Symphony of Spacetime: Tuning into EMRI Resonances with LISA

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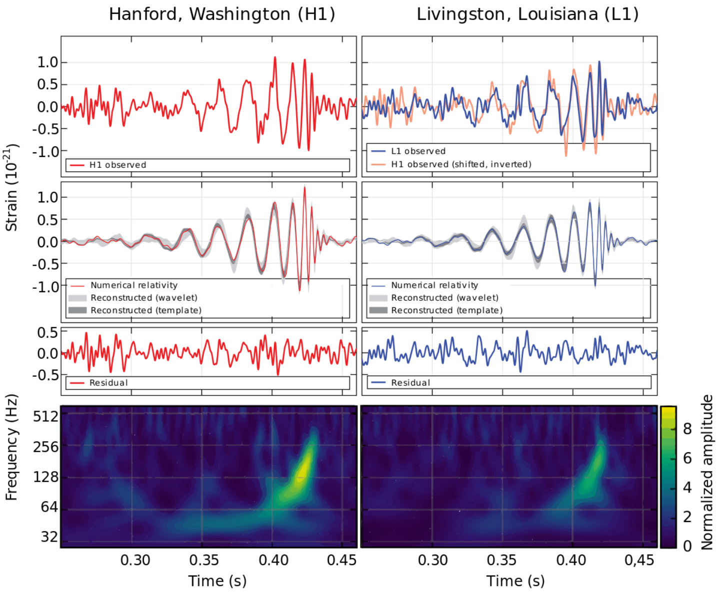
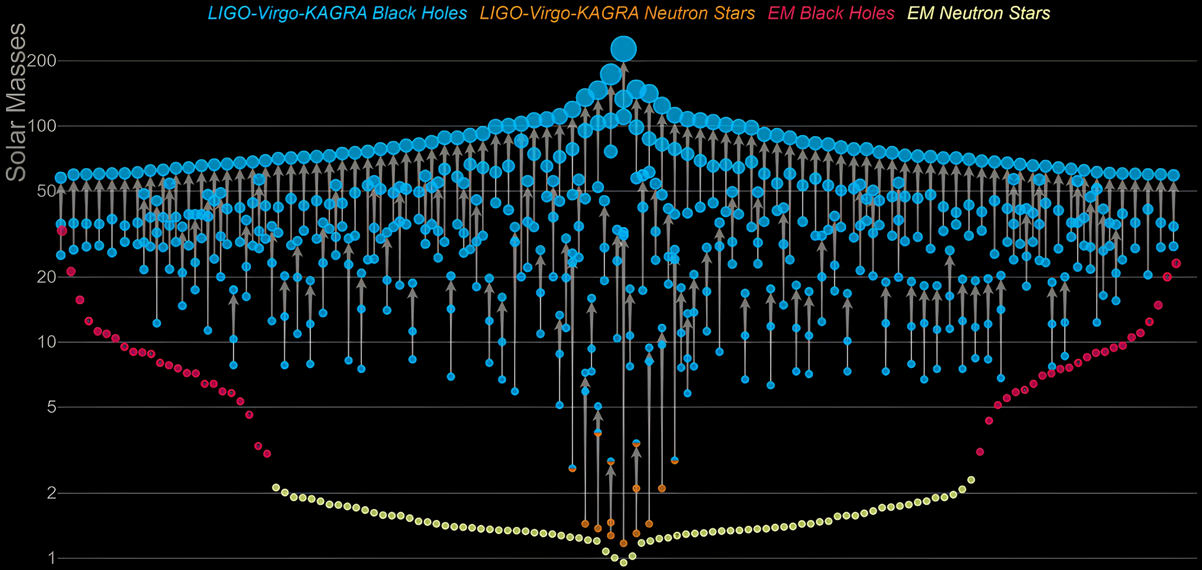


[LIGO and Virgo Collaboration, PRL (2016)]

LIGO/Virgo/KAGRA:

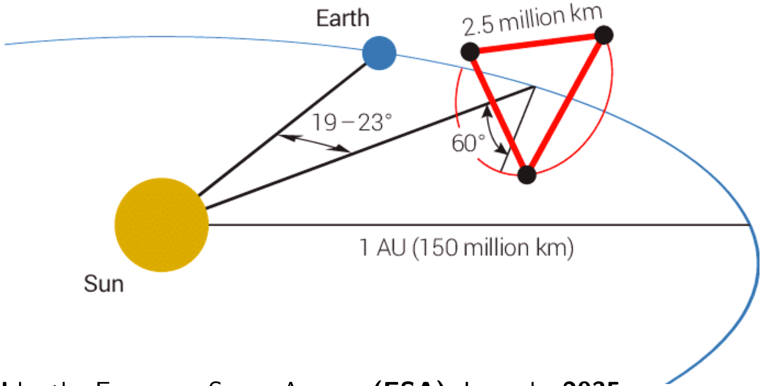
- ground-based detectors.
- 4th observation run.
- Increasing sensitivity.
- kHz band.
- 300+ detections.

Masses in the Stellar Graveyard



LIGO-Virgo-KAGRA | Aaron Geller | Northwestern

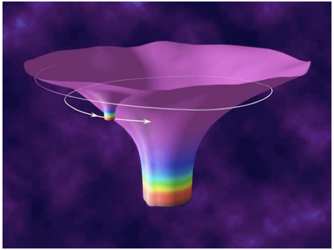
Laser Interferometer Space Antenna (LISA)



- **Adopted** by the European Space Agency **(ESA)**. Launch: **2035**.
- **Multiple targets:** supermassive black-hole binaries, **extreme-mass-ratio inspirals**, compact binaries in the Milky Way, tests of GR, nature of black holes, environmental effects, dark matter, ΛCDM, stochastic gravitational-wave background, inflation.
- **Greece plays an active role (through the ΛISA-GR Member Group)** in the **LISA Consortium**. [Karnesis+, IJMPD (2024)]

Extreme-mass-ratio inspirals

- **Extreme–mass–ratio inspirals (EMRIs): primary supermassive** black hole (M) + **secondary stellar-mass** compact object (μ).
- **Originate in** dense stellar clusters and **galactic cores**.
- The **mass ratio of EMRIs** (μ/M) span in the range $\sim \mathbf{10^{-7} - 10^{-4}}$.
- **Generate GWs** in the frequency range $\sim \mathbf{10^{-4} - 10^{-1}}$ **Hz**.
- Perform $\sim \mathbf{10^5}$ **revolutions in the strong-field regime**.
- **Visible for years** in the mHz band **(LISA)**.
- **Rich phenomenology**.



EMRI modeling: adiabatic approximation + NK scheme

Short-timescale evolution: geodesics

Stationary/axisymmetric spacetime: $ds^2 = g_{tt}dt^2 + 2g_{t\phi}dtd\phi + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\phi\phi}d\phi^2$

Geodesic equations: $\ddot{x}^\kappa + \Gamma^\kappa_{\lambda\nu}\dot{x}^\lambda\dot{x}^\nu = 0$

Stationarity: $\mathcal{E} = -\mu\left(g_{tt}\dot{t} + g_{t\phi}\dot{\phi}\right)$, **Axisymmetry:** $\mathcal{L}_z = \mu\left(g_{t\phi}\dot{t} + g_{\phi\phi}\dot{\phi}\right)$

Carter constant $\leftrightarrow$ separation of radial and polar motion $\leftrightarrow$ **integrability**

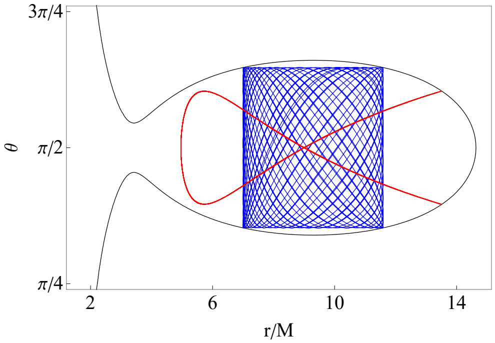
Non-integrability $\leftrightarrow$ **chaos** [Contopoulos, (2002)]

Long-timescale evolution: gravitational radiation reaction (made easy?)

Adiabatic approximation + numerical kludge (NK) scheme: Change in the momenta $(\mathcal{E},\, \mathcal{L}_z,\, \mathcal{Q})$ is small over a single orbit $\rightarrow$ **approximate the dissipative fluxes** with post-Newtonian formulae [Glampedakis+, PRD (2002), Gair+, PRD (2008), Apostolatos+, PRL (2009)]

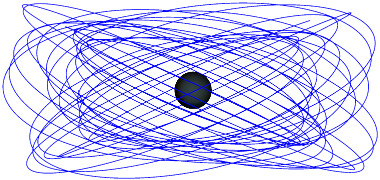
- **Short timescales:** orbit neglects radiative backreaction
- **Long timescales:** inspiral behaves like a **'flow' through successive geodesics**

(r,θ)-bound motion

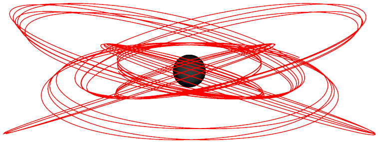


$3D$ Cartesian coordinates

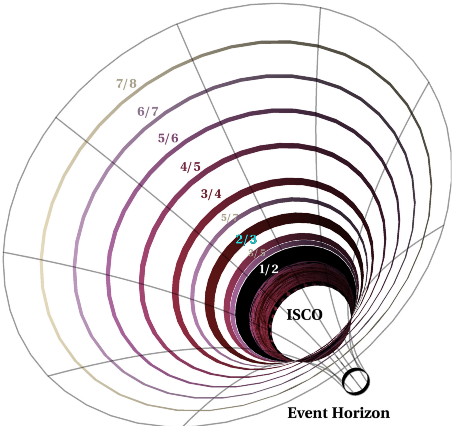
Generic orbit: $\omega_r/\omega_\theta =$ irrational



Resonant orbit: $\omega_r/\omega_\theta =$ rational



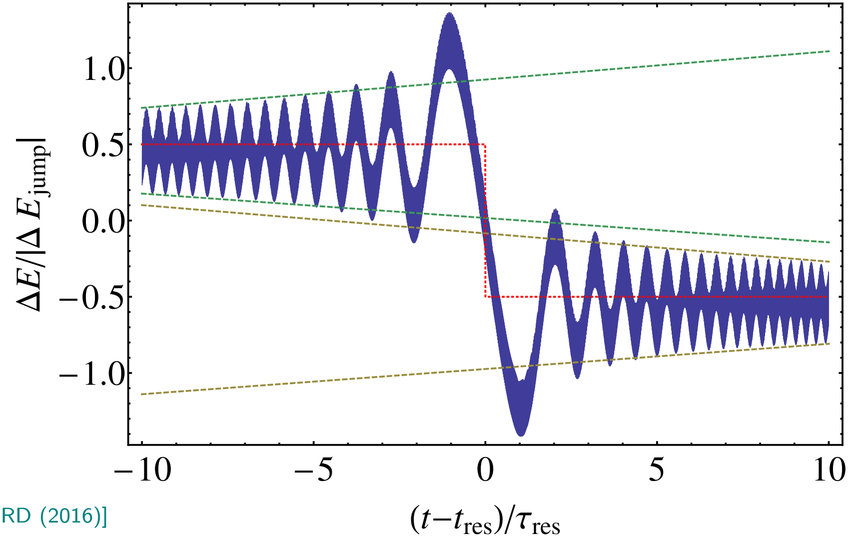
Resonances in Kerr EMRIs



[Brink, Geyer, Hinderer, PRL (2015)]

- **Resonances are ubiquitous** in EMRIs.
- Occur when **two orbital frequencies form a rational ratio**.
- Last for **dozens of cycles** in Kerr. [Ruangsri+, PRD (2014), Brink+, PRD (2015)]
- Lead to **resonant "kicks"** [Flanagan, Hinderer, PRL (2012), Berry+, PRD (2016)]
- **Impact parameter estimation** [Berry+, PRD (2016), Speri+, PRD (2021)]

Resonant kick in fluxes of a Kerr EMRI $(2:3)$



[Berry+, PRD (2016)]

Effective resonance model: including resonant effects in NK

Since the **leading-order dissipative component** of the self-force alone is **insufficient to model the effect of resonances**, one must find a way to **enhance the NK fluxes**.

The **effective rosenance model (ERM) simulates the resonant contribution to the fluxes** by adding to the NK the following **impulse function** $w(t)$: [Speri+, PRD (2021)]

$$\frac{dJ_i}{dt} = f_{\rm NK}\left[1 + \mathcal{C}_i\, w(t)\right], \qquad w(t) = \begin{cases} \dfrac{1+\cos\left[4\pi\left(\frac{t-t_0}{t_{\rm res}}\right)^2\right]}{\int_{-1/2}^{+1/2} 1+\cos[4\pi x^2]\, dx}, & \text{t} \in \left[t_{\rm start}, t_{\rm start} + t_{\rm res}\right], \\ 0, & \text{elsewhere}, \end{cases}$$

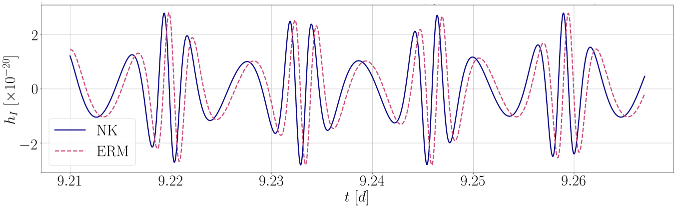
where $J_i = (\mathcal{E}, \mathcal{L}_z, \mathcal{Q})$, $\mathcal{C}_i = (\mathcal{C}_{\mathcal{E}}, \mathcal{C}_{\mathcal{L}_z}, \mathcal{C}_{\mathcal{Q}})$ are the **resonance coefficients**.

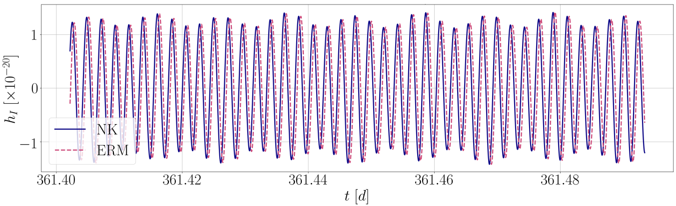
The **resonance duration** can be found by a Taylor expansion, resulting to

$$t_{\rm res} = \sqrt{\frac{4\pi}{|l^* \dot{\omega}_{r_0} + m^* \dot{\omega}_{\theta_0}|}},$$

where $l^* \omega_{r_0} + m^* \omega_{\theta_0} = 0$ at $t = t_0$ is the **resonance condition**. [Ruangsri+, PRD (2014)]

Dephasing between NK and NK+ERM ($2:3$ resonance)



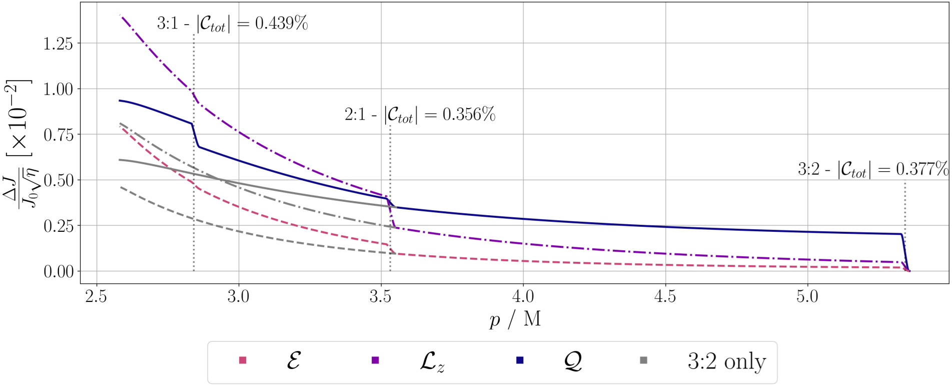


EMRI$_1$(top): $a = 0.80$, $p/M = 7.03$, $e = 0.40$, $\iota = 0.70$, $\mathcal{M}_{\rm top} = 0.24$.

EMRI$_2$(bottom): $a = 0.80$, $p/M = 8.46$, $e = 0.20$, $\iota = 1.10$, $\mathcal{M}_{\rm bottom} = 0.99$.

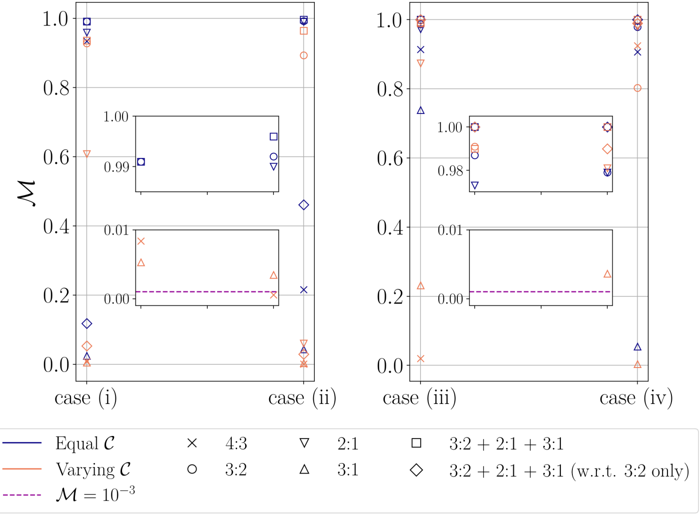
[Levati, Cardenas-Avendanõ, **KD**, Pani, PRD (2025)]

Multiple resonance-crossing with ERM: fluxes



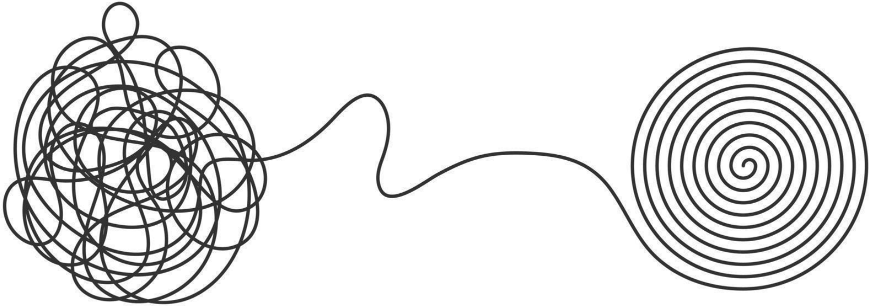
- Taking into account the $(\mathbf{3:2})$ **already affects the inspiral** and resulting GW. [Flanagan+, PRL (2012), Speri+, PRD (2021)]
- Taking into account **sub-dominant resonances affects the inspiral even more** when compared with the inspiral that only includes $(3:2)$. [Levati, Avendanõ, **KD**, Pani, PRD (2025)]

GW mismatch between NK and NK+multiple ERM



[Levati, Cardenas-Avendanõ, **KD**, Pani, PRD (2025)]

Chaos and Order



EMRIs and chaos: a non-Kerr primary

[**KD**, Suvorov, Kokkotas, PRD (2020)]

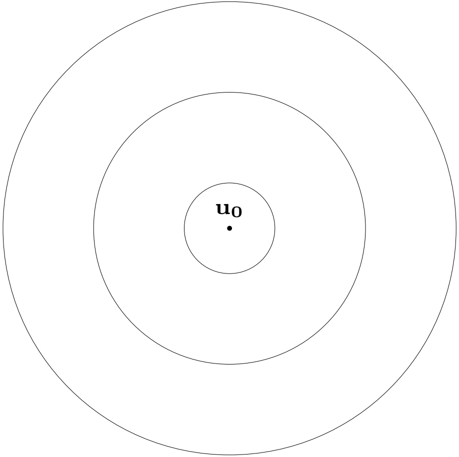
$$ds^2 = -\frac{\Sigma[(\alpha_Q/r)\,M^3 + \Delta - a^2 A(r)^2 \sin^2\theta]}{[(r^2+a^2) - a^2 A(r)\sin^2\theta]^2}dt^2 - \frac{2a[(r^2+a^2)A(r) - \Delta]\Sigma \sin^2\theta}{[(r^2+a^2) - a^2 A(r)\sin^2\theta]^2}dt d\phi$$
$$+ \frac{(\alpha_Q/r)\,M^3 + \Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + \frac{\Sigma \sin^2\theta\left[(r^2+a^2)^2 - a^2\Delta\sin^2\theta\right]}{[(r^2+a^2) - a^2 A(r)\sin^2\theta]^2}d\phi^2,$$

$$\Sigma = r^2 + a^2\cos^2\theta,\ \Delta = r^2 - 2Mr + a^2,\ A(r) = 1 + r^{-2}\alpha_{22}M^2.$$

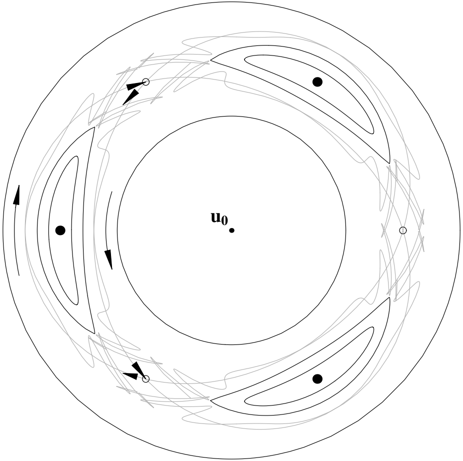
- α_Q controls **integrability**, $\alpha_Q \neq 0 \to$ **no Carter constant** (**"non-Kerr"**)
- α_{22} affects **frame-dragging**, $\alpha_Q = 0$ and $\alpha_{22} \neq 0 \to$ **Carter constant** (**"Kerr-like"**)
- $\alpha_Q = \alpha_{22} = 0 \to$ **Kerr metric**
- **Perturbed quadrupole moment:** $M_2 = -Ma^2 - \frac{1}{3}M^3\alpha_Q$

Poincaré map: Signatures of non-integrability

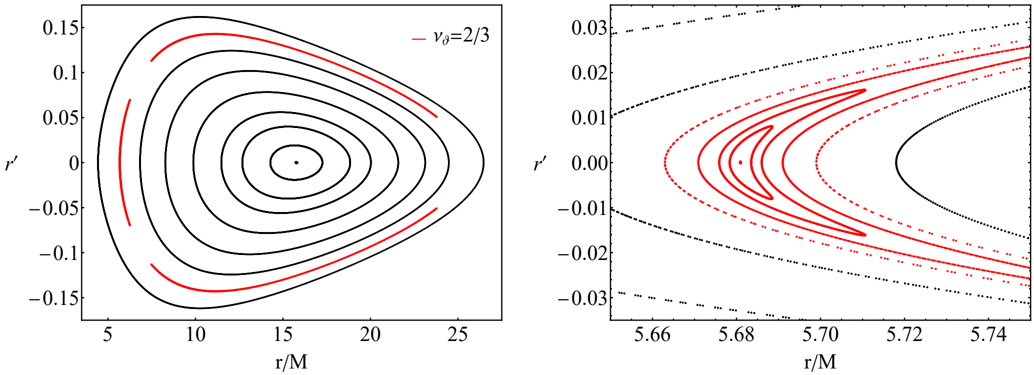
integrability



non-integrability



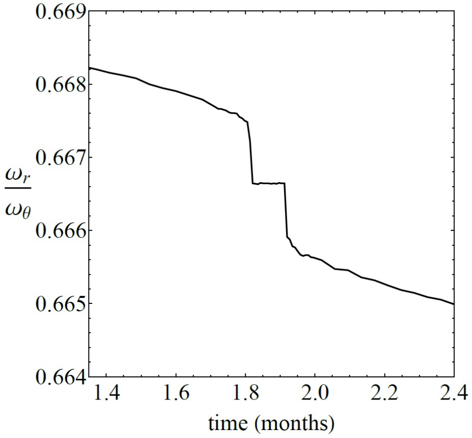
[Lukes-Gerakopoulos+, PRD (2010)]

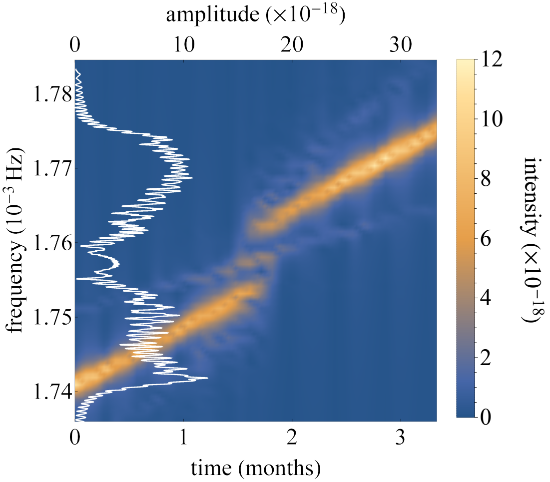


[**KD**, Kokkotas, GRG (2023)]

- Away from resonances, **the curves surround the central point of the map**.
- **Stable periodic points** are encapsulated by **nested islands**.
- Geodesics share the **same rational ratio $\nu_\vartheta = \omega_r/\omega_\theta$ throughout the island**.

Signatures of non-integrability in EMRIs



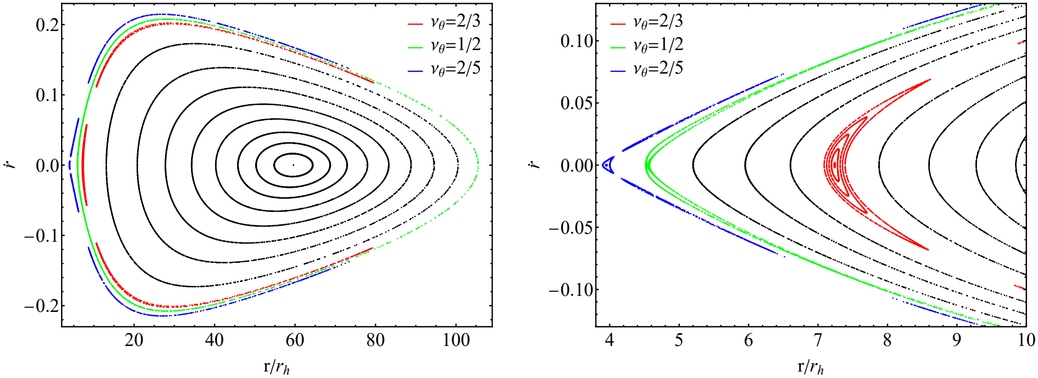


Average crossing time: $\sim$ **300 cycles**

[**KD**, Suvorov, Kokkotas, PRL (2021)]

Outro: Rotating BHs in astrophysical environments and chaos

- **First solution** of a general-relativistic **rotating BH within a matter halo**. [Fernandes, Cardoso, 2507.04389]
- **First geodesic analysis:** General-relativistic, rapidly-rotating BH within a compact matter halo → **environmentally-induced chaos!** [**KD**, Fernandes, 2508.20191]



[**KD**, Fernandes, 2508.20191]

Conclusions

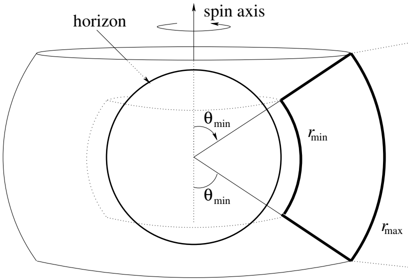
- **Thousands of EMRIs are expected** to be detected by LISA. [Gair+, J. Phys. (2017)]
- **Precise modeling of EMRI waveforms is still under construction** due to the **perplexity of gravitational self-force**. [Barack, CQG (2009)]
- **EMRI modeling approaches** (relativistic, or post-Newtonian) are in full effect and **constantly updated** to include various effects, such as **resonances**. [Katz+, PRD (2021), Ruangsri+, PRD (2014), Berry+, PRD (2016), Speri+, PRD (2021), Gupta+, PRD (2022)]
- Taking into account **the most dominant resonance** $(2:3)$ is not enough, **sub-dominant resonances introduce further dephasing**. [Levati, Cardenas-Avendanõ, **KD**, Pani, PRD (2025)]
- **Non-integrability** is **imprinted in EMRI GWs**. [**KD**, Suvorov, Kokkotas, PRL (2021), **KD**, Kokkotas, PRD (2021), **KD**, Kokkotas, GRG (2023), **KD**, Huez+, GRG (2023), **KD**, Pani+, PRD (2023)]
- **Can (non-)integrability de-circularize initially-spherical EMRIs?** [Eleni, **KD**, Apostolatos, Kokkotas, PRD (2024)] **(see talk by A. Eleni on Thursday to find out!)**
- **High-precision EMRI GW modeling** requires the inclusion of **resonances** and potential **chaotic phenomena** in order to perform **accurate parameter inference**.

Thanks for your attention!

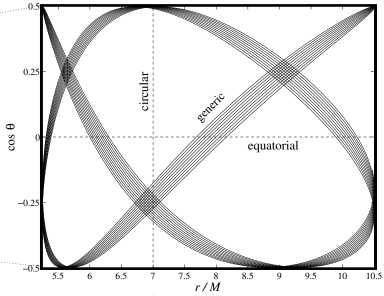
Appendix

LISA targets

- **Extreme–mass–ratio inspirals**
- Supermassive black hole binaries
- Ultra compact binaries in the Milky Way
- Stellar mass black hole binaries years before the merger
- Stochastic gravitational-wave background
- Unforeseen and unmodeled sources



[Drasco, Hughes, PRD (2006)]



Motion is characterized by three orbital frequencies: $\omega_r,\, \omega_\theta,\, \omega_\phi$

Initial parameters: $\{\mathcal{E},\, \mathcal{L}_z,\, \vec{x}(0),\, \dot{\vec{x}}(0)\}$ $\quad\longleftrightarrow\quad$ **Alternative parameters:** $\{e,\, p,\, \iota\}$

$$e = \frac{r_{\text{max}} - r_{\text{min}}}{r_{\text{max}} + r_{\text{min}}}, \qquad p = \frac{2 r_{\text{max}}\, r_{\text{min}}}{r_{\text{max}} - r_{\text{min}}}, \qquad \iota = \frac{\pi}{2} - \theta_{\text{min}}$$

EMRI evolution scheme

The calculation proceeds via the following steps:

1. Define the EMRI initial parameters M, μ, a and α_Q, α_{22}
2. Initialize the trajectory with E_0, $L_{z,0}$ and $(r(0), \theta(0), \dot{r}(0), \dot{\theta}(0))$
3. Calculate the orbital elements e, p, ι
4. Approximate the fluxes

$$\left\langle \frac{dE}{dt} \right\rangle_{\text{2PN}} = \frac{\mu}{M^2} f_E(e, p, \iota, M_2), \; \left\langle \frac{dL_z}{dt} \right\rangle_{\text{2PN}} = \frac{\mu}{M} f_L(e, p, \iota, M_2)$$ [Gair, Glampedakis, PRD (2006)]

5. Evolve the coupled (r, θ) geodesic equations with E, L_z changing adiabatically as:

$$E(t) = E_0 + \langle dE/dt \rangle t, \quad L_z(t) = L_{z,0} + \langle dL_z/dt \rangle t$$

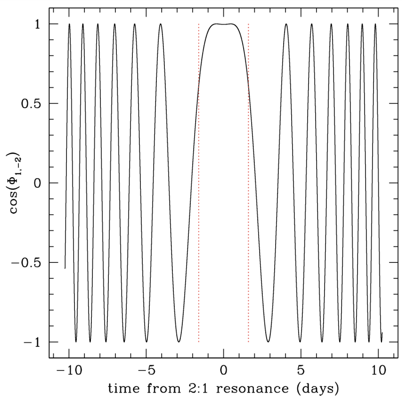
6. Update the fluxes every N_r cycles and repeat from step 2

Phase variables and resonance duration

$$\frac{dJ_k}{d\tau} \sim \dot{J}_k e^{-i\Phi_k(t)}$$

Resonance duration:

$$\Phi_k(t) = \Phi_k(t_{\rm res}) + (l^*\omega_\theta + m^*\omega_r)(t - t_{\rm res}) + \frac{1}{2}(l^*\dot{\omega}_\theta + m^*\dot{\omega}_r)(t - t_{\rm res})^2 + \ldots$$
$$\simeq \Phi_k(t_{\rm res}) + \frac{1}{2}(l^*\dot{\omega}_\theta + m^*\dot{\omega}_r)(t - t_{\rm res})^2.$$



[Ruangsri+, PRD (2014)]

Resonance	e	ι	p/M	$\mathrm{t_{res}}$ [d]	$\mathrm{T_{max}}$ [d]	$\lvert\mathcal{C}_{\mathcal{E}}\rvert$	$\lvert\mathcal{C}_{\mathcal{L}_z}\rvert$	$\lvert\mathcal{C}_{\mathcal{Q}}\rvert$	$\lvert\mathcal{C}_{\mathrm{tot}}\rvert$
Case (i)									
4:3	0.30	0.35	7.46	5.7	362	0.00001	0.00001	0.00006	0.00008
3:2	0.30	0.35	5.36	2.3	84	0.00102	0.00067	0.00208	0.00377
2:1	0.30	0.35	3.59	0.7	10	0.00131	0.00179	0.00046	0.00356
3:1	0.30	0.35	2.92	0.2	1	0.00059	0.00070	**0.00310**	0.00439
Case (ii)									
4:3	0.30	1.22	11.36	14.6	365	0.00003	0.00004	0.00002	0.00009
3:2	0.30	1.22	8.67	6.5	365	0.00303	0.00123	0.00123	0.00549
2:1	0.30	1.22	6.18	2.0	55	0.00004	0.00080	0.00002	0.00086
3:1	0.30	1.22	5.07	0.5	5	0.00008	0.00024	0.00033	0.00065
Case (iii)									
4:3	0.70	0.35	7.58	11.4	365	0.00001	0.00002	0.00023	0.00026
3:2	0.70	0.35	5.50	4.7	144	0.00127	0.00078	0.00210	0.00415
2:1	0.70	0.35	3.82	1.5	30	0.00167	0.00067	0.00357	0.00591
3:1	0.70	0.35	3.28	0.5	17	0.00026	0.00009	0.00035	0.00070
Case (iv)									
4:3	0.70	1.22	11.47	28.2	365	0.00047	**0.00060**	0.00002	0.00109
3:2	0.70	1.22	8.80	12.4	365	**0.01030**	0.00489	0.00261	0.01780
2:1	0.70	1.22	6.36	3.7	88	**0.00662**	0.00270	0.00494	0.01426
3:1	0.70	1.22	5.41	0.8	5	0.00125	0.00027	0.00126	0.00278

Table 1: Parameter-space location and resonance coefficients for the investigated orbital resonances. We use $a = 0.9$, $\eta = 10^{-5}$ and $M = 10^6 M_\odot$. [Flanagan+, PRD (2014), Levati, Cardenas-Avendanõ, **KD**, Pani, PRD (2025)]

Impact of non-integrability: orbital level

$\boldsymbol{w_n}$ **:** n^{th} crossing of the orbit through a surface of section

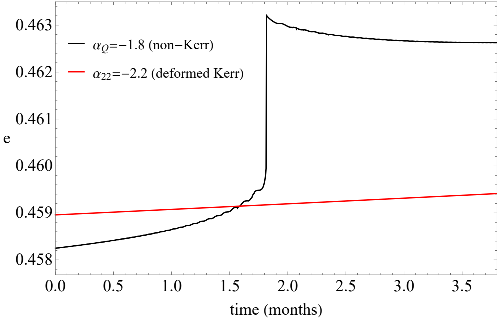
$\boldsymbol{\vartheta_n}$ **:** angle between two successive intersections at $\boldsymbol{w_n}$, $\boldsymbol{w_{n-1}}$

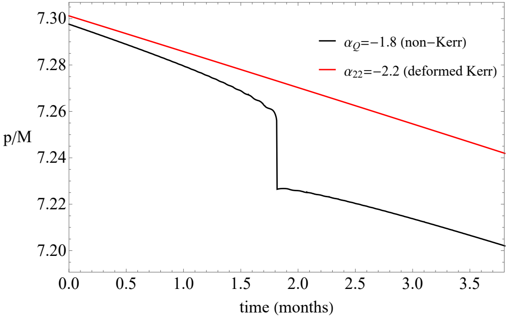
Rotation number:

$$\nu_{\theta,N} = \frac{1}{2\pi N}\sum_{i=1}^{N}\vartheta_i$$

- The limit $N \to \infty$ converges to $\nu_\theta = \omega_r/\omega_\theta$ [Contopoulos (2002)]
- Successive rotation numbers form a rotation curve
- The rotation curve of integrable systems is monotonous
- The rotation curve of non-integrable systems exhibits a plateau inside resonant islands

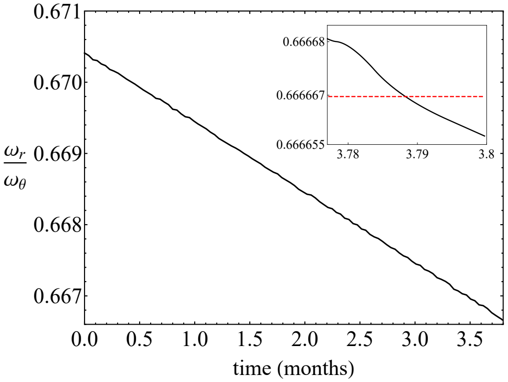
Orbital elements



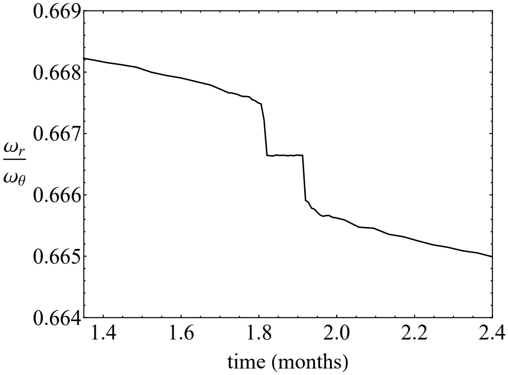


Orbital frequency evolution $(\mu/M = 10^{-6})$

deformed Kerr

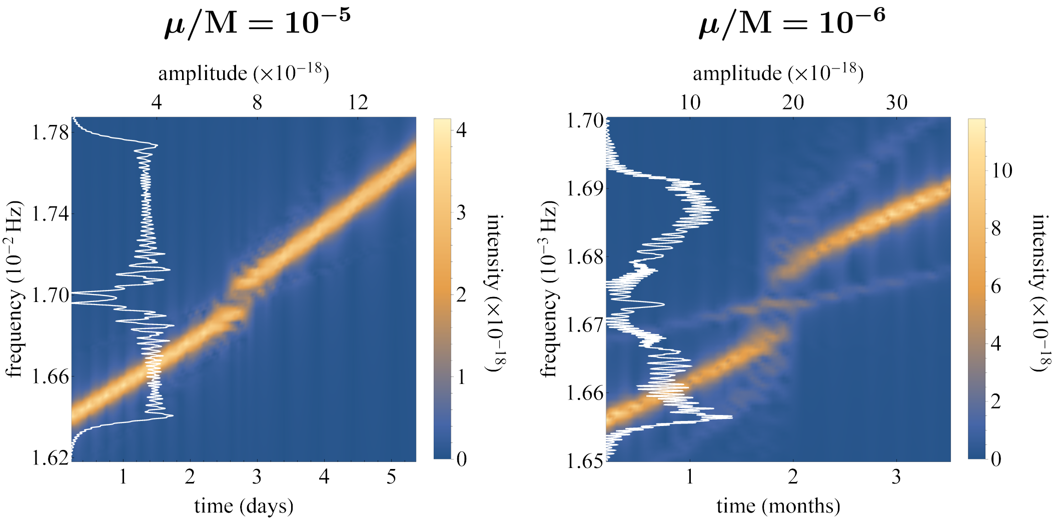


non-Kerr



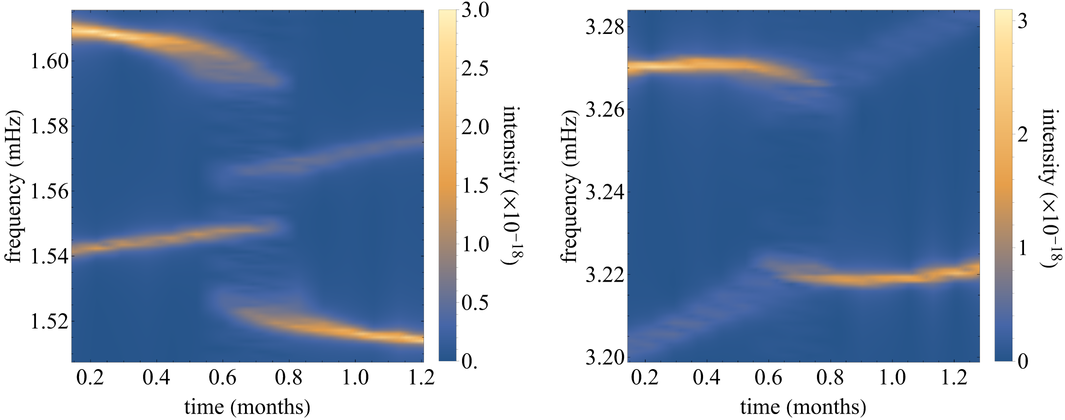
Average time spent in island: ~ 300 cycles

GW glitches in Manko-Novikov EMRIs: a generic feature of chaos



[**KD**, Kokkotas, PRD (2021)]

EMRIs with supermassive self-interacting rotating boson stars



[**KD**, Pani+, PRD (2023)]

Waveform modeling and LISA response

Numerical kludge waveforms: Combine exact particle trajectories with approximate expressions for gravitational-wave (GW) emission [Babak et al., PRD (2007)]

- ‘quick and dirty’ EMRI waveforms, perfectly-suited for phenomenology
- remarkable agreement with Teukolsky-based waveforms ($\sim 95\%$)

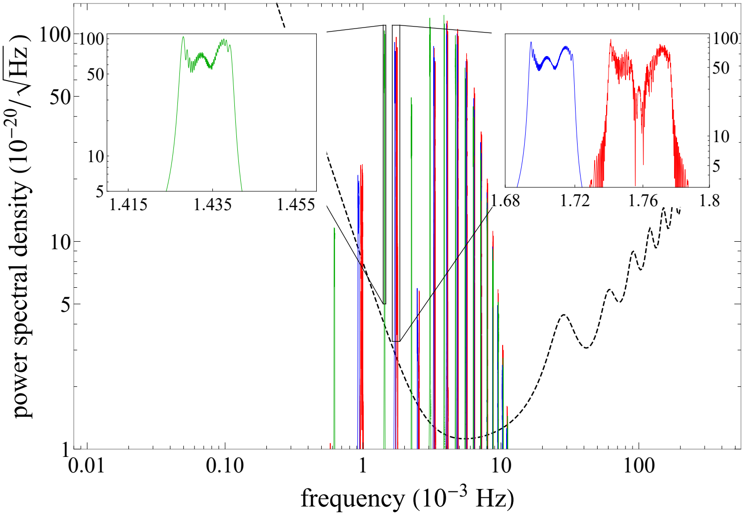
Einstein-quadrupole formula: $h_{ij}^{\mathrm{TT}} = \frac{2}{d}\frac{d^2 I_{ij}(Z^i(t))}{dt^2}$, $\mathbf{Z}(t)$: inspiral trajectory

Incoming GW: $h_{+,\times}(t) = \frac{2\mu}{d}\epsilon_{ij}^{+,\times}\left[\frac{Z^i(t)}{dt^2}Z^j(t) + \frac{Z^i(t)}{dt}\frac{Z^j(t)}{dt}\right]$

LISA response: $h_\alpha(t) \sim [F_\alpha^+(t)h_+(t) + F_\alpha^\times(t)h_\times(t)]$ [Barack, Cutler, PRD (2004)]

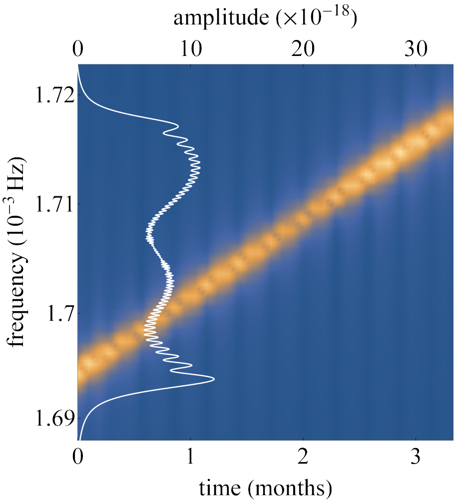
We use a **single-channel approach**, for simplicity, and **omit any noise** in the data stream.

Detectability and GW frequency evolution $(\mu/M = 10^{-6})$

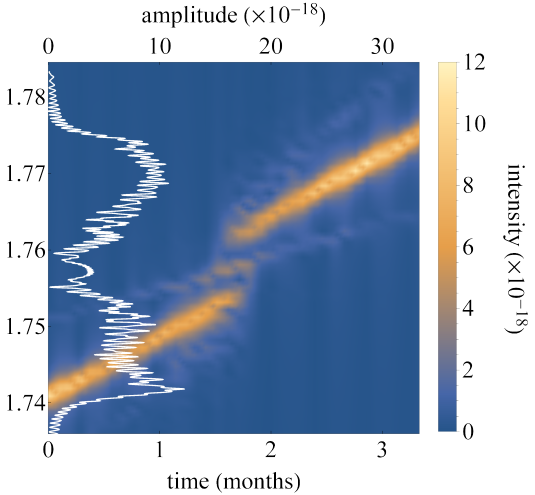


[KD, Suvorov, Kokkotas, PRL (2021)]

deformed Kerr

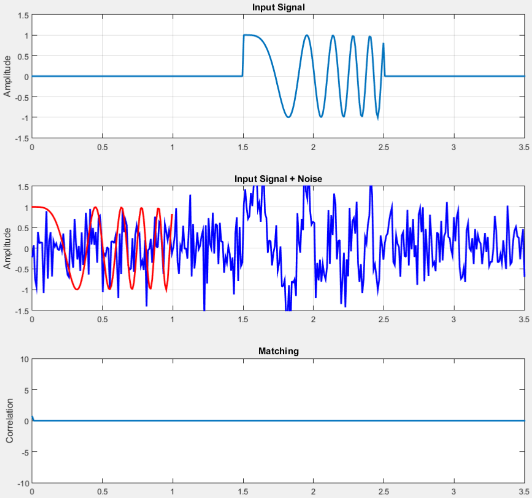


non-Kerr



[KD, Suvorov, Kokkotas, PRL (2021)]

Data analysis: Matched filtering



[Courtesy of: Vitor Cardoso]