Tracing Torsional Gravity Signatures in Infationary Observables

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Goal

- We search for signatures of torsional modified gravity in late- and early-time cosmological observations

- The advancing gravitational wave multi-messenger astronomy opens a new era towards investigating gravity.

Gravity and Cosmology

- A successful cosmological model must:

1) Describe the evolution of the universe at the background level
2) Describe the evolution of the universe at the perturbation level

- ΛCDM paradigm seems to succeed in both, at post-inflationary eras
- Open issues:
- 1) The cosmological-constant problem. Calculation of Λ gives a number 120 orders of magnitude larger than observed.

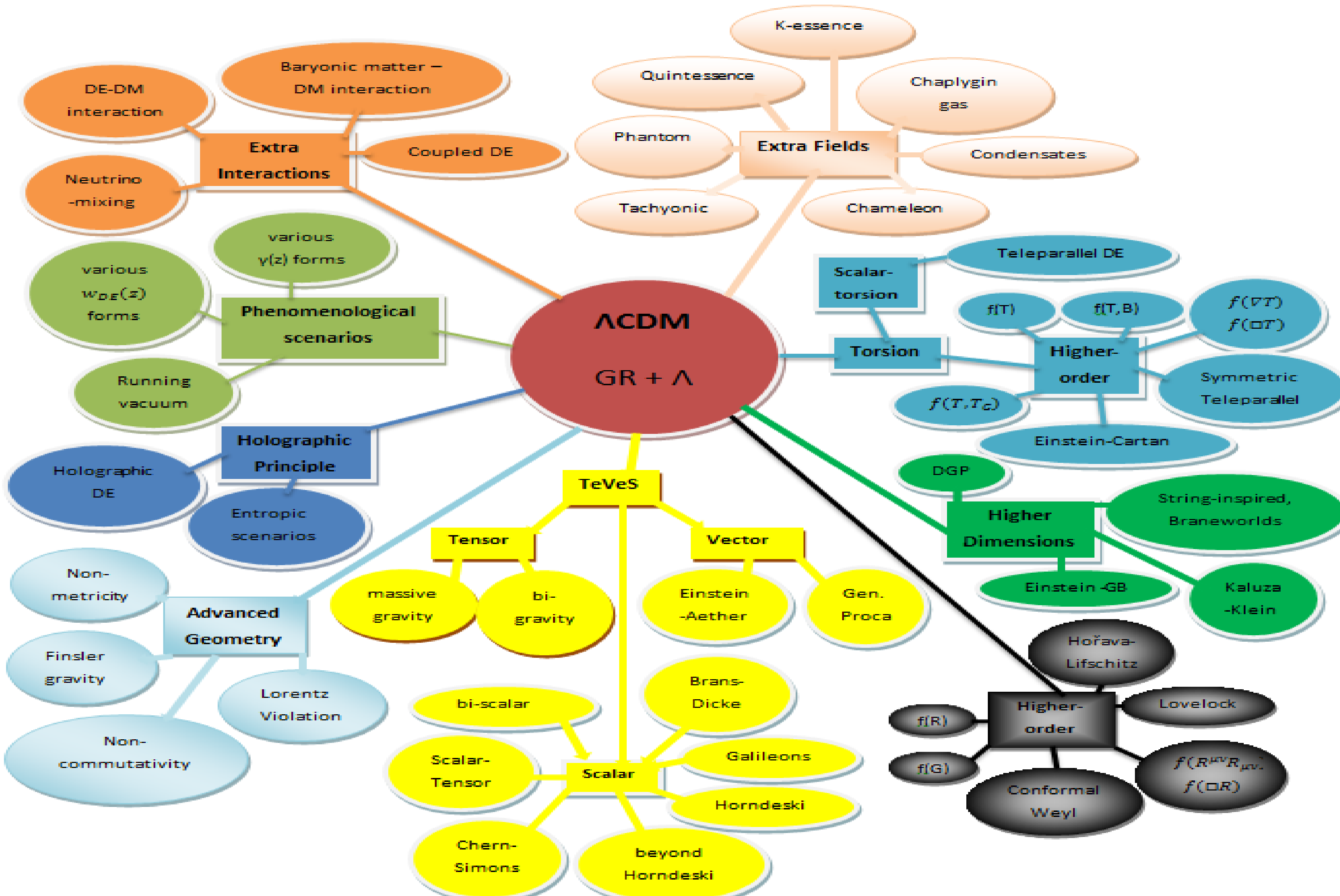
  Worst error in the ~~history of physics~~, ~~history of science~~, history

  2) How to describe primordial universe (inflation)

  3) Tensions with some data sets, e.g. H0 and fσ8 data

General Relativity is not renormalizable/quantizable

Modified Gravity



Torsional Gravity

- Einstein 1916: **General Relativity**:
  energy-momentum source of spacetime Curvature
  Levi-Civita connection: Zero Torsion

- Einstein 1928: **Teleparallel Equivalent of GR:**
  Weitzenbock connection: Zero Curvature

[Cai, Capozziello, De Laurentis, Saridakis, Rept.Prog.Phys. 79]

Teleparallel Equivalent of General Relativity (TEGR)

- Let's start from the simplest tosion-based gravity formulation, namely TEGR:
- Vierbeins e_A^μ : four linearly independent fields in the tangent space

$$g_{\mu\nu}(x) = \eta_{AB}\ e_\mu^A(x)\ e_\nu^B(x)$$

- Use curvature-less Weitzenböck connection instead of torsion-less Levi-Civita one: $\Gamma_{\nu\mu}^{\lambda\{W\}} = e_A^\lambda \partial_\mu e_\nu^A$
- Torsion tensor:

$$T_{\mu\nu}^\lambda = \Gamma_{\nu\mu}^{\lambda\{W\}} - \Gamma_{\mu\nu}^{\lambda\{W\}} = e_A^\lambda\left(\partial_\mu e_\nu^A - \partial_\nu e_\mu^A\right)$$

[Einstein 1928], [Pereira: Introduction to TG]

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- Lagrangian (imposing coordinate, Lorentz, parity invariance, and up to 2nd order in torsion tensor)

$$L \equiv T = \frac{1}{4} T^{\rho\mu\nu} T_{\rho\mu\nu} + \frac{1}{2} T^{\rho\mu\nu} T_{\nu\mu\rho} - T_{\rho\mu}^{\rho} T_{\nu}^{\nu\mu}$$

- **Completely equivalent** with GR at the level of equations

[Einstein 1928], [Hayaski,Shirafuji PRD 19], [Pereira: Introduction to TG]

f(T) Gravity and f(T) Cosmology

- f(T) Gravity: Simplest torsion-based modified gravity
- Generalize T to f(T)   (inspired by f(R))

$$S = \frac{1}{16\pi G}\int d^4x\, e\, \left[T + f(T)\right] + S_m$$

[Ferraro, Fiorini PRD 78], [Bengochea, Ferraro PRD 79]
[Linder PRD 82]

- Equations of motion:

$$e^{-1}\partial_\mu\left(e e_A^\rho S_\rho^{\mu\nu}\right)\left(1+f_T\right) - e_A^\lambda T^\rho_{\mu\lambda} S_\rho^{\nu\mu} + e_A^\rho S_\rho^{\mu\nu}\partial_\mu(T) f_{TT} - \frac{1}{4}e_A^\nu\left[T + f(T)\right] = 4\pi G e_A^\rho \mathrm{T}_\rho^{\nu\{\mathrm{EM}\}}$$

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- f(T) Cosmology: Apply in FRW geometry:

$$e_\mu^A = diag(1, a, a, a) \Rightarrow ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j$$ (not unique choice)

- Friedmann equations:

$$H^2 = \frac{8\pi G}{3}\rho_m - \frac{f(T)}{6} - 2 f_T H^2$$

$$\dot{H} = -\frac{4\pi G(\rho_m + p_m)}{1 + f_T - 12H^2 f_{TT}}$$

- Find easily

$$T = -6H^2$$

f(T) Cosmology: Background

- Effective Dark Energy sector:

$$\rho_{DE} = \frac{3}{8\pi G}\left[-\frac{f}{6} + \frac{T}{3} f_T\right]$$

$$w_{DE} = -\frac{f - Tf_T + 2T^2 f_{TT}}{[1 + f_T + 2Tf_{TT}][f - 2Tf_T]}$$

[Linder PRD 82]

- Interesting cosmological behavior: Acceleration, Inflation etc

Non-minimally coupled scalar-torsion theory

- In curvature-based gravity, apart from $R+f(R)$ one can use $R+\xi R\varphi$^2
- Let's do the same in torsion-based gravity:

$$S=\int d^4x\, e\left[\frac{T}{2\kappa^2}+\frac{1}{2}\left(\partial_\mu\varphi\partial^\mu\varphi+\xi T\varphi^2\right)-V(\varphi)+L_m\right]$$

[Geng, Lee, Saridakis, Wu PLB 704]

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[Geng, Lee, Saridakis, Wu PLB 704]

- Friedmann equations in FRW universe:

$$H^2=\frac{\kappa^2}{3}(\rho_m+\rho_{DE})$$

$$\dot{H}=-\frac{\kappa^2}{2}(\rho_m+p_m+\rho_{DE}+p_{DE})$$

with effective Dark Energy sector:

$$\rho_{DE}=\frac{\dot{\varphi}^2}{2}+V(\varphi)-3\xi H^2\varphi^2$$

$$p_{DE}=\frac{\dot{\varphi}^2}{2}-V(\varphi)+4\xi H\varphi\dot{\varphi}+\xi\left(3H^2+2\dot{H}\right)\varphi^2$$

- **Different** than non-minimal quintessence!

  (no conformal transformation in the present case)

[Geng, Lee, Saridakis,Wu PLB 704]

[Hohmann, Pfeifer, PRD 98]

Teleparallel Equivalent of Gauss-Bonnet and f(T,T_G) gravity

- In curvature-based gravity, one can use higher-order invariants like the Gauss-Bonnet one $G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\kappa\lambda}R^{\mu\nu\kappa\lambda}$
- Let's do the same in torsion-based gravity:
- Similar to $e\overline{R} = -eT + 2\left(eT_{\nu}^{\nu\mu}\right)_{,\mu}$ we construct $e\overline{G} = eT_G + tot.diverg$ with

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$$T_G = \left(K_{ea_2}^{a_1} K_b^{ea_2} K_{fc}^{a_3} K_d^{fa_4} - 2K_a^{a_1a_2} K_{eb}^{a_3} K_{fc}^{e} K_d^{fa_4} + 2K_a^{a_1a_2} K_{eb}^{a_3} K_f^{ea_4} K_{cd}^{f} + +2K_a^{a_1a_2} K_{eb}^{a_3} K_f^{ea_4} K_{c,d}^{f}\right)\delta_{a_1a_2a_3a_4}^{abcd}$$

- $f(\mathrm{T}, T_G)$ gravity:

$$S = \frac{1}{2\kappa^2}\int d^4x\, e\, \{T + f(T, T_G)\} + S_m$$

[Kofinas, Saridakis, PRD 90a]

[Kofinas, Saridakis, PRD 90b]

[Kofinas, Leon, Saridakis, CQG 31]

- **Different** from $f(R,G)$ and $f(T)$ gravities

Teleparallel Equivalent of Gauss-Bonnet and f(T,T_G) gravity

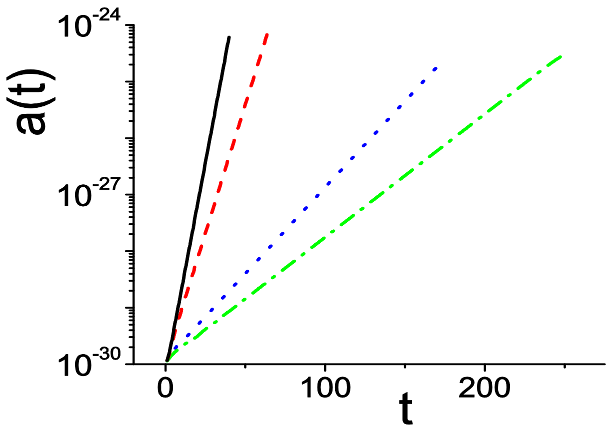
- Cosmological application:

$$\rho_{DE} = -\frac{1}{2\kappa^2}\left[f - 12H^2 f_T - T_G f_{T_G} + 24H^3 \dot{f}_{T_G}\right]$$

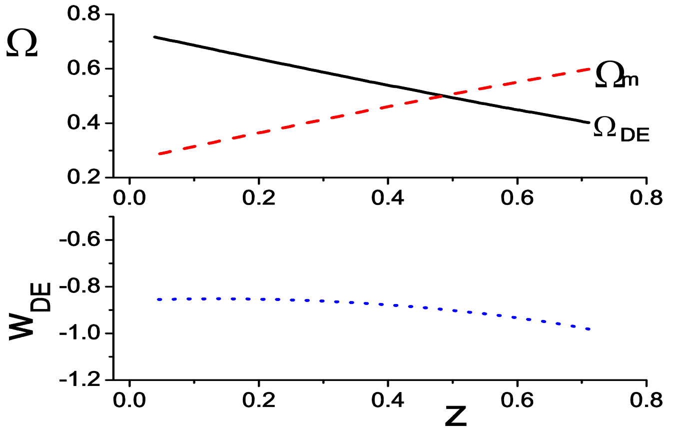
$$p_{DE} = \frac{1}{2\kappa^2}\left[f - 4\left(\dot{H} + 3H^2\right)f_T - 4H\dot{f}_T - T_G f_{T_G} + \frac{2}{3H} T_G \dot{f}_{T_G} + 8H^2 \ddot{f}_{T_G}\right]$$

$$T = 6H^2$$

$$T_G = 24H^2\left(\dot{H} + H^2\right)$$



$$f(T,T_G) = \alpha_1 T^2 + \alpha_2 T\sqrt{|T_G|}$$



$$f(T,T_G) = \beta_1\sqrt{T^2 + \beta_2 T_G}$$

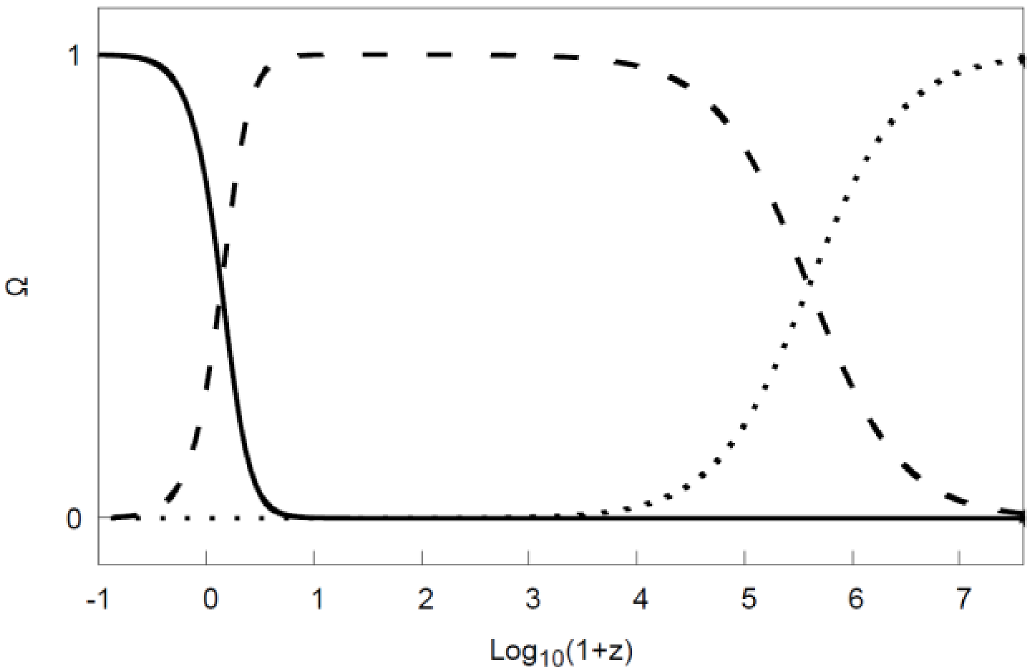
[Kofinas, Saridakis, PRD 90a]

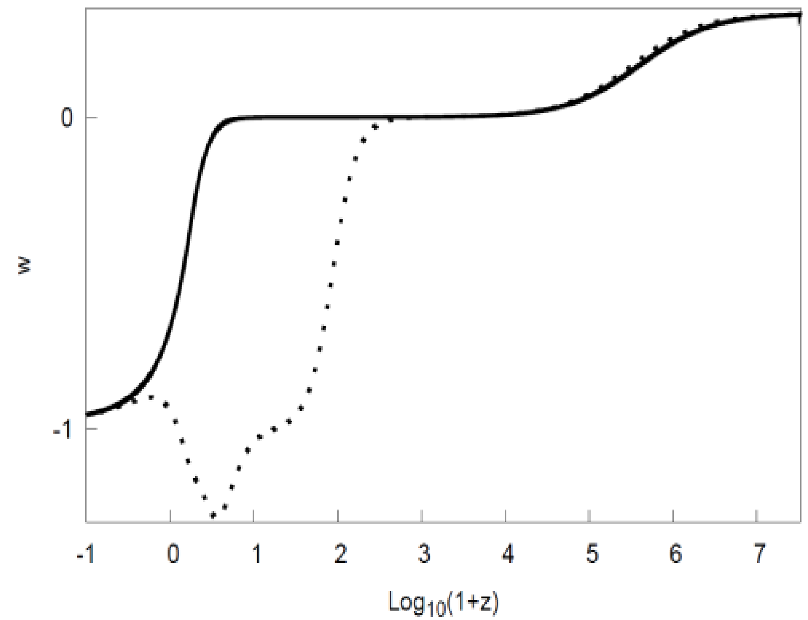
[Kofinas, Saridakis, PRD 90b]

[Kofinas, Leon, Saridakis, CQG 31]

Torsional Gravity with higher derivatives

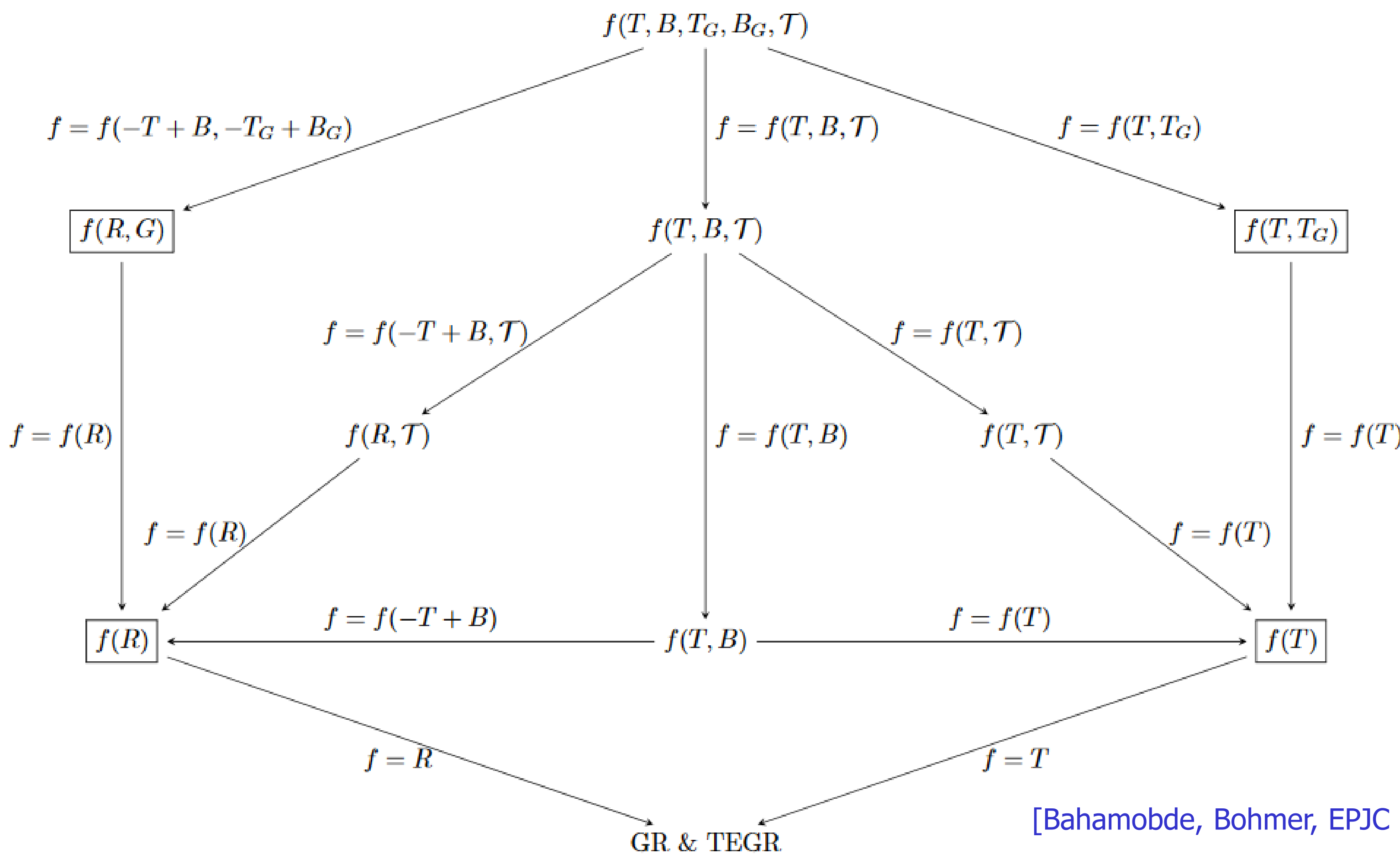
$$S = \frac{1}{2\kappa^2}\int d^4x\, e\, F\left(T,(\nabla T)^2,\Diamond T\right) + S_m(e^A_\mu,\Psi_m)$$





[Otalora, Saridakis, PRD 94]

Torsional Modified Gravity



[Bahamobde, Bohmer, EPJC 76]

Metric-Affine Modified Gravity

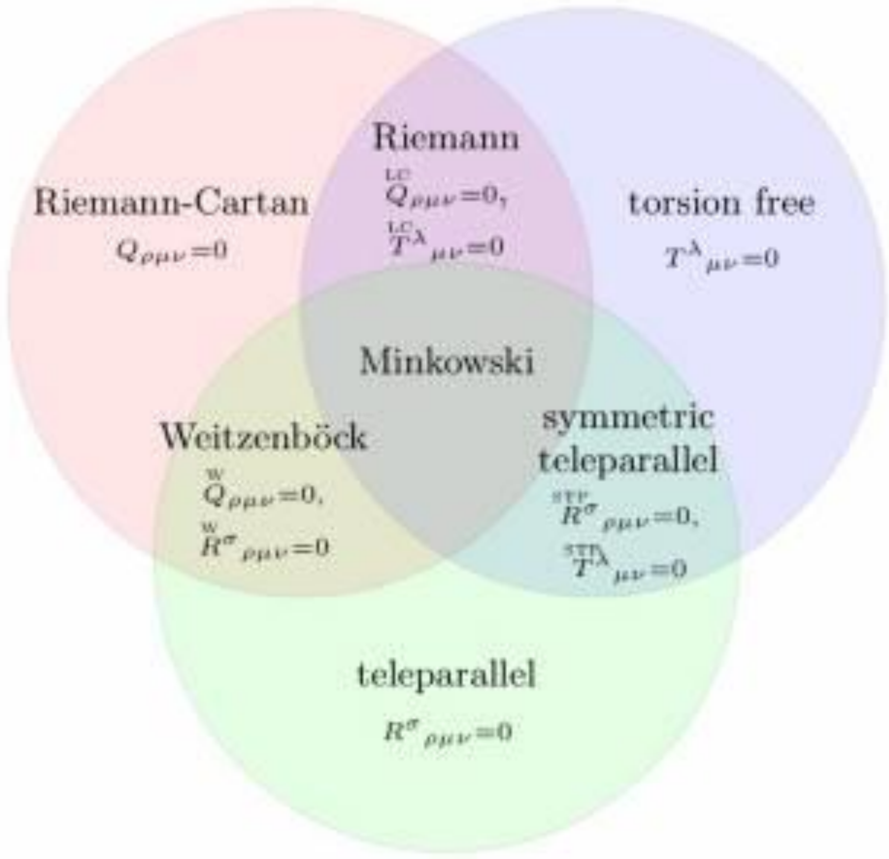


FIG. 1. Subclasses of metric-affine geometry, depending on the properties of connection.

Growth-index constraints on f(T) gravity

- Perturbations: $\ddot{\delta}_m + 2H\dot{\delta}_m = 4\pi G_{eff}\ \rho_m \delta_m$, clustering growth rate: $\frac{d\ \ln \delta_m}{d\ \ln a} = \Omega_m^{\gamma}(a)$
- γ(z): Growth index. $G_{eff} = \frac{1}{1+f'(T)}$

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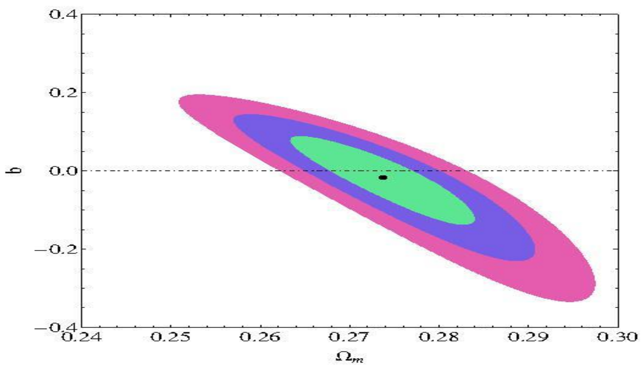
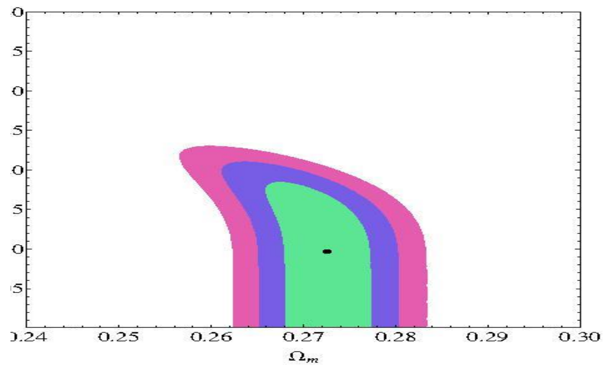
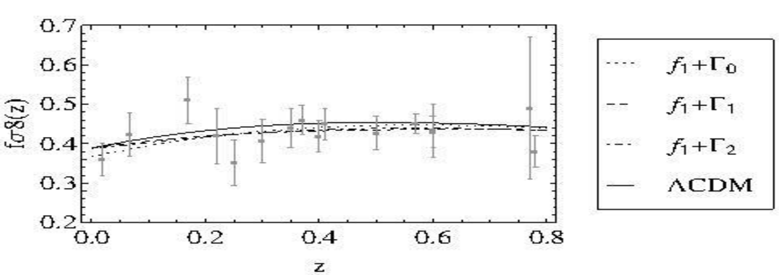
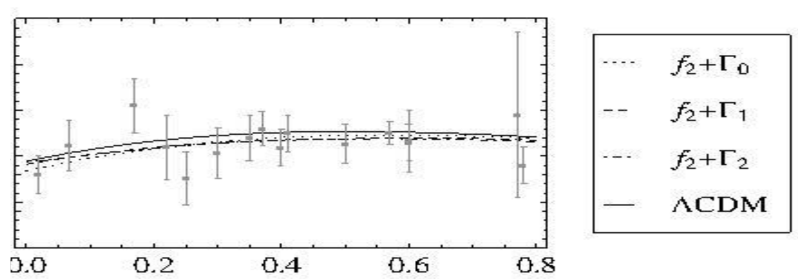






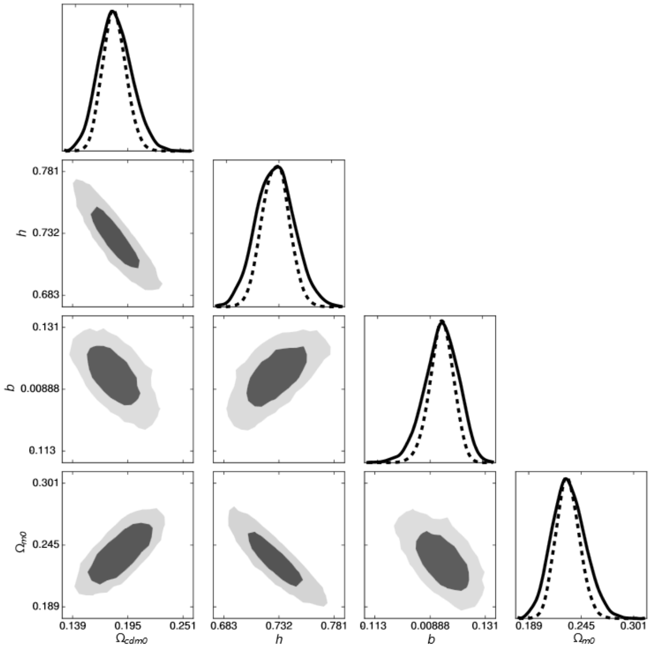


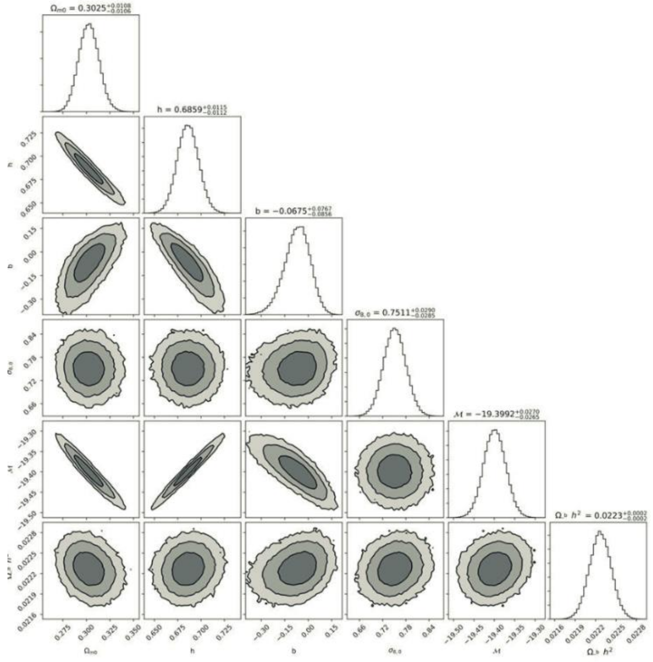
- Viable f(T) models are practically indistinguishable from ΛCDM.

[Nesseris, Basilakos, Saridakis, Perivolaropoulos, PRD 88]

[Nunes, Pan, Saridakis, JCAP 1608]

Observational Constraints on f(T) gravity





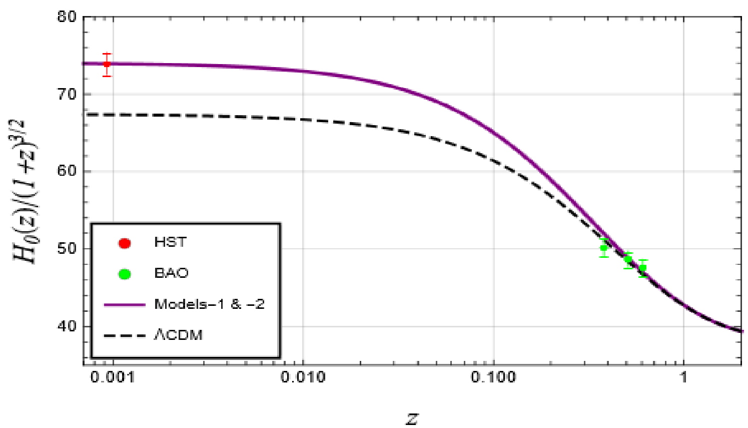
[Anagnostopoulos, Basilakos, Nesseriss, Saridakis JCAP08]

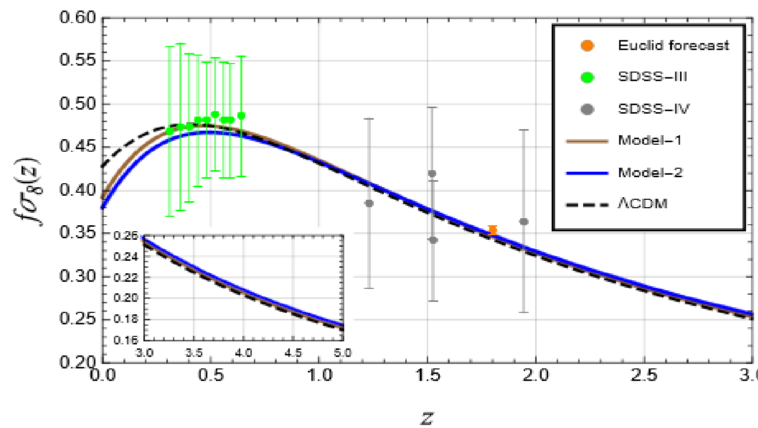
[Nunes, Pan, Saridakis, JCAP08]

[Anagnostopoulos, Basilakos, Saridakis PRD 100]

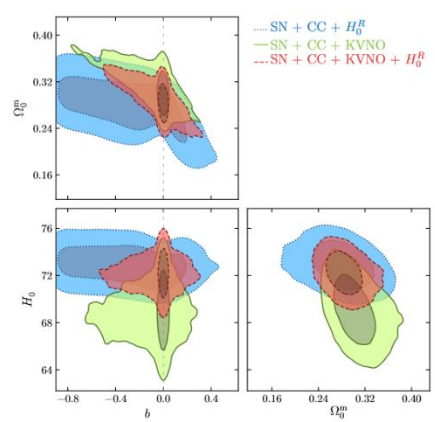
[Nunes, Bonilla, Pan, Saridakis, EPJC77]

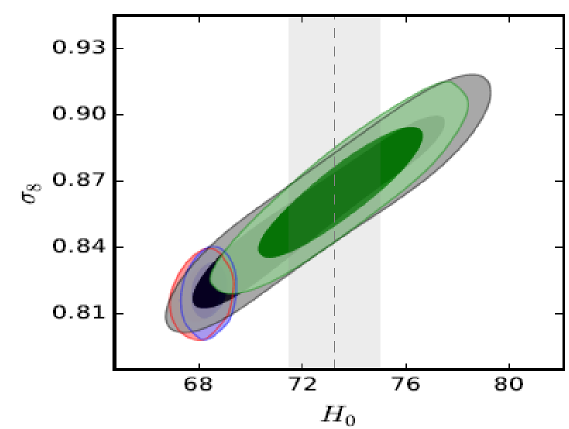
H0 and σ8 tension can be alleviated





Parameter	CMB + BAO	CMB + BAO + H_0
$10^2\omega_b$	$2.235^{+0.013}_{-0.013}$	$2.235^{+0.013}_{-0.013}$
ω_{cdm}	$0.1181^{+0.001}_{-0.001}$	$0.118^{+0.001}_{-0.001}$
$100\theta_s$	$1.041^{+0.00027}_{-0.00027}$	$1.041^{+0.00030}_{-0.00027}$
$\ln 10^{10}A_s$	$3.078^{+0.023}_{-0.023}$	$3.08^{+0.022}_{-0.022}$
n_s	$0.9678^{+0.0039}_{-0.0039}$	$0.9684^{+0.0039}_{-0.0044}$
τ_{reio}	$0.073^{+0.012}_{-0.013}$	$0.075^{+0.012}_{-0.012}$
n	$0.0043^{+0.0033}_{-0.0039}$	$0.0054^{+0.0020}_{-0.0020}$
$\log\alpha$	$10.00^{+0.081}_{-0.12}$	$10.03^{+0.06}_{-0.06}$
Ω_{F0}	$0.73^{+0.021}_{-0.028}$	$0.738^{+0.015}_{-0.015}$
H_0	$72.4^{+3.3}_{-4.1}$	$73.5^{+2.1}_{-2.1}$
σ_8	$0.855^{+0.023}_{-0.033}$	$0.866^{+0.02}_{-0.02}$
$\chi^2_{min}/2$	6480.48	6482.27

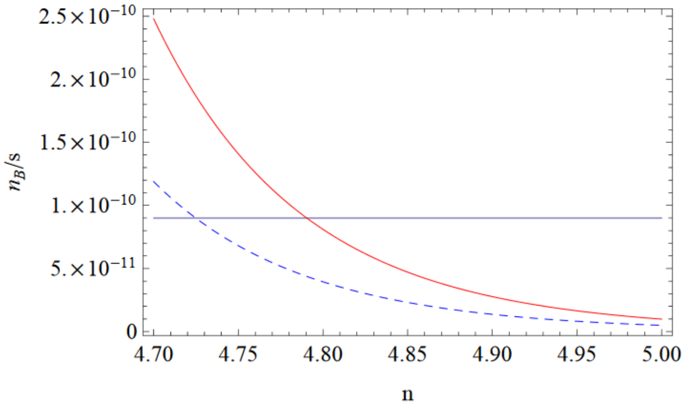




[Yang,Zhang,Chen,Cai,Li,Saridakis,Xue, PRD 101] [Said, Mifsud, Parkinson, Saridakis, Sultana, 2005.05368] [Nunes, JCAP 1805]

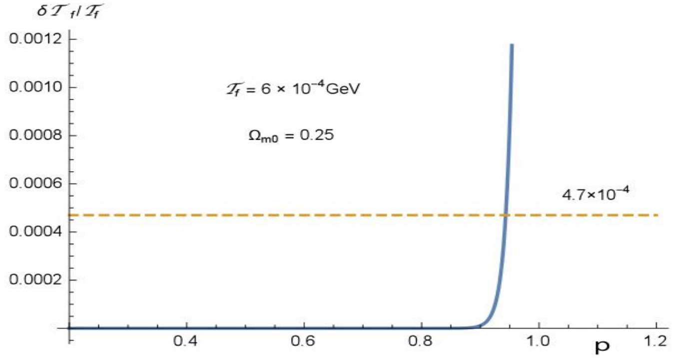
Baryogenesis and BBN constraints on f(T) gravity

- Baryon-anti-baryon asymmetry through CP violating term: $\frac{1}{M_*^2}\int d^4x\, e\left[\partial_\mu f(T)\right]J^\mu$



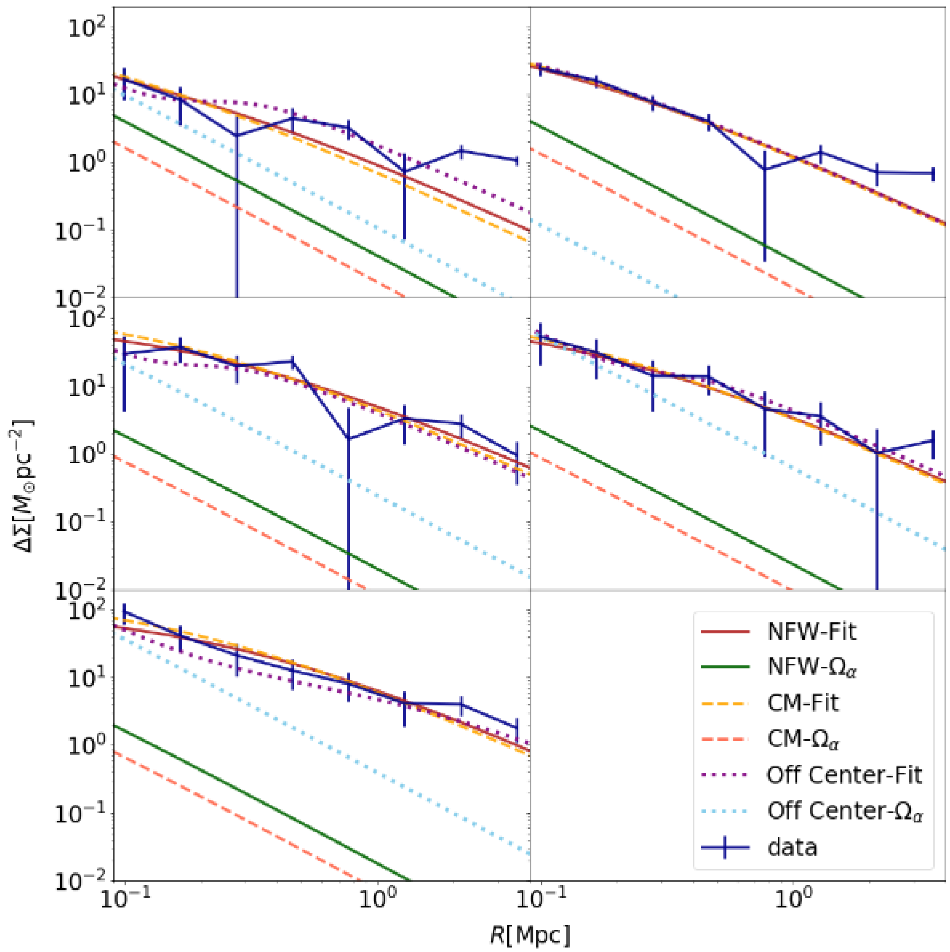
[Oikonomou, Saridakis, PRD 94]

- BBN constraints: $\frac{\delta T_f}{T_f} \approx \frac{\rho_T}{\rho}\frac{H_{GR}}{10qT_f^5}$



[Capozziello, Lambiase, Saridakis, EPJC77]

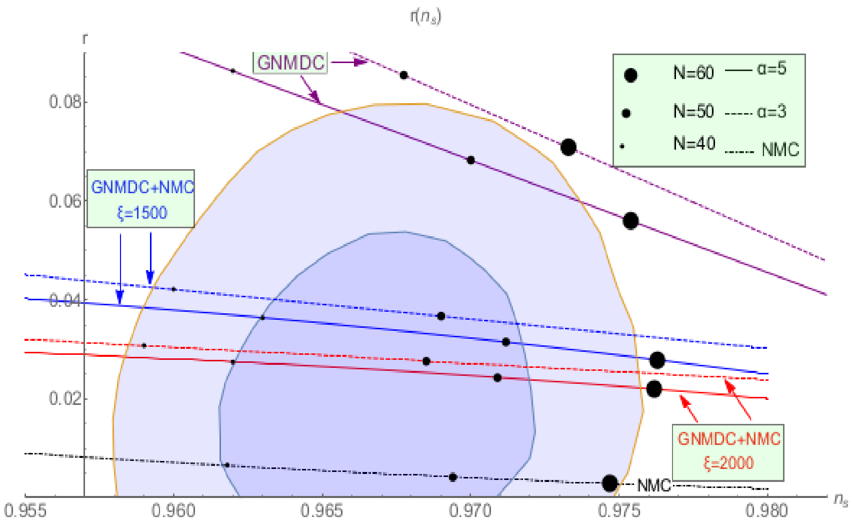
Galaxy-Galaxy lensing constraints on f(T) gravity

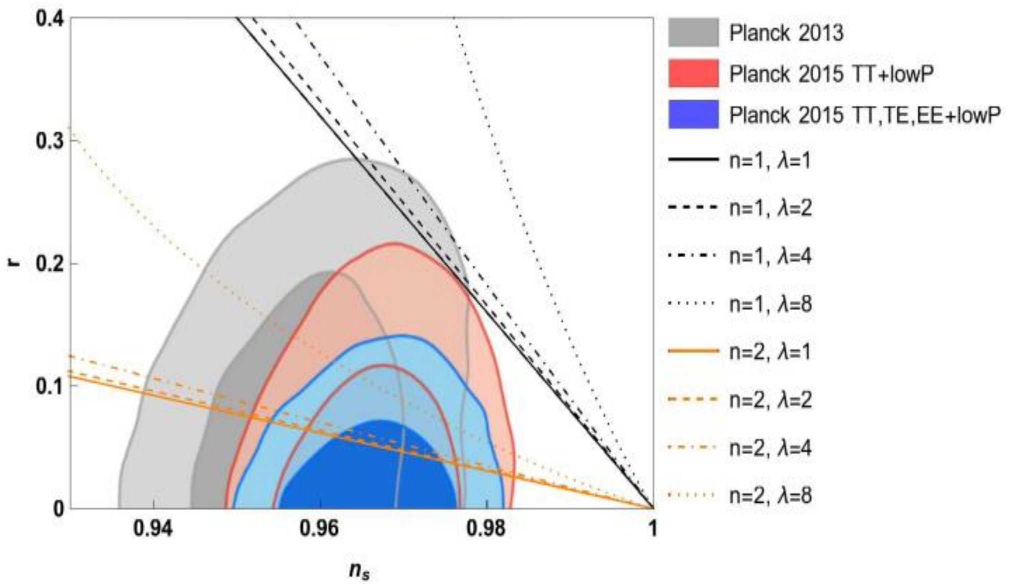


$$f(T) = -T - 2\Lambda/M_P^2 + \alpha T^\beta$$

[Chen. Luo, Cai, Saridakis, PRD 102]

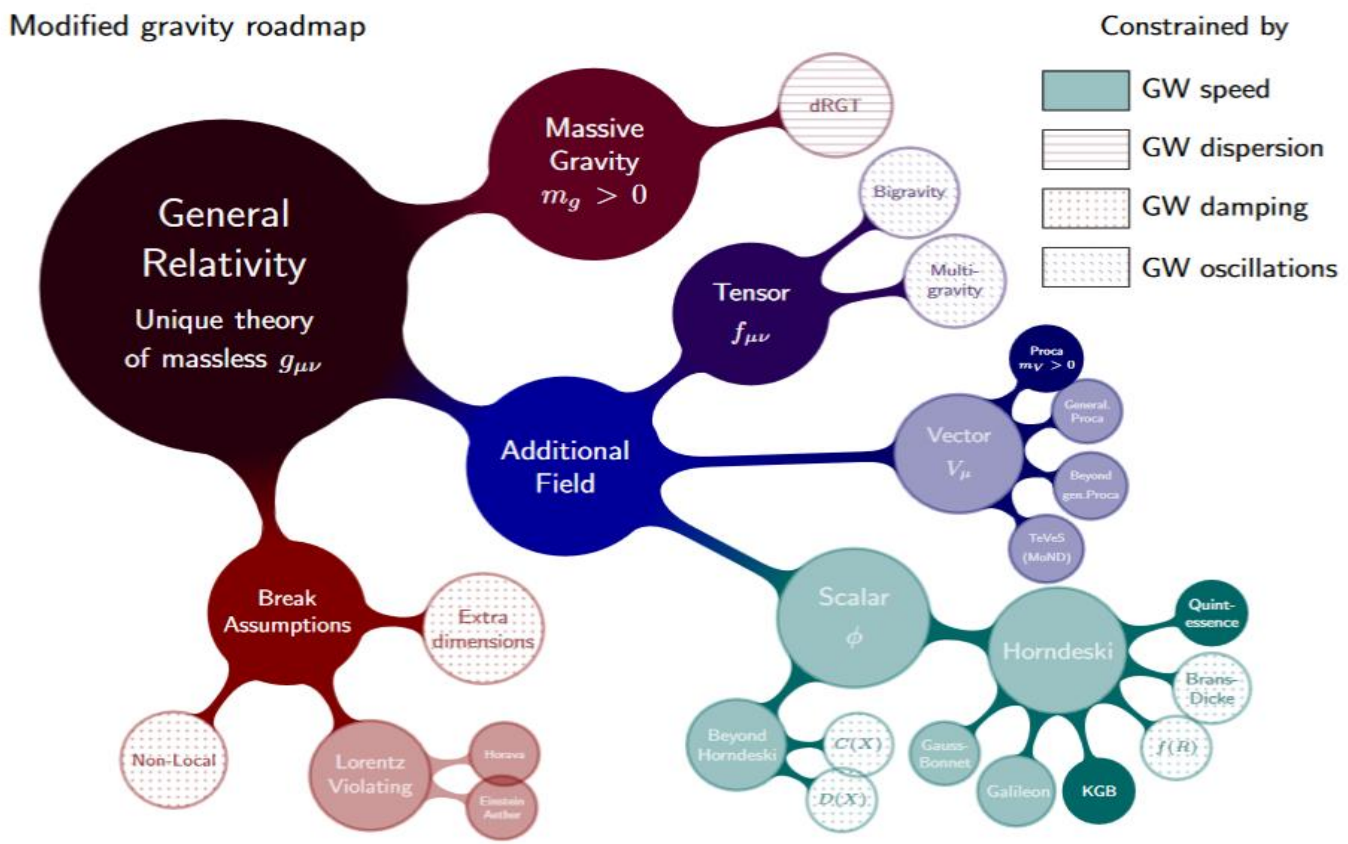
Inflation in f(T) and torsional gravity





- How can we distinguish between modified gravity theories?

Gravitational waves



[Ezquiaga, Zumalacarregui PRL 119]

Gravitational waves

- For tensor perturbations: $$g_{00} = -1\,, \quad g_{0i} = 0\,,$$ $$g_{ij} = a^2\left(\delta_{ij} + h_{ij} + \frac{1}{2}h_{ik}h_{kj}\right)$$

$$\boxed{\ddot{h}_{ij} + (3+\alpha_M)\dot{h}_{ij} + (1+\alpha_T)\frac{k^2}{a^2}h_{ij} = 0}$$

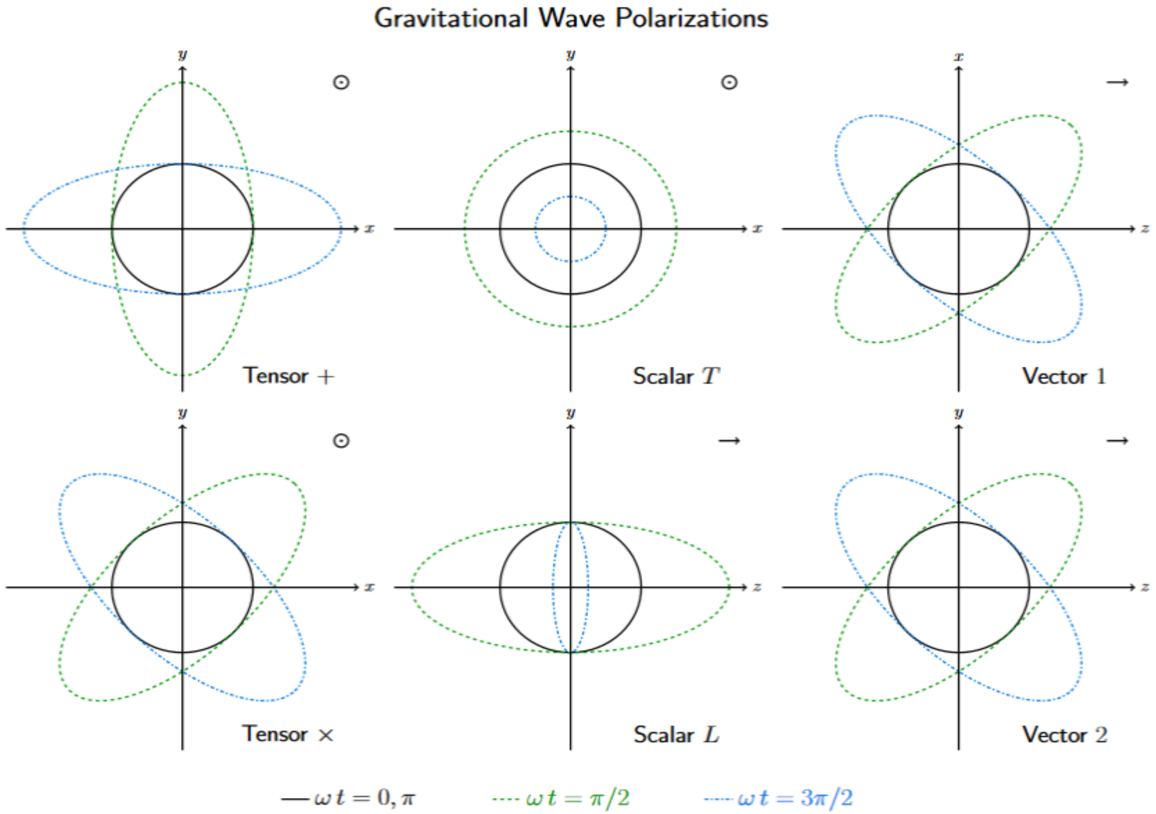
$$\alpha_M = \frac{d\log(M_*^2)}{d\log a} \qquad\qquad c_g^2 = (1+\alpha_T)$$

- $$h_{\rm GW} \sim h_{\rm GR} \quad \underbrace{e^{-\frac{1}{2}\int \nu\mathcal{H}d\eta}}_{\text{Affects amplitude}} \quad \underbrace{e^{ik\int(\alpha_T + a^2m^2/k^2)^{1/2}d\eta}}_{\text{Affects phase}}$$

[Ezquiaga, Zumalacarregui PRL 119]

Gravitational waves

- Polarizations:



[Ezquiaga, Zumalacarregui PRL 119]

The Effective Field Theory (EFT) approach

- The EFT approach allows to ignore the details of the underlying theory and write an action for the perturbations around a time-dependent background solution.
- One can systematically analyze the perturbations separately from the background evolution. [Arkani-Hamed, Cheng JHEP0405 (2004)]

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$$S = \int d^4x \Big\{ \sqrt{-g} \Big[\frac{M_P^2}{2} \Psi(t) R - \Lambda(t) - b(t) g^{00}$$ <- background

$$+ M_2^4 (\delta g^{00})^2 - \bar{m}_1^3 \delta g^{00} \delta K - \bar{M}_2^2 \delta K^2 - \bar{M}_3^2 \delta K_\mu^\nu \delta K_\nu^\mu$$ <- linear evolution of perturbations

$$+ m_2^2 h^{\mu\nu} \partial_\mu g^{00} \partial_\nu g^{00} + \lambda_1 \delta R^2 + \lambda_2 \delta R_{\mu\nu} \delta R^{\mu\nu} + \mu_1^2 \delta g^{00} \delta R \Big]$$ <- linear evolution of perturbations

$$+ \gamma_1 C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma} + \gamma_2 \epsilon^{\mu\nu\rho\sigma} C_{\mu\nu}{}^{\kappa\lambda} C_{\rho\sigma\kappa\lambda}$$ <- linear evolution of perturbations

$$+ \sqrt{-g} \Big[\frac{M_3^4}{3} (\delta g^{00})^3 - \bar{m}_2^3 (\delta g^{00})^2 \delta K + \ldots \Big] \Big\} ,$$ <- 2^{nd}-order evolution of perturbations

The functions Ψ(t), Λ(t), b(t), are determined by the background solution

[Gubitosi, Piazza, Vernizzi, JCAP1302]

The (EFT) approach to torsional gravity

- Application of the EFT approach to torsional gravity leads to include terms:
- i) Invariant under 4D diffeomorphisms: e.g. R,T multiplied by functions of time.
- ii) Invariant under spatial diffeomorphisms: e.g. $g^{00},\ R^{00}$ and T^0
- ii) Invariant under spatial diffeomorphisms: e.g. ${}^{(3)}R_{\mu\nu\rho\sigma}$, ${}^{(3)}T^{\rho}{}_{\mu\nu}$, $K_{\mu\nu}$, $\hat{K}_{\mu\nu}$

  the extrinsic torsion is defined as

$$\hat{K}_{\mu\nu} \equiv h^{\sigma}_{\mu}\hat{\nabla}_{\sigma}n_{\nu} = K_{\mu\nu} - \mathcal{K}^{\lambda}{}_{\nu\mu}n_{\lambda} + n_{\mu}\frac{1}{g^{00}}T^{00}{}_{\nu}\ ;$$

with n_{μ} the orthogonal to t=cont. surfaces unitary vector $n_{\mu} = \frac{\delta^0_{\mu}}{\sqrt{-g^{00}}}$

[Cai, Li, Saridakis, Xue, PRD 97], [Li, Cai, Cai, Saridakis, JCAP18]

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- Application of the EFT approach to torsional gravity leads to include terms:
- i) Invariant under 4D diffeomorphisms: e.g. R,T multiplied by functions of time.
- ii) Invariant under spatial diffeomorphisms: e.g. $g^{00},\ R^{00}$ and T^0
- ii) Invariant under spatial diffeomorphisms: e.g. ${}^{(3)}R_{\mu\nu\rho\sigma}$, ${}^{(3)}T^{\rho}{}_{\mu\nu}$, $K_{\mu\nu}$, $\hat{K}_{\mu\nu}$

  the extrinsic torsion is defined as

$$\hat{K}_{\mu\nu} \equiv h^{\sigma}_{\mu}\hat{\nabla}_{\sigma}n_{\nu} = K_{\mu\nu} - \mathcal{K}^{\lambda}{}_{\nu\mu}n_{\lambda} + n_{\mu}\frac{1}{g^{00}}T^{00}{}_{\nu}$$

  with n_{μ} the orthogonal to t=cont. surfaces unitary vector $n_{\mu} = \frac{\delta^0_{\mu}}{\sqrt{-g^{00}}}$

Using the projection operator h^{μ}_{ν} we can express ${}^{(3)}R_{\mu\nu\rho\sigma} = h^{\alpha}_{\mu}h^{\beta}_{\nu}h^{\gamma}_{\rho}h^{\delta}_{\sigma}R_{\alpha\beta\gamma\delta} - K_{\mu\rho}K_{\nu\sigma} + K_{\nu\rho}K_{\mu\sigma}$

$$h^{d}_{a}h^{c}_{b}h^{f}_{e}T^{e}{}_{dc} = {}^{(3)}T^{f}{}_{ab}$$

[Cai, Li, Saridakis, Xue, PRD 97], [Li, Cai, Cai, Saridakis, JCAP18]

The (EFT) approach to torsional gravity

- We perturb the previous tensors, and we finally obtain:

$$
\begin{aligned}
R^{(0)}_{\mu\nu\rho\sigma} &= f_1(t)g_{\mu\rho}g_{\nu\sigma} + f_2(t)g_{\mu\rho}n_\nu n_\sigma + f_3(t)g_{\mu\sigma}g_{\nu\rho} \\
&\quad + f_4(t)g_{\mu\sigma}n_\nu n_\rho + f_5(t)g_{\nu\sigma}n_\mu n_\rho \\
&\quad + f_6(t)g_{\nu\rho}n_\mu n_\sigma , \\
T^{(0)}_{\rho\mu\nu} &= g_1(t)g_{\rho\nu}n_\mu + g_2(t)g_{\rho\mu}n_\nu , \\
K^{(0)}_{\mu\nu} &= f_7(t)g_{\mu\nu} + f_8(t)n_\mu n_\nu , \\
\hat{K}^{(0)}_{\mu\nu} &= 0 \, .
\end{aligned}
$$

where the time-dependent functions are determined by the background solution.

[Cai, Li, Saridakis, Xue, PRD 97], [Li, Cai, Cai, Saridakis, JCAP18]

The (EFT) approach to torsional gravity

- Finally, the EFT action of torsional gravity becomes:

$$S=\int d^4x\sqrt{-g}\Big[\frac{M_P^2}{2}\Psi(t)R-\Lambda(t)-b(t)g^{00}+\frac{M_P^2}{2}d(t)T^0\Big]$$
$$+S^{(2)}\ ,$$

- The perturbation part contains:

  i) Terms present in curvature EFT action

  ii) Pure torsion terms such as δT^2 , $\delta T^0\delta T^0$ and $\delta T^{\rho\mu\nu}\delta T_{\rho\mu\nu}$

  iii) Terms that mix curvature and torsion, such as $\delta T\delta R$, $\delta g^{00}\delta T$, $\delta g^{00}\delta T^0$ and $\delta K\delta T^0$

[Cai, Li, Saridakis, Xue, PRD 97], [Li, Cai, Cai, Saridakis, JCAP18]

The (EFT) approach to f(T) gravity: Tensor Perturbations

- For tensor perturbations: $g_{00}=-1\ ,\quad g_{0i}=0\ ,$

$$g_{ij}=a^2\big(\delta_{ij}+h_{ij}+\frac{1}{2}h_{ik}h_{kj}\big)$$

i.e.

$$\bar{e}^0_\mu=\delta^0_\mu\ ,$$
$$\bar{e}^a_\mu=a\delta^a_\mu+\frac{a}{2}\delta^i_\mu\delta^{aj}h_{ij}+\frac{a}{8}\delta^i_\mu\delta^{ja}h_{ik}h_{kj}\ ,$$
$$\bar{e}^\mu_0=\delta^\mu_0\ ,$$
$$\bar{e}^\mu_a=\frac{1}{a}\delta^\mu_a-\frac{1}{2a}\delta^{\mu i}\delta^j_a h_{ij}+\frac{1}{8a}\delta^{i\mu}\delta^j_a h_{ik}h_{kj}$$

- We obtain:

$$^{(3)}R\approx-\frac{1}{4}a^{-2}(\partial_i h_{kl}\partial_i h_{kl})\ ,$$
$$K^{ij}K_{ij}\approx 3H^2+\frac{1}{4}\dot{h}_{ij}\dot{h}_{ij}\ ,$$
$$K\approx 3H\ ,$$
$$\boxed{T=T^{(0)}+O(h^2)=6H^2+O(h^2)}$$

- And finally:

$$S=\frac{M_P^2}{2}\int d^4x\sqrt{-g}\Big[\frac{f_T}{4}\big(a^{-2}\vec{\nabla}h_{ij}\cdot\vec{\nabla}h_{ij}-\dot{h}_{ij}\dot{h}_{ij}\big)+6H^2f_T-12H\dot{f}_T-T^{(0)}f_T+f(T^{(0)})\Big]$$

[Cai, Li, Saridakis, Xue, PRD 97]

The (EFT) approach to f(T) gravity: Scalar Perturbations

- For scalar perturbations:

$$g_{00} = -1 - 2\phi \ , \quad g_{0i} = 0 \ , \quad g_{ij} = a^2[(1-2\psi)\delta_{ij} + \partial_i\partial_j F]$$

i.e

$$e^0_\mu = \delta^0_\mu + \delta^0_\mu \phi + a\delta^i_\mu \partial_i \chi \ ,$$
$$e^a_\mu = a\delta^i_\mu \delta^a_i + \delta^0_\mu \delta^a_i \partial^i \mathcal{E} + a\delta^i_\mu \delta^a_j \left[\epsilon_{ijk}\partial_k \sigma - \psi\delta_{ij} + \frac{1}{2}\partial_i\partial_j F\right]$$

- So

$$T^0 = g^{0\mu} T^\nu{}_{\mu\nu} = -3H + 6H\phi + 3\dot{\psi} - 6H\phi^2 - 6\dot{\psi}\phi + \frac{1}{a}\partial_i\partial_i\chi - \frac{1}{2a}\partial_i\phi\partial_i\chi - \frac{3}{2a}\phi\partial_i\partial_i\chi - \frac{1}{2a}\partial_i\psi\partial_i\chi + \frac{1}{2a}\psi\partial_i\partial_i\chi$$

- Thus:

$$S = \int d^4x \Big[\frac{M_P^2}{2}\Big(-2af_T\partial_i\psi\partial_i\psi + 4af_T\partial_i\phi\partial_i\psi + 4a^2\dot{f}_T\partial_i\psi\partial_i\chi + 4\dot{f}_T a^2 H\partial_i\pi\partial_i\chi\Big) + a^3M^2\pi^2 - a^3\phi\delta\rho_m\Big]$$

[Li, Cai, Cai, Saridakis, JCAP18]

Gravitational waves in f(T) gravity

- Varying the action and going to Fourier space we get the equation for GWs:

$$\ddot{h}_{ij} + 3H(1-\beta_T)\dot{h}_{ij} + \frac{k^2}{a^2}h_{ij} = 0$$

with $\beta_T \equiv -\frac{\dot{f}_T}{3Hf_T}$

$$h_{\mu\nu}^{(1)} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 2\gamma_1^{(1)1} & {B_1}^2 \exp(ip_\mu x^\mu) & 0 \\ 0 & {B_1}^2 \exp(ip_\mu x^\mu) & -2\gamma_1^{(1)1} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

- An immediate result: The speed of GWs is equal to the speed of light!
- GW170817 constraints that

$$|c_g/c - 1| \leq 4.5 \times 10^{-16}$$

are trivially satisfied.

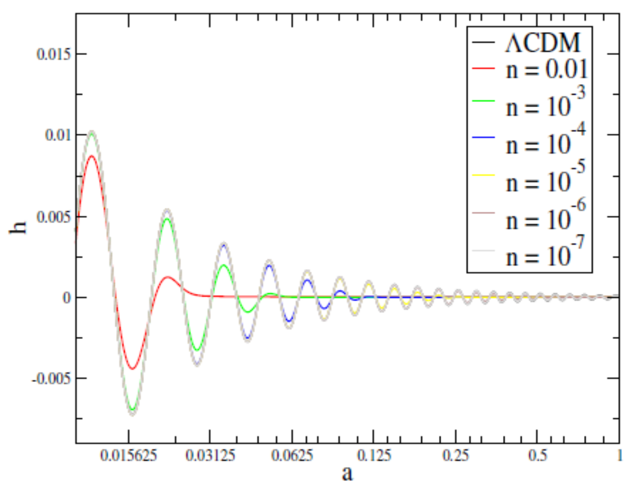
[Cai, Li, Saridakis, Xue, PRD 97]

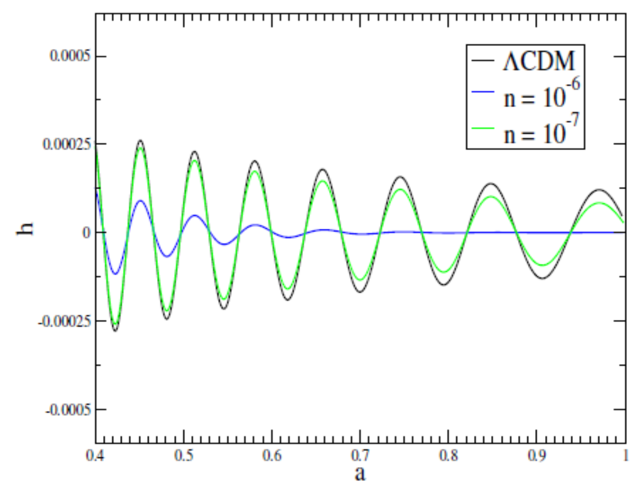
Gravitational waves in f(T) gravity

- Gw's propagation at cosmological scales: $h = e^{-\mathcal{D}} e^{-ik\Delta T} h_{GR}$

$\mathcal{D} = \frac{1}{2}\int \nu \mathcal{H} d\tau'$ (affects amplitude) $\Delta T = \int \left(1 - c_T - \frac{a^2\mu^2}{2k^2}\right) d\tau'$ (affects phase)

- In f(T) gravity:





[Cai, Li, Saridakis, Xue PRD 97]

[Farrugia, Said, Gakis, Saridakis, PRD 97]

[Soudi, Farrugia, Gakis, Said, Saridakis, PRD 100]

[Nunes, Pan, Saridakis, PRD98]

Gravitational Waves in Modified Teleparallel Theories

- $$S = \frac{1}{16\pi G}\int d^4x\, e\, f(T,B) + \int d^4x\, e\, \mathcal{L}_m$$ $$R = -T - 2\nabla^\mu T^\nu{}_{\mu\nu}$$

$$\begin{aligned} &- f_T G_{\mu\nu} + \left(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu\right) f_B \\ &+ \frac{1}{2} g_{\mu\nu}\left(f_B B + f_T T - f\right) \\ &+ 2S_\nu{}^\alpha{}_\mu \partial_\alpha \left(f_T + f_B\right) = 8\pi G\, \Theta_{\mu\nu} \end{aligned}$$

- $$g_{\mu\nu} = \eta_{\mu\nu} + h^{(1)}_{\mu\nu} + \mathcal{O}\left(h^{(2)}_{\mu\nu}\right)$$

$$h^{(1)}_{\mu\nu} = \begin{pmatrix} -2A\exp(ik_\mu x^\mu) - \frac{f^{(0)}_{BB}B^{(1)}}{f^{(0)}_T} & B_1\exp(ik_\mu x^\mu) & B_2\exp(ik_\mu x^\mu) & -2A\exp(ik_\mu x^\mu) \\ B_1\exp(ik_\mu x^\mu) & h_+ + \frac{f^{(0)}_{BB}B^{(1)}}{f^{(0)}_T} & h_\times & B_1\exp(ik_\mu x^\mu) \\ B_2\exp(ik_\mu x^\mu) & h_\times & -h_+ + \frac{f^{(0)}_{BB}B^{(1)}}{f^{(0)}_T} & B_2\exp(ik_\mu x^\mu) \\ -2A\exp(ik_\mu x^\mu) & B_1\exp(ik_\mu x^\mu) & B_2\exp(ik_\mu x^\mu) & -2A\exp(ik_\mu x^\mu) + \frac{f^{(0)}_{BB}B^{(1)}}{f^{(0)}_T} \end{pmatrix}$$

$$\begin{aligned} B^{(1)} &= -2\left(\nabla^\mu T^\nu{}_{\mu\nu}\right)^{(1)} = -2\eta^{\mu\rho}\partial_\rho T^{(1)\nu}{}_{\mu\nu} \\ &= 2\delta^\rho_b\left(\eta^{\mu\nu}\partial_\nu\partial_\rho\gamma^{(1)b}_\mu - \Box\gamma^{(1)b}_\rho\right) \end{aligned}$$

$$R^{(1)} = 2\delta^\rho_b\left(\eta^{\mu\nu}\partial_\nu\partial_\rho\gamma^{(1)b}_\mu - \Box\gamma^{(1)b}_\rho\right)$$

Hence, no further polarization modes in f(T), but further polarization modes in f(T,B) gravity!

[Farrugia, Said, Gakis, Saridakis, PRD 97]

Gravitational Waves in f(T,B) gravity

$$[\delta e^A{}_\mu] := \begin{bmatrix} \varphi & a\left(\partial_i\beta + \beta_i\right) \\ \delta^I{}_i\left(\partial^i b + b^i\right) & a\delta^{Ii}\left(-\psi\delta_{ij} + \partial_i\partial_j h + 2\partial_{(i}h_{j)} + \frac{1}{2}h_{ij} + \epsilon_{ijk}\left(\partial^k\sigma + \sigma^k\right)\right) \end{bmatrix}$$

$$[\delta g_{\mu\nu}] = \begin{bmatrix} -2\varphi & a\left(\partial_i(b-\beta) + (b_i - \beta_i)\right) \\ a\left(\partial_i\left(b-\beta\right) + \left(b_i - \beta_i\right)\right) & 2a^2\left(-\psi\delta_{ij} + \partial_i\partial_j h + 2\partial_{(i}h_{j)} + \frac{1}{2}h_{ij}\right) \end{bmatrix}$$

We get:

$$\ddot{h}_{ij} + (3+\nu)H\dot{h}_{ij} + \frac{k^2}{a^2}h_{ij} = 0$$

with $\nu = \frac{1}{H}\frac{\dot{f}_T}{f_T}$ $\qquad c_T^2 = 1$

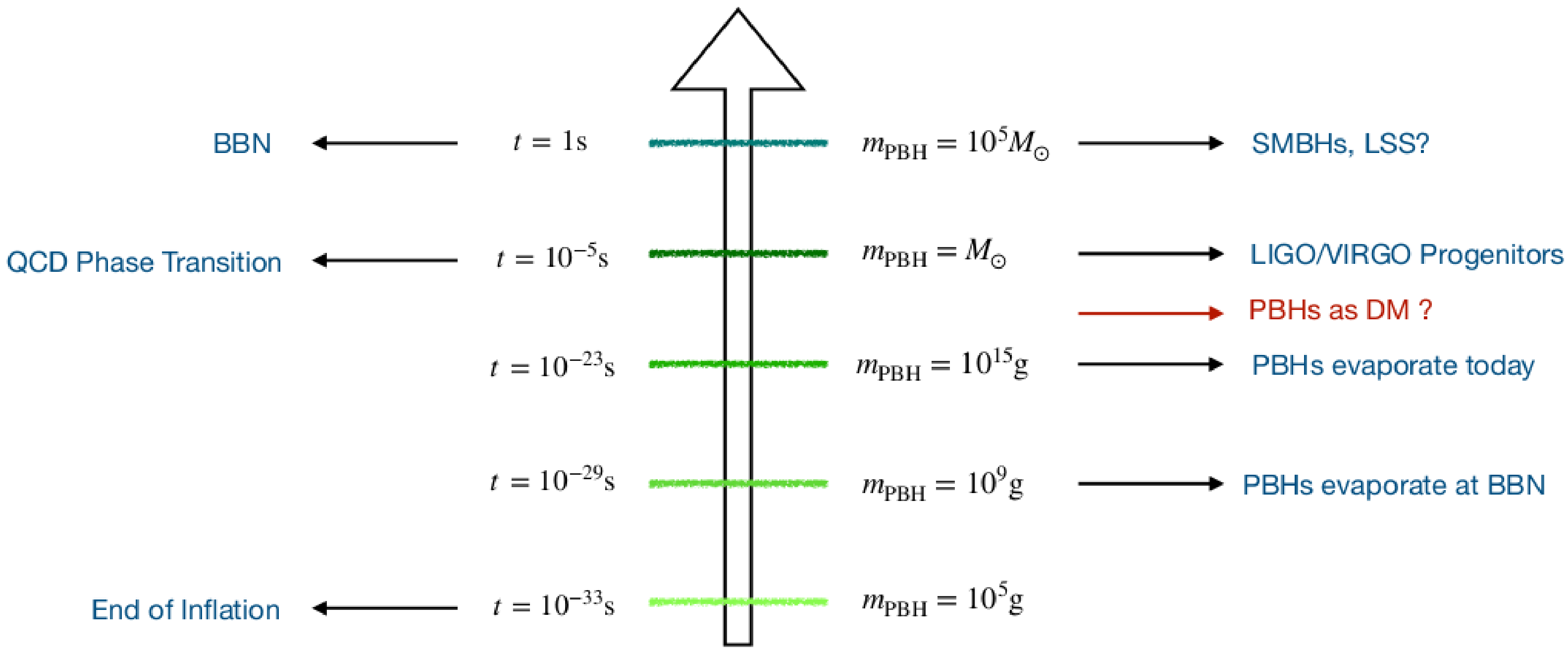
Stability conditions: $f_T < 0$, $f_{BB} < 0$

[Bahamonde, Gakis Kiorpelidi, Koivisto,Said, Saridakis, EPJC81]

Primordial Black Holes (PBHs)

- Primordial Black Holes (PBHs) are formed out of the **collapse of enhanced energy density perturbations** upon horizon reentry of the typical size of the collapsing overdensity region. This happens when $\delta > \delta_c(w \equiv p/\rho)$ [Carr - 1975].

$$m_{\rm PBH} = \gamma M_{\rm H} \propto H^{-1} \text{ where } \gamma \sim \mathrm{O}(1)$$



See for reviews in [Carr et al.- 2020, Sasaki et al - 2018, Clesse et al. - 2017]

PBHs and Gravitational Waves

PBHs and GWs

- 1) **Primordial induced GWs** generated through second order gravitational effects: $\mathscr{L}^{(3)}_{\Phi,h} \ni h\Phi^2$, [Bugaev - 2009, Kohri & Terada - 2018].

  ↙ GWs ↘ PBHs

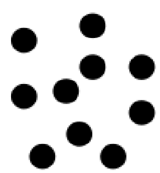
- 2) **Relic Hawking radiated gravitons** from PBH evaporation [Anantua et al. - 2008, Dong et al. - 2015].

- 3) **GWs** emitted **by PBH mergers** [Eroshenko - 2016, Raidal et al. - 2017].

- 4) **GWs induced** at second order **by PBHs themselves** [Papanikolaou et al. - 2020].

PBHs and Gravitational Potential

The PBH Matter Field



→ { **Poisson Statistics** [Desjacques & Riotto - 2018, Ali-Haimoud - 2018]

Same mass [Dizgah, Franciolini & Riotto - 2019] }

$$P_{\delta_{\rm PBH}}(k) \equiv \langle | \delta_k^{\rm PBH} |^2 \rangle = \frac{4\pi}{3}\left(\frac{\bar{r}}{a}\right)^3 = \frac{4\pi}{3k_{\rm UV}^3}, \quad \text{where} \quad k < k_{\rm UV} = \frac{a}{\bar{r}}$$

$\rho_{\rm PBH}$ **is inhomogeneous**

$\rho_{\rm tot}$ is **homogeneous**

} $\delta_{\rm PBH}$ can be seen as an **isocurvature perturbation.**

$\Omega_{\rm PBH} = \rho_{\rm PBH}/\rho_{\rm tot} \propto a^{-3}/a^{-4} \propto a \Rightarrow$ **the isocurvature perturbation,** $\delta_{\rm PBH}$ **will convert during the PBHD era to a curvature perturbation** $\zeta_{\rm PBH}$**, associated to a PBH gravitational potential** Φ**.**

$$\mathcal{P}_\Phi(k) = \frac{2}{3\pi}\left(\frac{k}{k_{\rm UV}}\right)^3 \left(5 + \frac{4}{9}\frac{k^2}{k_{\rm d}^2}\right)^{-2}$$

Scalar Induced Gravitational Waves

- Choosing as the gauge for the GW frame the Newtonian gauge, the metric is written as

$$\mathrm{d}s^2 = a^2(\eta)\left\{-(1+2\Phi)\mathrm{d}\eta^2 + \left[(1-2\Phi)\delta_{ij} + \frac{h_{ij}}{2}\right]\mathrm{d}x^i\mathrm{d}x^j\right\}$$

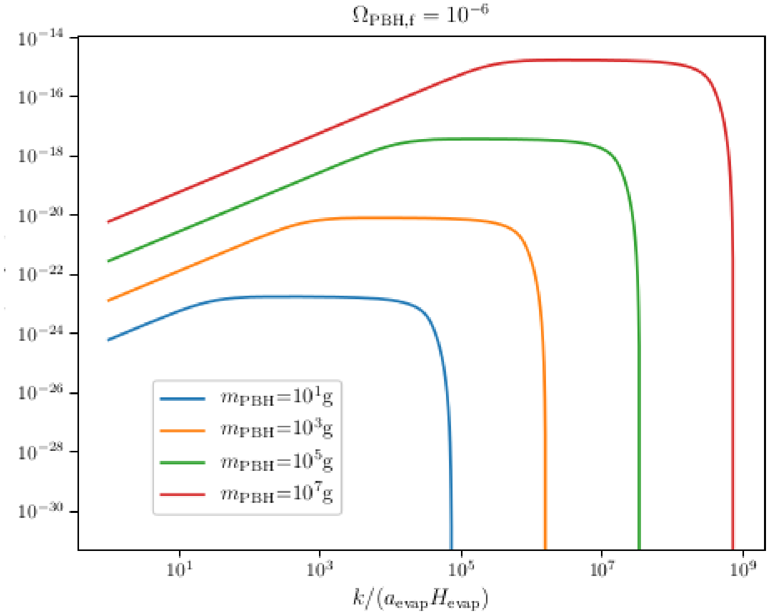
- The equation of motion for the Fourier modes, $h_{\vec{k}}$, read as:

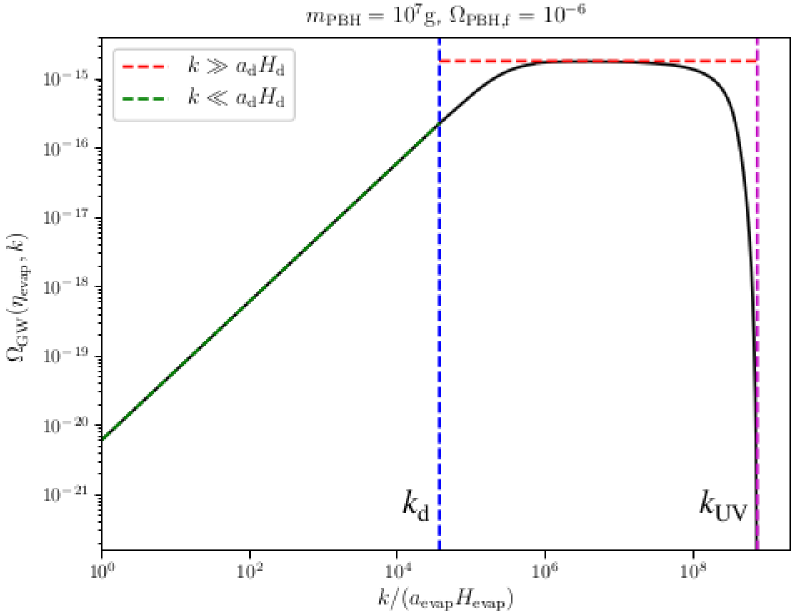
$$h^{s,\prime\prime}_{\vec{k}} + 2\mathcal{H}h^{s,\prime}_{\vec{k}} + k^2 h^{s}_{\vec{k}} = 4S^{s}_{\vec{k}}$$

- The source term, $S_{\vec{k}}$ can be recast as:

$$S^s_{\vec{k}} = \int \frac{\mathrm{d}^3\vec{q}}{(2\pi)^{3/2}} e^s_{ij}(\vec{k})q_i q_j \left[2\Phi_{\vec{q}}\Phi_{\vec{k}-\vec{q}} + \frac{4}{3(1+w)}(\mathcal{H}^{-1}\Phi'_{\vec{q}} + \Phi_{\vec{q}})(\mathcal{H}^{-1}\Phi'_{\vec{k}-\vec{q}} + \Phi_{\vec{k}-\vec{q}})\right]$$

Gravitational Wave Spectrum

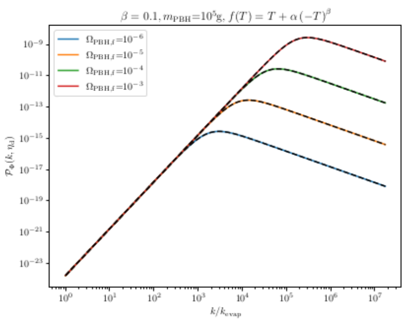




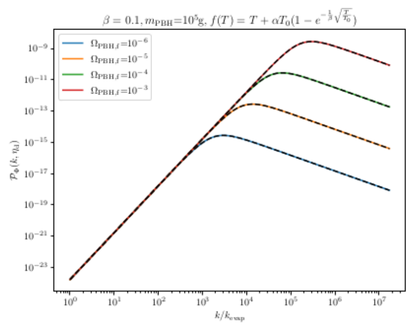
$$\Omega_{\rm GW}(\eta_{\rm evap},k) \simeq 10^{19}\left(\frac{g_{\rm eff}}{100}\right)^{-2/3}\left(\frac{m_{\rm PBH}}{10^9{\rm g}}\right)^{4/3}\Omega_{\rm PBH,f}^{16/3}\times\begin{cases}\frac{k}{k_{\rm d}} & \text{for} \quad k \ll \mathscr{H}_{\rm d}\\ 8 & \text{for} \quad k \gg \mathscr{H}_{\rm d}\end{cases}$$

- One identifies a broken power law for the GW spectrum. Two scales enter in the problem, $k_{\rm d} = \mathscr{H}_{\rm d}$ and $k_{\rm UV} = a_{\rm f}H_{\rm f}\Omega_{\rm PBH,f}^{1/3}$.

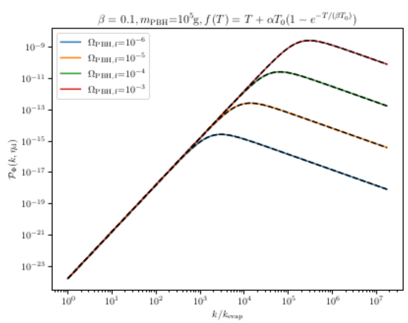
Modified Gravity Signatures



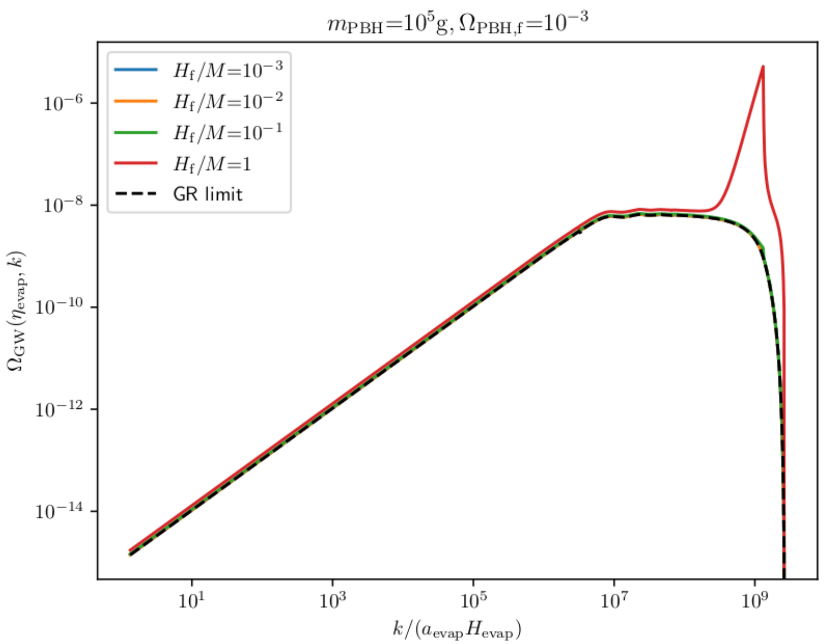
(a) The power-law model



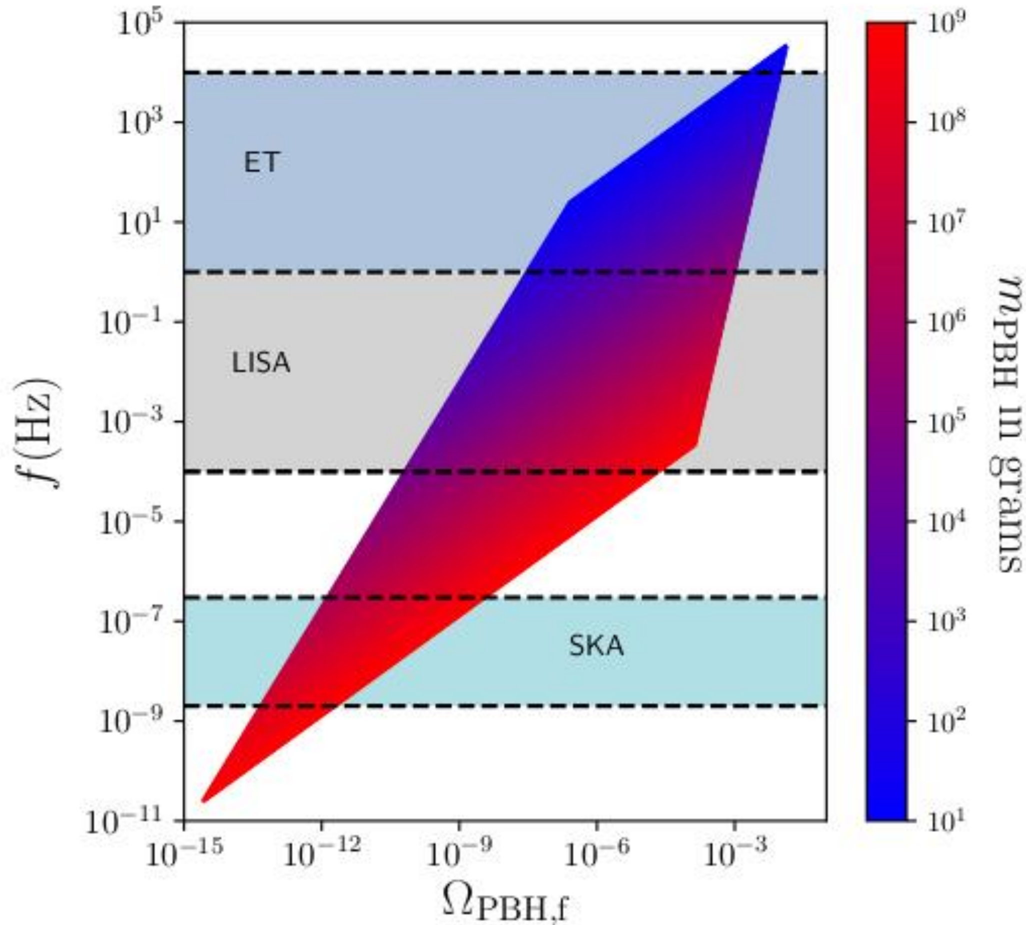
(b) The square-root exponential model



(c) The exponential model



Gravitational Wave Astronomy



Conclusions - Outlook

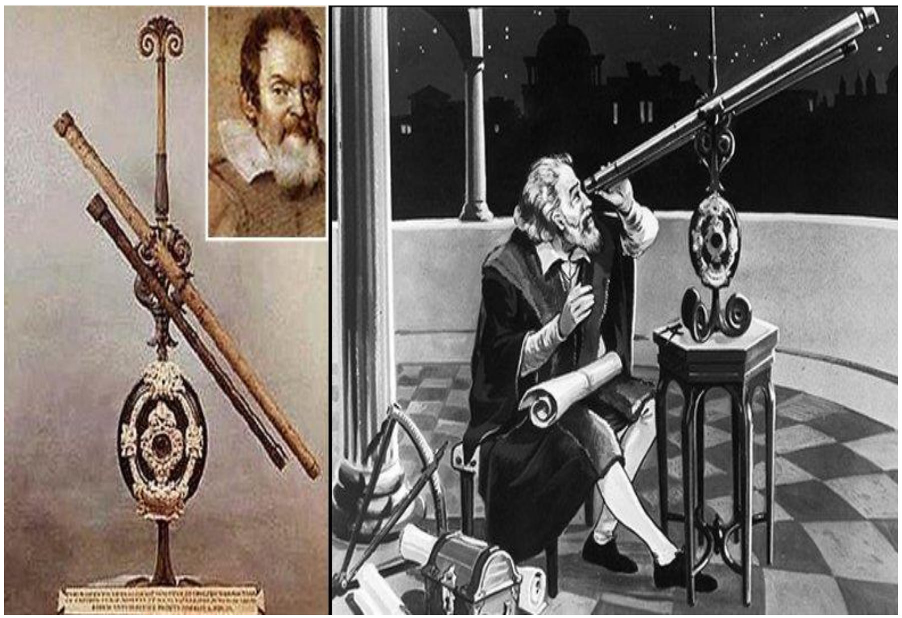
- Torsional modified gravity is theoretically robust and leads to very efficient cosmology at both background and perturbation levels.

- f(T) gravity, f(T,TG), f(T,B), f(Q) gravity, Symmetric teleparallel gravity, and modified teleparallel gravity in general, can be distinguishable in infation-related data

  [Saridakis, Cai, Capozziello, Said, Bahamonde, Koivisto, Ren, Zhao, Wong, Ilyas, Zhu, Zheng, Yan, Zhang, Chen, Zhang, Luo, Khurshudyan, Marciano, Krssak, Odintsov, Nojiri, Nunes, Toporensky, Basilakos, Anagnostopouos, Kofinas, Dialektopoulos, Gakis, Palikaris, Iosifidis, Kiorpelidi, Chatzifotis, Asimakis]

- vi) Get prepared for the huge flow of data that will come!

Multi-messenger Astronomy Era!



EM observations: 400 years

GW observations: 5 years

"Those that do not know geometry are not allowed to enter".
Front Door of Plato's Academy

THANK YOU!

Curvature and Torsion

- Vierbeins e_A^{μ} : four linearly independent fields in the tangent space

  $$g_{\mu\nu}(x) = \eta_{AB}\ e_{\mu}^{A}(x)\ e_{\nu}^{B}(x)$$

- Connection: ω_{ABC}
- Curvature tensor: $R^{A}_{B\mu\nu} = \omega^{A}_{B\nu,\mu} - \omega^{A}_{B\mu,\nu} + \omega^{A}_{C\mu}\omega^{C}_{B\nu} - \omega^{A}_{C\nu}\omega^{C}_{B\mu}$
- Torsion tensor: $T^{A}_{\mu\nu} = e^{A}_{\nu,\mu} - e^{A}_{\mu,\nu} + \omega^{A}_{B\mu}e^{B}_{\nu} - \omega^{A}_{B\nu}e^{B}_{\mu}$
- Levi-Civita connection and Contorsion tensor: $\omega_{ABC} = \Gamma_{ABC} + K_{ABC}$

  $$K_{ABC} = \frac{1}{2}\left(T_{CAB} - T_{BCA} - T_{ABC}\right) = -K_{BAC}$$

- Curvature and Torsion Scalars: $R = \overline{R} + T - 2\left(T_{\nu}^{\nu\mu}\right)_{;\mu}$

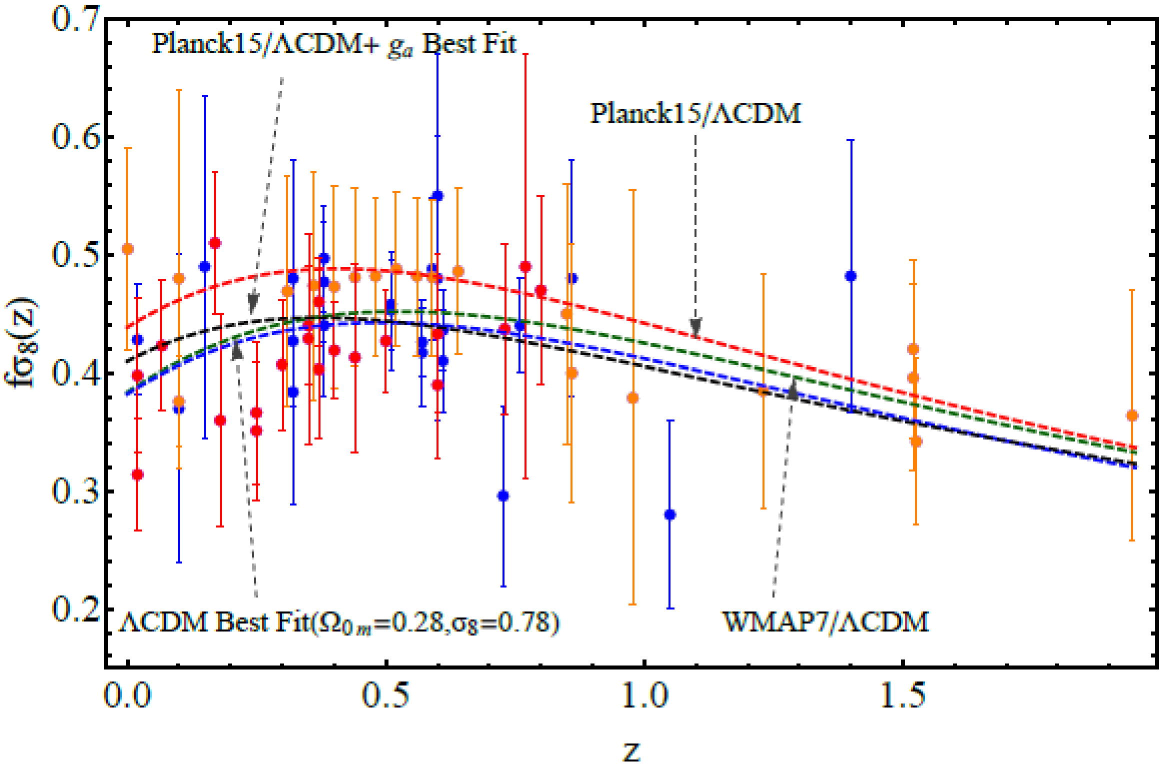
$$R = g^{\mu\nu}R_{\mu\nu} = g^{\mu\nu}R^{\rho}_{\mu\rho\nu}$$

$$T = \frac{1}{4}T^{\rho\mu\nu}T_{\rho\mu\nu} + \frac{1}{2}T^{\rho\mu\nu}T_{\nu\mu\rho} - T^{\rho}_{\rho\mu}T^{\nu\mu}_{\nu}$$

Tension1 – fσ8

- Tension between the data and Planck/ΛCDM. The data indicate a lack of "gravitational power" in structures on intermediate-small cosmological scales.

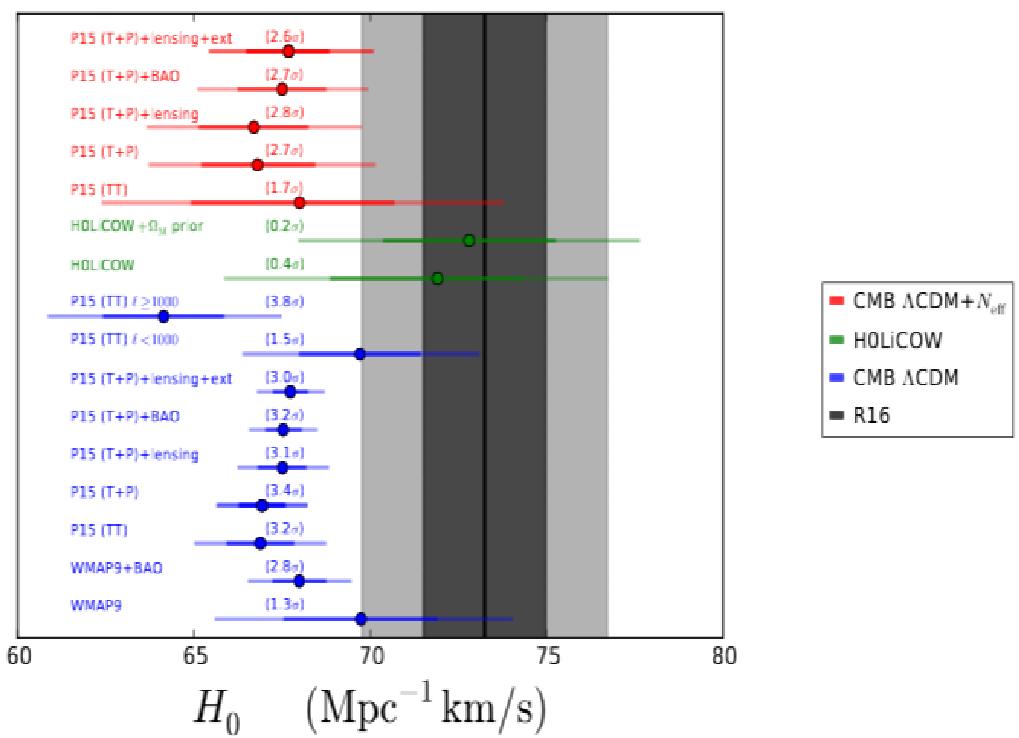
Parameter	Planck15/ΛCDM [12]	WMAP7/ΛCDM [45]
$\Omega_b h^2$	0.02225 ± 0.00016	0.02258 ± 0.00057
$\Omega_c h^2$	0.1198 ± 0.0015	0.1109 ± 0.0056
n_s	0.9645 ± 0.0049	0.963 ± 0.014
H_0	67.27 ± 0.66	71.0 ± 2.5
Ω_{0m}	0.3156 ± 0.0091	0.266 ± 0.025
w	-1	-1
σ_8	0.831 ± 0.013	0.801 ± 0.030

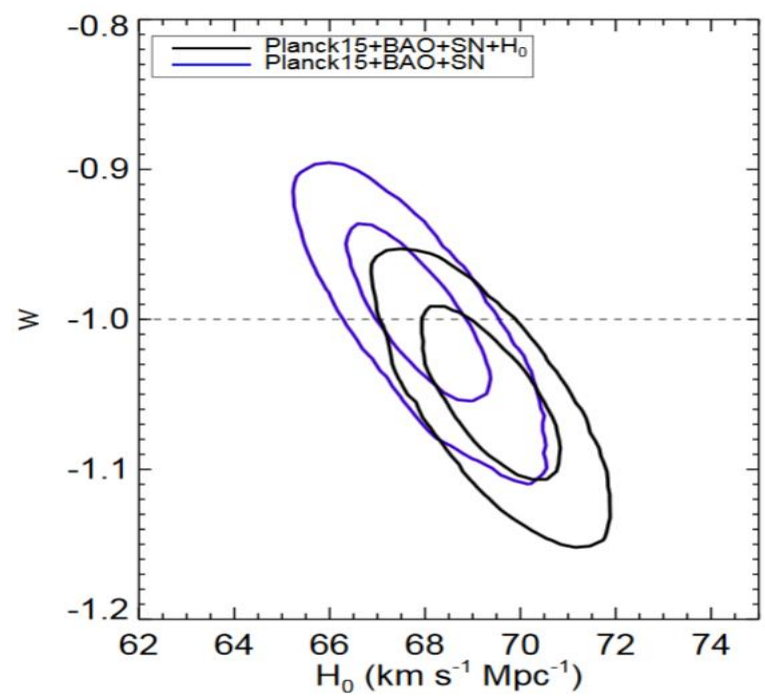


[Kazantzidis, Perivolaropoulos, PRD97]

Tension2 – H0

- Tension between the data (direct measurements) and Planck/ΛCDM (indirect measurements). The data indicate a lack of "gravitational power".





[Bernal, Verde, Riess, JCAP1610]

[Riess et al, Astrophys.J 826]