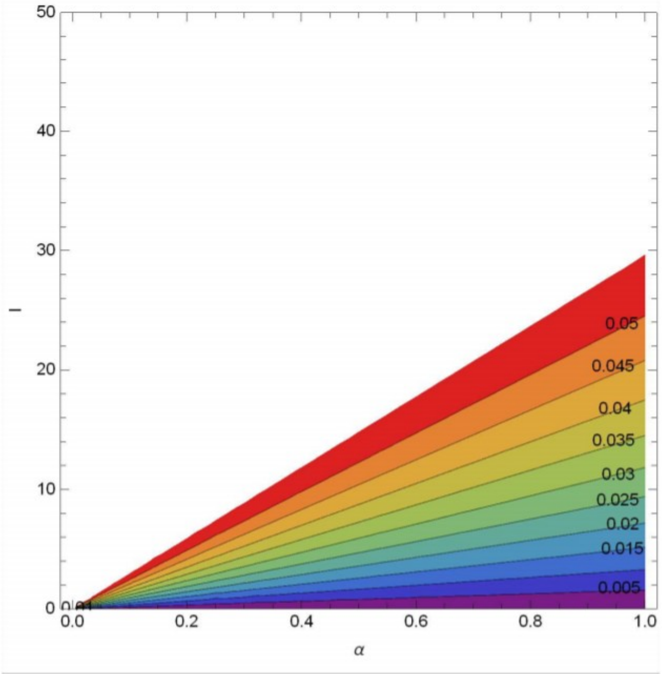
T – Model (m = 1)

$$V(\phi) = \Lambda^4 \tanh^2\left(\frac{\kappa\phi}{\sqrt{6l}}\right)$$

$$l = [10^{-2}, 10^4]$$

$$n_s = -\frac{18l}{3l + 4\alpha N^2 + 2lN\sqrt{\frac{12\alpha}{l} + 9}} + \frac{2\alpha\left(\operatorname{csch}^2\left(\frac{1}{2}\cosh^{-1}\left(\frac{l\sqrt{\frac{12\alpha}{l}+9}+4\alpha N}{3l}\right)\right) - 2\right)\operatorname{sech}^2\left(\frac{1}{2}\cosh^{-1}\left(\frac{l\sqrt{\frac{12\alpha}{l}+9}+4\alpha N}{3l}\right)\right)}{3l} + 1$$

Its value is really close to 0.9667 for all values of α



$$r = \frac{48l}{3l + 4\alpha N^2 + 2lN\sqrt{\frac{12\alpha}{l} + 9}}$$

The constraints for α are

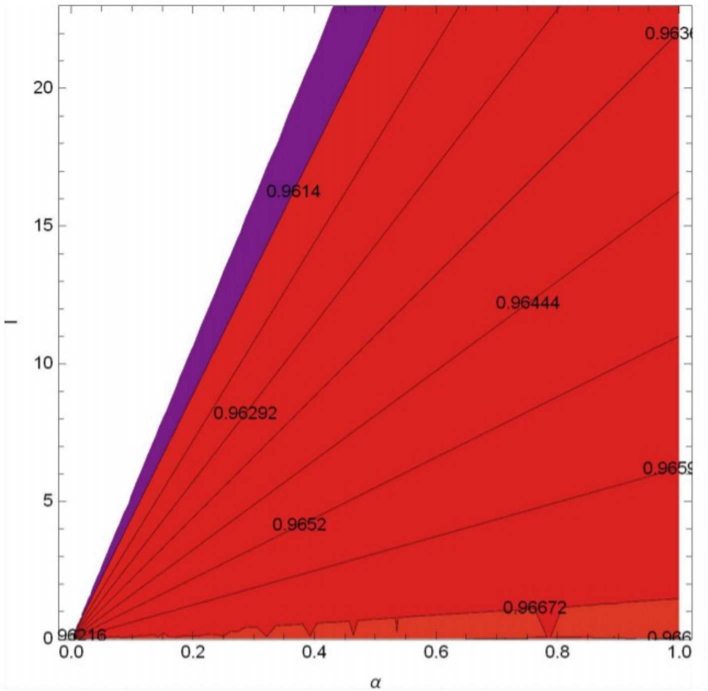
$$0.03376l < \alpha \leq 1$$

- The maximum value of l is narrowed down.

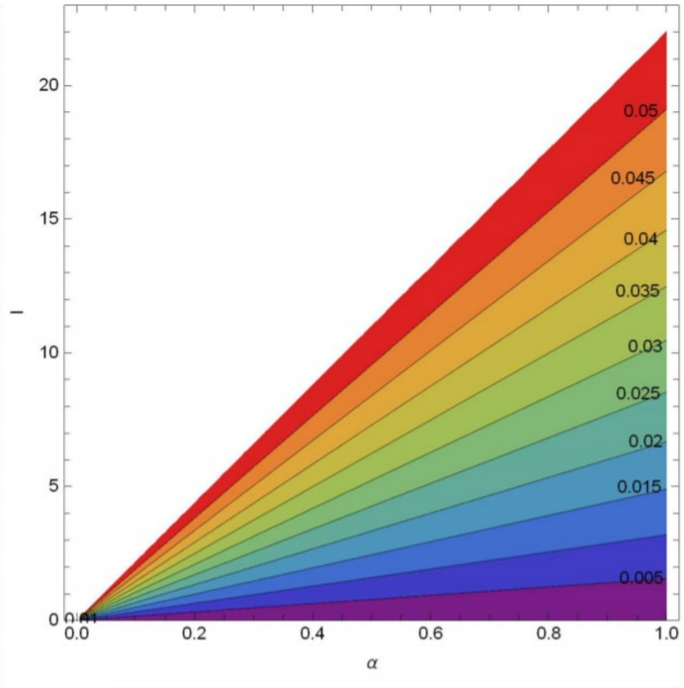
T – Model (m = 2)

$$V(\phi) = \Lambda^4 \tanh^4\left(\frac{\kappa\phi}{\sqrt{6l}}\right)$$

$$l = [10^{-2}, 10^4]$$



$$n_s = -\frac{18l}{3l+4\alpha N^2 + lN\sqrt{\frac{48\alpha}{l}+9}} + \frac{4\alpha\left(3\operatorname{csch}^2\left(\frac{1}{2}\cosh^{-1}\left(\frac{l\sqrt{\frac{48\alpha}{l}+9}+8\alpha N}{3l}\right)\right)-2\right)\operatorname{sech}^2\left(\frac{1}{2}\cosh^{-1}\left(\frac{l\sqrt{\frac{48\alpha}{l}+9}+8\alpha N}{3l}\right)\right)}{3l}+1$$



$$r = \frac{48l}{3l+4\alpha N^2 + lN\sqrt{\frac{48\alpha}{l}+9}}$$

The constraints for α are

$$0.04538l \le \alpha \le 30l\ ,\ l = [10^{-2}, 0.03]$$

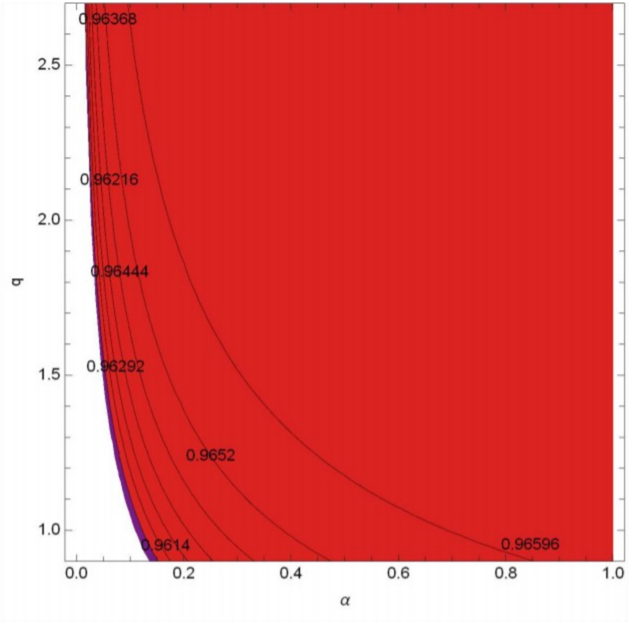
$$0.0453846l \le \alpha \le 1\ ,\ l = [0.033, 22.034]$$

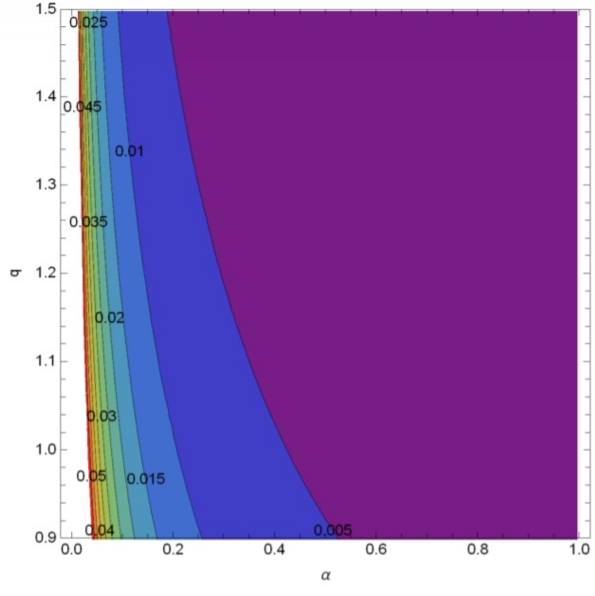
- Again, a maximum value for l has been imposed.

Potential with exponential tails model

$$V(\phi) = \Lambda^4(1 - e^{-\kappa q\phi})$$

$$q = [10^{-3}, 10^{3}]$$





The constraints for α are

$$0.993/q^2 \leq \alpha \leq 1\ ,\ q = [0.9967, 1.0059]$$

$$0.993/q^2 \leq \alpha \leq 1.012/q^2\ ,\ q = [1.0059, 10^3]$$

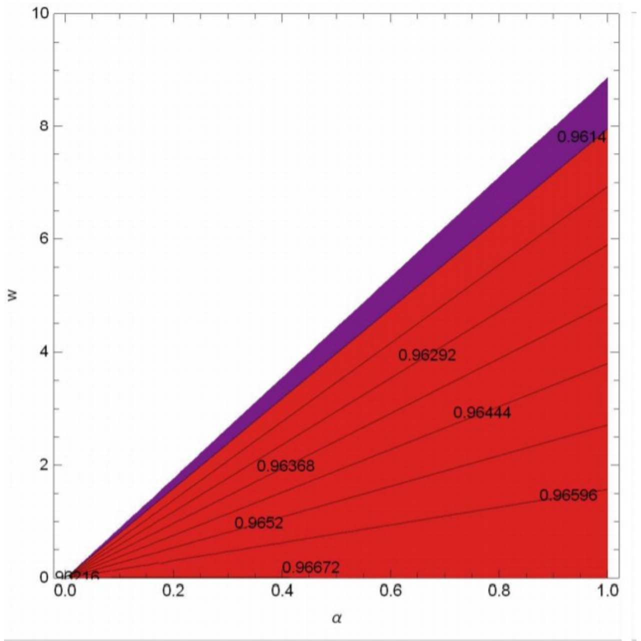
$$n_s = -\frac{4\alpha q}{\sqrt{2}\sqrt{\alpha} + 2\alpha q N} - \frac{12}{\left(2\sqrt{\alpha} q N + \sqrt{2}\right)^2} + 1$$

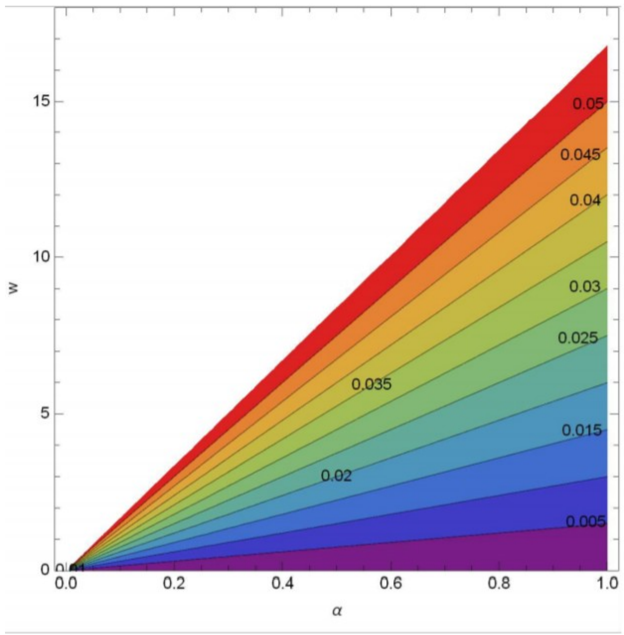
$$r = \frac{32}{\left(2\sqrt{\alpha} q N + \sqrt{2}\right)^2}$$

E – Model (n = 1)

$$V(\phi) = \Lambda^4 \left(1 - e^{-\frac{\sqrt{\frac{2}{3}}\kappa\phi}{\sqrt{w}}}\right)^2$$

$$w = [10^{-2}, 10^4]$$





The constraints for α are

$$0.0949w \leq \alpha \leq 1\ ,\ \ w = [10^{-2}, 10.538]$$

- The range of w is severely restricted.

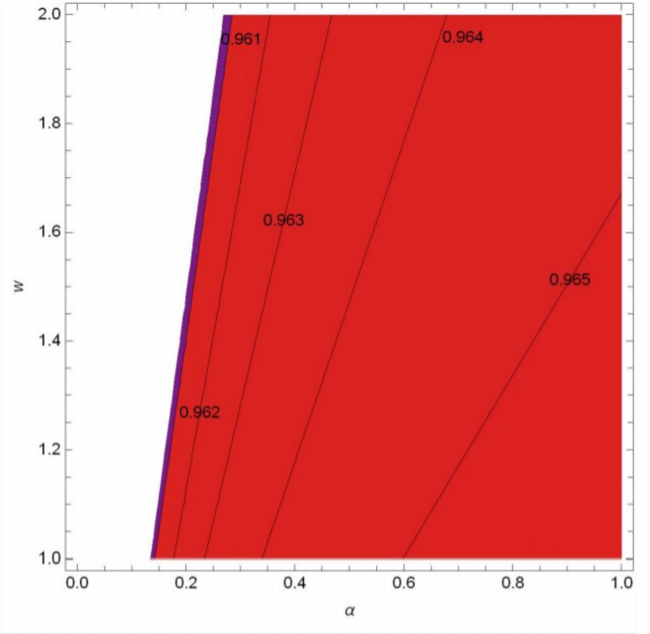
$$n_s \simeq \frac{\sqrt{3}\sqrt{\alpha}\sqrt{w} - 3w + \alpha N^2 - 2\alpha N}{\alpha N^2}$$

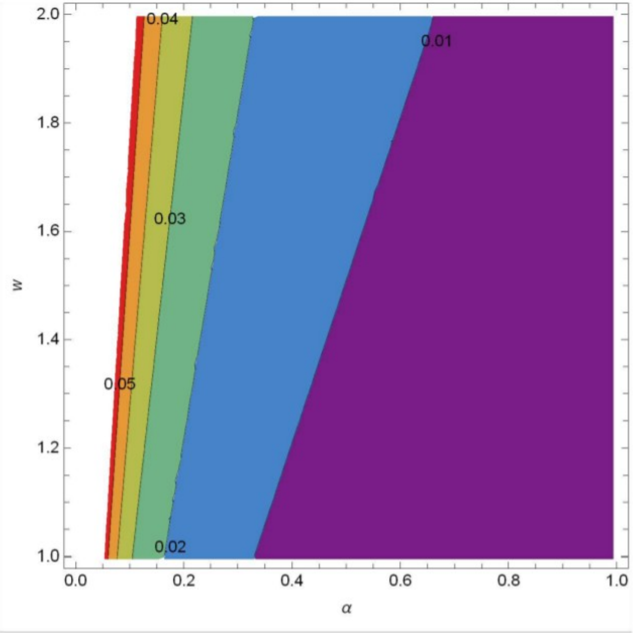
$$r \simeq \frac{12w}{\alpha N^2}$$

E – Model (n = 2)

$$V(\phi) = \Lambda^4 \left(1 - e^{-\frac{\sqrt{\frac{2}{3}}\kappa\phi}{\sqrt{w}}}\right)^4$$

$$w = [10^{-2}, 10^4]$$





$$n_s \simeq \frac{\frac{\sqrt{3}\sqrt{w}}{\sqrt{\alpha}} - \frac{9w}{4\alpha}}{N^2} - \frac{2}{N} + 1$$

$$r \simeq \frac{12w}{\alpha N^2}$$

The constraints for α are

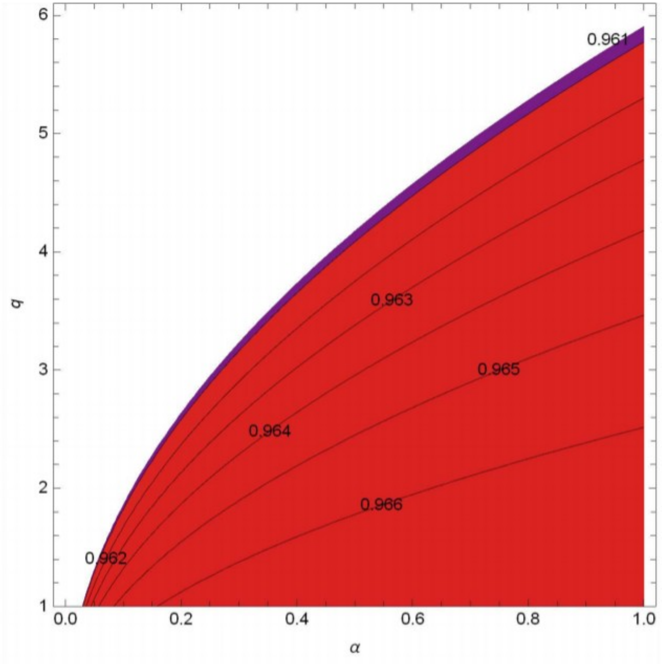
$$0.0817w \le \alpha \le 1 \;,\; w = [10^{-2}, 12.24]$$

- Again, the range of w is quite limited.

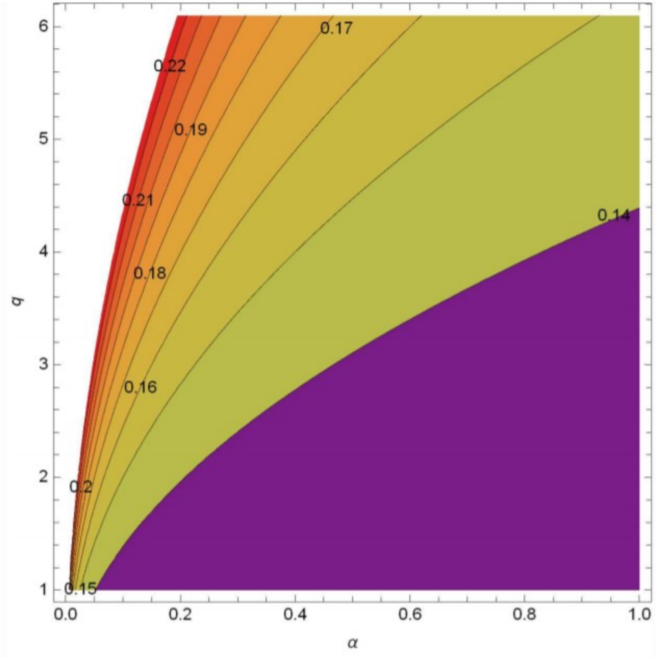
Hilltop Quadratic Model

$$V(\phi) = \Lambda^4\left(1 - \frac{\kappa^2\phi^2}{q^2}\right)$$

$$q = [10^{0.3}, 10^{4.85}]$$



$$n_s \simeq \frac{2\sqrt{\alpha\left(\alpha+2q^2\right)} - 3q^2 + \alpha\left(4N^2 - 8N + 2\right)}{4\alpha N^2}$$



$$r \simeq -\frac{2\left(\sqrt{\alpha\left(\alpha+2q^2\right)} + \alpha - q^2 - 4\alpha N\right)}{\alpha N^2}$$

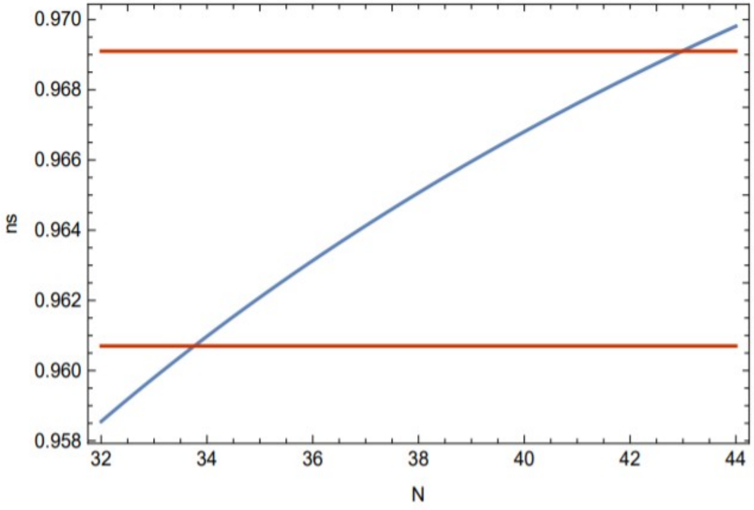
The constraint for α is

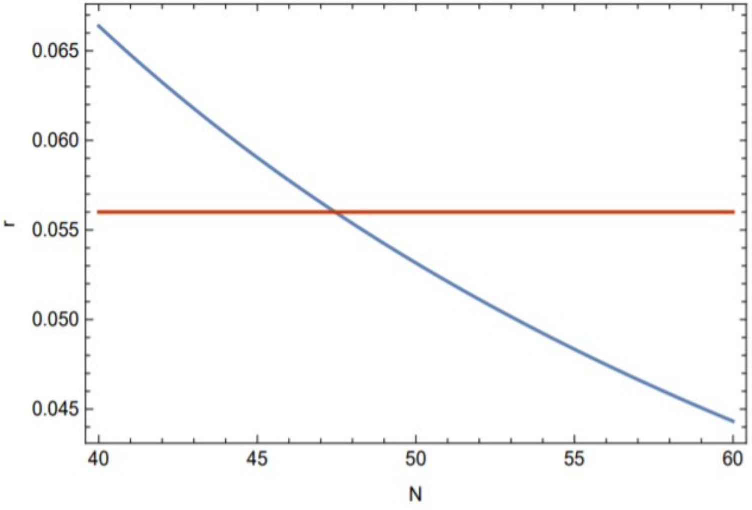
$$\frac{\alpha}{q^2} \geq 0.0286389$$

From the plot of r, we see that this model is not viable for our theory.

Power – law Potential (I)

$$V(\phi) = \frac{\lambda \phi^{2/3}}{\kappa^{10/3}}$$





- The observational quantities are now functions only of the e-foldings number N.

$$n_s \simeq \frac{3}{8N^2} - \frac{3}{2N} + 1$$

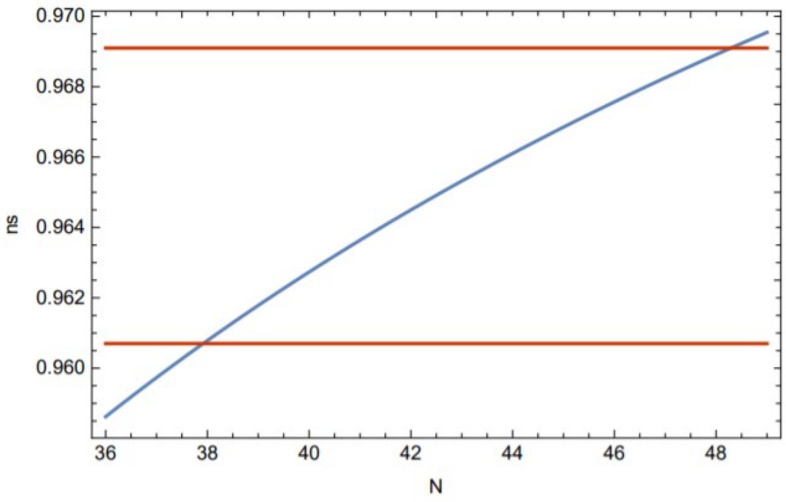
$$r \simeq \frac{4N-1}{N^2}$$

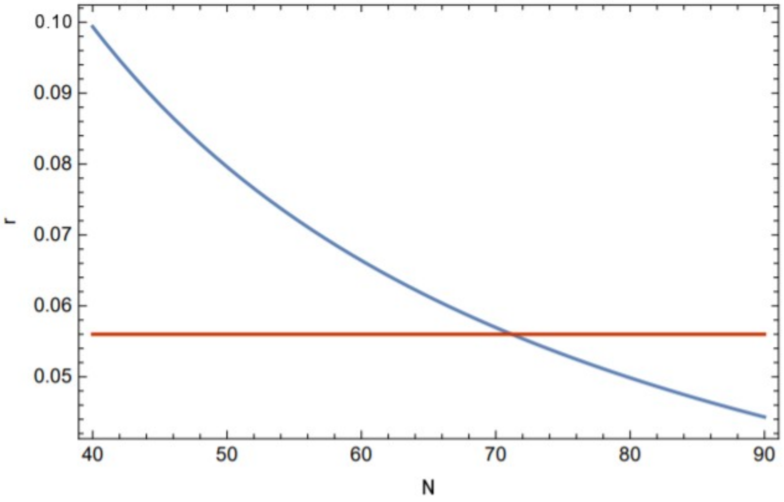
(Inflation ends at N = [50 , 60])

- Planck contraints are not satisfied simultaneously so from the plots, the model is a not viable one.

Power – law Potential (II)

$$V(\phi) = \frac{\lambda\phi}{\kappa^3}$$





- Again, Planck constraints are not satisfied simultaneously.

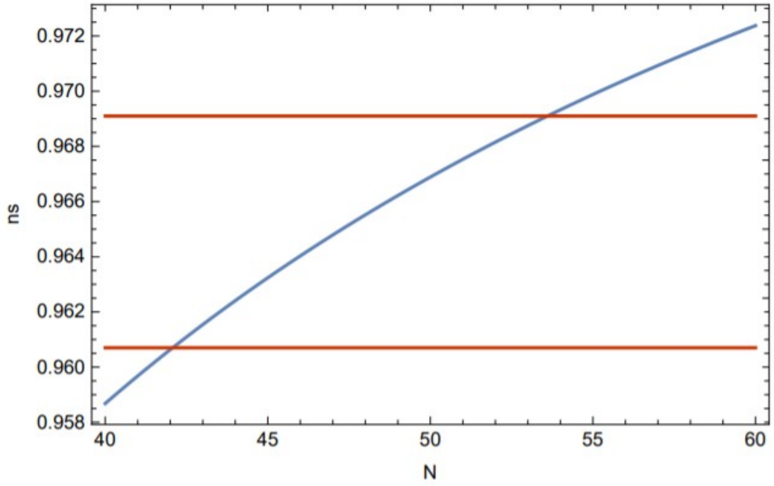
$$n_s \simeq \frac{3}{8N^2} - \frac{3}{2N} + 1$$

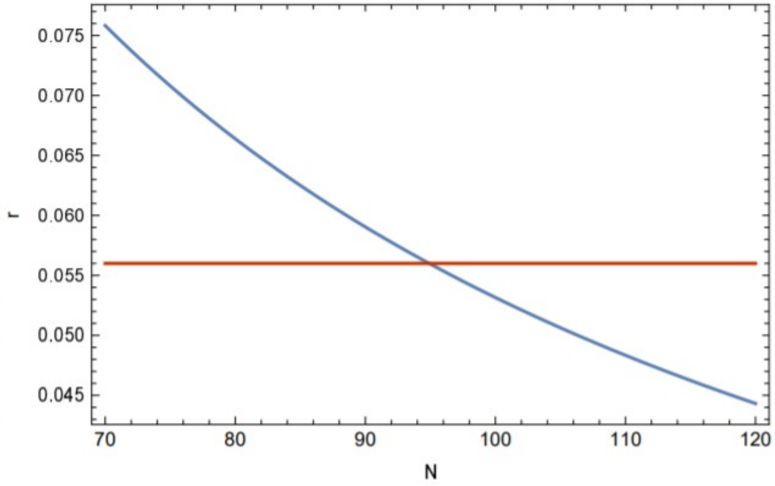
$$r \simeq \frac{4N-1}{N^2}$$

- Not a viable model for our theory.

Power – law Potential (III)

$$V(\phi) = \frac{\lambda \phi^{4/3}}{\kappa^{8/3}}$$





- Like before, Planck constraints are not satisfied simultaneously.

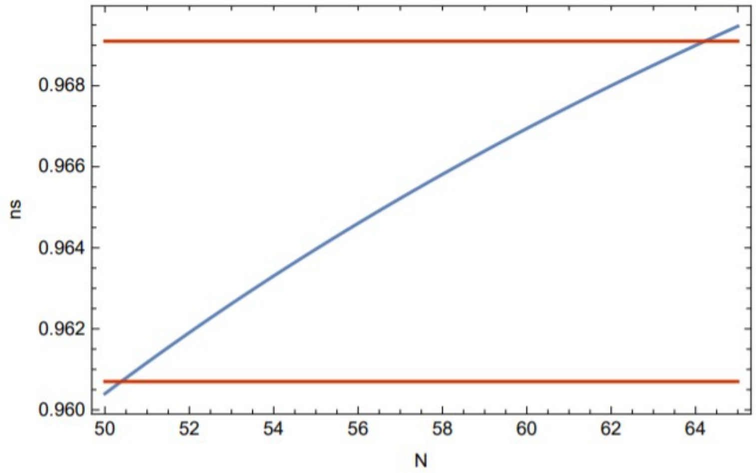
$$n_s \simeq \frac{5}{9N^2} - \frac{5}{3N} + 1$$

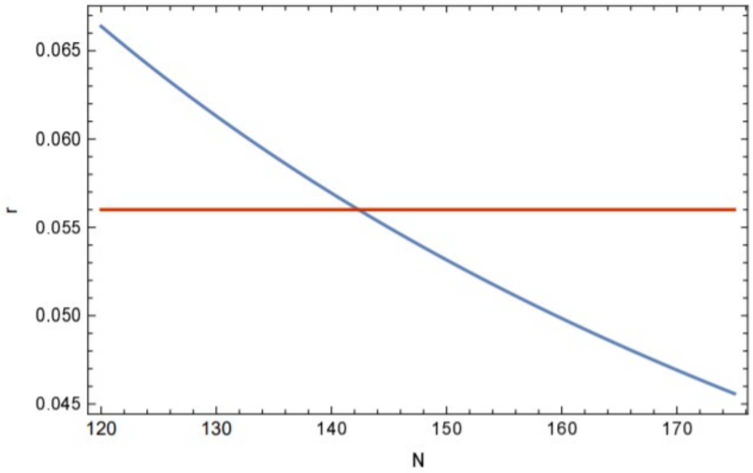
$$r \simeq \frac{16(3N-1)}{9N^2}$$

- Yet another not viable power-law model.

Power – law Potential (IV)

$$V(\phi) = \frac{\lambda\phi^2}{k^2}$$





$$n_s \simeq \frac{(N-1)^2}{N^2}$$

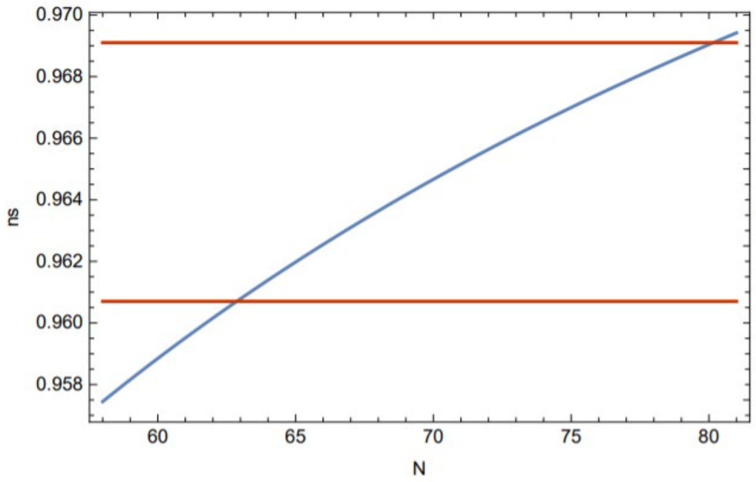
$$r \simeq \frac{8N-4}{N^2}$$

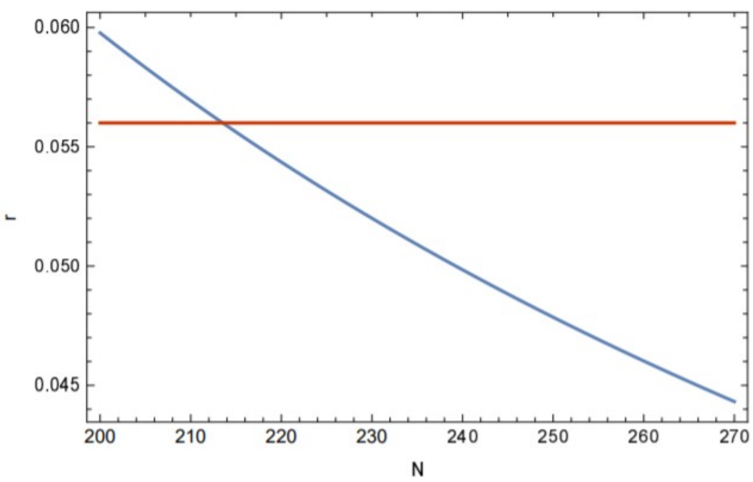
- Planck constraints not satisfied simultaneously.

- Another not viable model for our theory.

Power – law Potential (V)

$$V(\phi) = \frac{\lambda \phi^3}{\kappa}$$





- As in the previous cases, Planck constraints are not satisfied simultaneously.

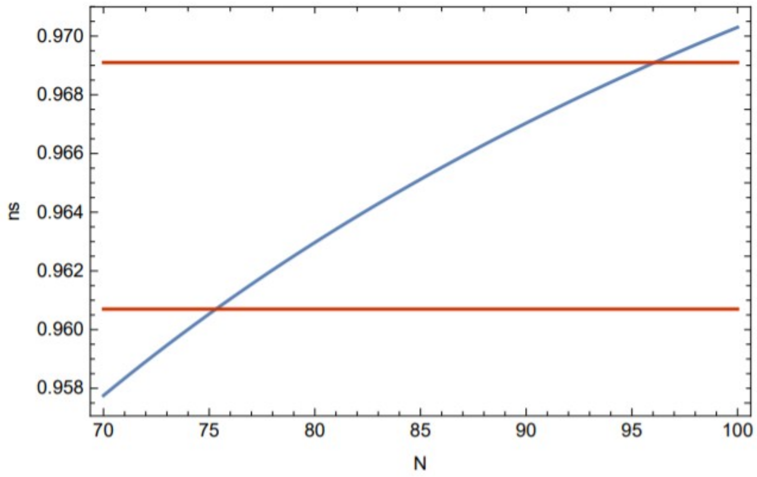
$$n_s \simeq \frac{4N - 7}{4N + 3}$$

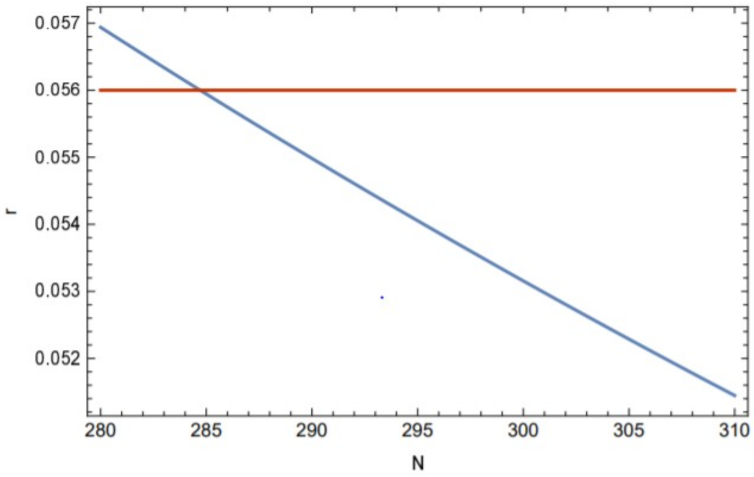
$$r \simeq \frac{3(4N - 3)}{N^2}$$

- Another not viable model.

Power – law Potential (VI)

$$V(\phi) = \lambda\phi^4$$





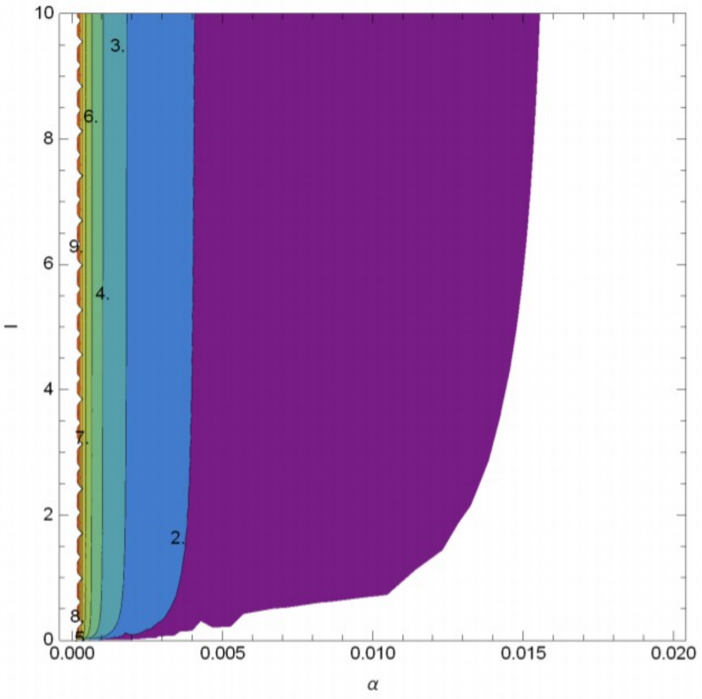
- Just like before, Planck constraints are not satisfied simultaneously.

$$n_s \simeq \frac{3}{N^2} - \frac{3}{N} + 1$$

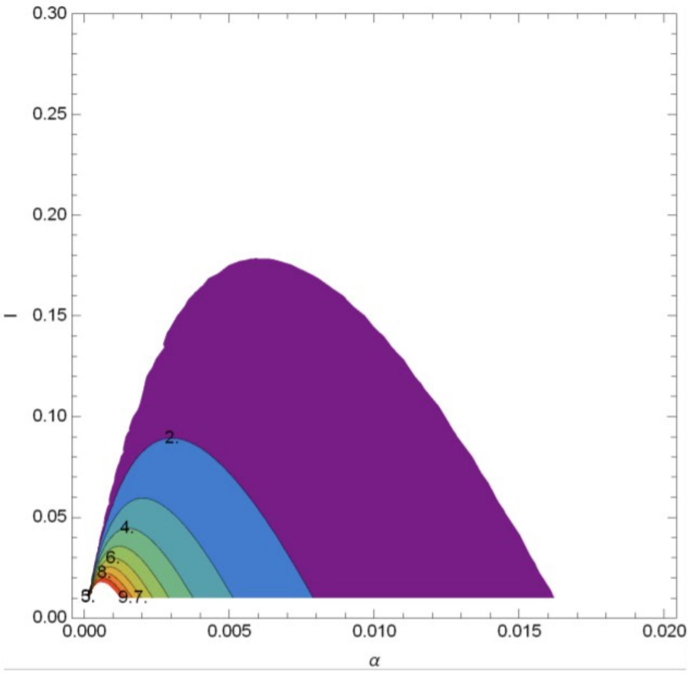
$$r \simeq 16\left(\frac{1}{N} - \frac{1}{N^2}\right)$$

- None of the Power – law models has been proven a viable one.

Swampland for T – Model (m = 1)



$$\frac{V'(\phi_i)}{V(\phi_i)} = \frac{2\sqrt{\frac{2}{3}}\kappa \operatorname{csch}\left(\kappa \cosh^{-1}\left(\frac{l\sqrt{\frac{12\alpha}{l}+9}+4\alpha N}{3l}\right)\right)}{\sqrt{l}}$$



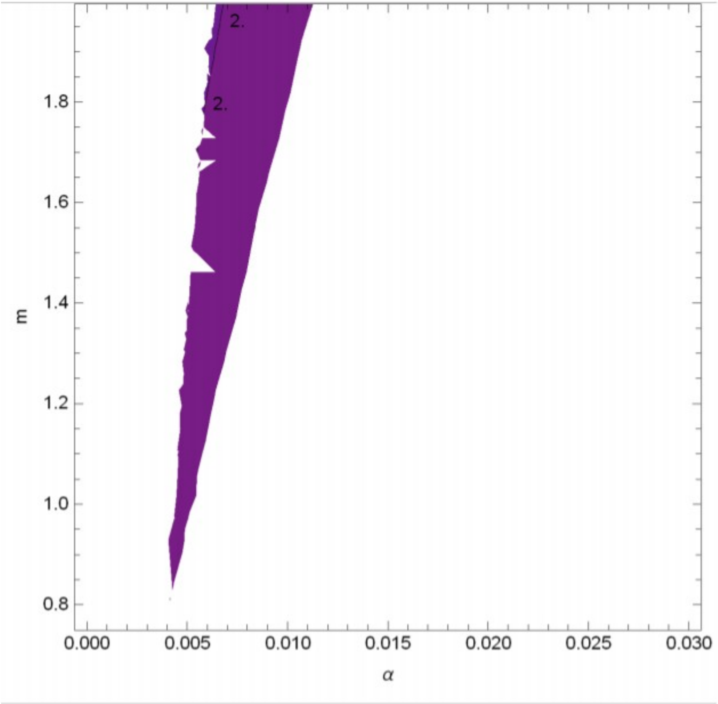
$$-\frac{V''(\phi_i)}{V(\phi_i)} = -\frac{l\sqrt{\frac{12\alpha}{l}+9}+240\alpha-6l}{3\alpha\left(40l\sqrt{\frac{12\alpha}{l}+9}+4800\alpha+l\right)}$$

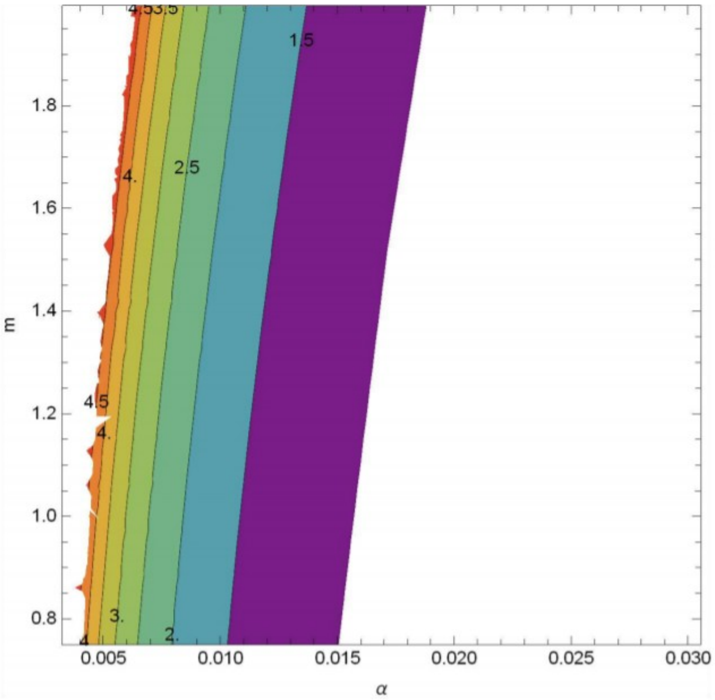
The range of α is

$$\alpha = [0, 0.008]$$

- The range is really narrow.

Swampland for D – Brane (p = 4)





$$\frac{V'(\phi_i)}{V(\phi_i)} = -\frac{4\kappa m^4}{m^4 \sqrt[6]{2\, 2^{4/5}\alpha^{3/5}m^{24/5}+24\alpha m^4 N} - \left(2\, 2^{4/5}\alpha^{3/5}m^{24/5}+24\alpha m^4 N\right)^{5/6}}$$

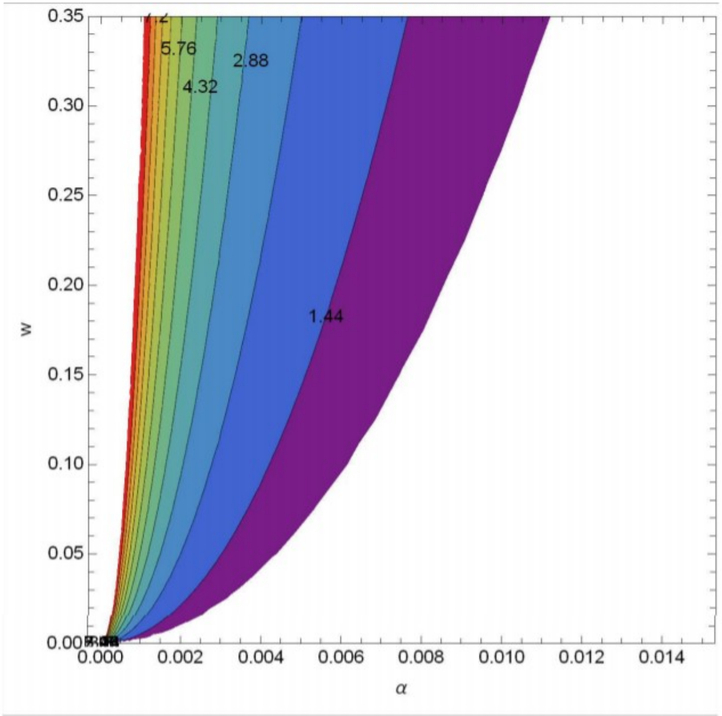
$$-\frac{V''(\phi_i)}{V(\phi_i)} = \frac{20\kappa^2}{-\sqrt[3]{2\, 2^{4/5}\alpha^{3/5}m^{24/5}+24\alpha m^4 N} + 2\, 2^{4/5}\alpha^{3/5}m^{4/5}+24\alpha N}$$

The ranges for α and m are

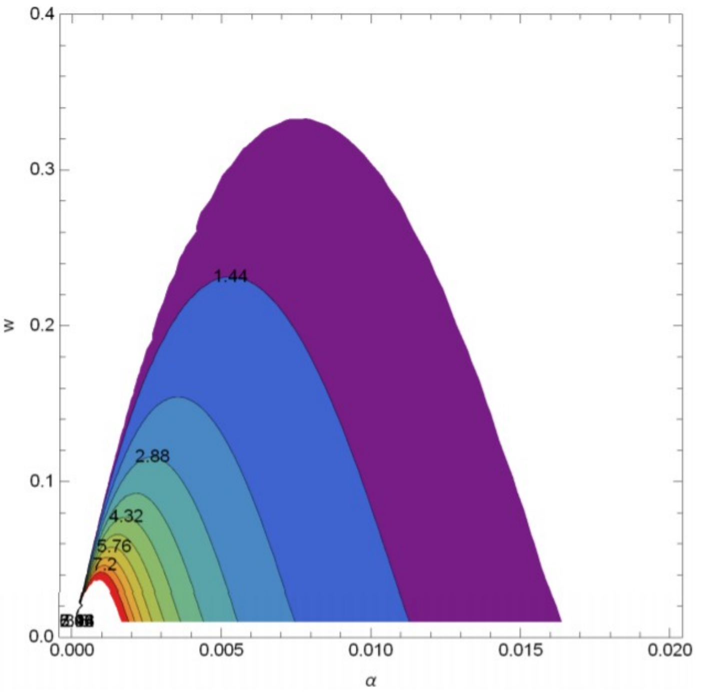
$$\begin{aligned} \alpha &= [0, 0.03] \\ m &= [0.75, 1.99526] \\ 512.802\alpha - 0.318 &< m \\ m &< 164.067\alpha + 0.146 \end{aligned}$$

- We can also evaluate relations between the two parameters.

Swampland for E – Model (n = 1)



$$\frac{V'(\phi_i)}{V(\phi_i)} = \frac{\sqrt{6}\kappa\sqrt{w}}{\sqrt{3}\sqrt{\alpha}\sqrt{w} + 2\alpha N}$$



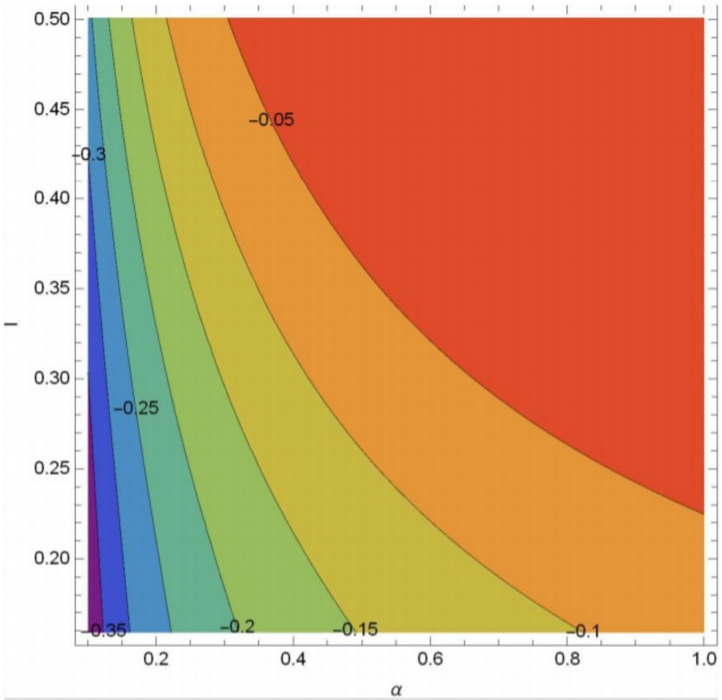
$$-\frac{V''(\phi_i)}{V(\phi_i)} = \frac{\kappa^2\left(2\sqrt{3}\sqrt{\alpha}\sqrt{w} + 4\alpha N - 3w\right)}{\alpha\left(2\sqrt{\alpha}N + \sqrt{3}\sqrt{w}\right)^2}$$

The ranges for α and w are

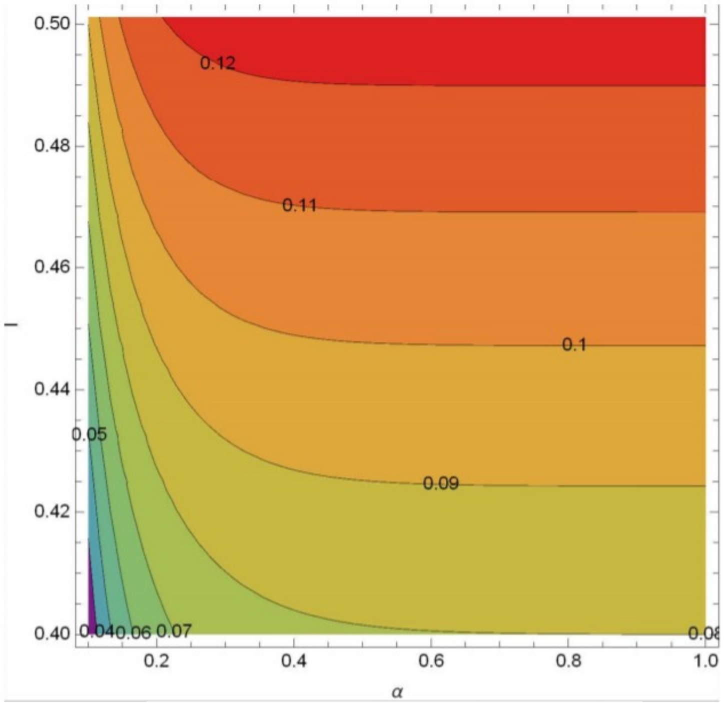
$$\begin{aligned} \alpha &= [0, 0.05138] \\ w &= [0.01, 0.05138] \\ w &> 3366.41\alpha^2 + 0.01 \end{aligned}$$

- One of the cases we can also find relations between the parameters.

Swampland for Natural Inflation



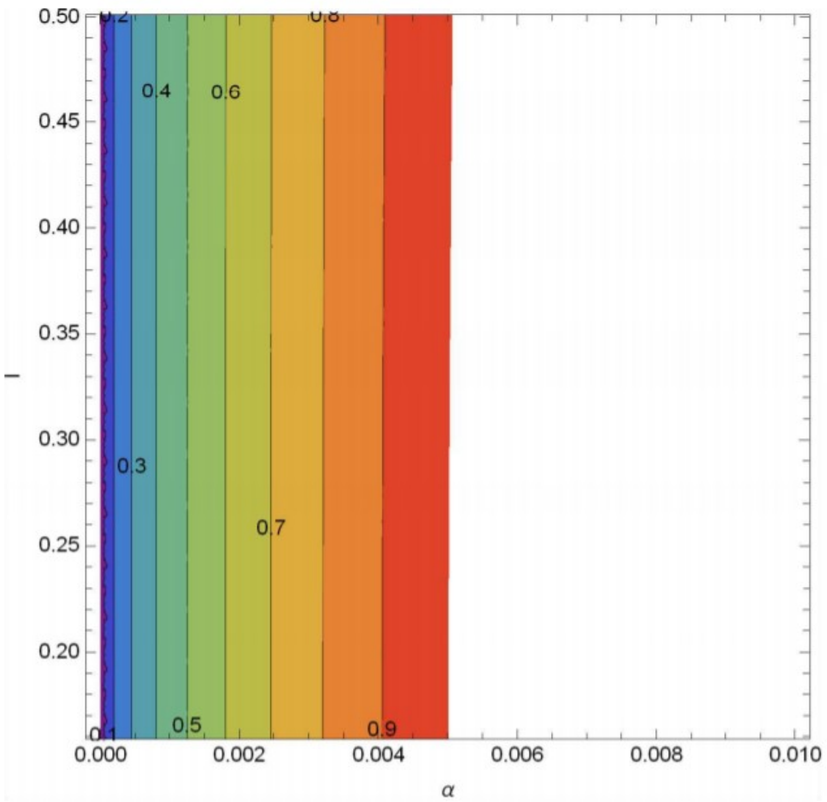
$$\frac{V'(\phi_i)}{V(\phi_i)} = -\frac{l \sin\left(2\sin^{-1}\left(\frac{\sqrt{2}e^{-30\alpha l^2}}{\sqrt{\alpha}\, l \sqrt{\frac{2}{\alpha l^2}+1}}\right)\right)}{\cos\left(2\sin^{-1}\left(\frac{\sqrt{2}e^{-30\alpha l^2}}{\sqrt{\alpha}\, l\sqrt{\frac{2}{\alpha l^2}+1}}\right)\right)+1}$$



$$-\frac{V''(\phi_i)}{V(\phi_i)} = \frac{l^2 \cos\left(2\sin^{-1}\left(\frac{\sqrt{2}e^{-30\alpha l^2}}{\sqrt{\alpha}\, l \sqrt{\frac{2}{\alpha l^2}+1}}\right)\right)}{\cos\left(2\sin^{-1}\left(\frac{\sqrt{2}e^{-30\alpha l^2}}{\sqrt{\alpha}\, l\sqrt{\frac{2}{\alpha l^2}+1}}\right)\right)+1}$$

- Up till now, none of the Swampland criteria is satiasfied.

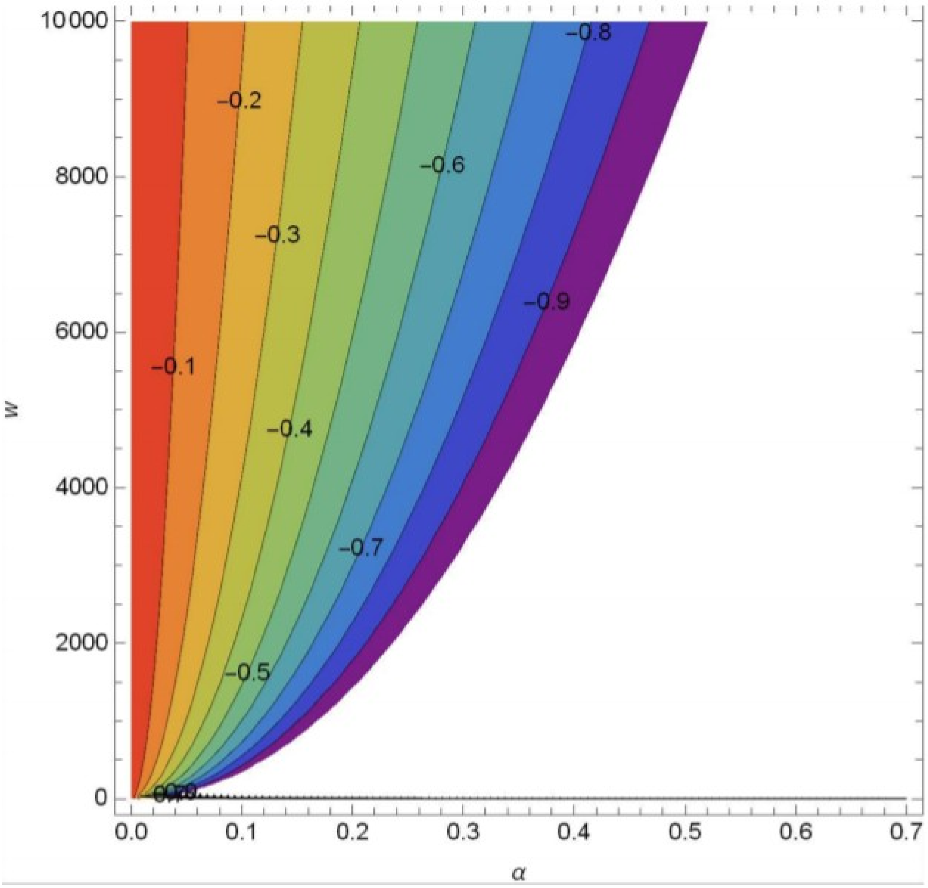
Swampland for Natural Inflation



$$\Delta\phi = \frac{2\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{\alpha}l}\right)}{\kappa l} - \frac{2\sin^{-1}\left(\frac{\sqrt{2}e^{-\frac{1}{2}\alpha l^2 N}}{\sqrt{\alpha}l\sqrt{\frac{2}{\alpha l^2}+1}}\right)}{\kappa l}$$

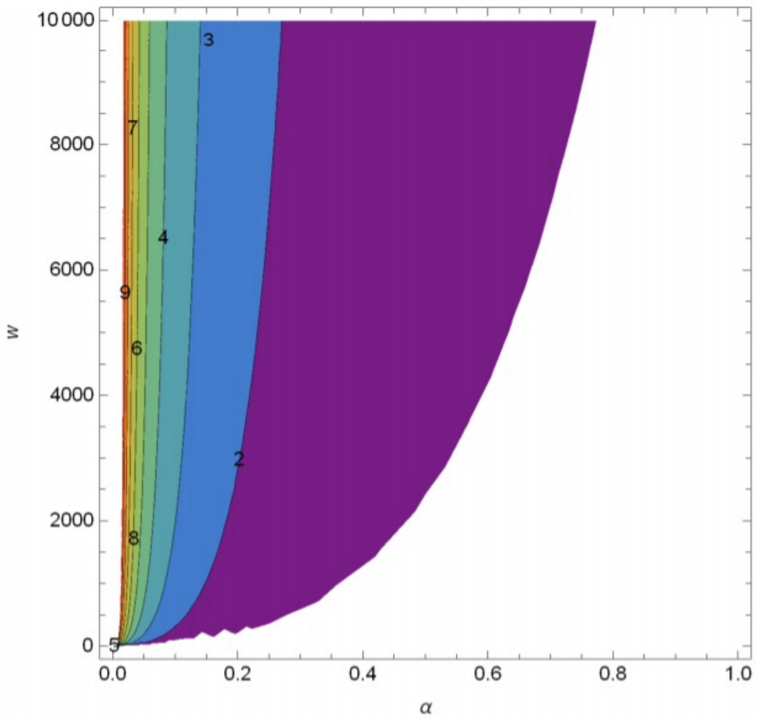
- The Swampland criteria are not satisfied for this potential.

Swampland for E – Model (n = 2)

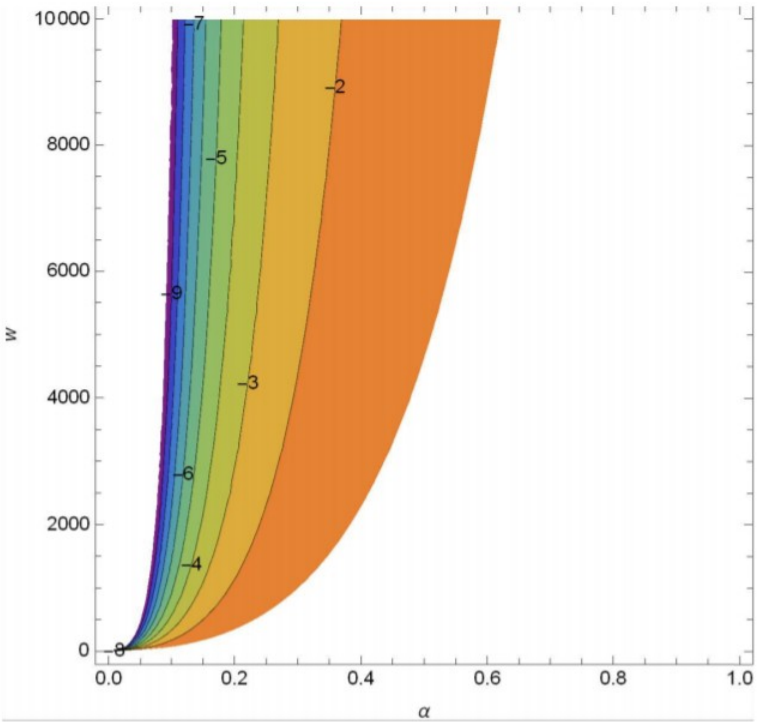


$$\Delta\phi = \sqrt{\frac{3w}{2}} \ln \frac{1 + \frac{4\sqrt{\alpha}}{\sqrt{3w}}}{1 + \frac{4\sqrt{\alpha}}{\sqrt{3w}} + \frac{8N\alpha}{3w}}$$

Swampland for E – Model (n = 2)



$$\frac{V'(\phi_i)}{V(\phi_i)} = 4\sqrt{\frac{2}{3}}\frac{1}{1+\frac{4\sqrt{\alpha}}{\sqrt{3w}}+\frac{8N\alpha}{3w}}\frac{\frac{1}{\sqrt{w}}}{1-\frac{1}{1+\frac{4\sqrt{\alpha}}{\sqrt{3w}}+\frac{8N\alpha}{3w}}}$$



$$-\frac{V''(\phi_i)}{V(\phi_i)} = \frac{8}{3w}\frac{\frac{4\sqrt{\alpha}}{\sqrt{3w}}+\frac{8N\alpha}{3w}-3}{3w\left(\frac{4\sqrt{\alpha}}{\sqrt{3w}}+\frac{8N\alpha}{3w}\right)^2}$$

The ranges for α and w are

$$\alpha = [0, 0.52]$$
$$w = [0.01, 10000]$$

Conclusions

- Most models are compatible with the Planck data while some comply with the Swampland criteria, too.

- The Swampland criteria restrict severely the range of α.

Appendix (I)

(Flat FRW metric assumed)

- From Einstein eqs, we get $\frac{3\alpha}{\kappa^2}H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi)$ and $\frac{2\alpha}{\kappa^2}\dot{H} = -\dot{\phi}^2$

- From $\frac{|\dot{H}|}{H^2} \ll 1$ we define $\epsilon_1 = -\frac{\dot{H}}{H^2} = \alpha\epsilon$ and from $|\ddot{\phi}| \ll 3H|\dot{\phi}|$ we define

$\epsilon_2 = \frac{\ddot{\phi}}{H\dot{\phi}} = -\alpha\eta + \epsilon_1$ where $\epsilon = \frac{1}{2\kappa^2}\frac{V'^2}{V^2}$ and $\eta = \frac{1}{\kappa^2}\frac{V''}{V}$

(The conditions for slow-roll inflation are $\epsilon_1 \ll 1$ and $\epsilon_2 \ll 1$)

- The spectral index and tensor-to-scalar-ratio are defined as $n_s - 1 = -4\epsilon_1 - 2\epsilon_2$ and $r = 8\kappa^2\frac{\dot{\phi}^2}{H^2}$

In our case, $n_s = 1 + 2\alpha\eta - 6\alpha\epsilon$ and $r = 16\alpha\epsilon.$

Appendix (II)

- From the de Sitter criterion, $\left|\frac{V'}{V}\right| = \sqrt{2\epsilon} \geq 1$

- For the rescaled case, the slow-roll has to do with ε1, not ε.

- We can choose an appropriately small value of α so that both slow-roll conditions and Swampland criteria hold true.

THANK YOU FOR YOUR TIME!