Dimming of light in general-relativistic cosmology

On the possibility of apparent dark energy effects

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Outline

Summary of the evidence of dark energy in cosmology

Friedmann-Lemaître-Robertson-Walker (FLRW) result on distances
when the strong energy condition is satisfied:
Milne bound on distances

Distances and redshifts for general geometries

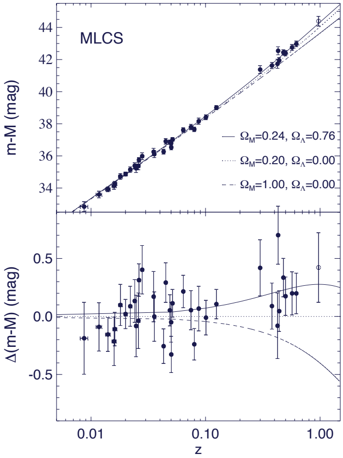
Geometrical condition for super-Milne distances

Discuss how this condition can be satisfied without introducing dark energy or any other energy component that violates the strong energy condition

Evidence for dark energy in cosmology

⋆ Supernovae

Dark energy introduced by Einstein, 1917. Seen with supernovae lightcurves in 1998 by Perlmutter, et al.; Riess, et al.



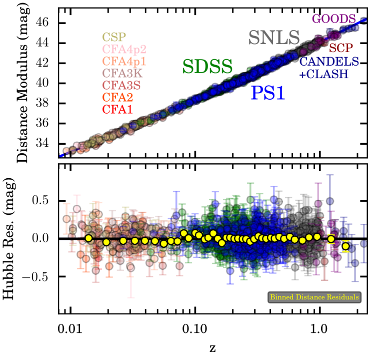
Riess A. G., et al., 1998, Astron. J., 116, 1009. Distance residuals for competing FLRW cosmologies.

Evidence for dark energy in cosmology

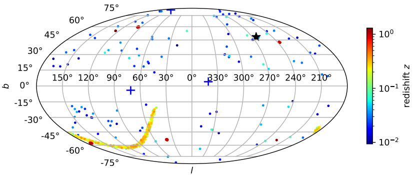
⋆ Supernovae

Modern supernova Ia datasets :

Improved redshift-range and angular coverage



D. M. Scolnic et al. (2018): The Hubble diagram for the Pantheon sample of 1048 SNe Ia. Top: distance modulus for each SN; bottom: residuals to the best fit cosmology.



A. Borderies (2021) Sky distribution of SNe Ia in the Joint Lightcurve Analysis sample in galactic coordinates (l, b). The color scale represents the redshift z. The star correspond to the CMB dipole direction.

Evidence for dark energy in cosmology

◦ Complementary probes

- Cosmic microwave background, large scales: Angular position of first peak in the temperature power spectrum.
- Baryon acoustic oscillations: Size of sound horizon scale seen in present epoch matter distribution (galaxy/ quasar correlations).
- Cosmic microwave background, small angles: Lensing; integrated Sachs-Wolfe effect.
- Cosmic chronometers: Direct measurement of expansion rate from passively evolving galaxies.
- Redshift drift (future measurement): Measurements of small real-time changes in cosmological redshifts.

Luminosity distance and angular diameter distance

Luminosity distance, d_L: Distance defined by the square-root of the ratio of intrinsic luminosity and flux observed from an astronomical object.

The greater the luminosity distance, the greater the dimming of light from the emitter to the observer.

Angular diameter distance, d_A: Distance defined by the square-root of the area of the emitting source divided by its angular size in the telescope.

The greater the angular diameter distance, the greater the focusing of the photons in the bundle when passing from the emitter to the observer.

Etherington's reciprocity theorem: $d_L = (1+z)^2 d_A$

Bound on distances in FLRW universe models

In FLRW space-times it is a known result that distances cannot exceed those of the Milne model, when the matter source obeys the strong energy condition

Intuitively: Ordinary matter and radiation tends to decrease distances between objects. Thus any presence of matter or photons will reduce distances relative to the empty Milne universe

$$
\begin{aligned}
H_o d_A &\leq \frac{1}{2}\left(1 - \frac{1}{(1+z)^2}\right) \qquad \text{(Milne bound)} \\
H_o d_L &\leq \frac{1}{2}\left((1+z)^2 - 1\right)
\end{aligned}
$$

We say that space-times that violate this Milne bound have *super-Milne dimming of light*

What happens generically in general-relativistic space-times?

Matter and radiation are still acting to decrease distances in space-times.

But photons are going through structures and they will carry information about these.

Sometimes photon propagation through structures can produce signatures similar to dark energy in the redshift–distance observations.

Simple example: Observers in the center of a void region (region that is almost free of matter). When photons propagate towards the observer, they are going from a region of slow expansion to fast expansion. This can mimic dark energy in the measurements of the observer.
Pascual-Sanchez J. F. (1999); Célérier M.-N. (2000)

The general universe models

AH, Phys.Rev.D 107 (2023) 10, L101301 arXiv:2212.05568

Assume: 1) A Lorentzian space-time with a Levi-Cevita connection.
2) Light is described within the geometrical optics limit: light is particles that follow geodesic paths.

Let a family of world lines exist to which emitters and observers of light (stars/galaxies) belong. We collectively name these world lines the cosmological congruence description.

Let λ be the affine parameter along the light ray.

Let E be the energy of the light measured by the emitters/observers.

Introduce useful *effective cosmological parameters.*

Effective Hubble parameter: $H \equiv \frac{\frac{\mathrm{d}a}{\mathrm{d}t}}{a} \;\rightarrow\; \mathfrak{H} \equiv \frac{\mathrm{d}E^{-1}}{\mathrm{d}\lambda}$

Effective deceleration parameter: $q \equiv -1 - \frac{\frac{\mathrm{d}H}{\mathrm{d}t}}{H^2} \;\rightarrow\; \mathfrak{Q} \equiv -1 - \frac{1}{E}\frac{\frac{\mathrm{d}\mathfrak{H}}{\mathrm{d}\lambda}}{\mathfrak{H}^2}$

⋆ $\mathfrak{H}$ and $\mathfrak{Q}$ show up naturally in model-independent approaches to cosmography AH, JCAP05(2021)008 [arXiv:2010.06534]
R. Maartens et al. [arXiv:2312.09875]

Distance–redshift relation in general universe models

Use the introduced variables on the observers lightcone, λ (affine parameter), E (photon energy), $\mathfrak{H}$ (Effective cosmological Hubble parameter), $\mathfrak{Q}$ (effective cosmological deceleration parameter) to write distances and redshifts to stars/galaxies measured by the observer.

z: Redshift of light from the emitter to the observer.

$$\frac{1-\frac{1}{(1+z)^2}}{2} = \mathfrak{H}_o E_o(\lambda_o - \lambda)\left(1 + \frac{E_o(\lambda_o - \lambda)\langle\!\langle \mathfrak{Q}\mathfrak{H}^2 \rangle\!\rangle}{2\mathfrak{H}_o}\right)$$

Left hand side is the dimensionless Milne distance.
Right hand side contains the effective cosmological parameters and affine distance $E_o(\lambda_o - \lambda)$ from the observer to the emitter.

Double average of effective deceleration parameter $\mathfrak{Q}$ along light ray:

$$\langle\!\langle \mathfrak{Q}\mathfrak{H}^2 \rangle\!\rangle \equiv \frac{\int_\lambda^{\lambda_o} \mathrm{d}\lambda' \int_{\lambda'}^{\lambda_o} \mathrm{d}\lambda'' \mathfrak{Q}\mathfrak{H}^2}{\frac{1}{2}(\lambda_o - \lambda)^2} \,.$$

Interpretation of effective cosmological parameters

⋆ The effective Hubble parameter

Multipole expansions of $\mathfrak{H}$ in direction of incoming light e^μ Ellis et al. (1984) :

$$\mathfrak{H} = \frac{1}{3}\theta - e^\mu a_\mu + e^\mu e^\nu \sigma_{\mu\nu} \,,$$

$$9 \text{ dof} \begin{cases} \theta & : \text{expansion of observer congruence} \\ a^\mu & : \text{4–acceleration of observer congruence} \\ \sigma_{\mu\nu} & : \text{shear of observer congruence} \end{cases}$$

Interpretation of effective cosmological parameters

⋆ The effective deceleration parameter

Multipole expansions of $\mathfrak{Q}$ in direction of incoming light e^{μ}:

$$\mathfrak{Q} = -1 - \frac{\overset{0}{\mathfrak{q}} + e^{\mu}\overset{1}{\mathfrak{q}}_{\mu} + e^{\mu}e^{\nu}\overset{2}{\mathfrak{q}}_{\mu\nu} + e^{\mu}e^{\nu}e^{\rho}\overset{3}{\mathfrak{q}}_{\mu\nu\rho} + e^{\mu}e^{\nu}e^{\rho}e^{\kappa}\overset{4}{\mathfrak{q}}_{\mu\nu\rho\kappa}}{\mathfrak{H}^2(\boldsymbol{e})}\,, \qquad 16 \text{ idof}$$

$$\overset{0}{\mathfrak{q}} \equiv \frac{1}{3}\frac{\mathrm{d}\theta}{\mathrm{d}\tau} + \frac{1}{3}D_{\mu}a^{\mu} - \frac{2}{3}a^{\mu}a_{\mu} - \frac{2}{5}\sigma_{\mu\nu}\sigma^{\mu\nu}$$

$$\overset{1}{\mathfrak{q}}_{\mu} \equiv -h^{\nu}_{\mu}\frac{\mathrm{d}a_{\nu}}{\mathrm{d}\tau} - \frac{1}{3}D_{\mu}\theta + a^{\nu}\omega_{\mu\nu} + \frac{9}{5}a^{\nu}\sigma_{\mu\nu} - \frac{2}{5}D_{\nu}\sigma^{\nu}_{\ \mu}$$

$$\overset{2}{\mathfrak{q}}_{\mu\nu} \equiv h^{\alpha}_{\mu}h^{\beta}_{\nu}\frac{\mathrm{d}\sigma_{\alpha\beta}}{\mathrm{d}\tau} + D_{\langle\mu}a_{\nu\rangle} + a_{\langle\mu}a_{\nu\rangle} - 2\sigma_{\alpha(\mu}\omega^{\alpha}_{\ \nu)} - \frac{6}{7}\sigma_{\alpha\langle\mu}\sigma^{\alpha}_{\ \nu\rangle}$$

$$\overset{3}{\mathfrak{q}}_{\mu\nu\rho} \equiv -D_{\langle\mu}\sigma_{\nu\rho\rangle} - 3a_{\langle\mu}\sigma_{\nu\rho\rangle}$$

$$\overset{4}{\mathfrak{q}}_{\mu\nu\rho\kappa} \equiv 2\sigma_{\langle\mu\nu}\sigma_{\rho\kappa\rangle}$$

D_{μ}: spatial derivative , $\langle\rangle$: trace-free part of spatial tensor

Interpretation of effective cosmological parameters

⋆ The effective deceleration parameter

Multipole expansions of $\mathfrak{Q}$ in direction of incoming light e^μ:

$$\mathfrak{Q} = -1 - \frac{\overset{0}{\mathfrak{q}} + e^\mu \overset{1}{\mathfrak{q}}_\mu + e^\mu e^\nu \overset{2}{\mathfrak{q}}_{\mu\nu} + e^\mu e^\nu e^\rho \overset{3}{\mathfrak{q}}_{\mu\nu\rho} + e^\mu e^\nu e^\rho e^\kappa \overset{4}{\mathfrak{q}}_{\mu\nu\rho\kappa}}{\mathfrak{H}^2(\boldsymbol{e})} \, ,$$

$$\begin{aligned}
\overset{0}{\mathfrak{q}} &\equiv \frac{1}{3}\frac{\mathrm{d}\theta}{\mathrm{d}\tau} + \frac{1}{3} D_\mu a^\mu - \frac{2}{3} a^\mu a_\mu - \frac{2}{5} \sigma_{\mu\nu}\sigma^{\mu\nu} \\
\overset{1}{\mathfrak{q}}_\mu &\equiv -h^\nu_\mu \frac{\mathrm{d}a_\nu}{\mathrm{d}\tau} - \frac{1}{3} D_\mu \theta + a^\nu \omega_{\mu\nu} + \frac{9}{5} a^\nu \sigma_{\mu\nu} - \frac{2}{5} D_\nu \sigma^\nu_{\ \mu} \\
\overset{2}{\mathfrak{q}}_{\mu\nu} &\equiv h^\alpha_\mu h^\beta_\nu \frac{\mathrm{d}\sigma_{\alpha\beta}}{\mathrm{d}\tau} + D_{\langle\mu} a_{\nu\rangle} + a_{\langle\mu} a_{\nu\rangle} - 2\sigma_{\alpha(\mu}\omega^\alpha_{\ \nu)} - \frac{6}{7} \sigma_{\alpha\langle\mu}\sigma^\alpha_{\ \nu\rangle} \\
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\overset{4}{\mathfrak{q}}_{\mu\nu\rho\kappa} &\equiv 2\sigma_{\langle\mu\nu}\sigma_{\rho\kappa\rangle}
\end{aligned}$$

$\frac{1}{3}\frac{\mathrm{d}\theta}{\mathrm{d}\tau}$: local acceleration of length scales;
only non-vanishing term in the comoving FLRW limit.

The effective deceleration parameter $\mathfrak{Q}$ does not directly measure deceleration of length scales!

Distance–redshift relation in general universe models

⋆ Imposing energy conditions

Assuming the null energy condition: $k^\mu k^\nu R_{\mu\nu} \geq 0$,
$R_{\mu\nu}$: Ricci curvature, k^μ: any null vector field.

Through Sachs focusing equation for light

$$\frac{\mathrm{d}^2 d_A}{\mathrm{d}\lambda^2} = -\mathcal{F} d_A \,, \qquad \mathcal{F} \equiv \frac{1}{2}\hat{\sigma}^{\mu\nu}\hat{\sigma}_{\mu\nu} + \frac{1}{2}k^\mu k^\nu R_{\mu\nu}$$

This immediately leads to bound for angular diameter distance, d_A:

$$E_o(\lambda_o - \lambda) \geq d_A \,.$$

This leads to

$$\begin{aligned} \frac{1 - \frac{1}{(1+z)^2}}{2} &= \mathfrak{H}_o E_o(\lambda_o - \lambda)\left(1 + \frac{E_o(\lambda_o - \lambda)\langle\!\langle \mathfrak{Q}\mathfrak{H}^2 \rangle\!\rangle}{2\mathfrak{H}_o}\right) \\ &\geq \mathfrak{H}_o d_A \left(1 + \frac{E_o(\lambda_o - \lambda)\langle\!\langle \mathfrak{Q}\mathfrak{H}^2 \rangle\!\rangle}{2\mathfrak{H}_o}\right) \end{aligned}$$

Distance–redshift relation in general universe models

⋆ Imposing energy conditions

Assuming the strong energy condition: $u^\mu u^\nu R_{\mu\nu} \geq 0$,
$R_{\mu\nu}$: Ricci curvature, u^μ: Any time-like vector field.

In the FLRW limit where $\mathfrak{Q} \to q \equiv -1 - \frac{\frac{\mathrm{d}H}{\mathrm{d}t}}{H^2}$, the strong energy condition immediately implies that $\mathfrak{Q} \geq 0 \Rightarrow \langle\!\langle \mathfrak{Q}\mathfrak{H}^2 \rangle\!\rangle \geq 0$.

Generally, the effective deceleration parameter is *not* constrained by the strong energy condition.

$$\mathfrak{Q} = -1 - \frac{\overset{0}{\mathfrak{q}} + e^\mu \overset{1}{\mathfrak{q}}_\mu + e^\mu e^\nu \overset{2}{\mathfrak{q}}_{\mu\nu} + e^\mu e^\nu e^\rho \overset{3}{\mathfrak{q}}_{\mu\nu\rho} + e^\mu e^\nu e^\rho e^\kappa \overset{4}{\mathfrak{q}}_{\mu\nu\rho\kappa}}{\mathfrak{H}^2(\boldsymbol{e})},$$

$$\overset{0}{\mathfrak{q}} \equiv \frac{1}{3}\frac{\mathrm{d}\theta}{\mathrm{d}\tau} + \frac{1}{3} D_\mu a^\mu - \frac{2}{3} a^\mu a_\mu - \frac{2}{5} \sigma_{\mu\nu}\sigma^{\mu\nu}$$

$$\overset{1}{\mathfrak{q}}_\mu \equiv -h^\nu_\mu \frac{\mathrm{d}a_\nu}{\mathrm{d}\tau} - \frac{1}{3} D_\mu \theta + a^\nu \omega_{\mu\nu} + \frac{9}{5} a^\nu \sigma_{\mu\nu} - \frac{2}{5} D_\nu \sigma^\nu_{\ \mu}$$

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$$\overset{3}{\mathfrak{q}}_{\mu\nu\rho} \equiv -D_{\langle\mu}\sigma_{\nu\rho\rangle} - 3a_{\langle\mu}\sigma_{\nu\rho\rangle}$$

$$\overset{4}{\mathfrak{q}}_{\mu\nu\rho\kappa} \equiv 2\sigma_{\langle\mu\nu}\sigma_{\rho\kappa\rangle}$$

Distance–redshift relation in general universe models

⋆ Imposing energy conditions

It is possible to have $\langle\!\langle \mathfrak{Q}\mathfrak{H}^2 \rangle\!\rangle \leq 0$ while satisfying the strong energy condition. The Milne bound $\mathfrak{H}_o d_A \leq \frac{1}{2}\left(1 - \frac{1}{(1+z)^2}\right)$ could consequently be broken in such space-times.

This happens for instance in certain Lemaître-Tolman-Bondi void models with the observer placed close to the center.

$$\mathfrak{Q} = -1 - \frac{\overset{0}{\mathfrak{q}} + e^{\mu}\overset{1}{\mathfrak{q}}_{\mu} + e^{\mu}e^{\nu}\overset{2}{\mathfrak{q}}_{\mu\nu} + e^{\mu}e^{\nu}e^{\rho}\overset{3}{\mathfrak{q}}_{\mu\nu\rho} + e^{\mu}e^{\nu}e^{\rho}e^{\kappa}\overset{4}{\mathfrak{q}}_{\mu\nu\rho\kappa}}{\mathfrak{H}^2(\boldsymbol{e})} ,$$

$$\overset{0}{\mathfrak{q}} \equiv \frac{1}{3}\frac{\mathrm{d}\theta}{\mathrm{d}\tau} + \frac{1}{3}D_{\mu}a^{\mu} - \frac{2}{3}a^{\mu}a_{\mu} - \frac{2}{5}\sigma_{\mu\nu}\sigma^{\mu\nu}$$

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Conclusions

The outlined results present a way to classify space-times that have (apparent) dark energy signatures in the predicted distance–redshift graph.

A particular geometrical integral constraint must be systematically violated in order to produce the observed excess-dimming of light.

Systematic violation of $\langle\langle \mathfrak{Q}\mathfrak{H}^2 \rangle\rangle \geq 0$ is necessary for the observer to measure super-Milne distances to stars/galaxies.

This violation can happen *without* violating the strong energy condition. Contrary to intuition from FLRW physics.

Questions of interest

Exact solutions where one can quantify cancellation effects along lightbeams.

Numerical simulations with ray tracing.

Why do we not observe super-Milne distances relativistic cosmology-simulations (unless in the trivial case where a dark energy like source is present)?

Can we find ways of violating any underlying symmetries/conditions in simulations that will give us super-Milne dimming?

Luminosity distance cosmography, **general geometry**

A. Heinesen, JCAP05(2021)008 [arXiv:2010.06534]

Isotropic FLRW cosmography $\rightarrow$ cosmography without symmetries

Important papers J. Kristian and R. K. Sachs (1966), S. Seitz, P. Schneider and J. Ehlers (1994), C. Clarkson and O. Umeh (2011), C. Clarkson, G. F. R. Ellis, A. Faltenbacher, R. Maartens, O. Umeh, J. P. Uzan (2012)

The luminosity distance Hubble law:

$$d_L = \frac{1}{\mathfrak{H}_o} z + \frac{1-\mathfrak{Q}_o}{2\mathfrak{H}_o} z^2 + \frac{-1+3\mathfrak{Q}_o^2+\mathfrak{Q}_o-\mathfrak{J}_o+\mathfrak{R}_o}{6\mathfrak{H}_o} z^3 + \mathcal{O}(z^4)$$

$$H \rightarrow \mathfrak{H}\,, \qquad q \rightarrow \mathfrak{Q}\,, \qquad j \rightarrow \mathfrak{J}\,, \qquad \Omega_k \rightarrow \mathfrak{R}$$

Generalised cosmological parameters $\mathfrak{H}\,, \mathfrak{Q}\,, \mathfrak{J}\,, \mathfrak{R}$ vary with the point of observation and the line of sight.

Luminosity distance cosmography, **general geometry**

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The luminosity distance Hubble law for a general congruence of observers and emitters in a general space-time:

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$$\mathfrak{H} = -\frac{\frac{\mathrm{d}E}{\mathrm{d}\lambda}}{E^2}\,, \qquad \mathfrak{Q} = -1 - \frac{1}{E}\frac{\frac{\mathrm{d}\mathfrak{H}}{\mathrm{d}\lambda}}{\mathfrak{H}^2}\,,$$

$$\mathfrak{R} = 1 + \mathfrak{Q} - \frac{1}{2E^2}\frac{k^\mu k^\nu R_{\mu\nu}}{\mathfrak{H}^2}\,, \qquad \mathfrak{J} = \frac{1}{E^2}\frac{\frac{\mathrm{d}^2\mathfrak{H}}{\mathrm{d}\lambda^2}}{\mathfrak{H}^3} - 4\mathfrak{Q} - 3$$

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$\frac{\mathrm{d}E}{\mathrm{d}\lambda}$: rate of change of photon energy, E, along null ray.

$\frac{\mathrm{d}}{\mathrm{d}\lambda} \equiv k^\mu \nabla_\mu$: Derivative along photon 4-momentum, k^μ.

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$k^\mu k^\nu R_{\mu\nu}$: Ricci focusing term.

Redshift drift in a general cosmology

A. Heinesen, Phys. Rev. D **103** (2021), 023537 [arXiv:2011.10048]

Redshift drift in a general space-time (the *exact* signal):

$$\dot{z} = (1+z)\mathfrak{H}_o - \mathfrak{H}_e + \mathcal{S}_{e\to o}$$

$$\text{(FLRW: } \dot{z} = (1+z)H_o - H_e)$$

$\mathcal{S}_{e\to o}$: zero in FLRW universe models and represents anisotropies and inhomogeneities along the null congruence

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Redshift drift as a probe of the strong energy condition

A. Heinesen, Phys. Rev. D **103** (2021) L081302 [arXiv:2102.03774]

Redshift drift in a general space-time (the *exact* signal):

$$\dot{z} = (1+z)\mathfrak{H}_o - \mathfrak{H}_e + \mathcal{S}_{e\to o} = E_e \int_{\lambda_e}^{\lambda_o} d\lambda\, \Pi$$

$$(\text{FLRW: } \dot{z} = (1+z)H_o - H_e \qquad = E_e \int_{\lambda_e}^{\lambda_o} d\lambda\, \ddot{a}/a)$$

Redshift drift as a probe of the strong energy condition

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$$\Pi = \Pi^o + e^\mu \Pi^{\boldsymbol{e}}_\mu + d^\mu \Pi^{\boldsymbol{d}}_\mu + e^\mu e^\nu \Pi^{\boldsymbol{ee}}_{\mu\nu} + e^\mu d^\nu \Pi^{\boldsymbol{ed}}_{\mu\nu} + e^\mu e^\nu e^\rho \Pi^{\boldsymbol{eee}}_{\mu\nu\rho} + e^\mu e^\nu e^\rho e^\kappa \Pi^{\boldsymbol{eeee}}_{\mu\nu\rho\kappa}$$

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e^μ: Direction of incoming light as seen by the observer

$d^\mu \equiv h^\mu_\nu e^\alpha \nabla_\alpha e^\nu$: spatially projected acceleration vector of e^μ

Redshift drift as a probe of the strong energy condition

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$$\Pi^o \equiv -\tfrac{1}{3} u^\mu u^\nu R_{\mu\nu} + \tfrac{1}{3} D_\mu a^\mu - \tfrac{1}{3} a^\mu a_\mu - d^\mu d_\mu - \tfrac{3}{5}\sigma^{\mu\nu}\sigma_{\mu\nu} - \omega^{\mu\nu}\omega_{\mu\nu}$$

Redshift drift as a probe of the strong energy condition

Redshift drift in a general space-time (the *exact* signal):

$$\dot{z} = (1+z)\mathfrak{H}_o - \mathfrak{H}_e + \mathcal{S}_{e\to o} = E_e \int_{\lambda_e}^{\lambda_o} d\lambda \, \Pi$$

$$\text{(FLRW: } \dot{z} = (1+z)H_o - H_e = E_e \int_{\lambda_e}^{\lambda_o} d\lambda \, \ddot{a}/a)$$

$$\Pi = \Pi^o + e^\mu \Pi^{\boldsymbol{e}}_\mu + d^\mu \Pi^{\boldsymbol{d}}_\mu + e^\mu e^\nu \Pi^{\boldsymbol{ee}}_{\mu\nu} + e^\mu d^\nu \Pi^{\boldsymbol{ed}}_{\mu\nu} + e^\mu e^\nu e^\rho \Pi^{\boldsymbol{eee}}_{\mu\nu\rho} + e^\mu e^\nu e^\rho e^\kappa \Pi^{\boldsymbol{eeee}}_{\mu\nu\rho\kappa}$$

$$\Pi^o \equiv -\frac{1}{3} u^\mu u^\nu R_{\mu\nu} + \frac{1}{3} D_\mu a^\mu - \frac{1}{3} a^\mu a_\mu - d^\mu d_\mu - \frac{3}{5}\sigma^{\mu\nu}\sigma_{\mu\nu} - \omega^{\mu\nu}\omega_{\mu\nu}$$

$u^\mu u^\nu R_{\mu\nu}$: Ricci focusing term;
only non-vanishing term in FLRW limit

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Now assume that $a^\mu = 0$, then

$\Pi^o \leq 0$ if $u^\mu u^\nu R_{\mu\nu} \geq 0$ (Strong Energy Condition)

$\Rightarrow \dot{z} \leq 0$ if $u^\mu u^\nu R_{\mu\nu} \geq 0$ if monopole, Π^o, is dominating

Positivity of redshift drift = detection of SEC violation (Dark energy)

Redshift drift cosmography

A. Heinesen [arXiv:2107.08674]

Redshift drift series expansion to first order in redshift:

$$\dot{z} = -\mathfrak{d}_0 \mathfrak{H}_0 z + \mathcal{O}(z^2)$$

$$(\text{FLRW: } \dot{z} = -q_o H_o z + \mathcal{O}(z^2))$$

$$H \to \mathfrak{H} = \frac{1}{3}\theta - e^{\mu} a_{\mu} + e^{\mu} e^{\nu} \sigma_{\mu\nu} \,, \qquad 1 \text{ dof} \to 9 \text{ dof}$$

$$q \to \mathfrak{d} = \frac{-\kappa^{\mu}\kappa_{\mu} - \Sigma^{o} - e^{\mu}\Sigma^{\boldsymbol{e}}_{\mu} - e^{\mu}e^{\nu}\Sigma^{\boldsymbol{ee}}_{\mu\nu} - e^{\mu}\kappa^{\nu}\Sigma^{\boldsymbol{e\kappa}}_{\mu\nu}}{\mathfrak{H}^2} \,, \qquad 1 \text{ dof} \to 12 \text{ indep. dof}$$

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The generalised deceleration parameter $\mathfrak{d} \neq \mathfrak{Q}$

$\kappa^\mu \equiv h^\mu_\nu u^\alpha \nabla_\alpha e^\nu$: position *drift*. Change of angular position on the sky of the source in an unrotated reference frame

e^μ_o is immediately known. κ^μ_o requires precise measurements.

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$$\Sigma^{\boldsymbol{ee}}_{\mu\nu} \equiv a_{\langle\mu} a_{\nu\rangle} + D_{\langle\mu} a_{\nu\rangle} - u^\rho u^\sigma C_{\rho\mu\sigma\nu} - \frac{1}{2} h^\alpha_{\langle\mu} h^\beta_{\nu\rangle} R_{\alpha\beta} \,,$$

$$\Sigma^{\boldsymbol{e\kappa}}_{\mu\nu} \equiv 2(\sigma_{\mu\nu} - \omega_{\mu\nu}) \,.$$

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$u^\mu u^\nu R_{\mu\nu}$: Ricci focusing term

$u^\rho u^\sigma C_{\rho\mu\sigma\nu}$: Electric part of Weyl tensor; (Analogous to Newtonian tidal tensor: gradient of gravitational acceleration)