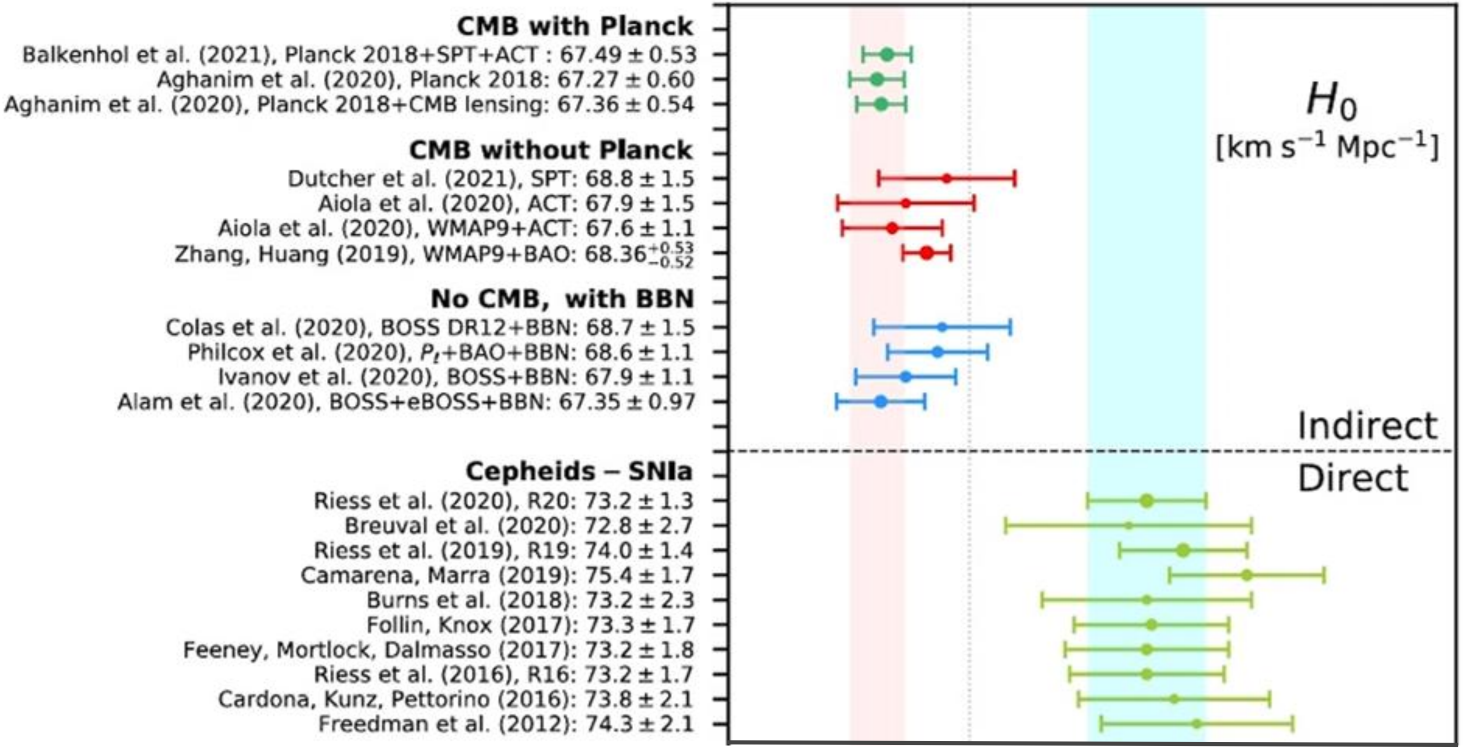
A radical solution to the Hubble tension problem

based on arXiv:2404.08586, with Neil Hyatt

Timothy Clifton, University of London

The Hubble tension



[credit: *Di Valentino et al, CQG 38, 153001, 2021* (abridged)]

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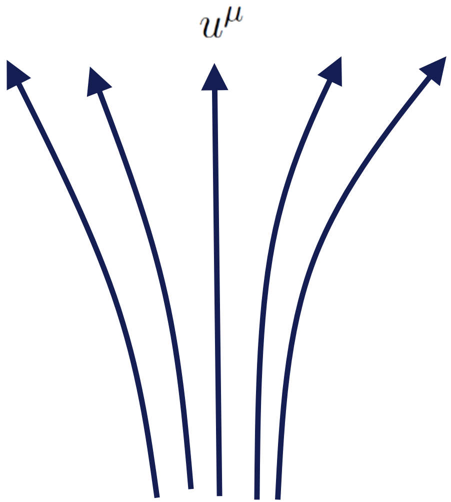
Cosmology without FRW

Cosmology without FRW

Consider a congruence of time-like curves with 4-velocity $\boldsymbol{u}$:



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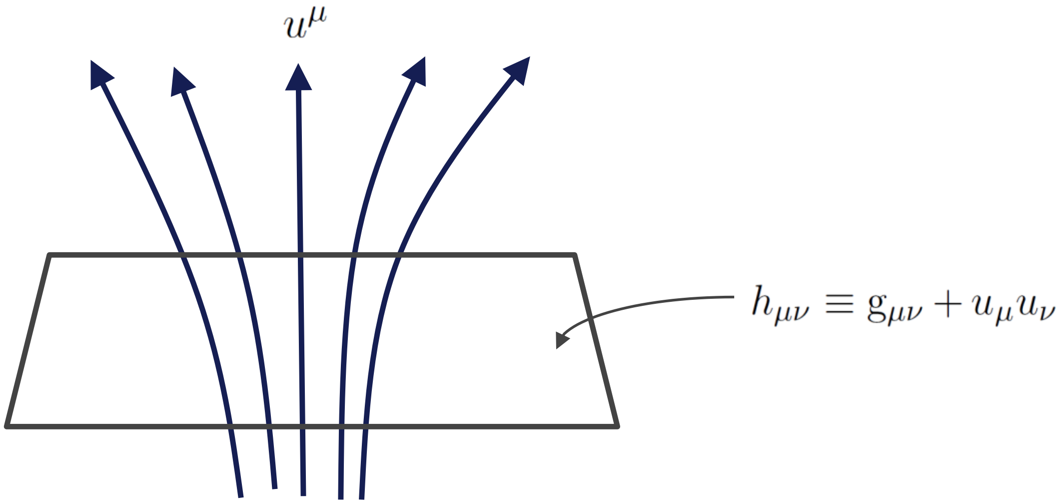

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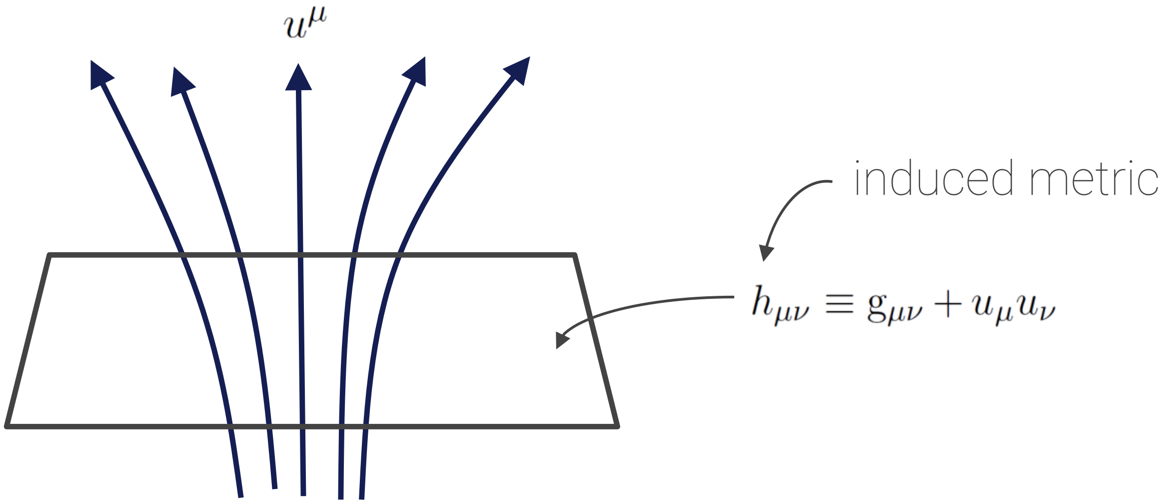

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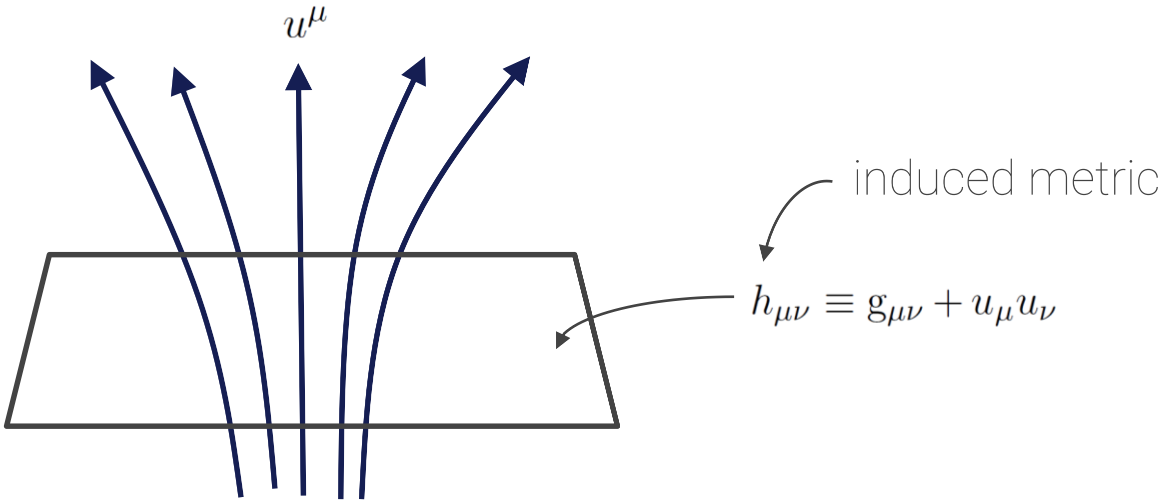

u^μ

induced metric

$h_{\mu\nu} \equiv g_{\mu\nu} + u_\mu u_\nu$

➡ $$\nabla_\nu u_\mu = -u_\nu \dot{u}_\mu + \frac{1}{3}\vartheta h_{\mu\nu} + \varsigma_{\mu\nu}$$

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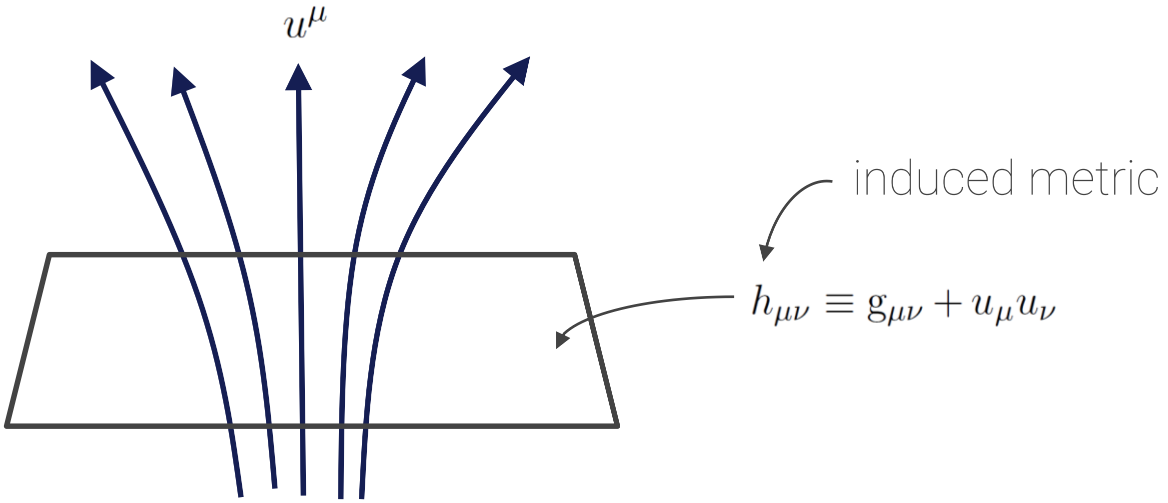

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acceleration expansion shear

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$$\mathcal{H}^2 = \frac{8\pi G}{3}\mu - \frac{\mathcal{R}}{6} + \frac{\varsigma^2}{3} + \frac{\Lambda}{3}$$

Cosmology without FRW

Equations governing the expansion of ***u***:

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These look like the Friedmann equations, but assume no symmetry!

[Ehlers, Gen. Rel. Grav. 25, 1225, 1993; Ellis & van Elst, arXiv:gr-qc/9812046]

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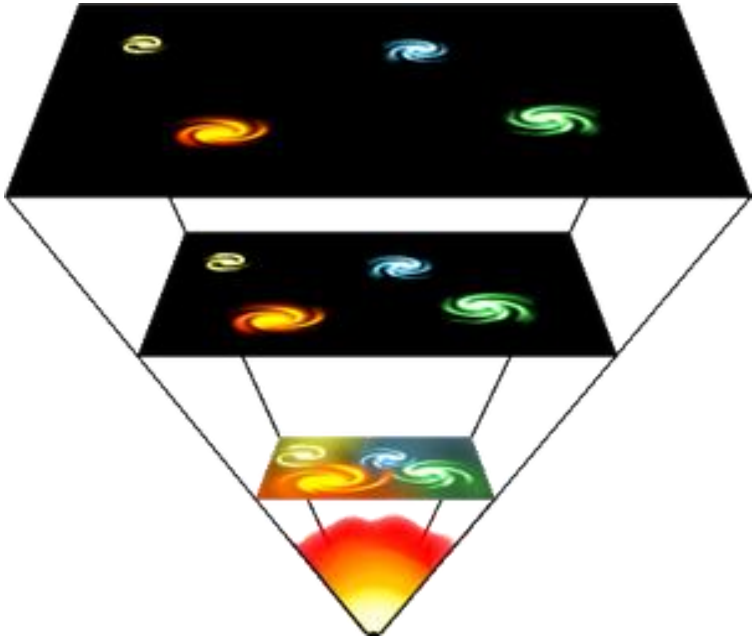
How to describe an extended space:



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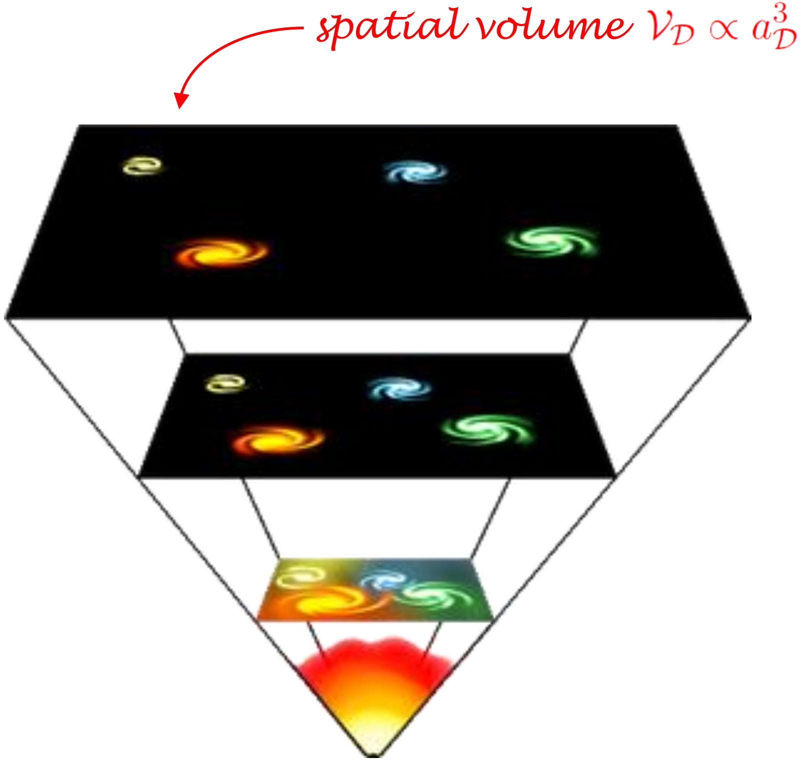

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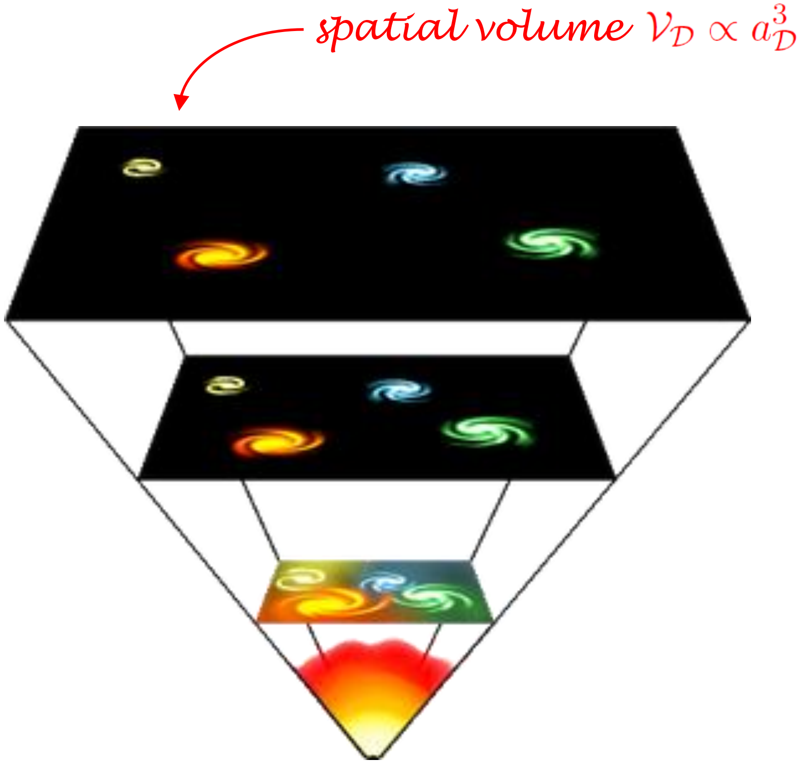
How to describe an extended space:

Define an average: $\langle \mathcal{S} \rangle \equiv \frac{\int_{\mathcal{D}} \sqrt{h}\, \mathcal{S}\, d^3x}{\int_{\mathcal{D}} \sqrt{h}\, d^3x}$

spatial volume $V_{\mathcal{D}} \propto a_{\mathcal{D}}^3$



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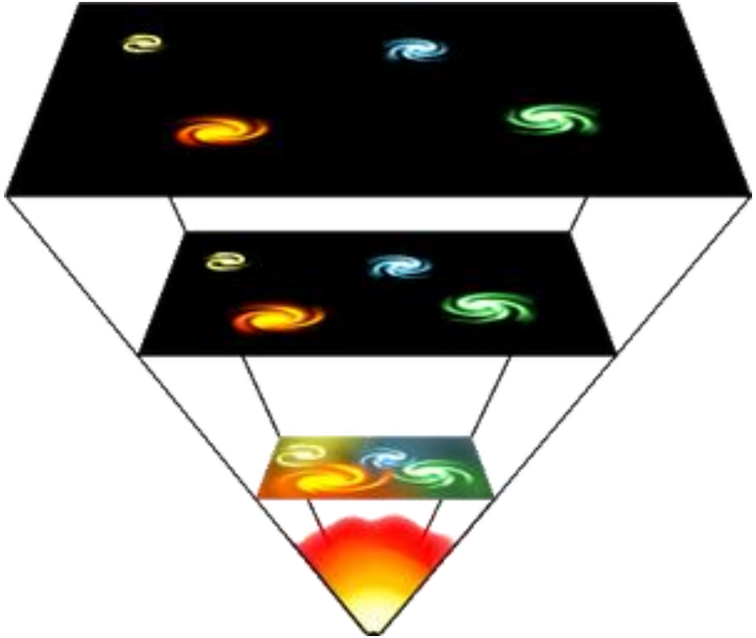

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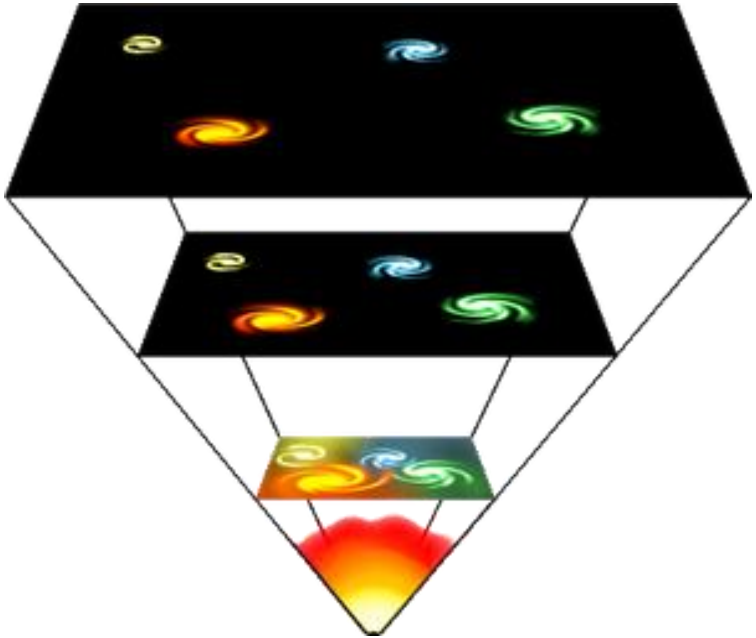

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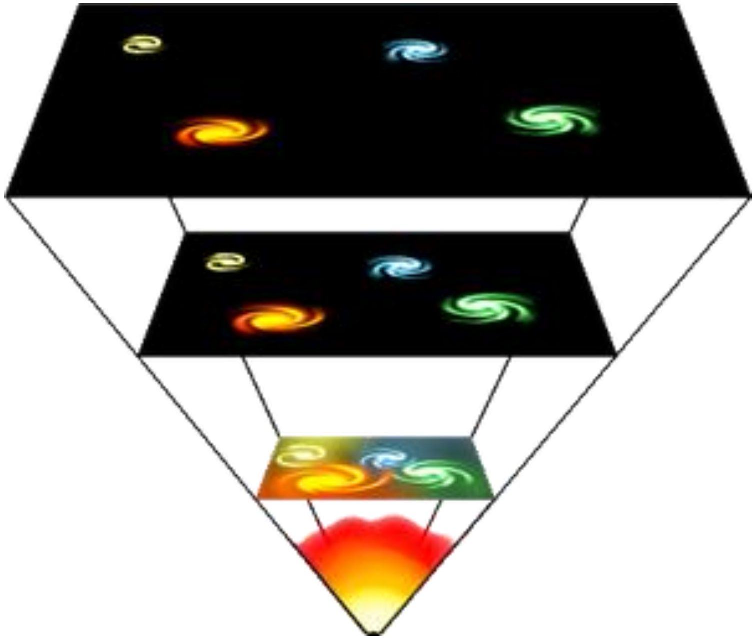

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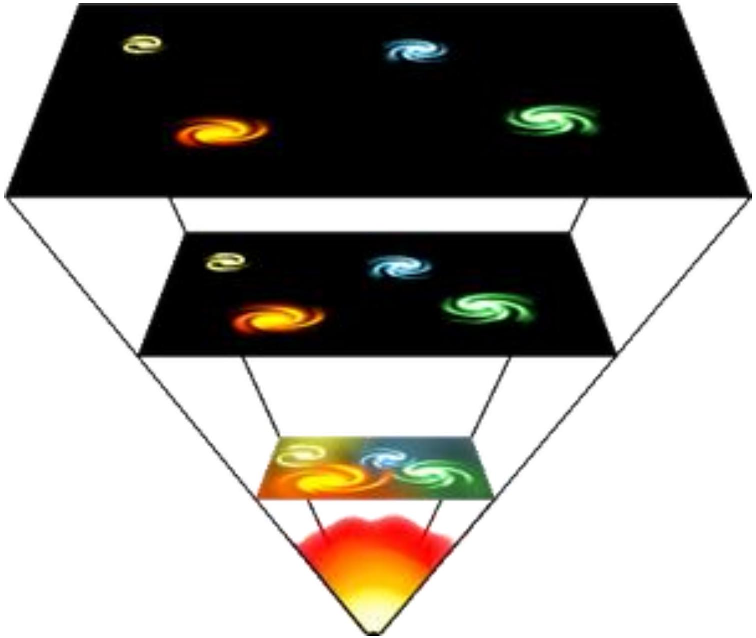
where $\mathcal{Q} = \frac{2}{3}\left(\langle \vartheta^2 \rangle - \langle \vartheta \rangle^2\right) - 2\langle \varsigma^2 \rangle$, $\langle \mu \rangle \propto a_{\mathcal{D}}^{-3}$

[Buchert Gen. Rel. Grav. 32, 105, 2000]



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Applying to local equations:

Friedmann with an extra term!

for dust

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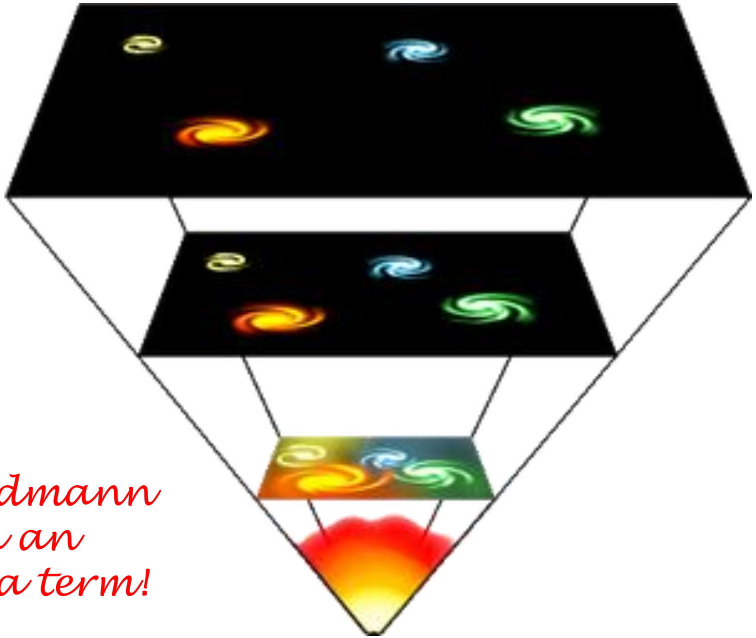

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Cosmology without FRW

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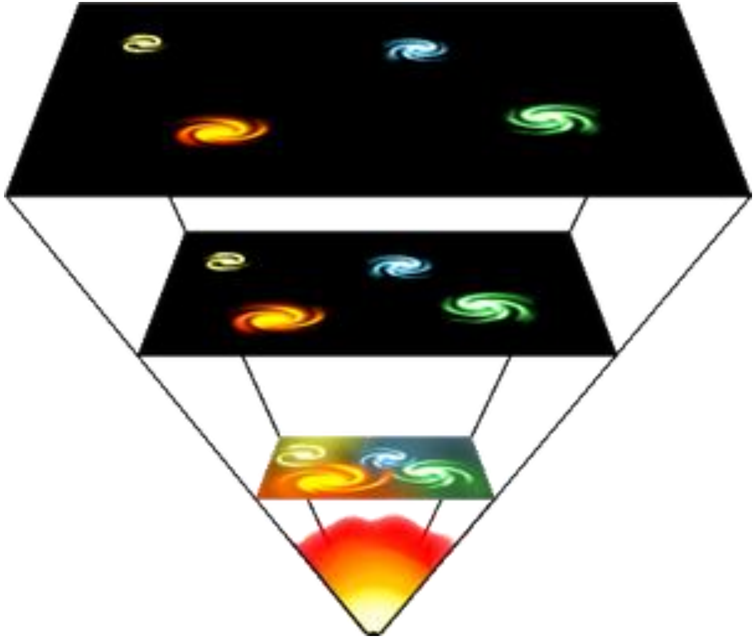

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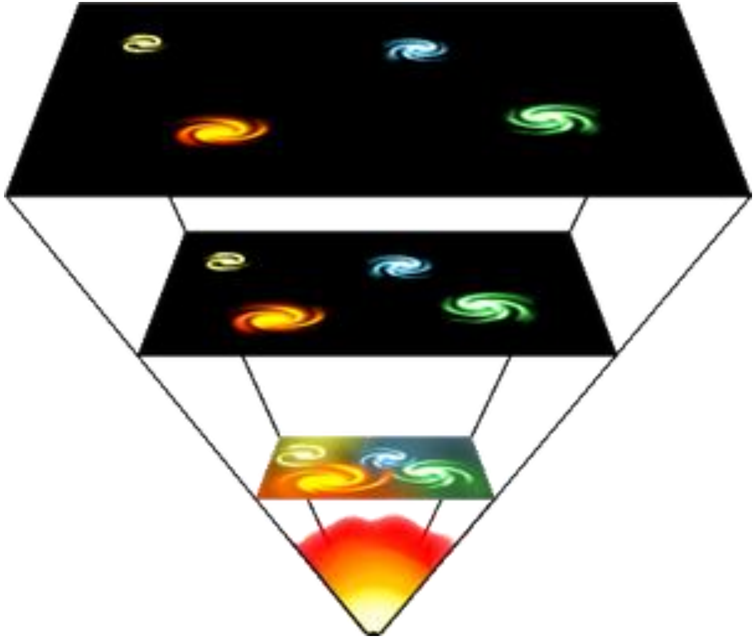
Notes:

- in general, there are *no* FRW models that have the same expansion and curvature as these models.



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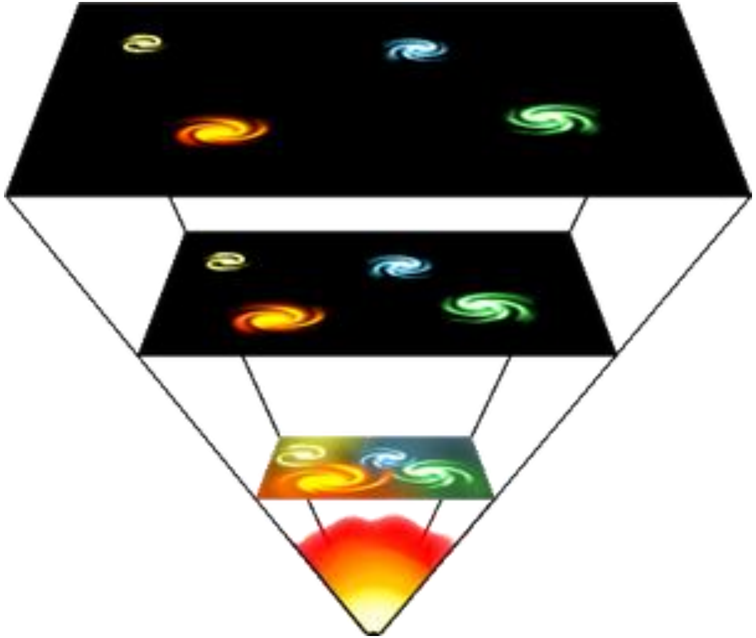

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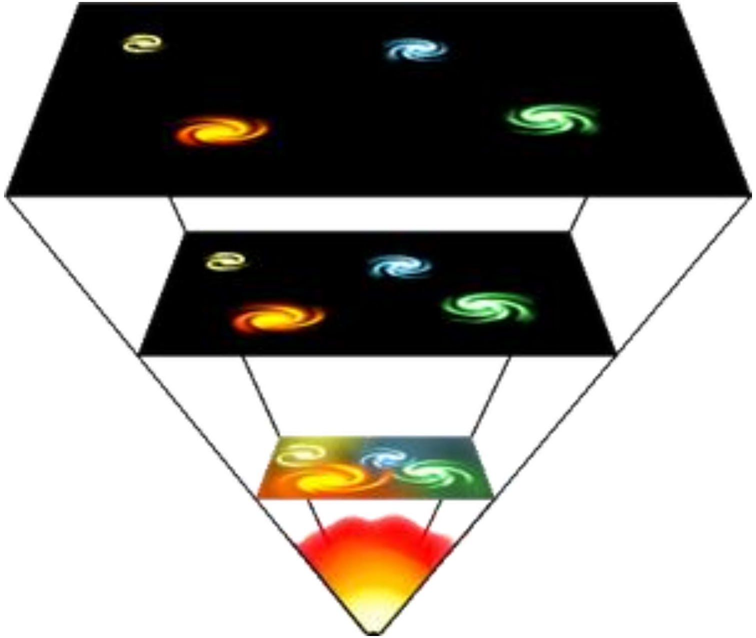
[for proof, see TC and Hyatt arXiv:2404.08586]



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Observations without FRW

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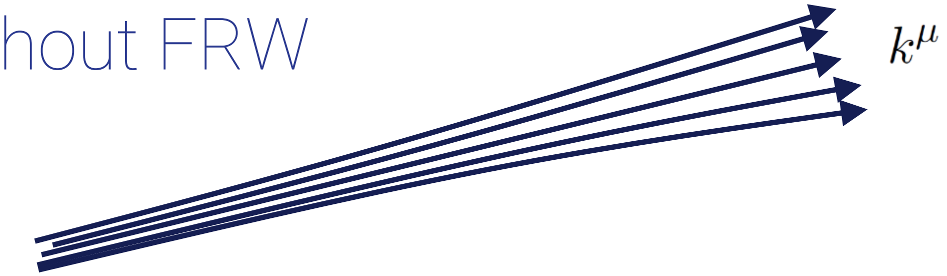

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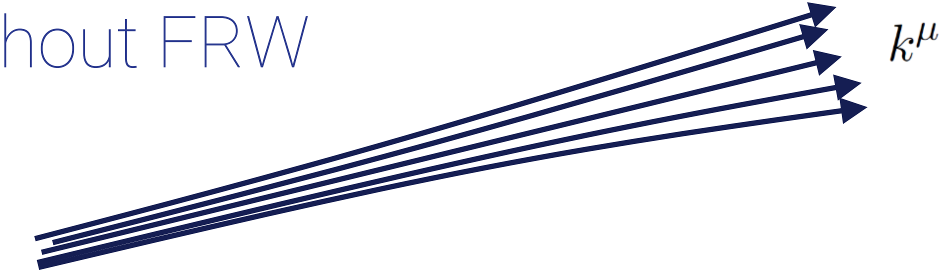

The expansion obeys:

$$\frac{d\tilde{\theta}}{d\lambda} + \frac{1}{2}\tilde{\theta}^2 + 2\tilde{\sigma}^2 = -R_{\mu\nu}k^\mu k^\nu$$

[*Sachs, Proc. Roy. Soc. Lond. A 264, 309 1961*]

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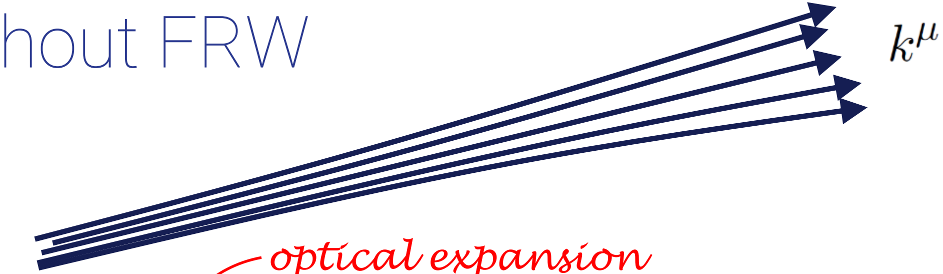

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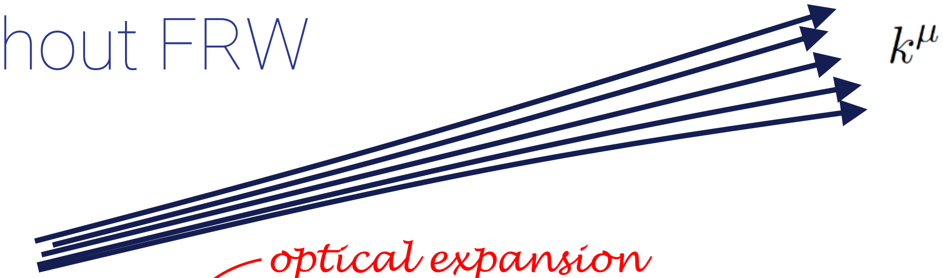

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⇒

$$D_A \propto \exp\left(\frac{1}{2}\int \mathrm{d}\lambda\tilde{\theta}\right)$$

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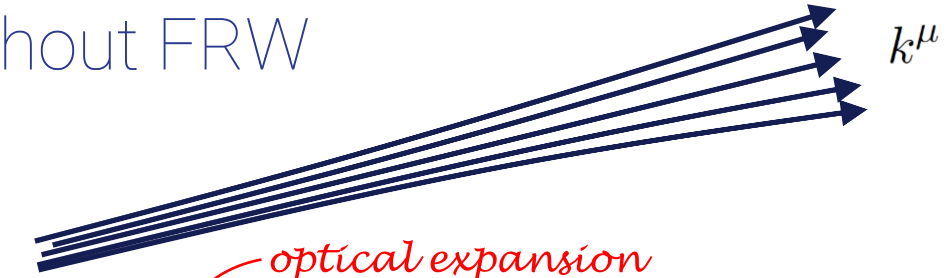

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⇒ $$D_A \propto \exp\left(\frac{1}{2}\int \mathrm{d}\lambda\tilde{\theta}\right)$$ & $$D_L = (1+z)^2 D_A$$ [*Etherington, GRG 39, 1055, 2007*]

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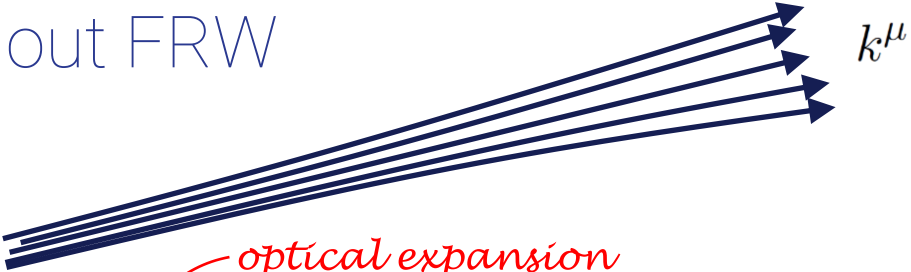

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With redshifts: $1+z=\dfrac{(-u_\mu k^\mu)_\mathrm{s}}{(-u_\nu k^\nu)_\mathrm{o}}$

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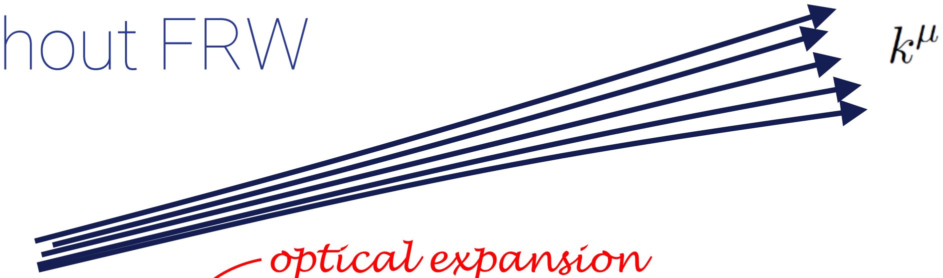

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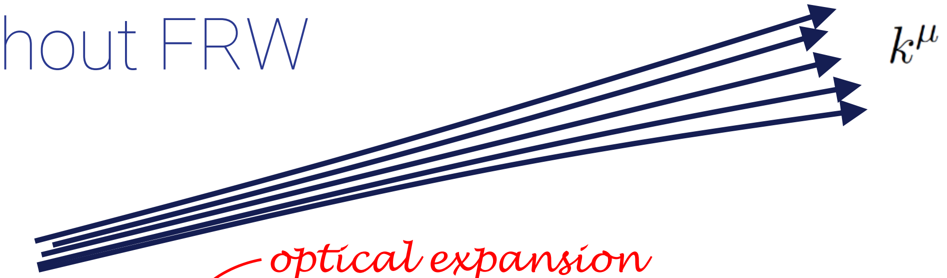
Observations without FRW

For a bundle of light rays:



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shear

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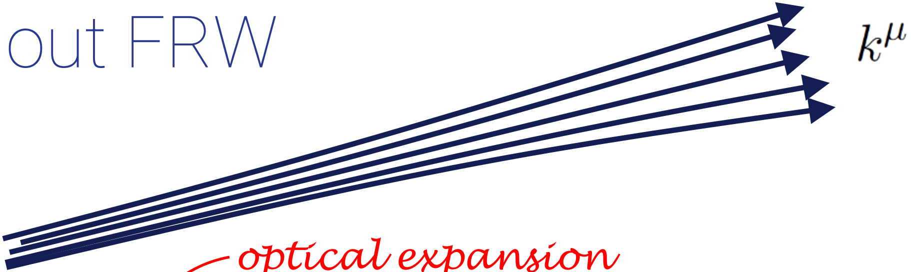

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all valid in any space-time!

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[Räsänen, JCAP 02, 011, 2009]

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[*see TC and Hyatt arXiv:2404.08586*]

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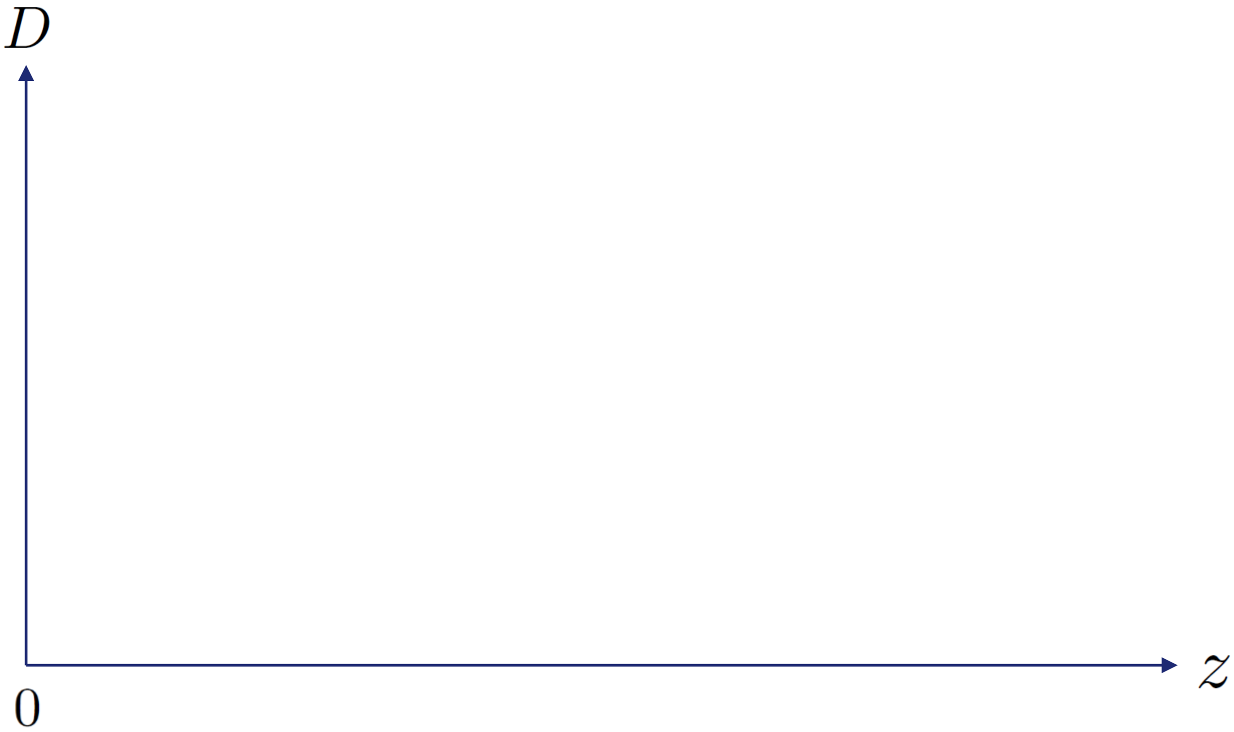

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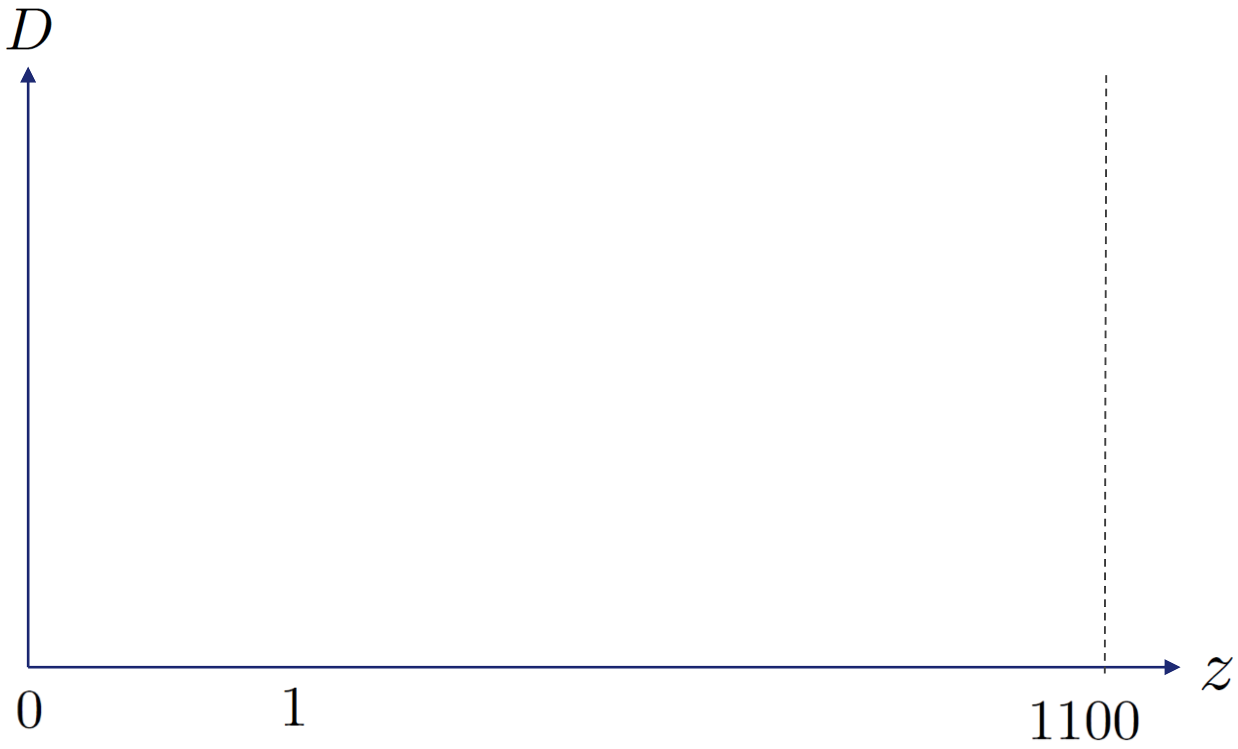

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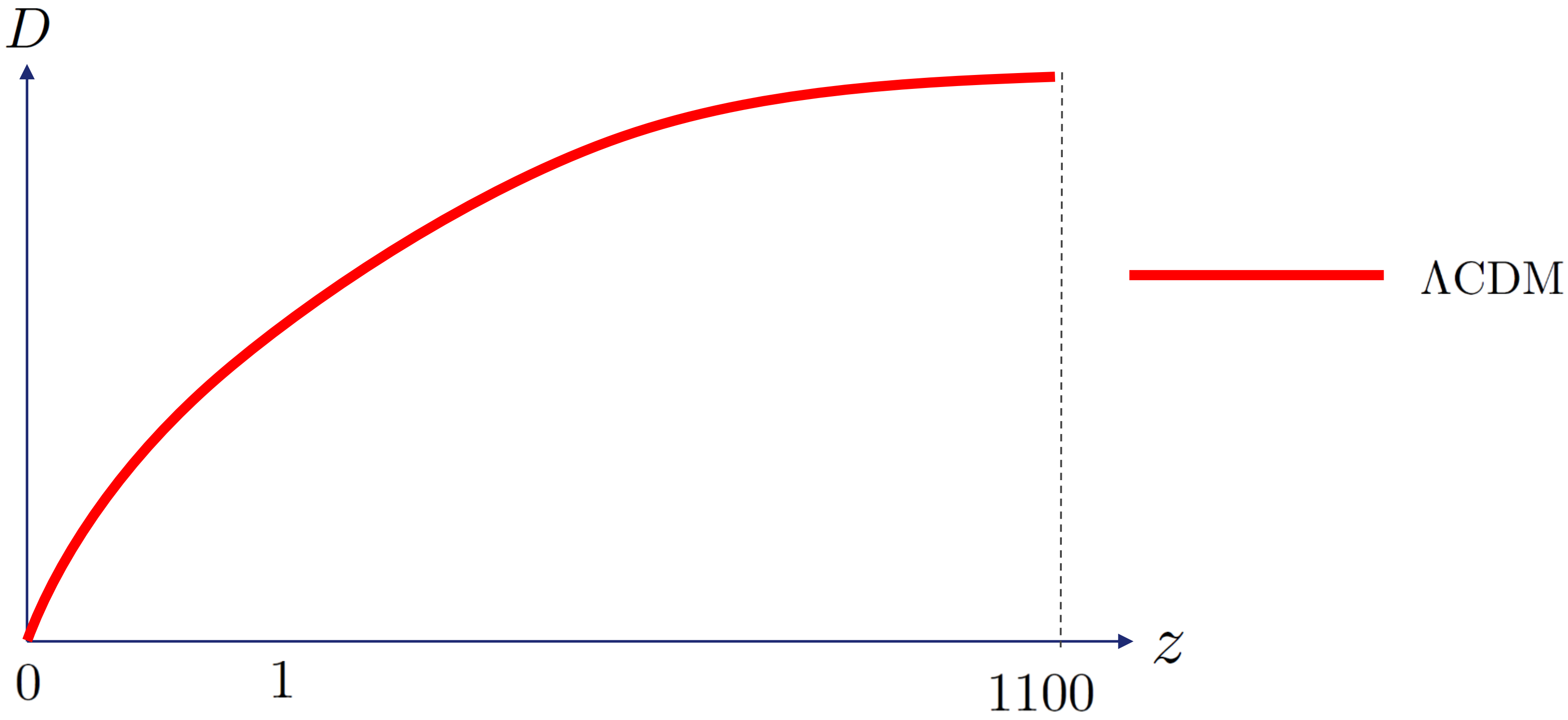

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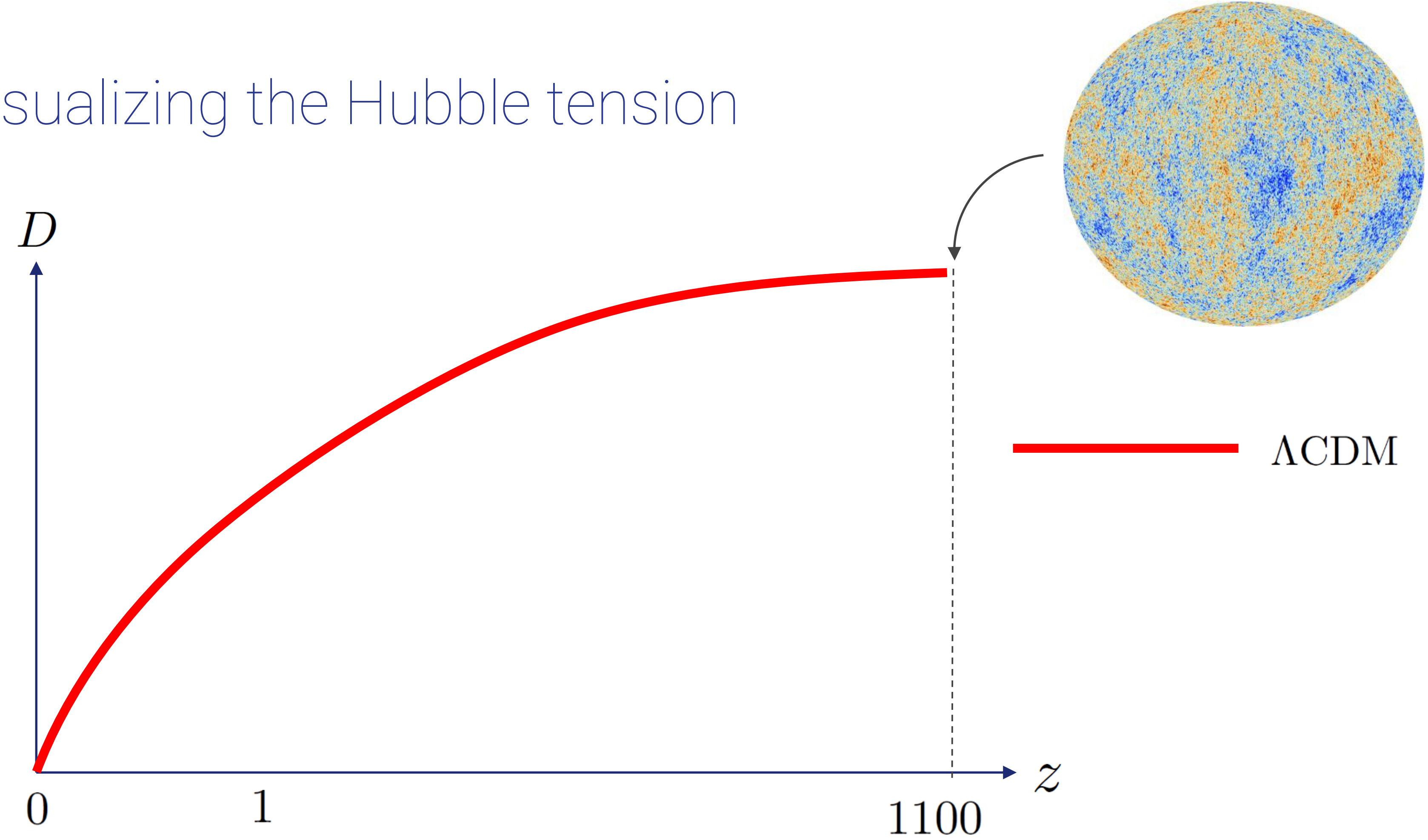

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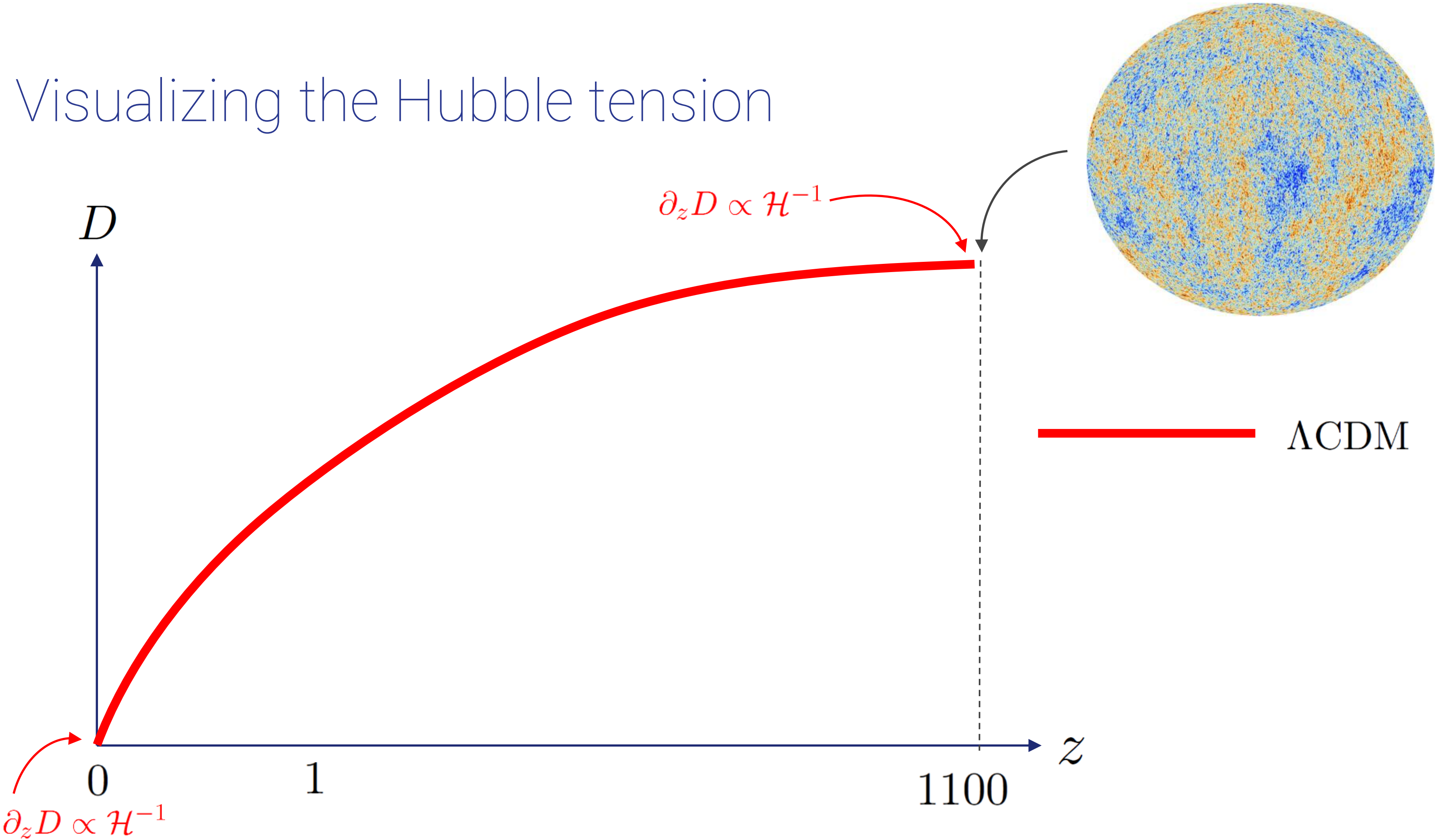

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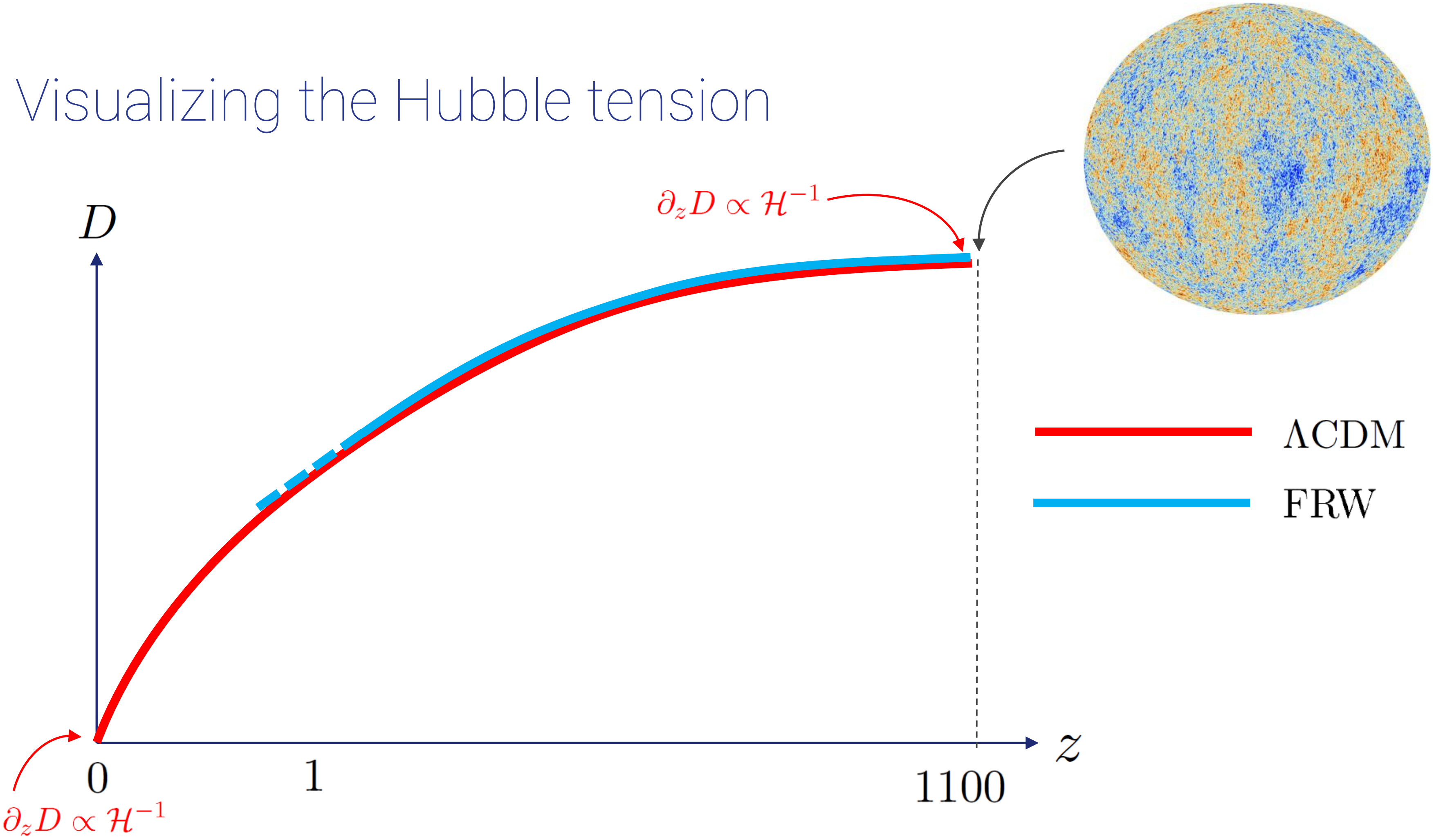

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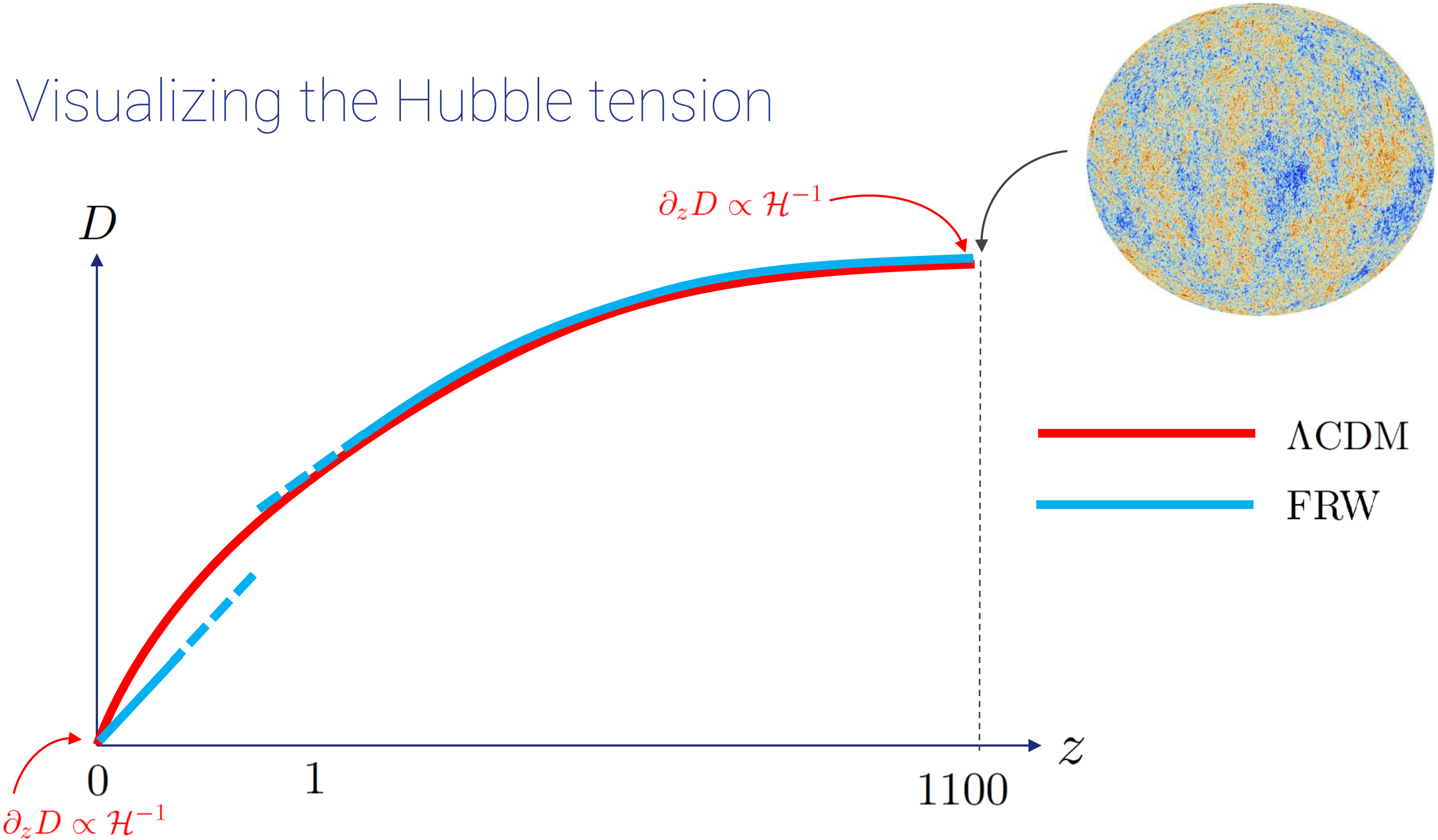

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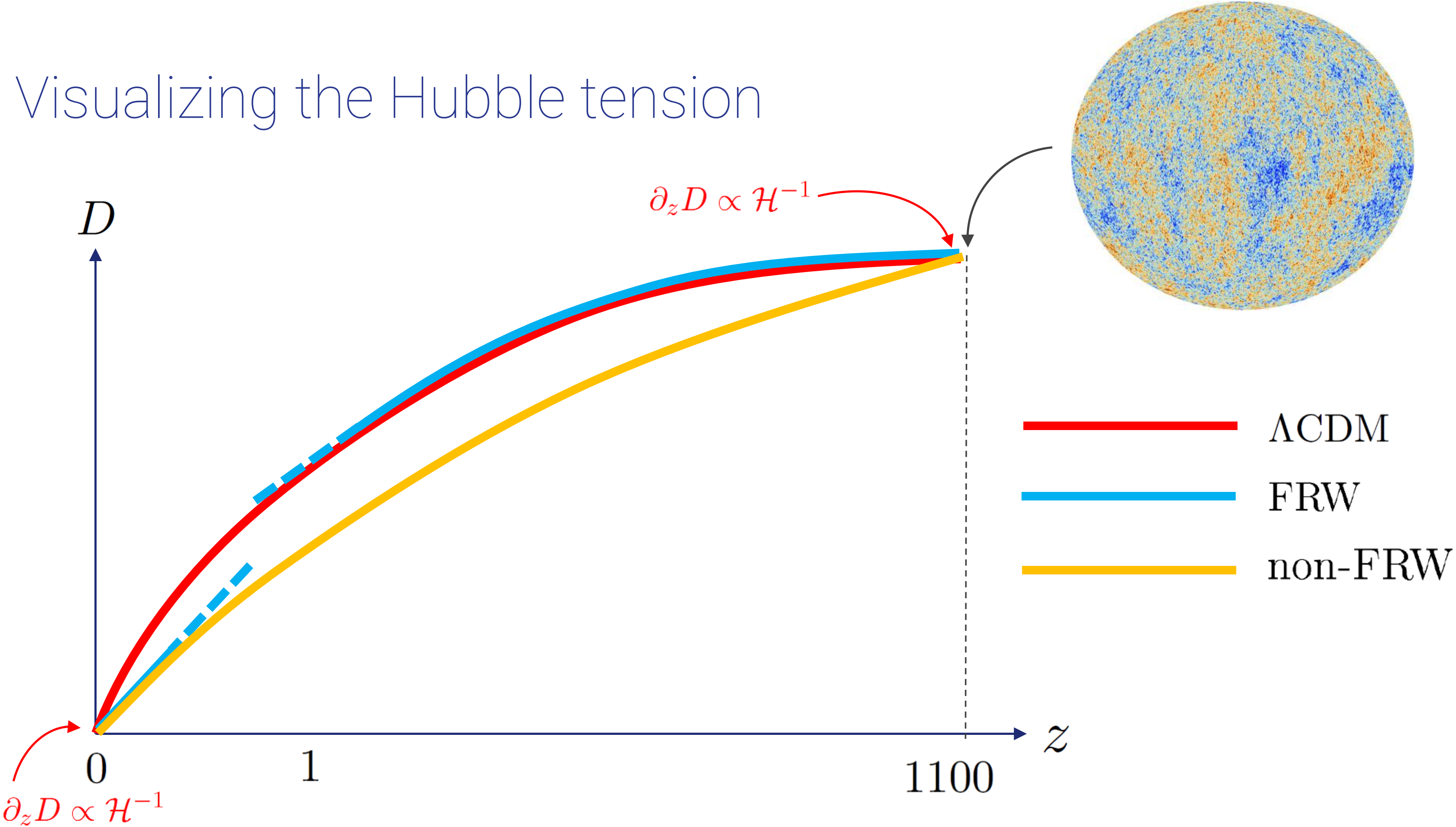

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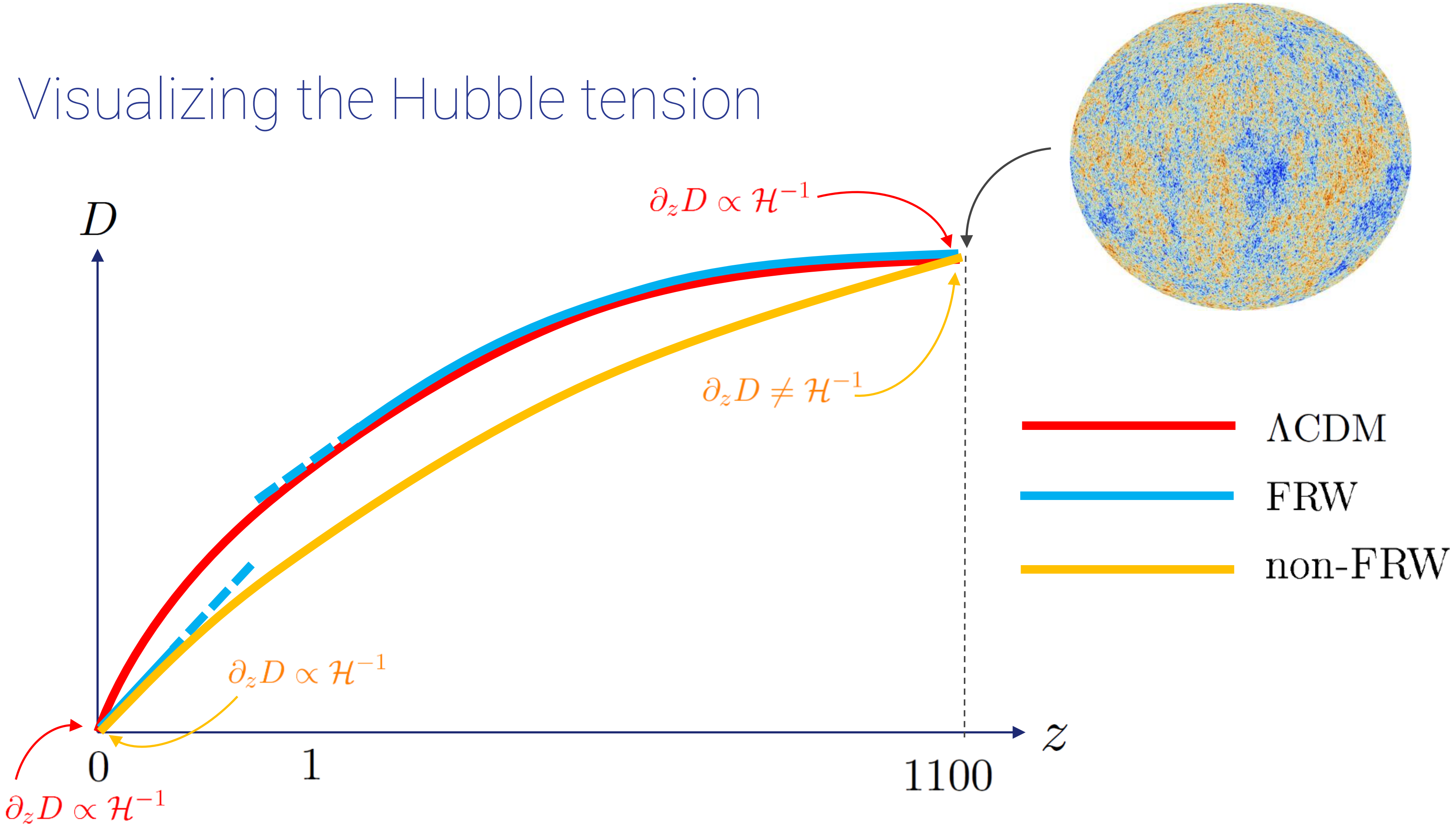

Some example universes

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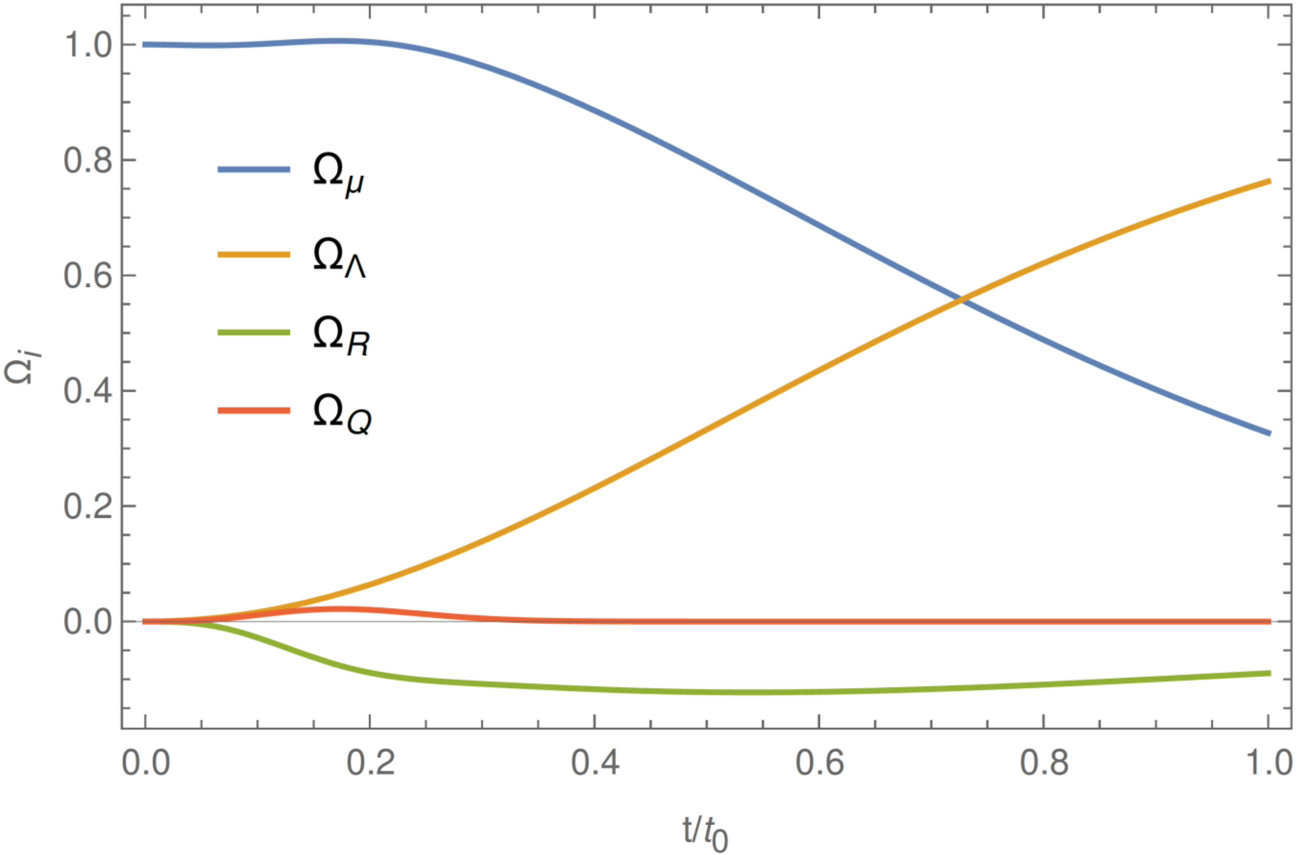

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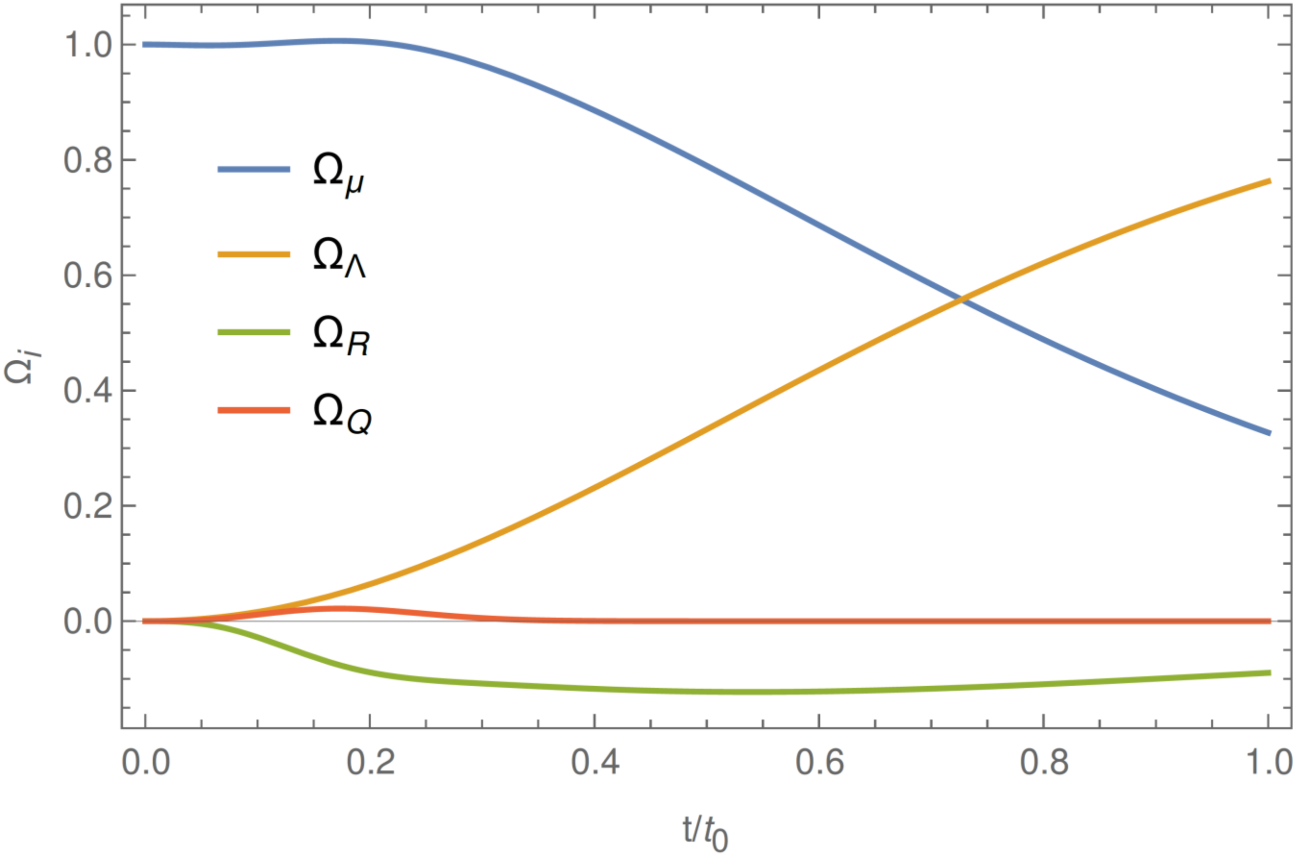

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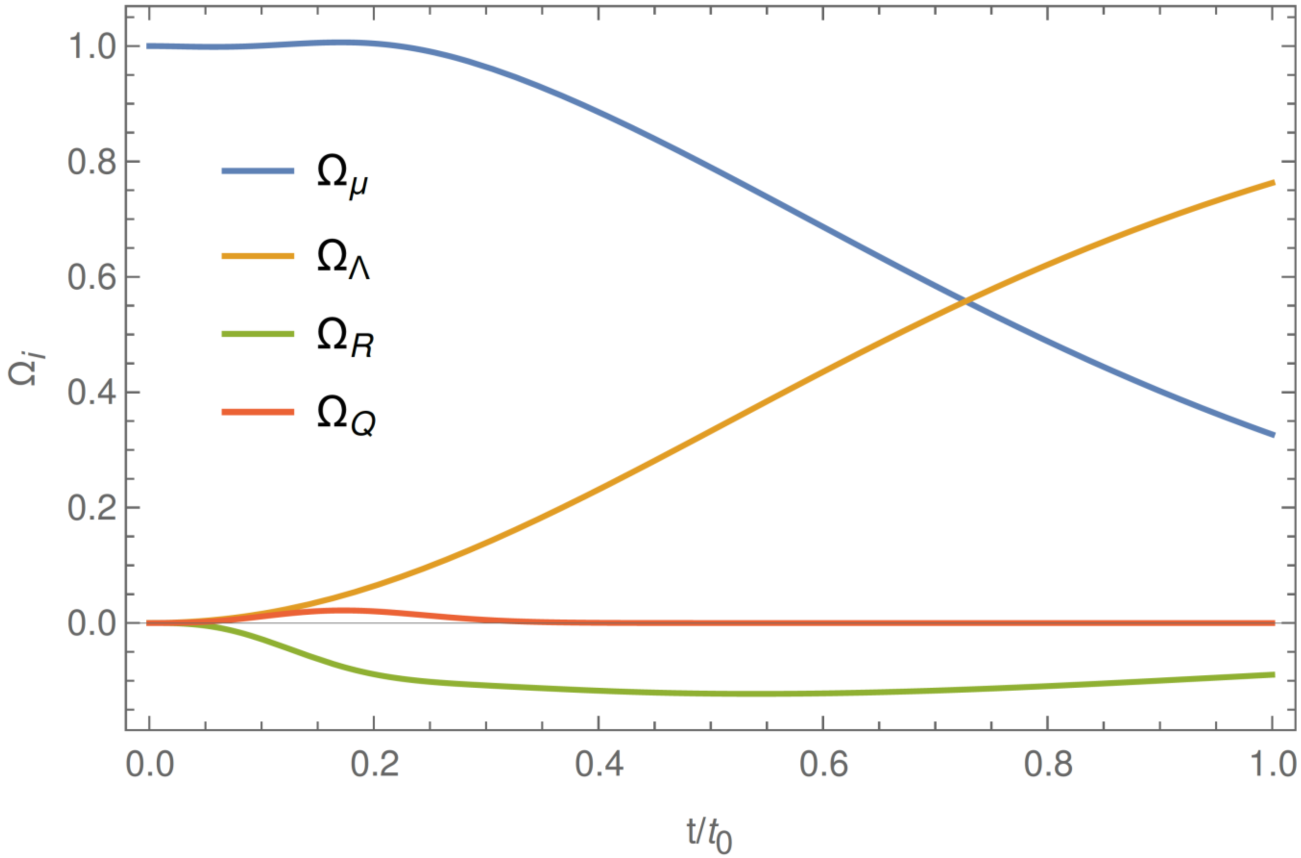
model (a)



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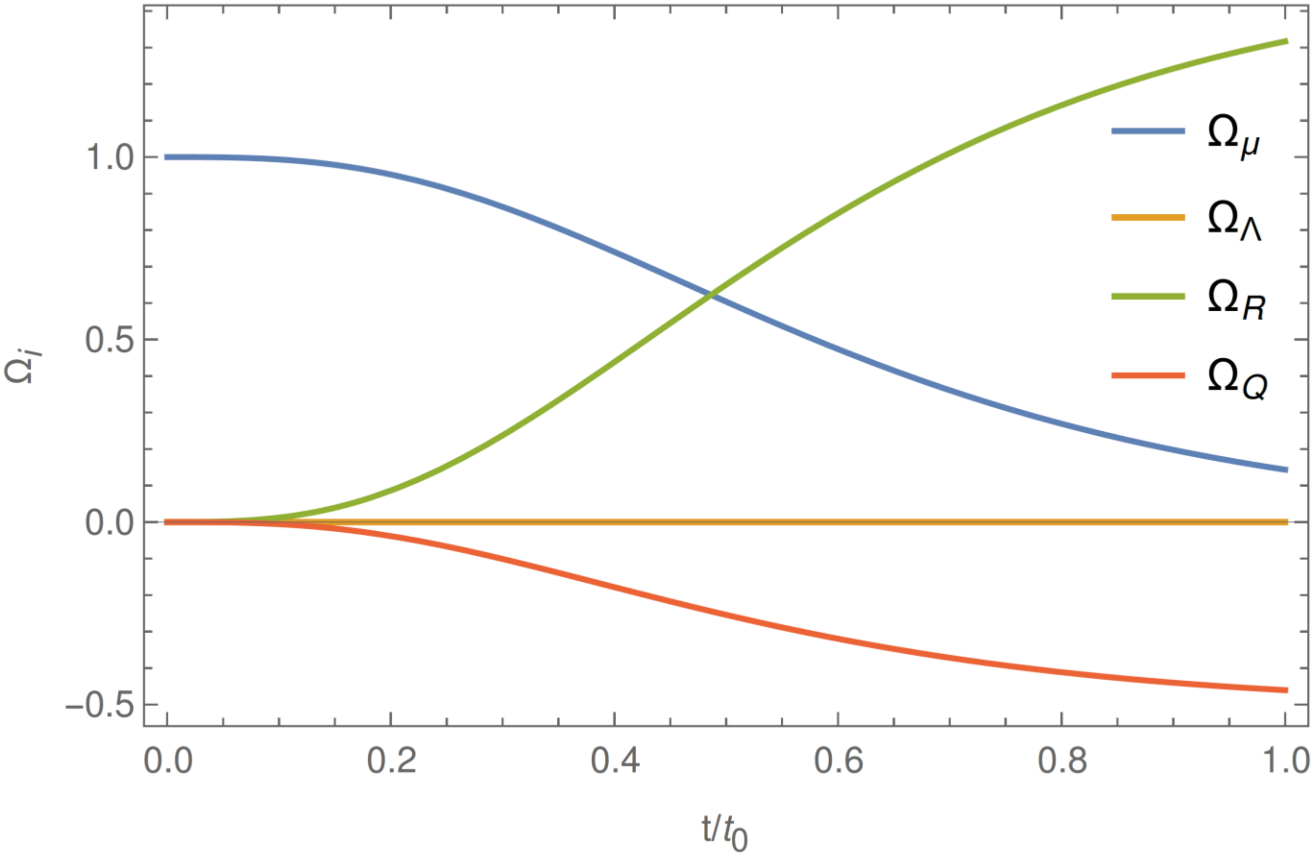

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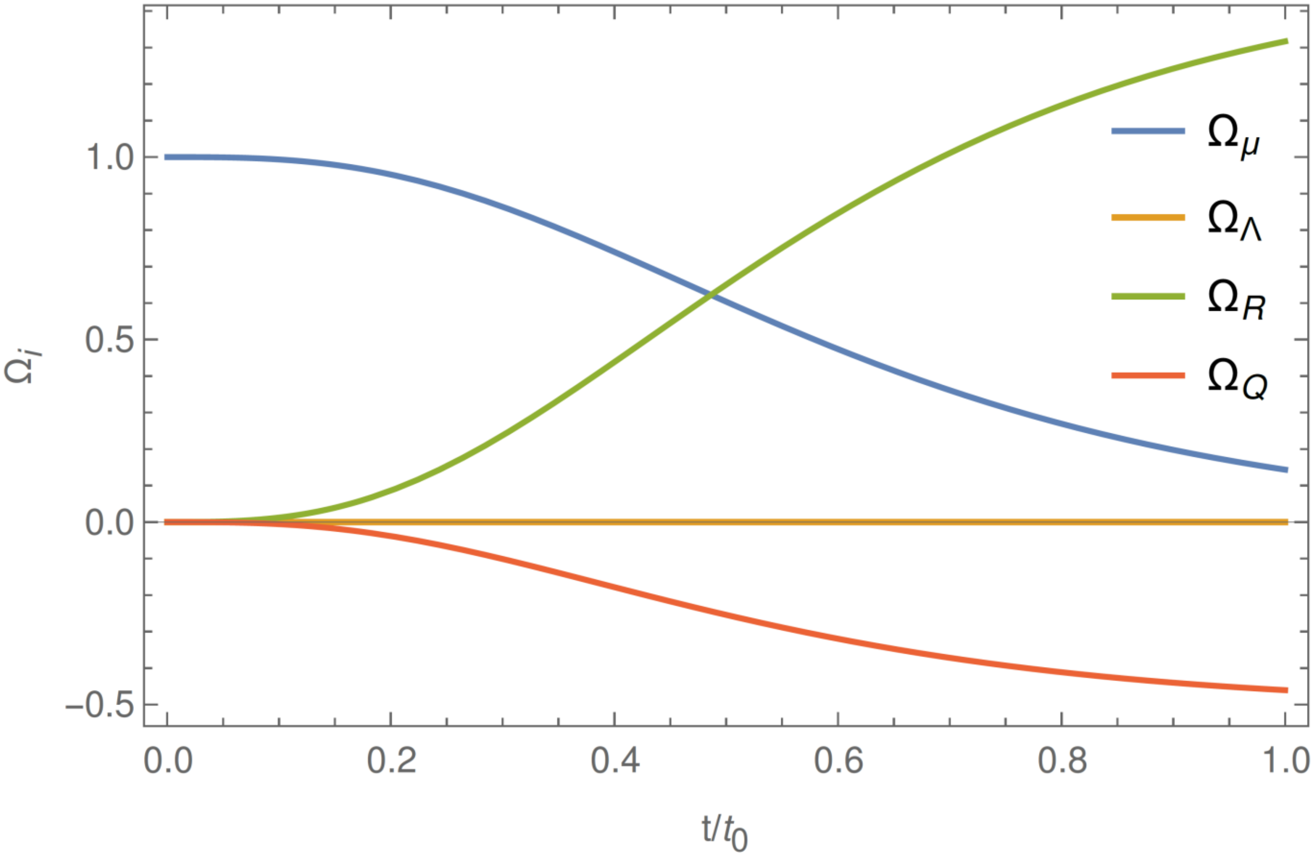

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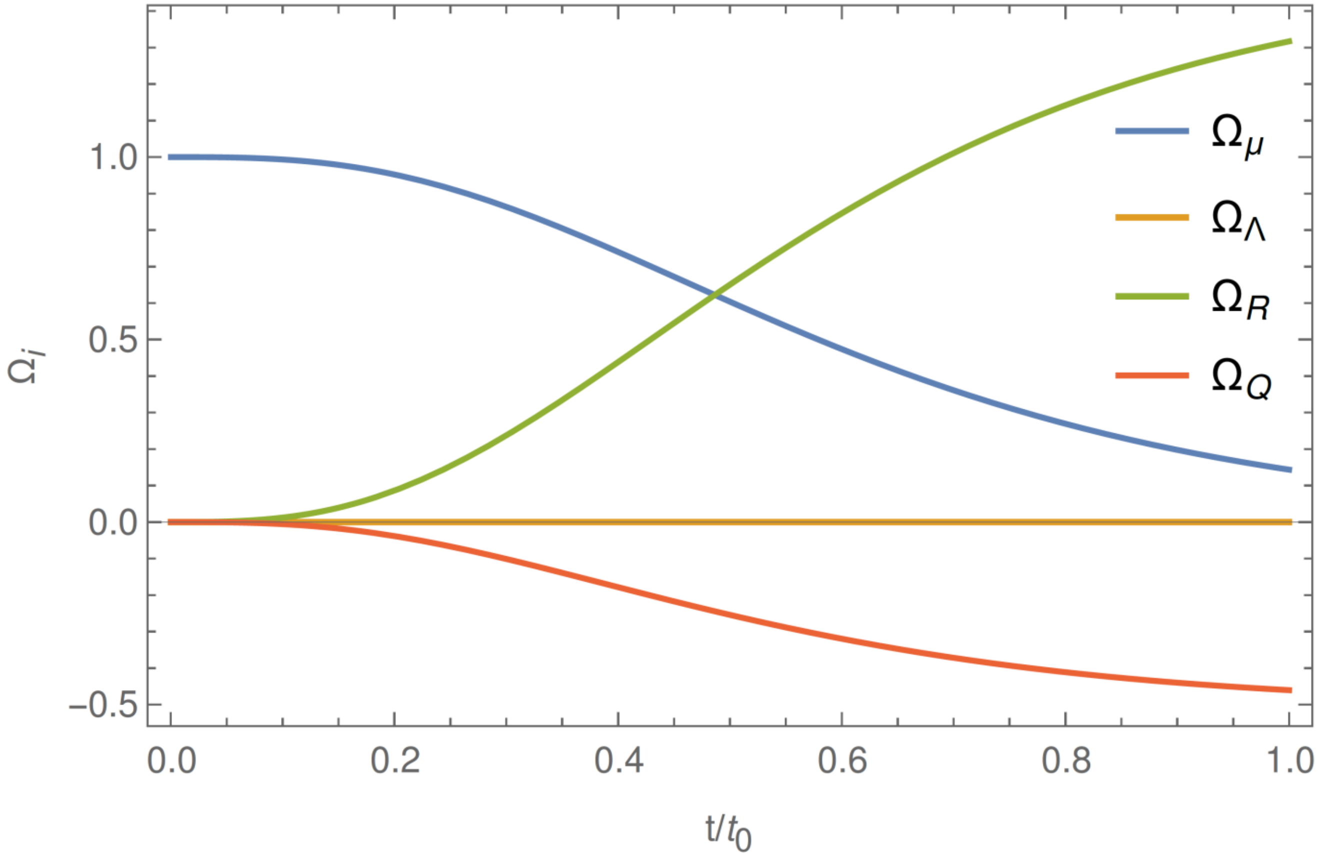
model (b)



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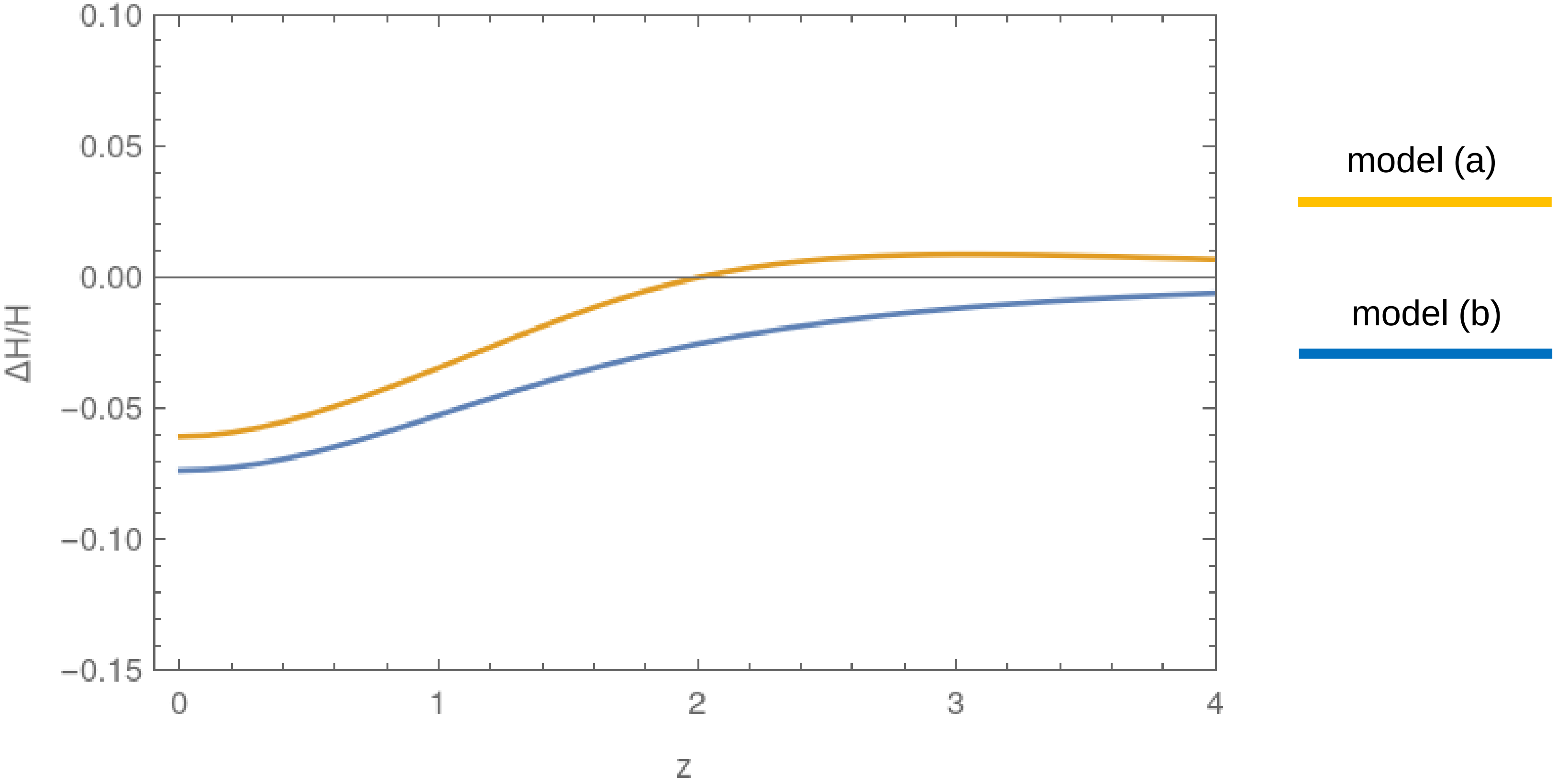
Difference from FRW



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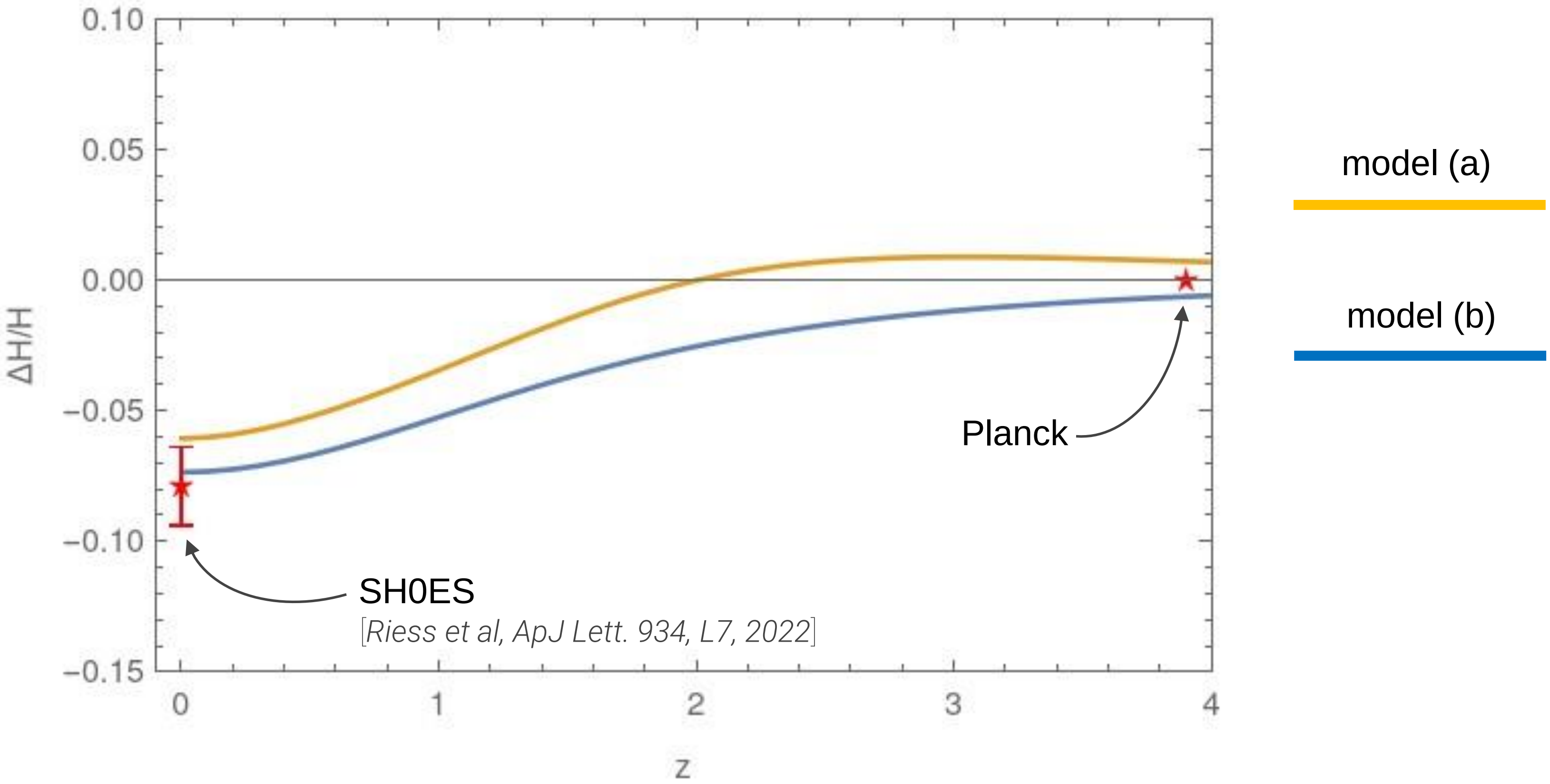

model (a)

model (b)

Planck

SH0ES

[*Riess et al, ApJ Lett. 934, L7, 2022*]

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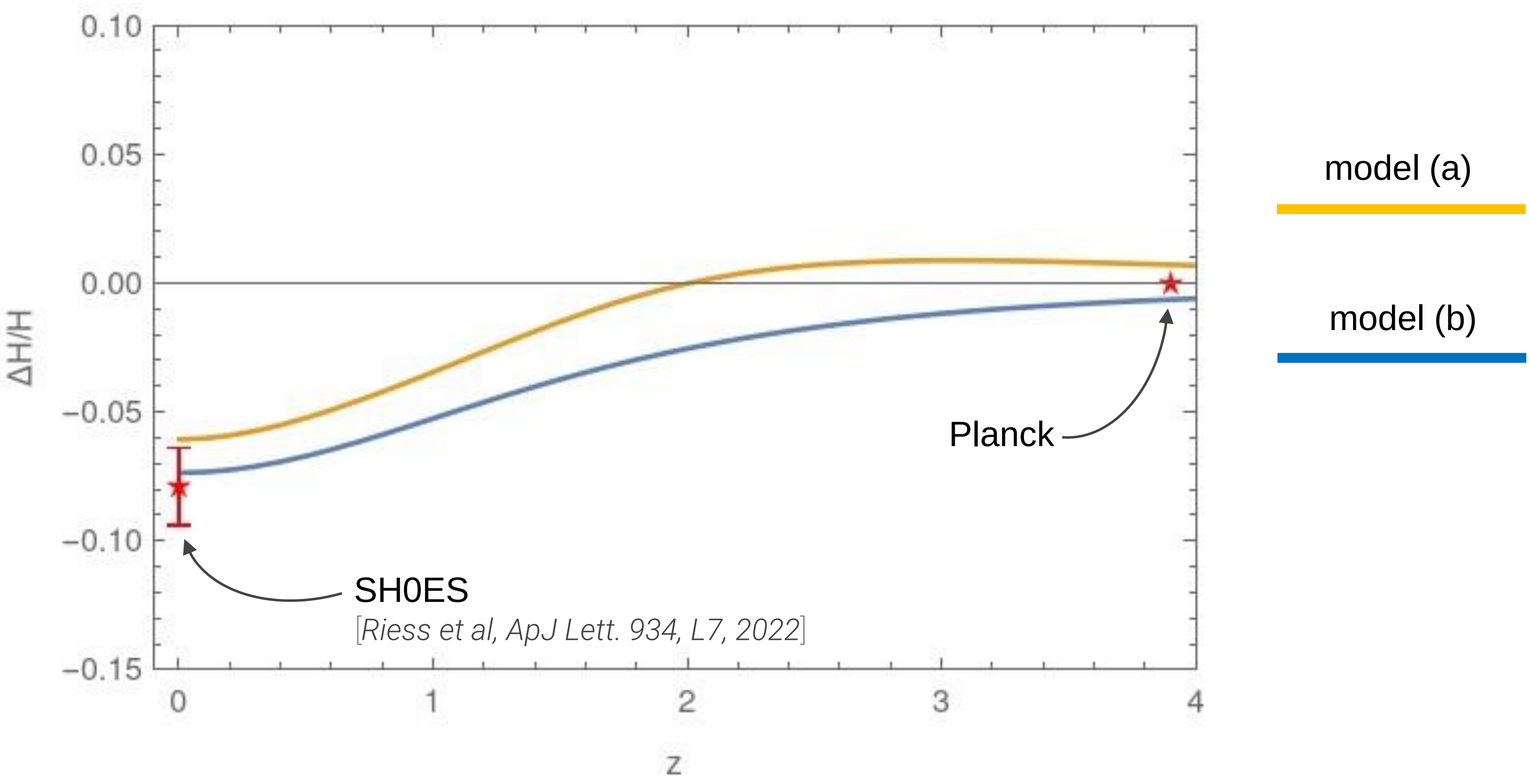
comoving sound horizon

$$\boxed{\frac{\Delta H}{H}(z) \simeq \alpha_{\parallel}(z) - 1}$$

Back to difference from FRW



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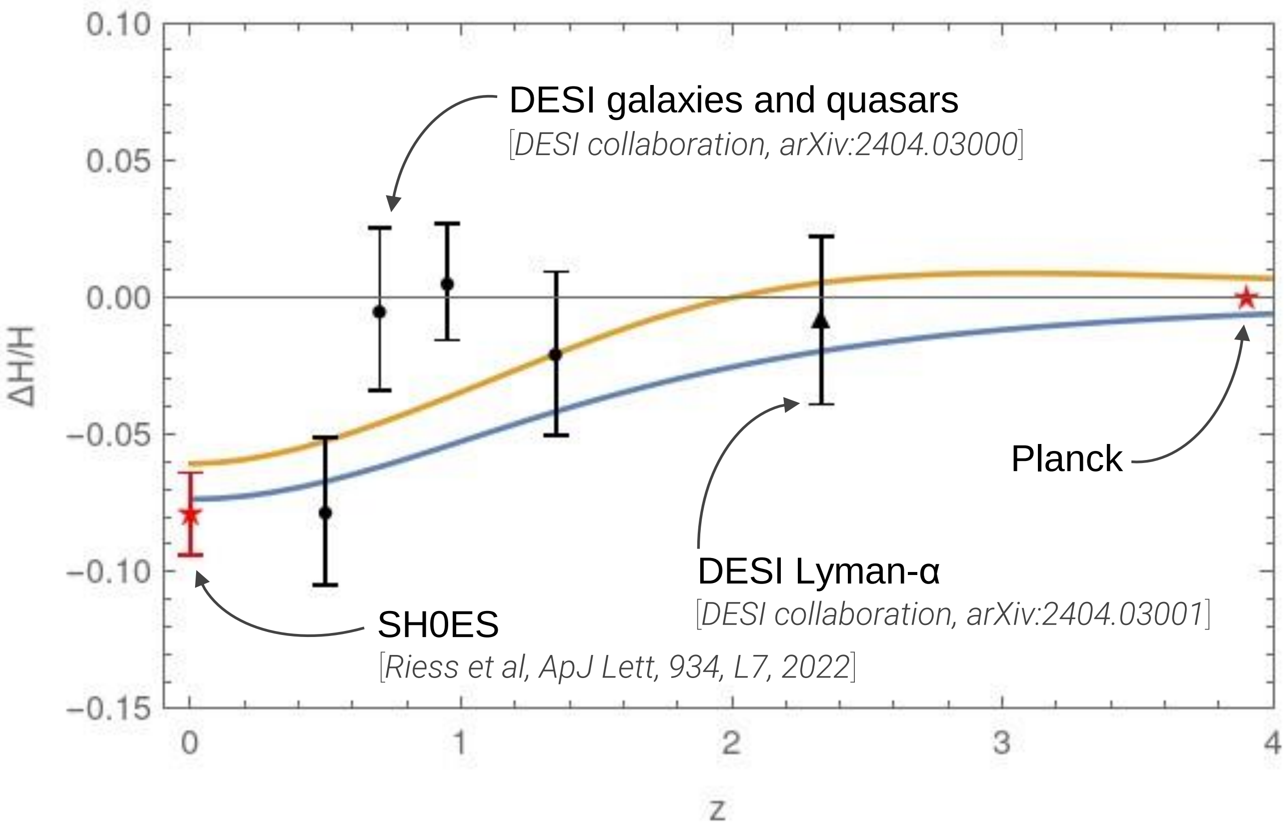

DESI galaxies and quasars

[DESI collaboration, arXiv:2404.03000]

Planck

DESI Lyman-α

[DESI collaboration, arXiv:2404.03001]

SH0ES

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model (a) 🟨

model (b) 🟦

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Thank you