NEB21 2/9/2025

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MINISTRY OF EDUCATION, YOUTH AND SPORTS

Extensions of GR and cosmological dark matter

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FORTE

The Wolfson Foundation*

THE ROYAL SOCIETY

erc European Research Council Established by the European Commission

GR + SM incompatible with observations

GR + SM incompatible with observations → New particle (dark matter)

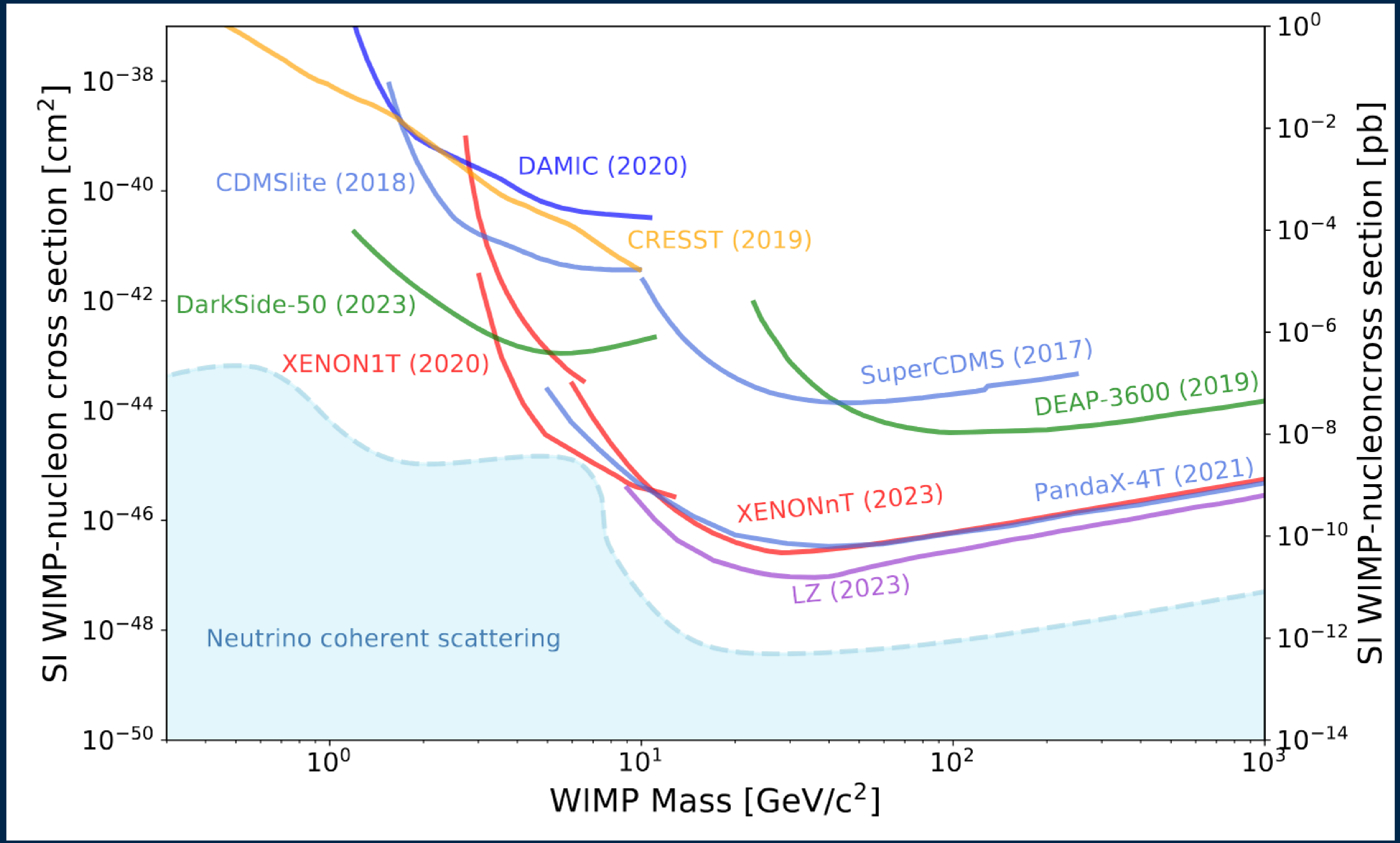
GR + SM incompatible with observations → New particle (dark matter)

GR + SM incompatible with observations → Non-GR corrections in the IR

Earth & the Solar System

Earth & the Solar System

Baudis & Profumo, PDG 2024



Earth & the Solar System

C. Will, Liv. Reviews. Rel.

Parameter	Remarks	Limit
$\gamma - 1$	Cassini tracking	2.3×10^{-5}
	VLBI	2×10^{-4}
$\beta - 1$	$J_{2\odot} = (2.2 \pm 0.1) \times 10^{-7}$	8×10^{-5}
	$\eta_N = 4\beta - \gamma - 3$ assumed	2.3×10^{-4}
ξ	millisecond pulsars	4×10^{-9}
α_1	Lunar laser ranging	10^{-4}
	PSR J1738+0333	4×10^{-5}
α_2	millisecond pulsars	2×10^{-9}
α_3	pulsar $\dot{P}$ statistics	4×10^{-20}
ζ_1	combined PPN bounds	2×10^{-2}
ζ_2	$\ddot{P}_p$ for PSR 1913+16	4×10^{-5}
ζ_3	lunar acceleration	10^{-8}
ζ_4	not independent	—

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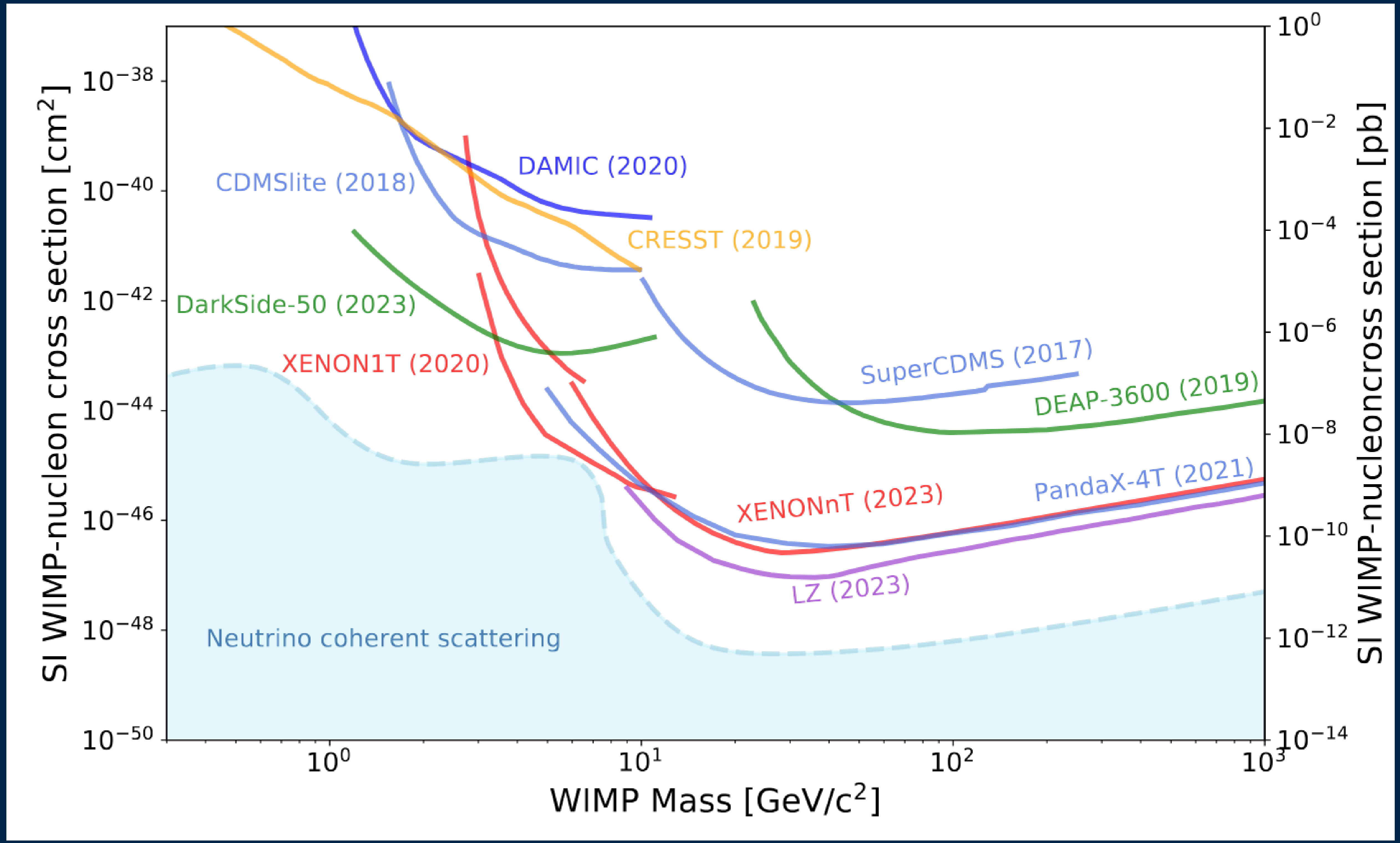

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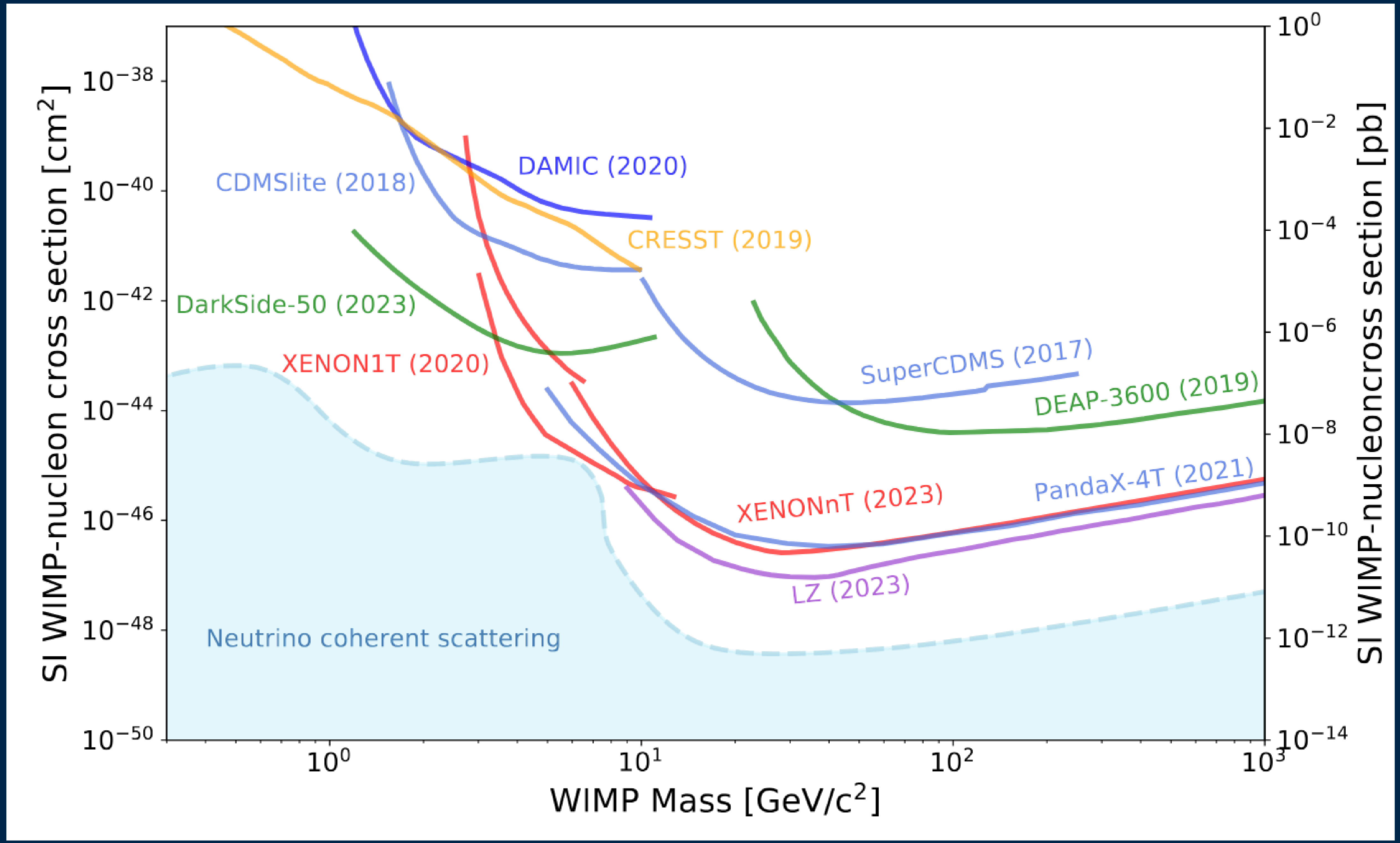


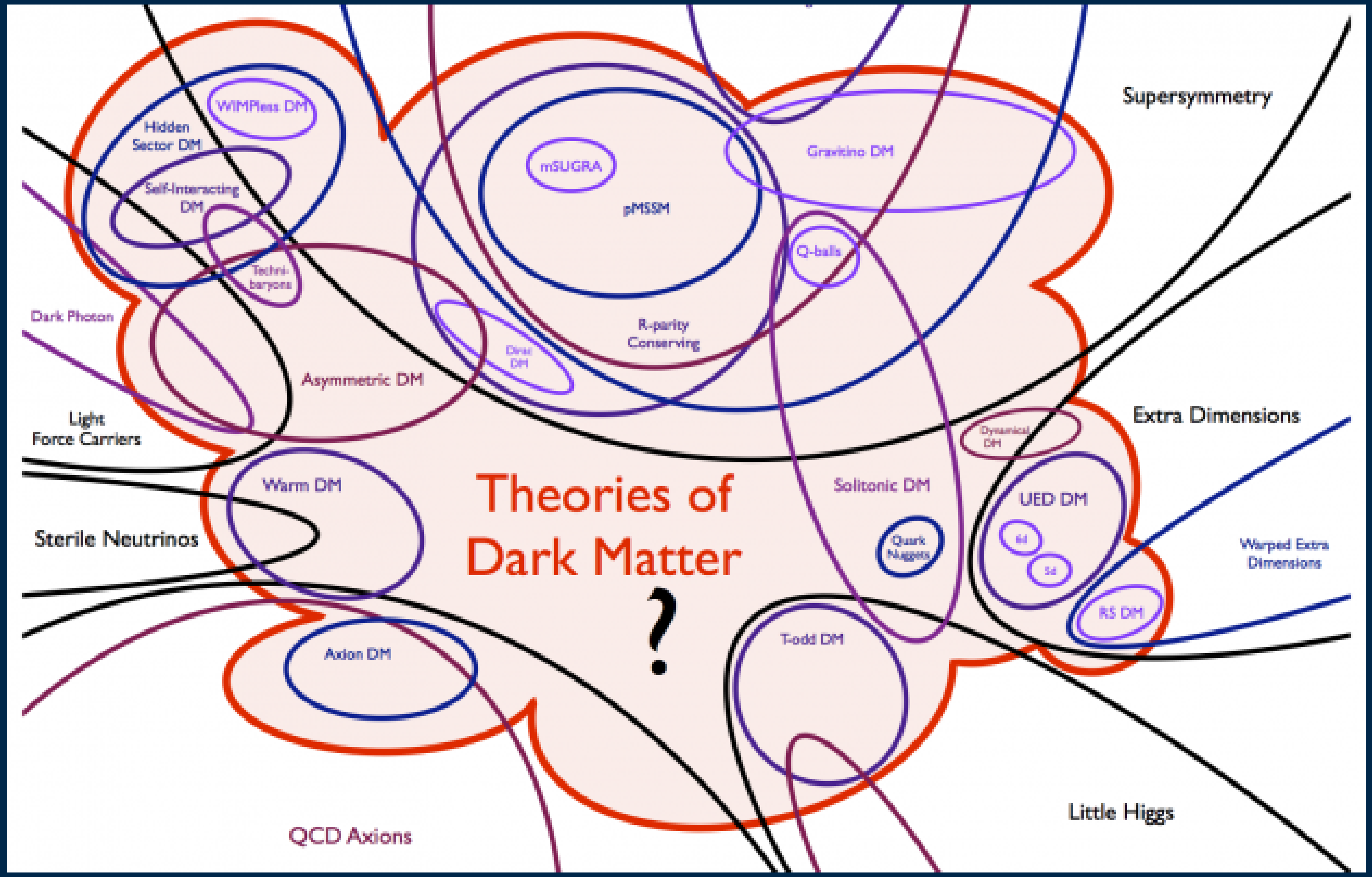
No deviation from GR yet

So far: NULL RESULT

No DM particle yet

Dark Matter

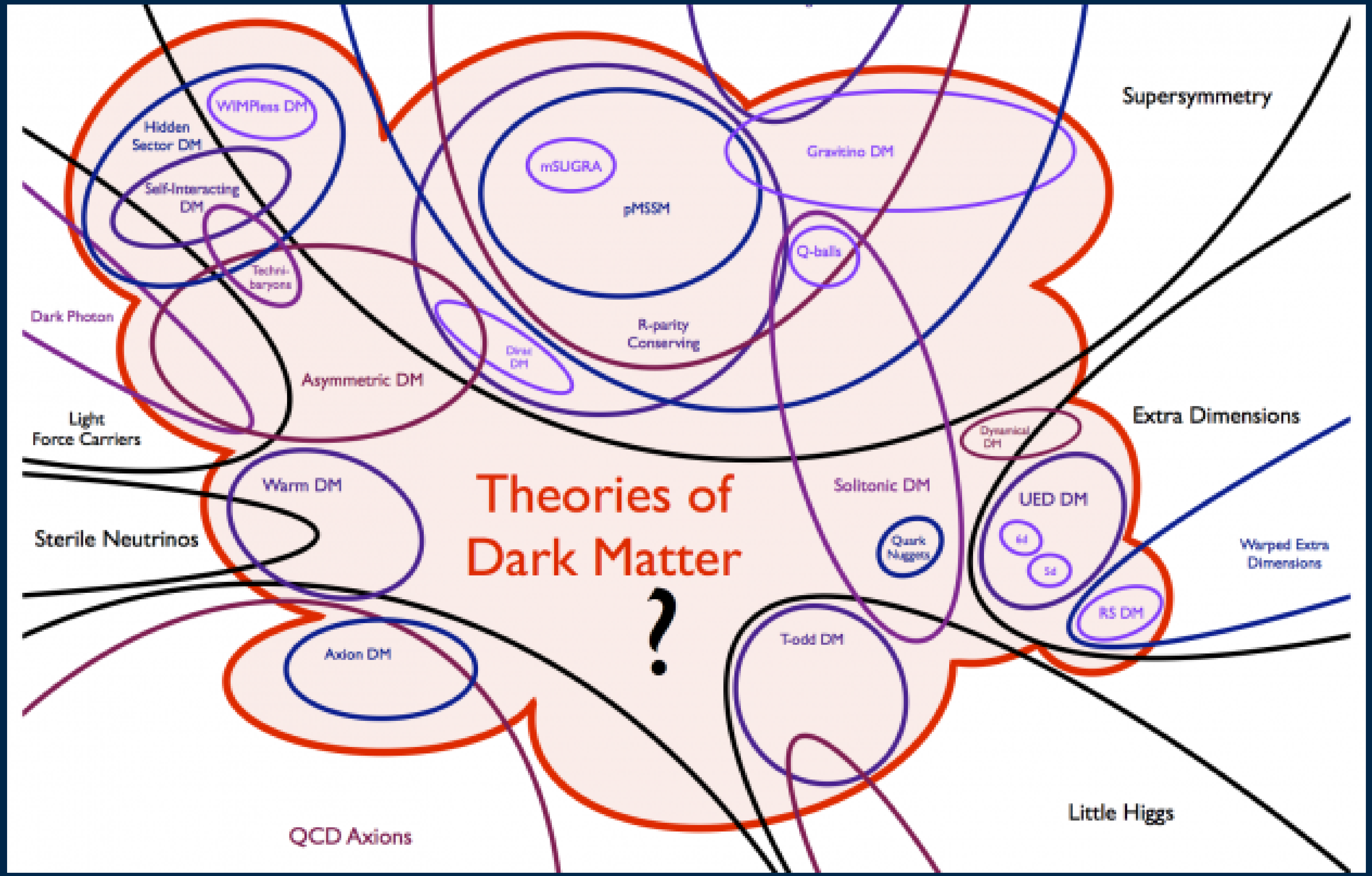
Plethora of particle DM models



Tim Tait’s Venn diagram

Dark Matter

Plethora of particle DM models



Tim Tait's Venn diagram

Particle-based

e.g. SUSY

Wave-based

e.g. ultralight axions

Dark Matter

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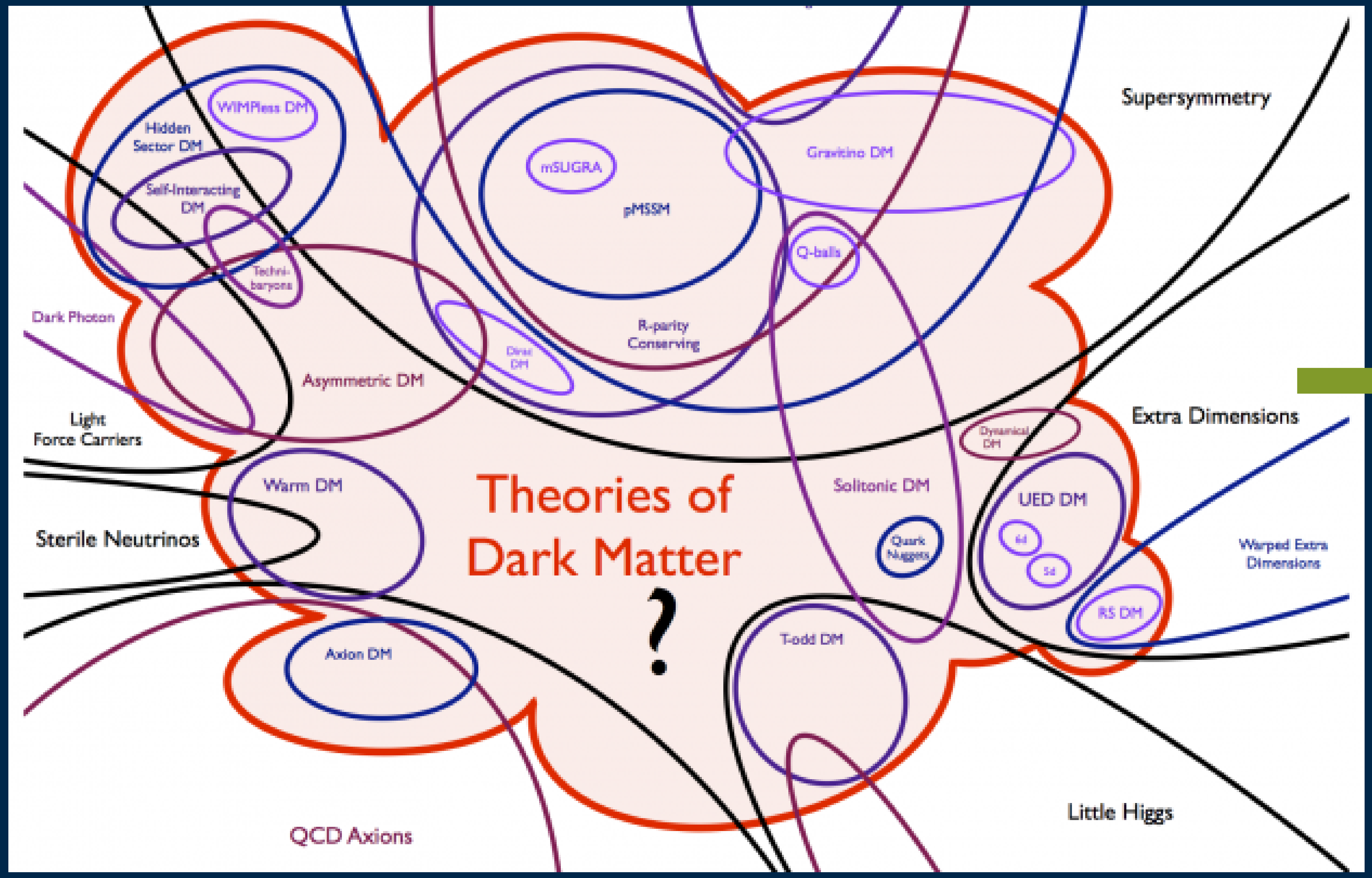


Tim Tait's Venn diagram

Particle-based

e.g. SUSY

Wave-based

e.g. ultralight axions

On galactic scales ($\sim Mpc$) and above

Distribution function (d.f.) $f(t, \vec{x}, \vec{u})$

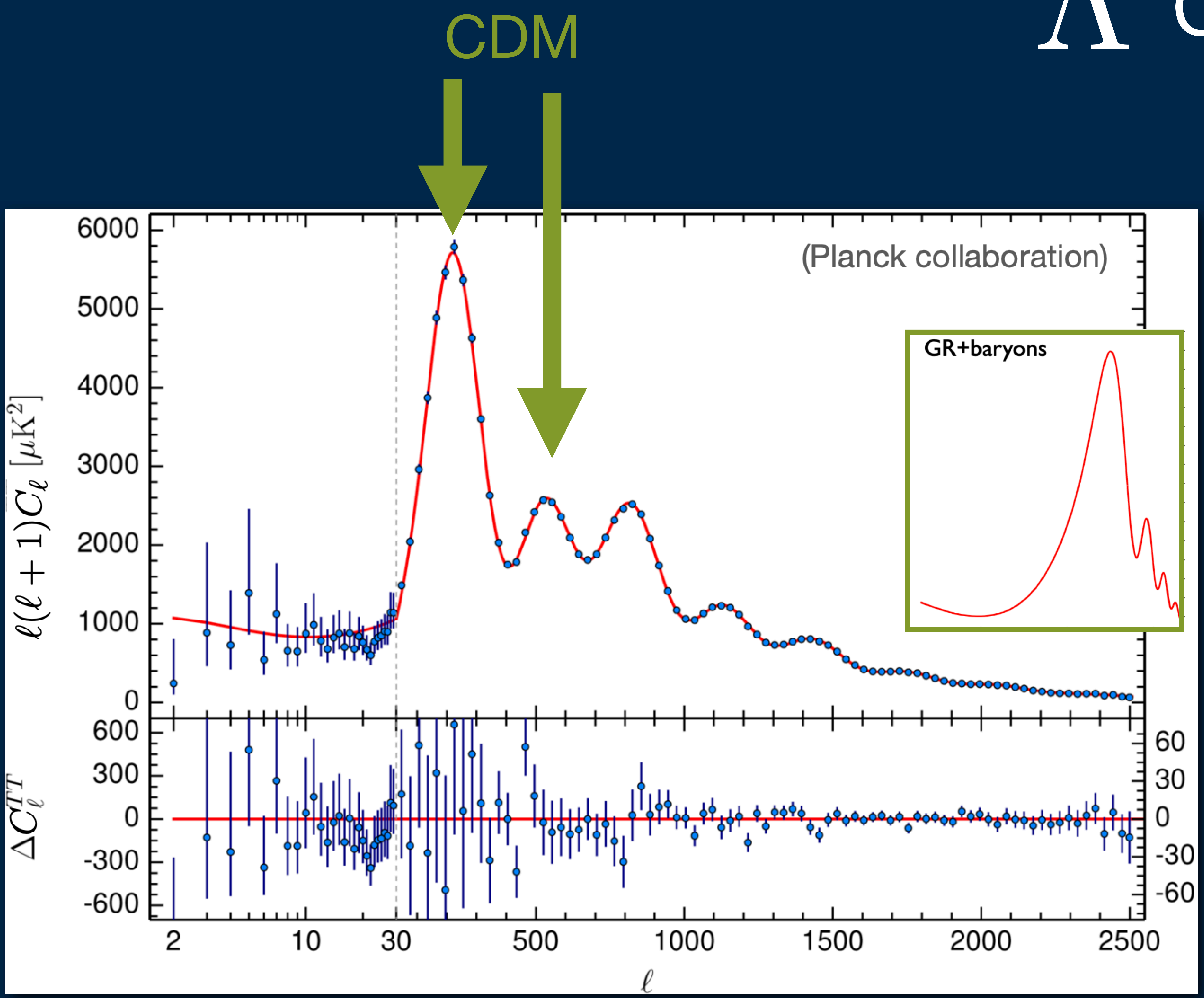
- Evolves in phase-space
- Boltzmann equation

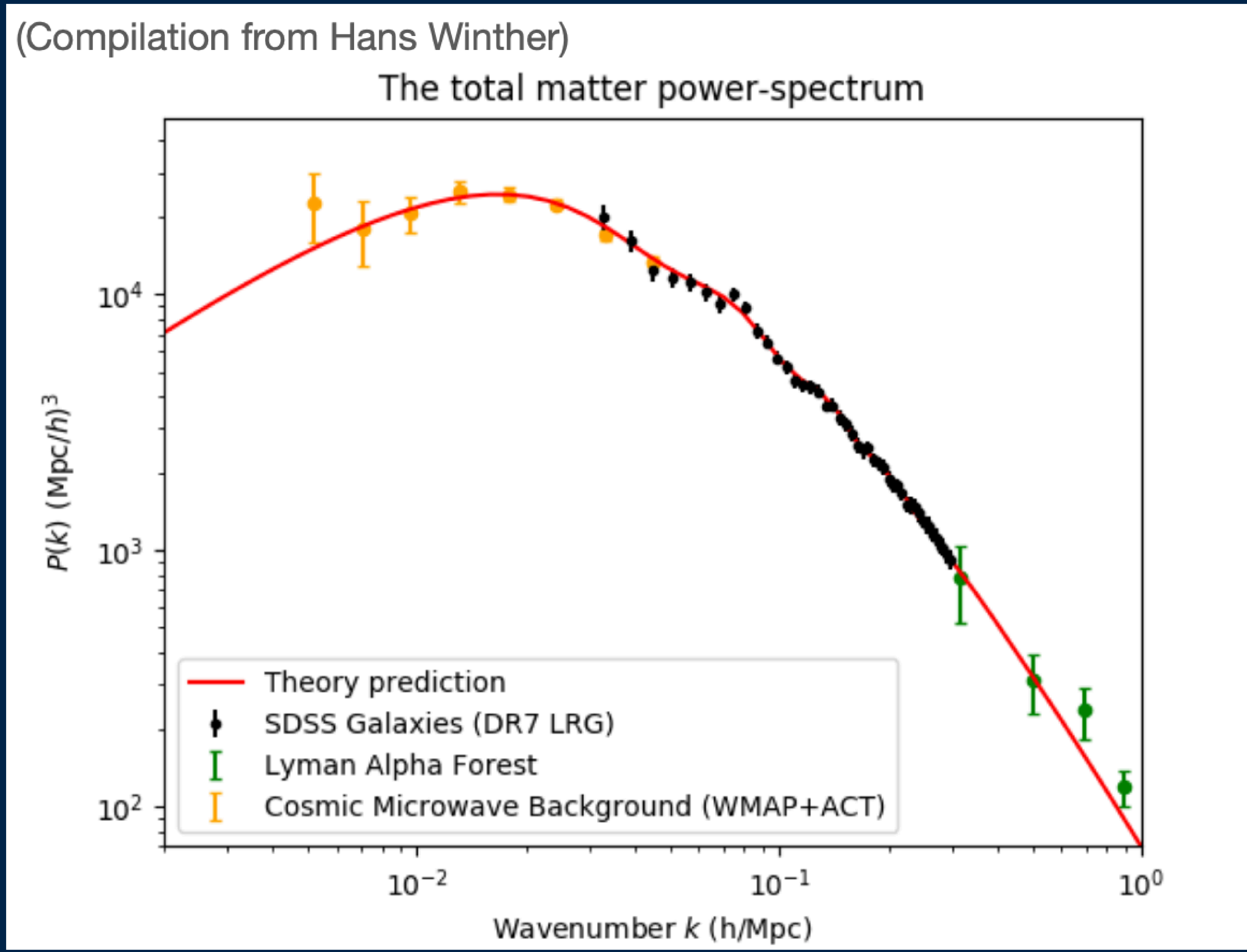
$$\frac{\partial f}{\partial t} + \vec{u} \cdot \vec{\nabla}_x f + \vec{\nabla}_x \Phi \cdot \vec{\nabla}_u f = 0$$

A simpler (cosmological) model emerges

Cold Dark Matter: CDM

Λ CDM: Superb fit on linear scales

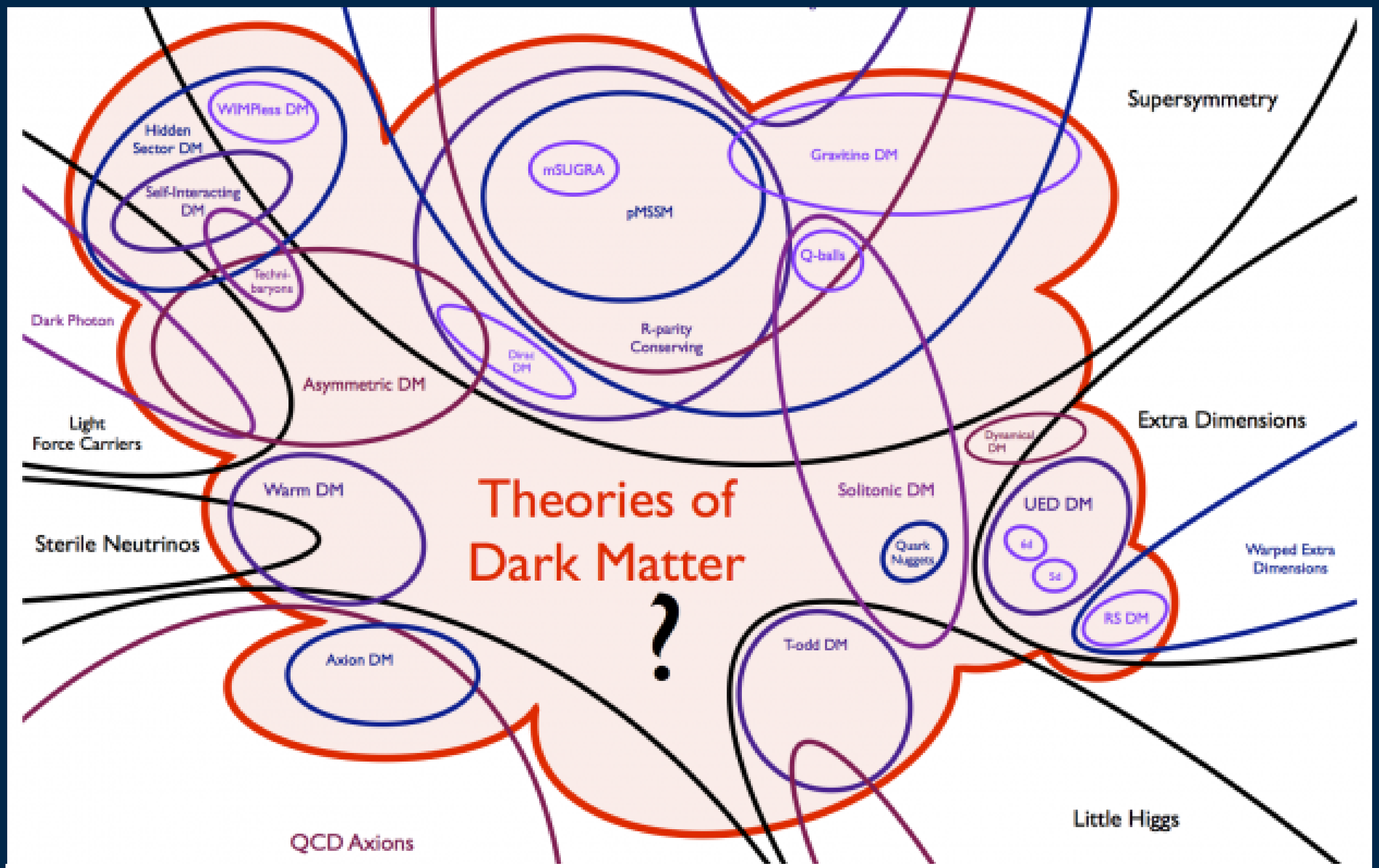




Particle/Wave DM $\xrightarrow{L \gtrsim 1Mpc}$ CDM $\xrightarrow{L \gtrsim 100Mpc}$ Dust

Density $\rho \sim a^{-3}$

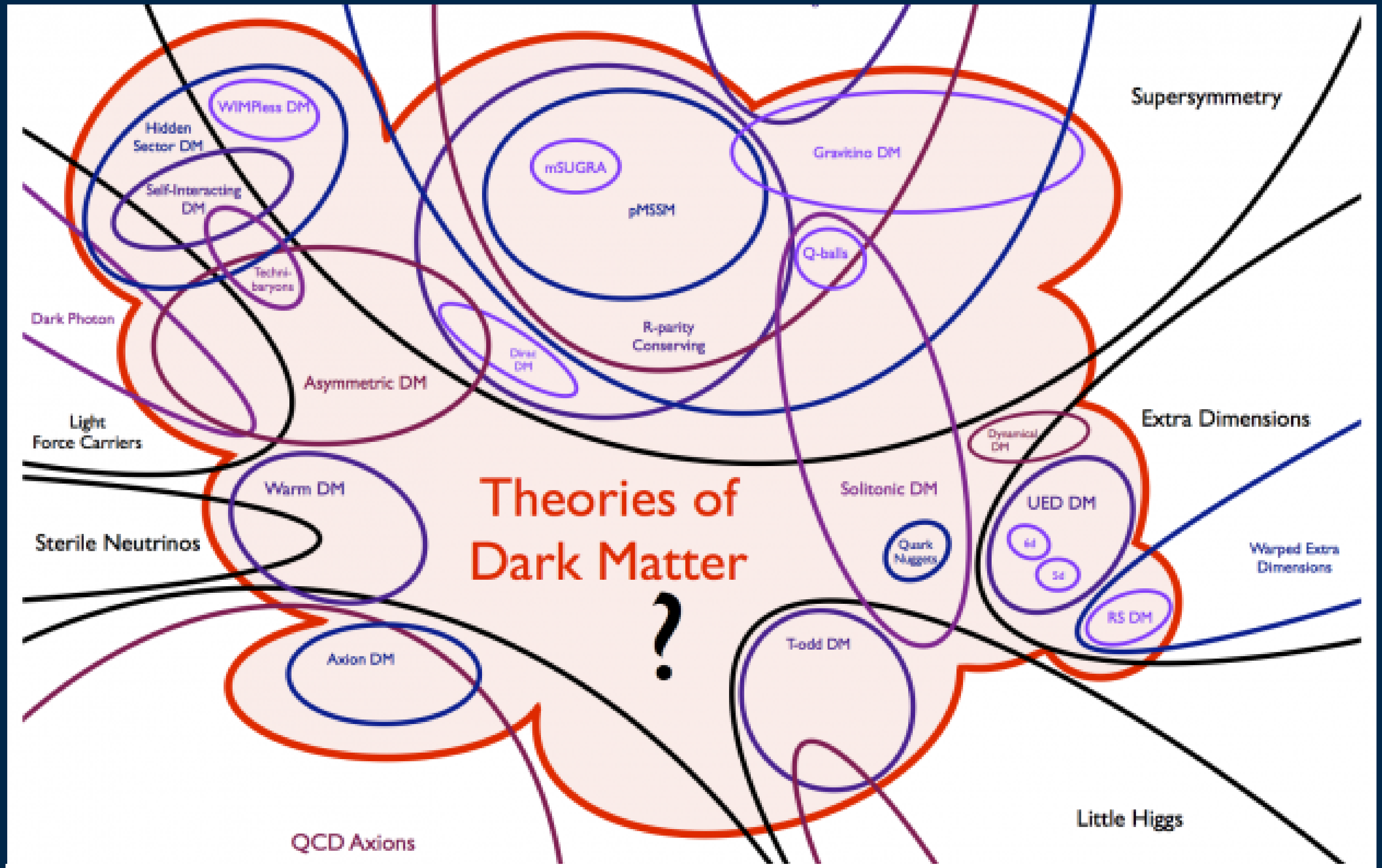
Density fluctuation $\delta\rho \sim a^{-2}$ (for pure dust)



Tim Tait's DM Venn diagram

$L \gtrsim 100 Mpc$

ΛCDM

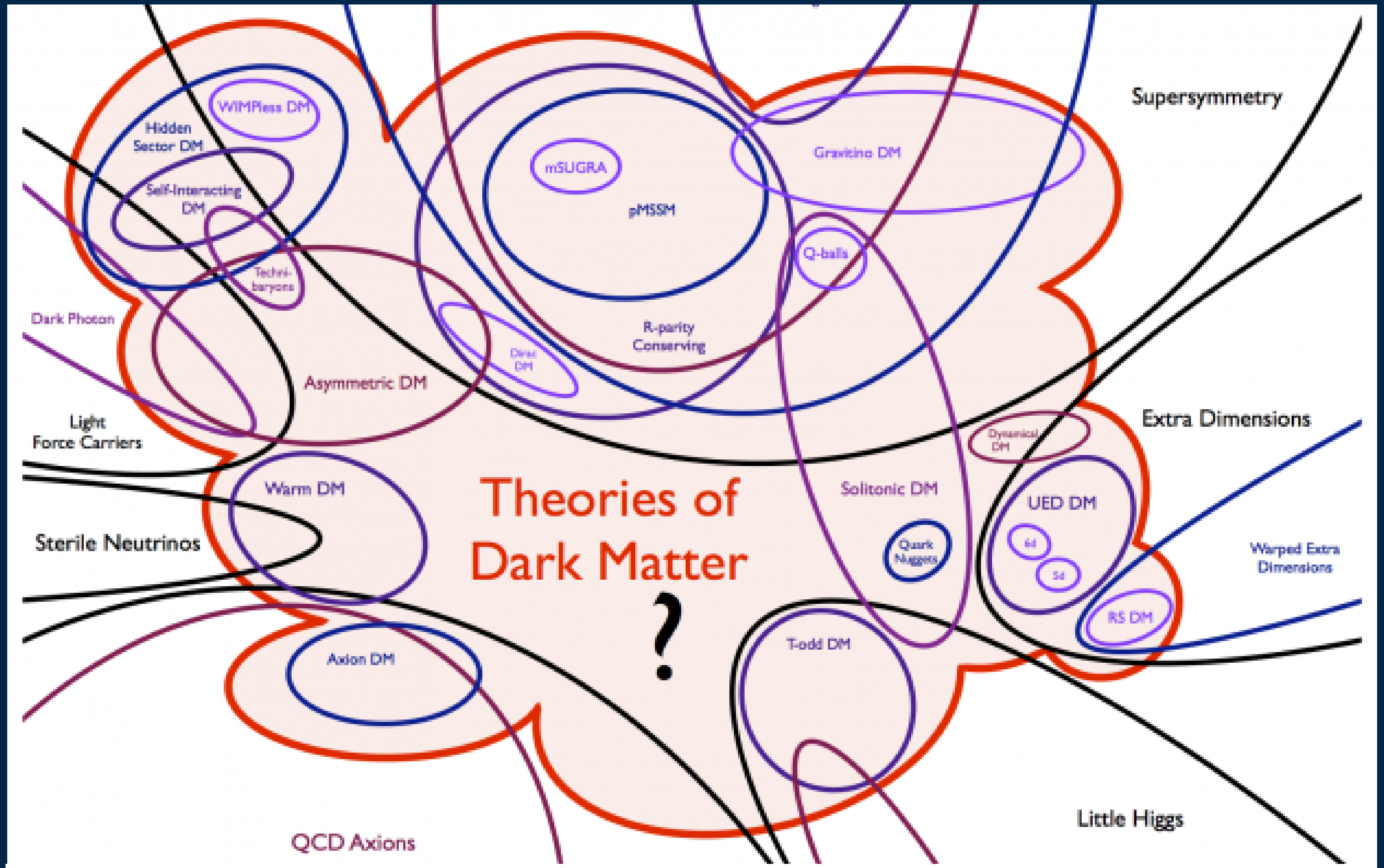


Tim Tait's DM Venn diagram

?

$L \gtrsim 100 Mpc$

ΛCDM



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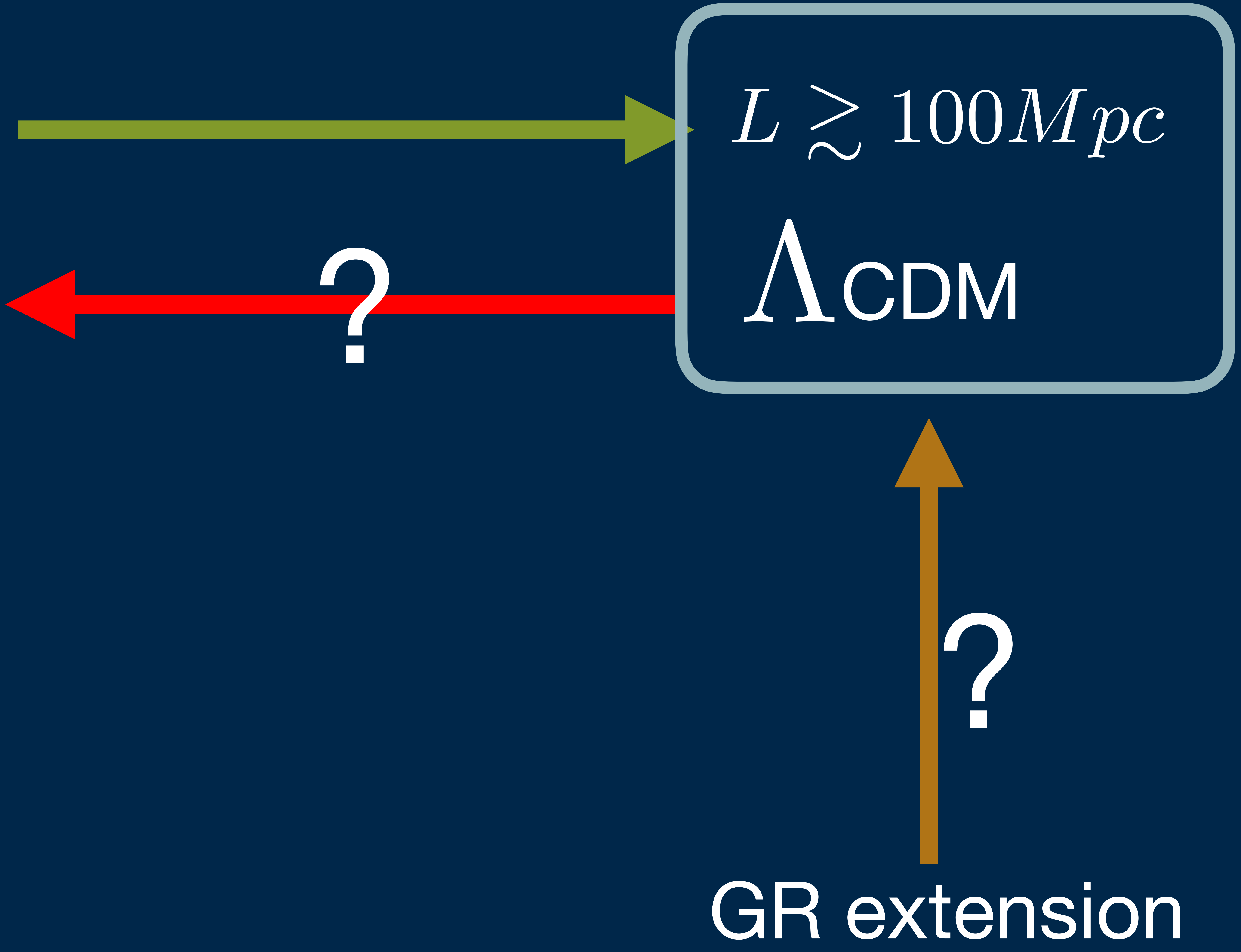

GR extensions

Lovelock's theorem

The only

- local,
- diffeomorphism invariant action,
- which leads to 2nd order field equations
- and which depends only on a metric

is a linear combination of the Einstein-Hilbert action with a cosmological constant up to a total derivative.

GR:

$$S = \frac{1}{16\pi G}\int d^4x\,[R - 2\Lambda] + S_m[g, \psi^A]$$

GR extensions

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The only

- local,
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→ New degrees of freedom: New fields

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GR:

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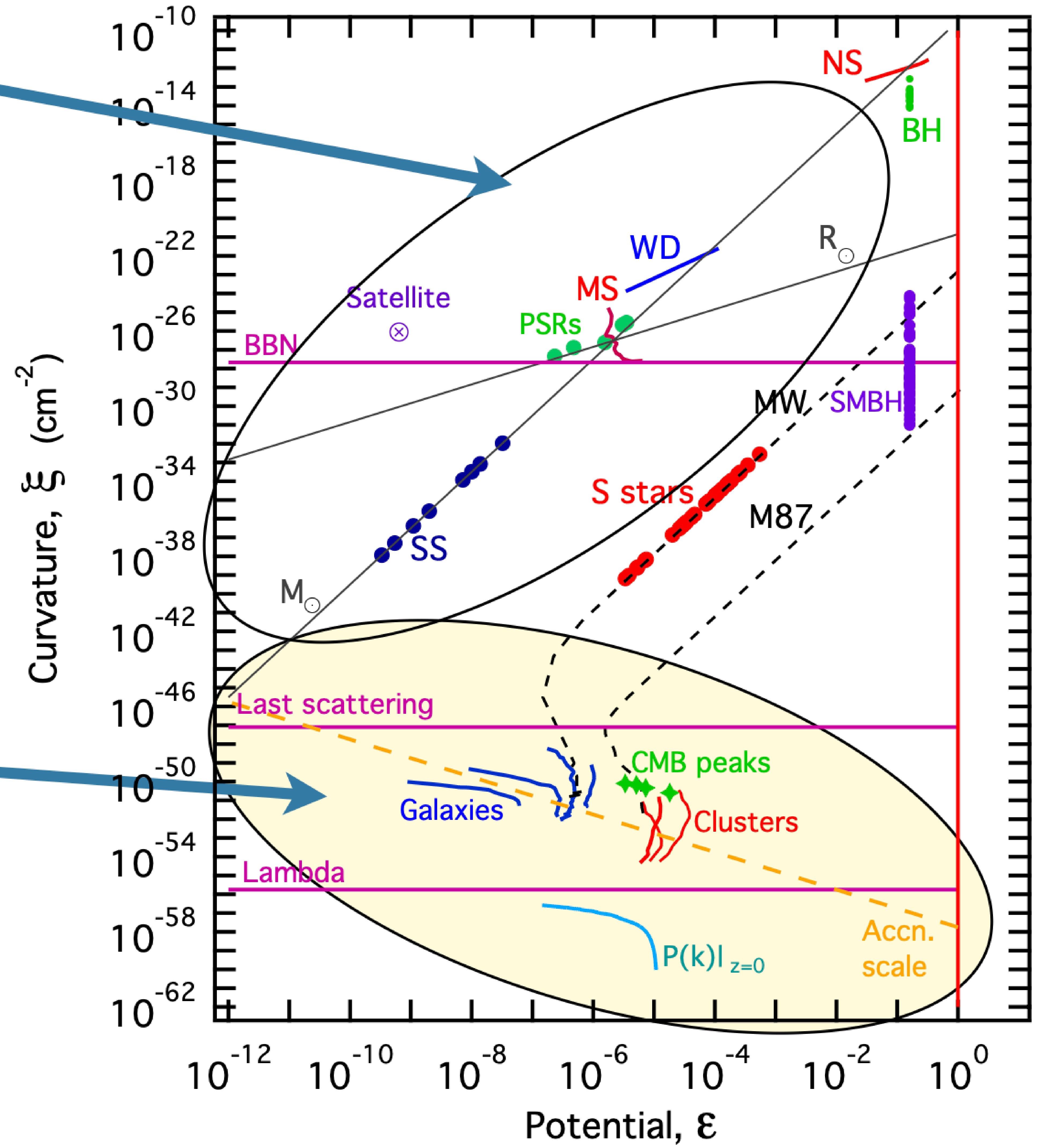
corrections to GR

T. Baker, D. Psaltis and C.S., Astrophys. J. 802, 63 (2015)

- GR experimentally tested
- Deviations at $10^{-4} - 10^{-8}$ level
- New dof suppressed



- Need DM
- GR not tested
- New dof active

Lovelock's theorem:

new (long-range) fields.

-- Galileons

-- Horndeski

-- Gen. Proca

-- dRGT massive gravity

-- bi-gravity

-- Einstein-Aether

-- Ghost Condensates

New dof from extension of GR: FRLW background

see CS, PRD79, 123527 (2009)

RW metric $ds^2 = -dt^2 + a^2 d\vec{x}^2$

Additional dof $\chi_I(t)$

Generalized Friedman equation: $f_{\text{very complicated}}(a, H, \chi_I, \dot{\chi}_I, \rho_b, \ldots) = 0$

$$3H^2 = 8\pi G \sum_I \rho_I + 8\pi G \rho_{new}$$

consequence of Bianchi identity $\dot{\rho}_{new} + 3H(1 + w_{new})\rho_{new} = 0$

Fluctuations: $\delta G_{\mu\nu} = 8\pi G \delta T^{(known)}_{\mu\nu} + 8\pi G \delta T^{(new)}_{\mu\nu}$

$\delta\left(\nabla_\mu T^{\mu}{}_{\nu}{}^{(new)}\right) = 0$

Fluid with generic density, velocity, pressure, shear

GDM - generalized dark matter

$w(a), \quad c_s^2(k,a), \quad c_{vis}^2(k,a),$

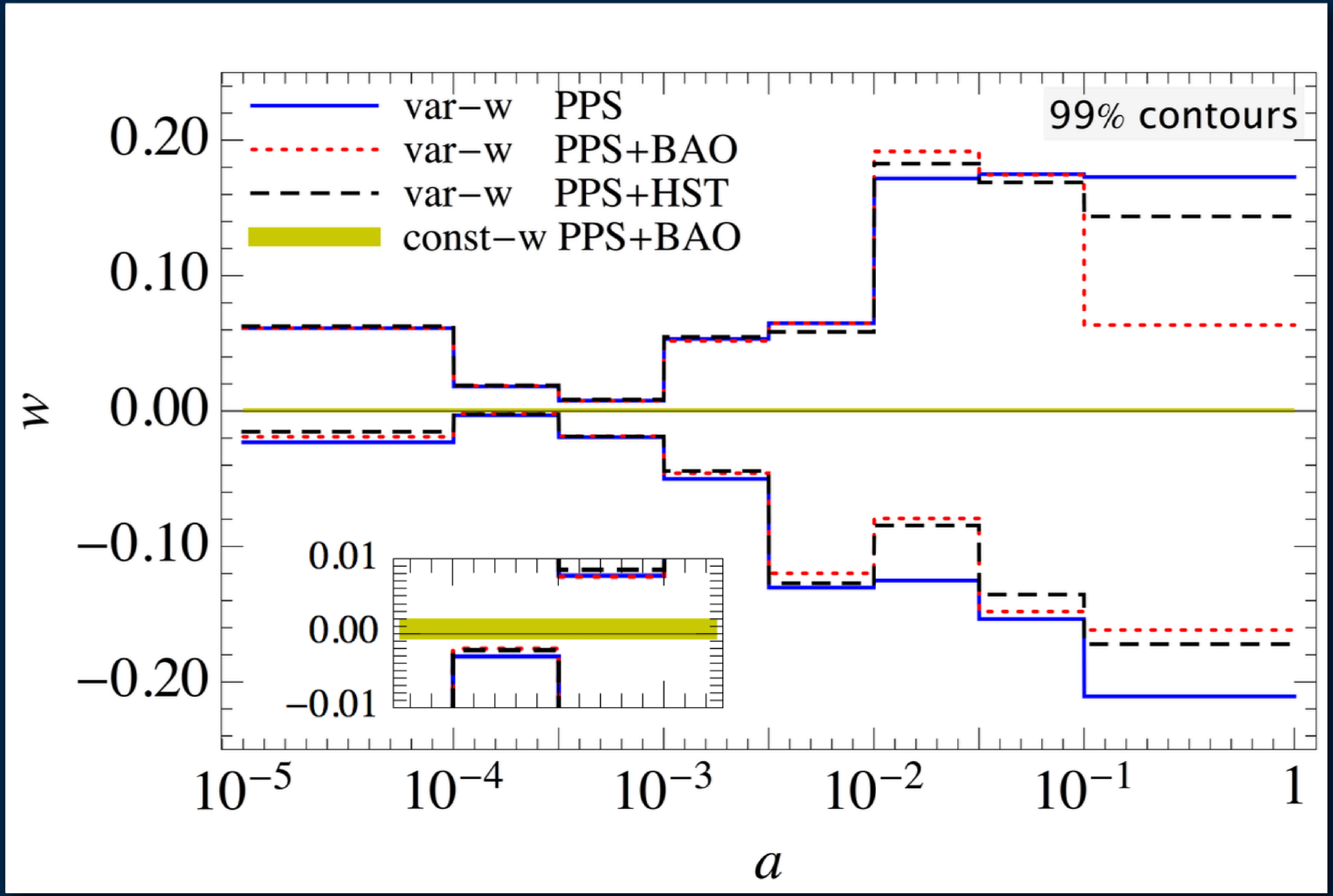
W. Hu, ApJ 506, 485 (1998)

Fairly general (but not unique)

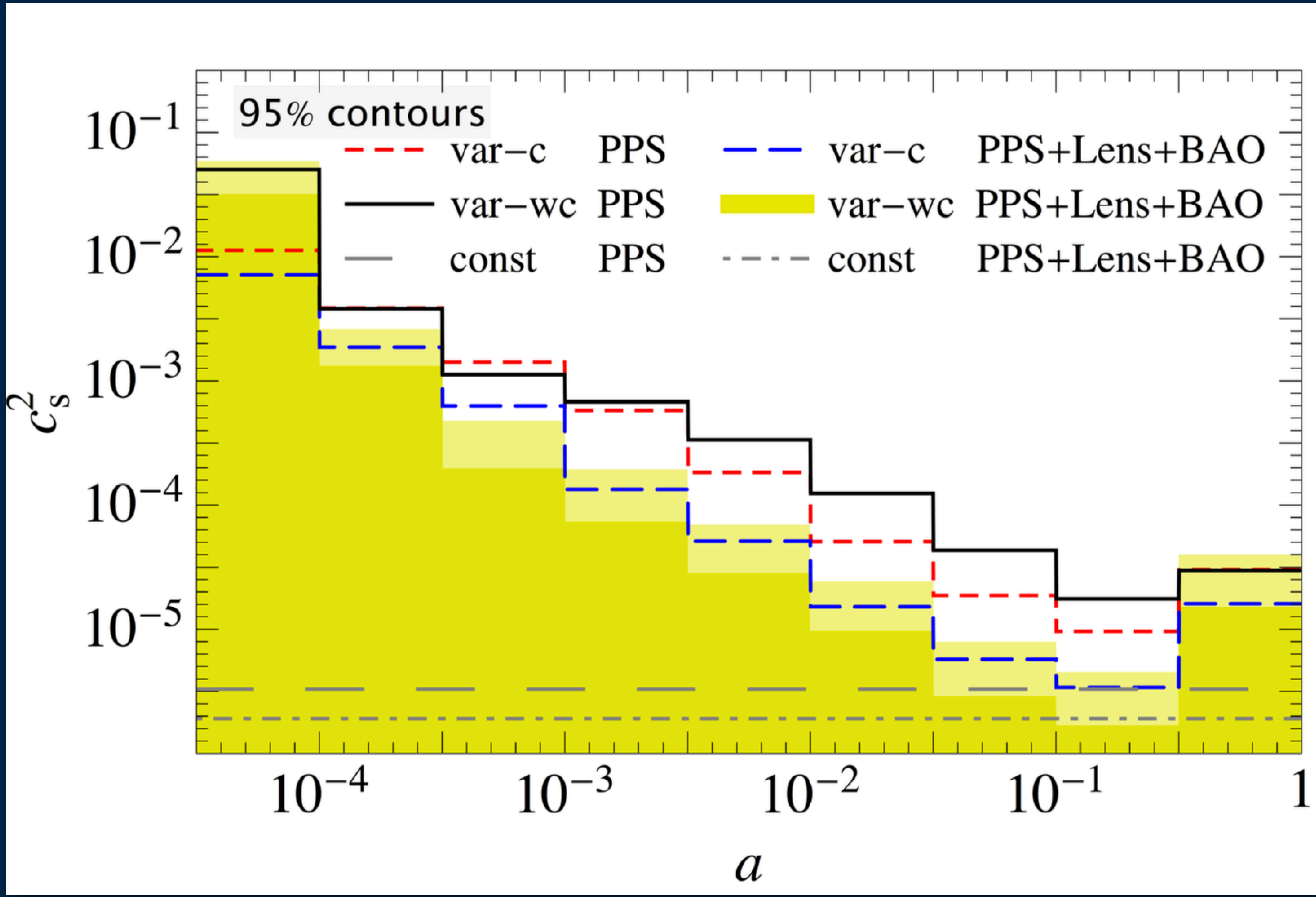
FLRW: ALL MODELS including Extensions of GR

Linear Fluctuations: Most MODELS including many Extensions of GR

CDM: $w = 0 \Leftrightarrow \rho \propto a^{-3}$ $\quad c_s = c_{vis} = 0$



D. Thomas, M. Kopp, C.S., S. Ilic, PRL 120, 221102 (2018



S. Ilic, M. Kopp, C.S., D. Thomas, PRD 104, 043520 (2021)

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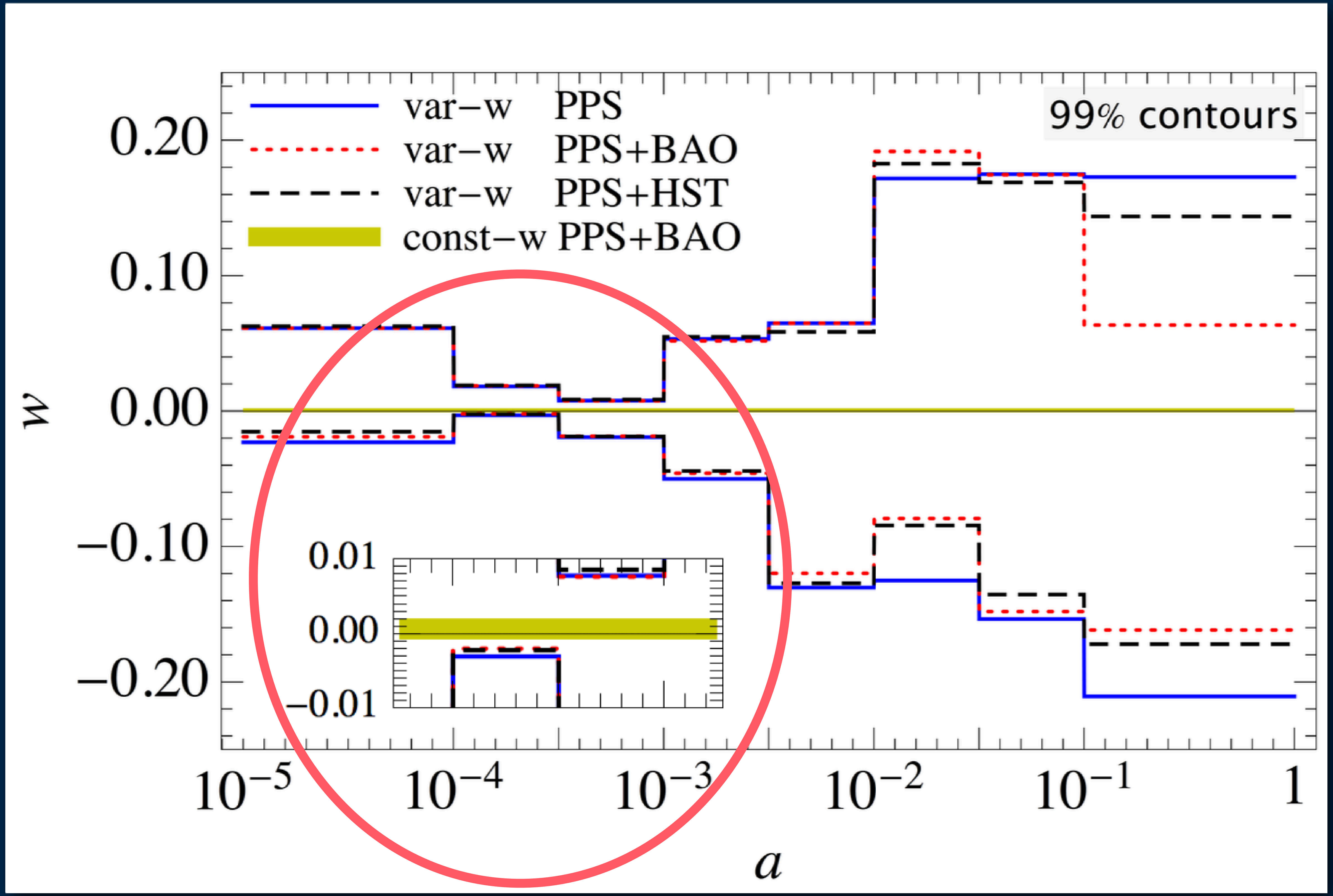

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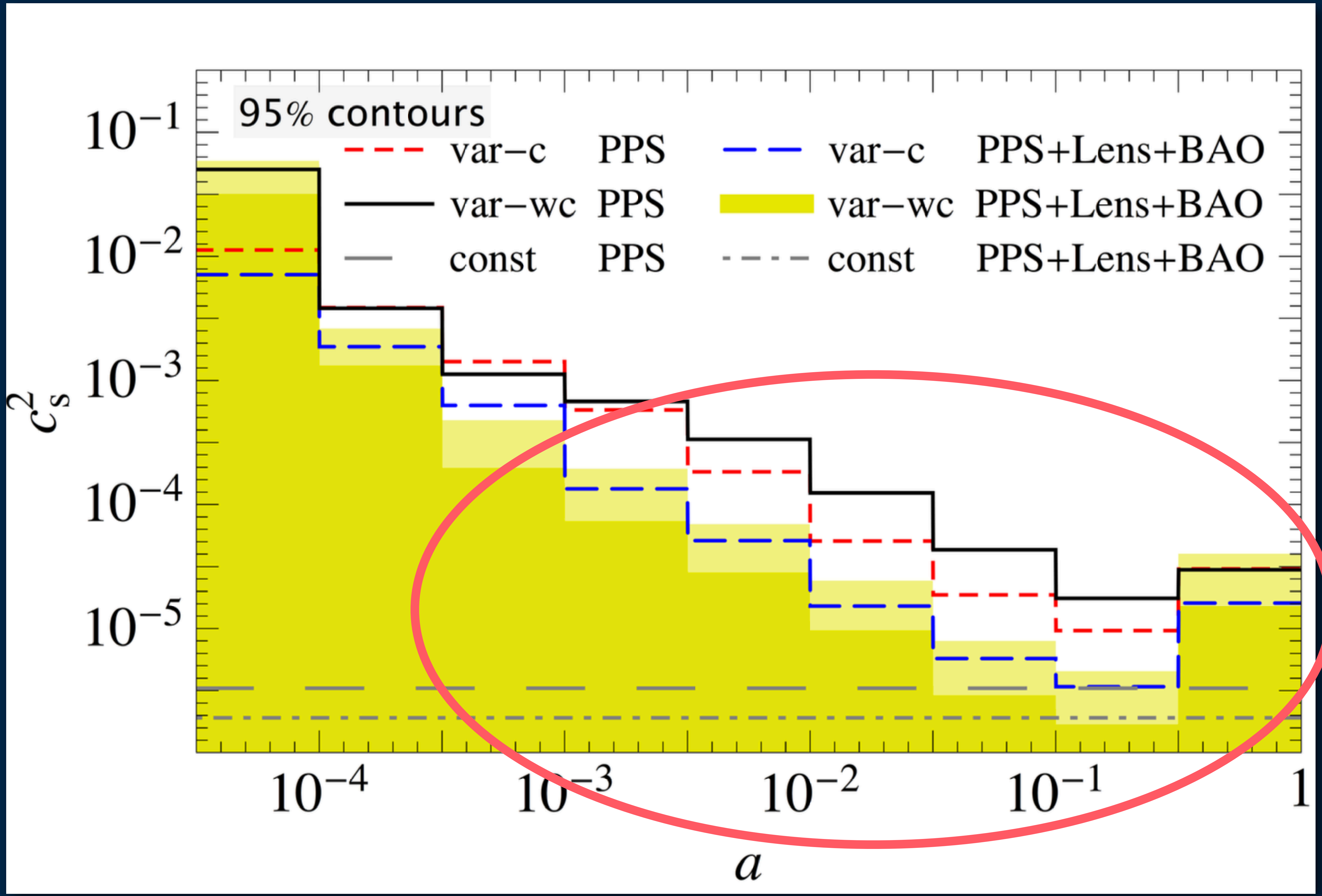

S. Ilic, M. Kopp, C.S., D. Thomas, PRD 104, 043520 (2021)

Can (approximately) ΛCDM

emerge on linear scales from new gravitational dof?

Can (approximately) Λ CDM

emerge on linear scales from new gravitational dof?

YES

CASE STUDY

Eddington-Born-Infeld theory

(Banados 2009)

Modelled after Eddington action $$S[\Gamma^c_{ab}] = \frac{1}{8\pi G\Lambda}\int d^4x\sqrt{-\det[R_{ab}]}$$ (Eddington 1924)

$$I[g_{\mu\nu}, C^\alpha_{\mu\nu}] = \frac{1}{16\pi G}\int d^4x\left[\sqrt{-g}R + \frac{2}{\alpha\ell^2}\sqrt{|g_{\mu\nu} - \ell^2 K_{\mu\nu}|}\right]$$

2nd connection

curvature of $C^\alpha_{\mu\nu}$

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$C^\alpha_{\mu\nu}$: 2nd connection

$K_{\mu\nu}$: curvature of $C^\alpha_{\mu\nu}$

Introduce 2nd metric $q_{\mu\nu}$, related to $C^\alpha_{\mu\nu}$ → bigravity (Isham, Salam, Strathdee 1971)

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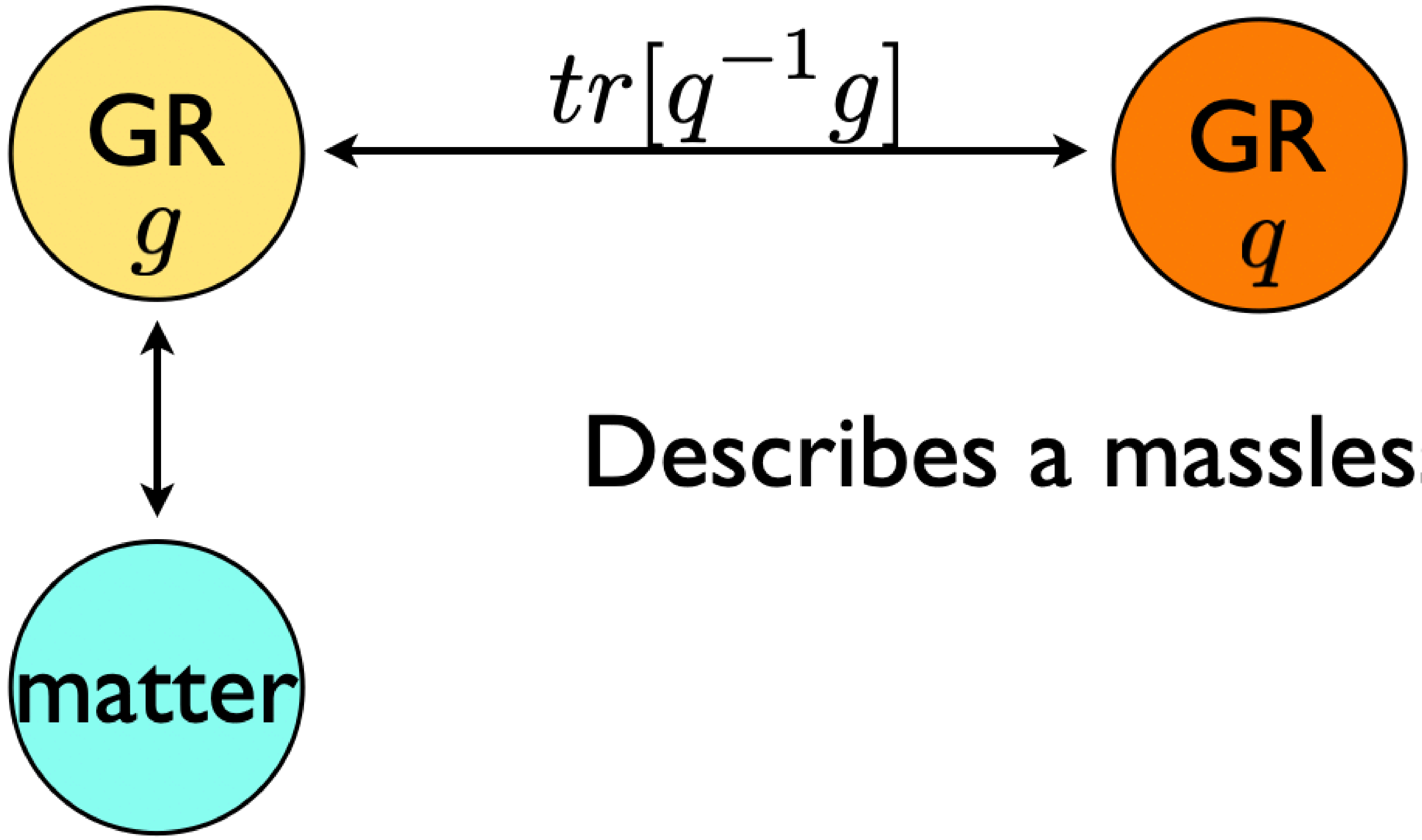


two metrics

two sectors of GR

Describes a massless and a massive graviton

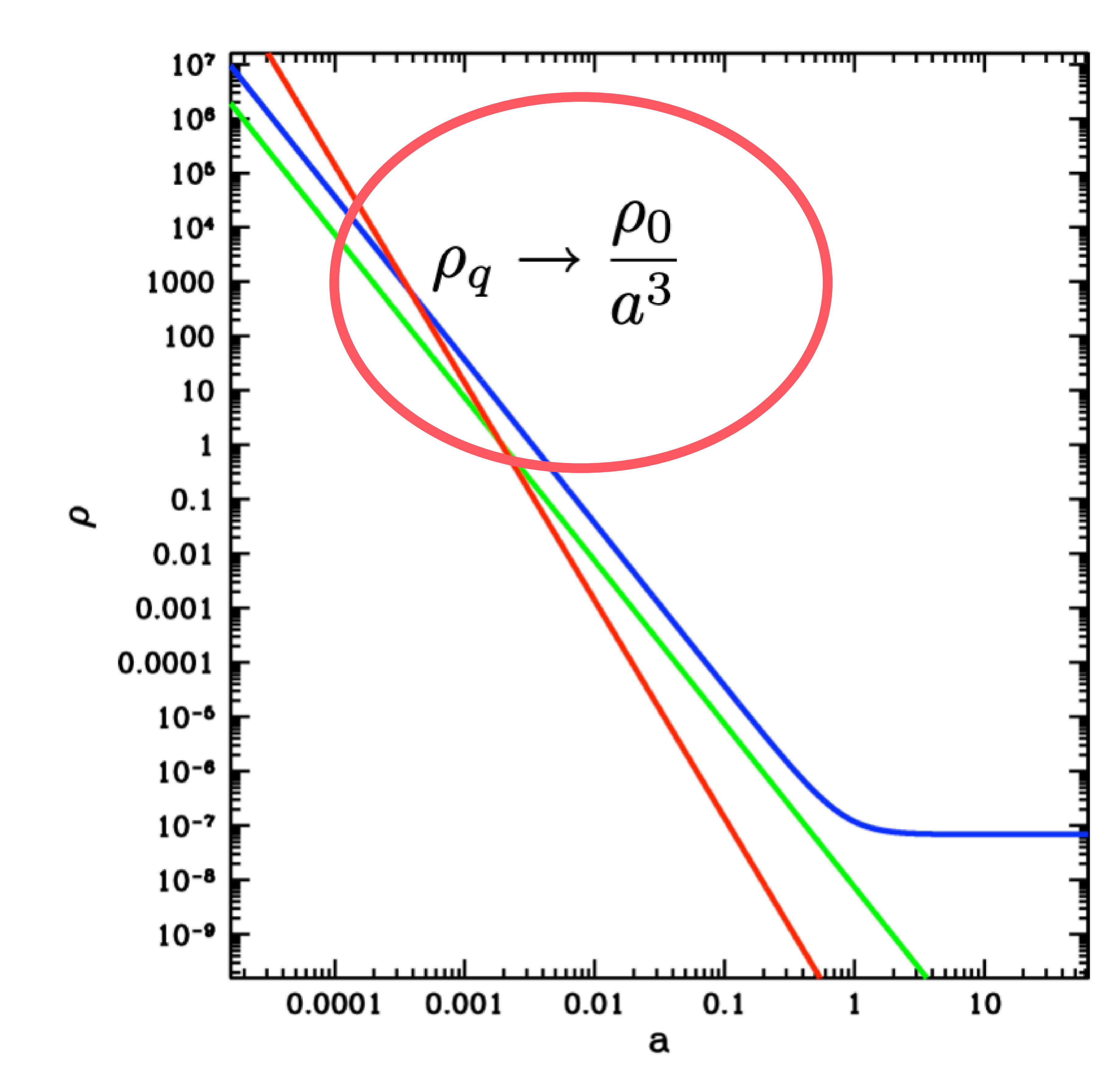
$$m \sim H_0 \sim 10^{-33} eV$$

$$ds_g^2 = a^2\left[-d\tau^2 + d\vec{x}\cdot d\vec{x}\right]$$

(M. Banados, P. Ferreira, C.S. 2009)

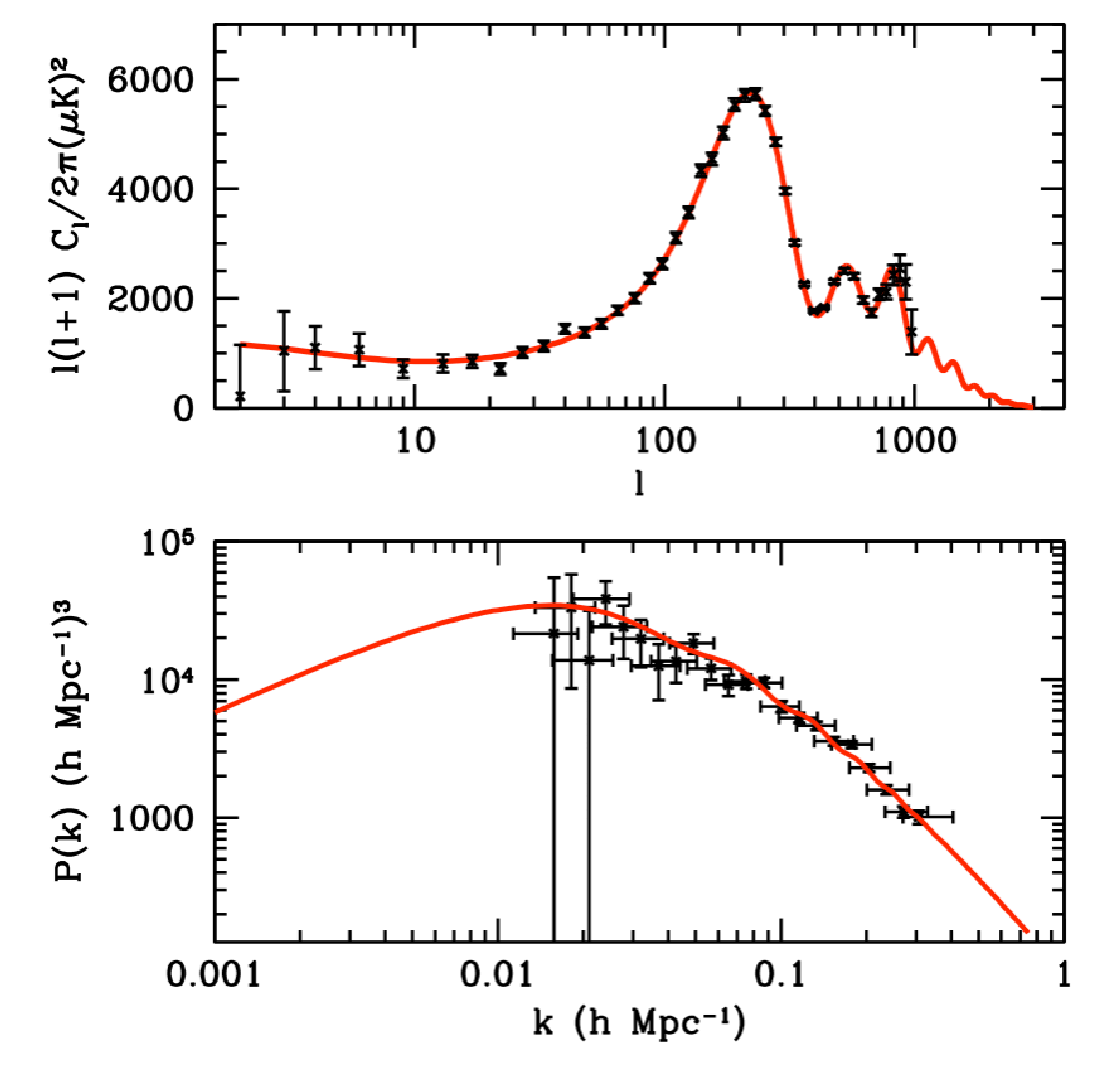
$$ds_q^2 = a^2\left[-X^2 d\tau^2 + Y^2 d\vec{x}\cdot d\vec{x}\right]$$

map $X, Y \rightarrow \rho_q, P_q$



$\rho_q \longrightarrow \frac{\rho_0}{a^3}$

$\rho_q \longrightarrow$ const



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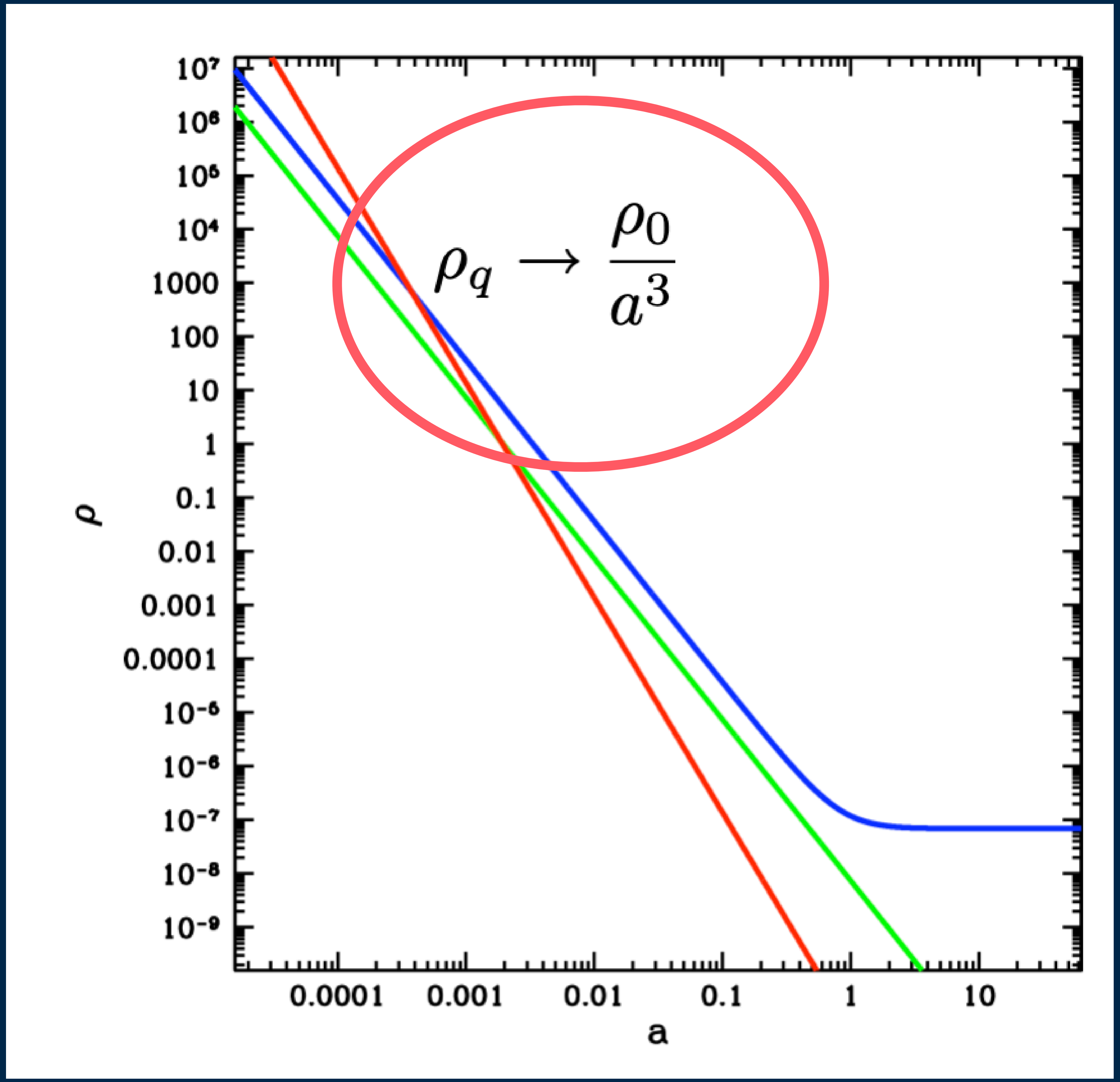
Non-linear dynamics has not been studied



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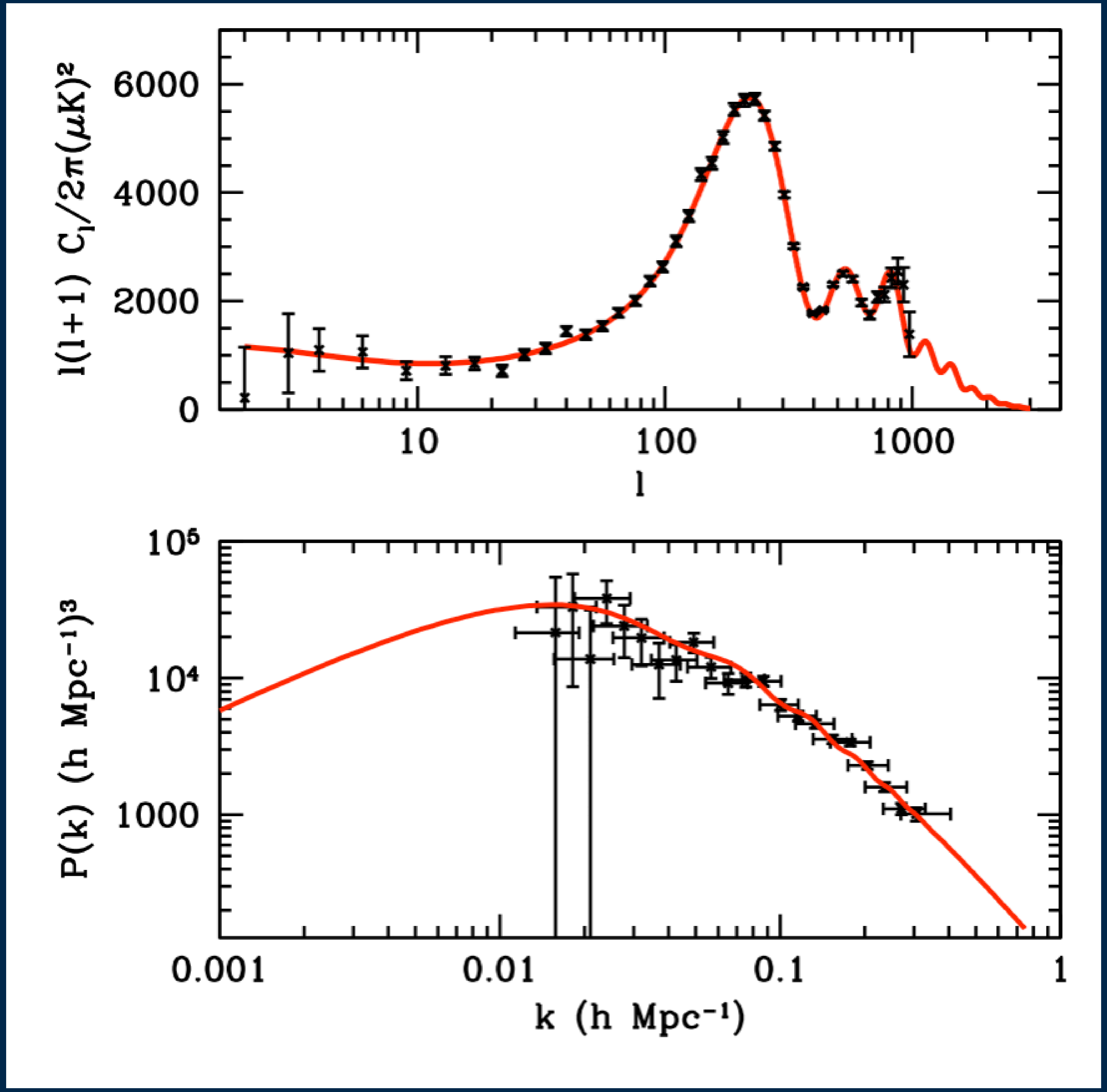


EBI not the only example

might have ghosts

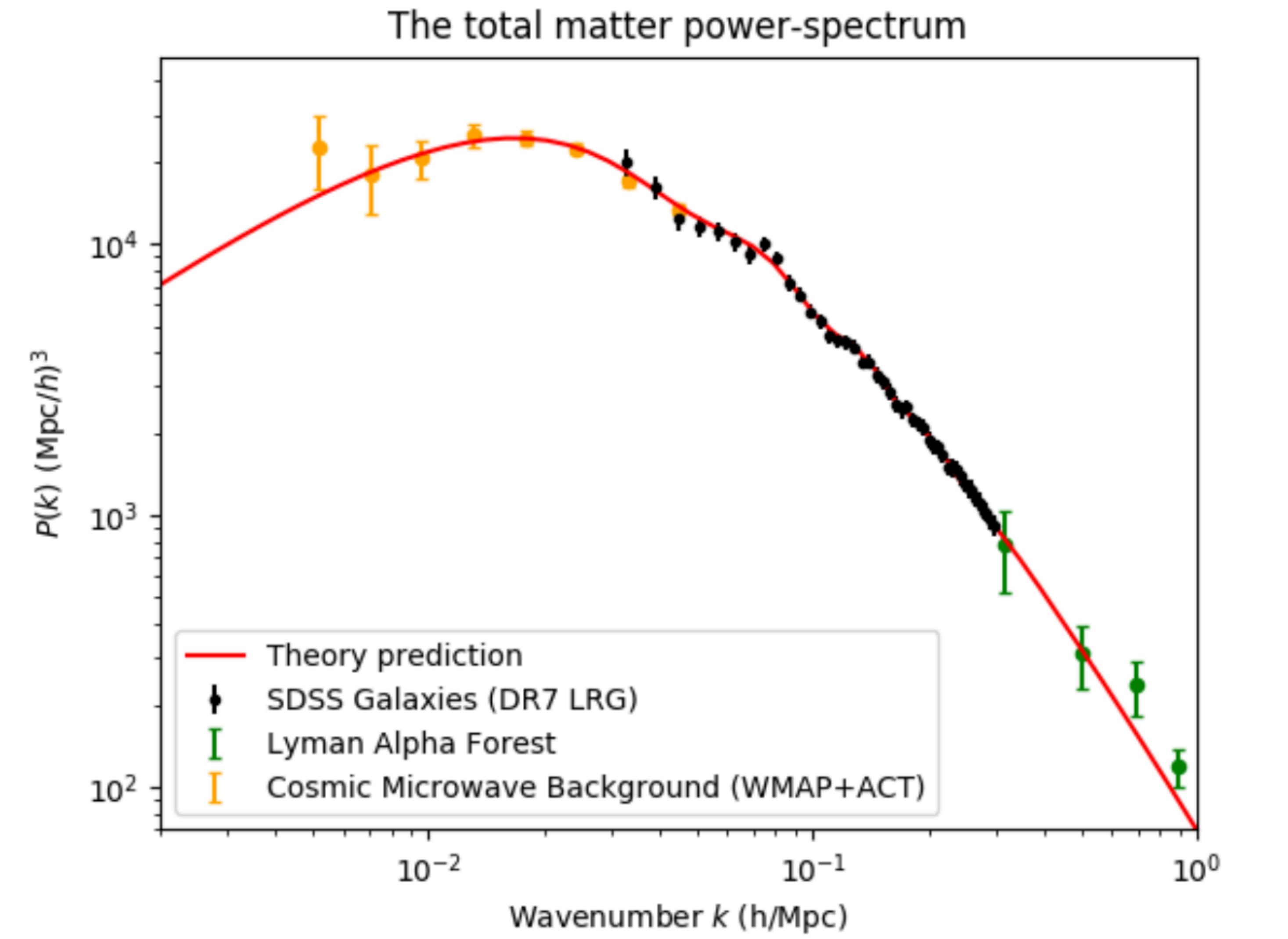
(probably) will not fit smaller scales

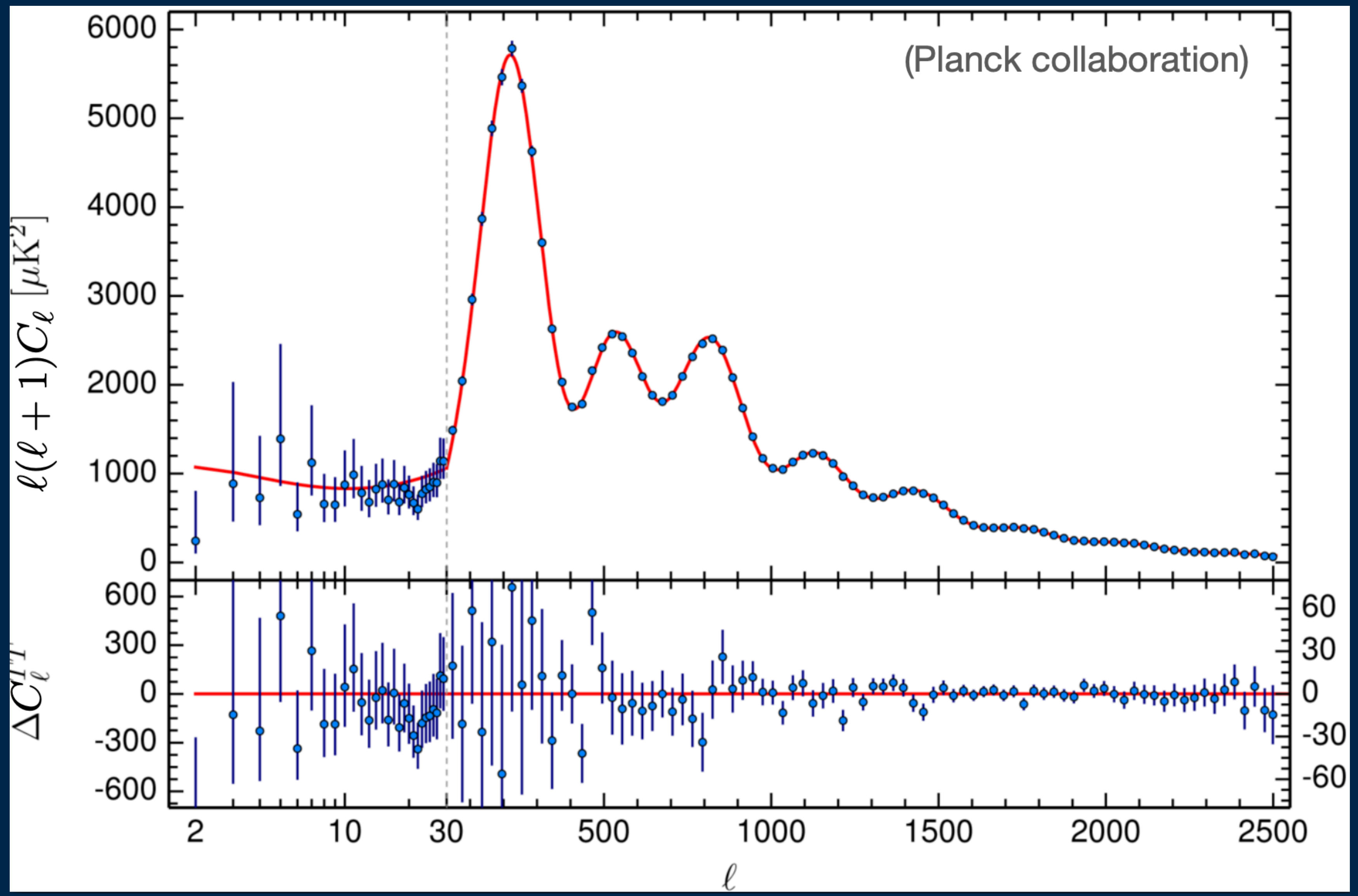
Are there other models?

Cold dark matter + cosmological constant: ΛCDM model

Superb fit on scales $\gtrsim Mpc$

(Compilation from Hans Winther)

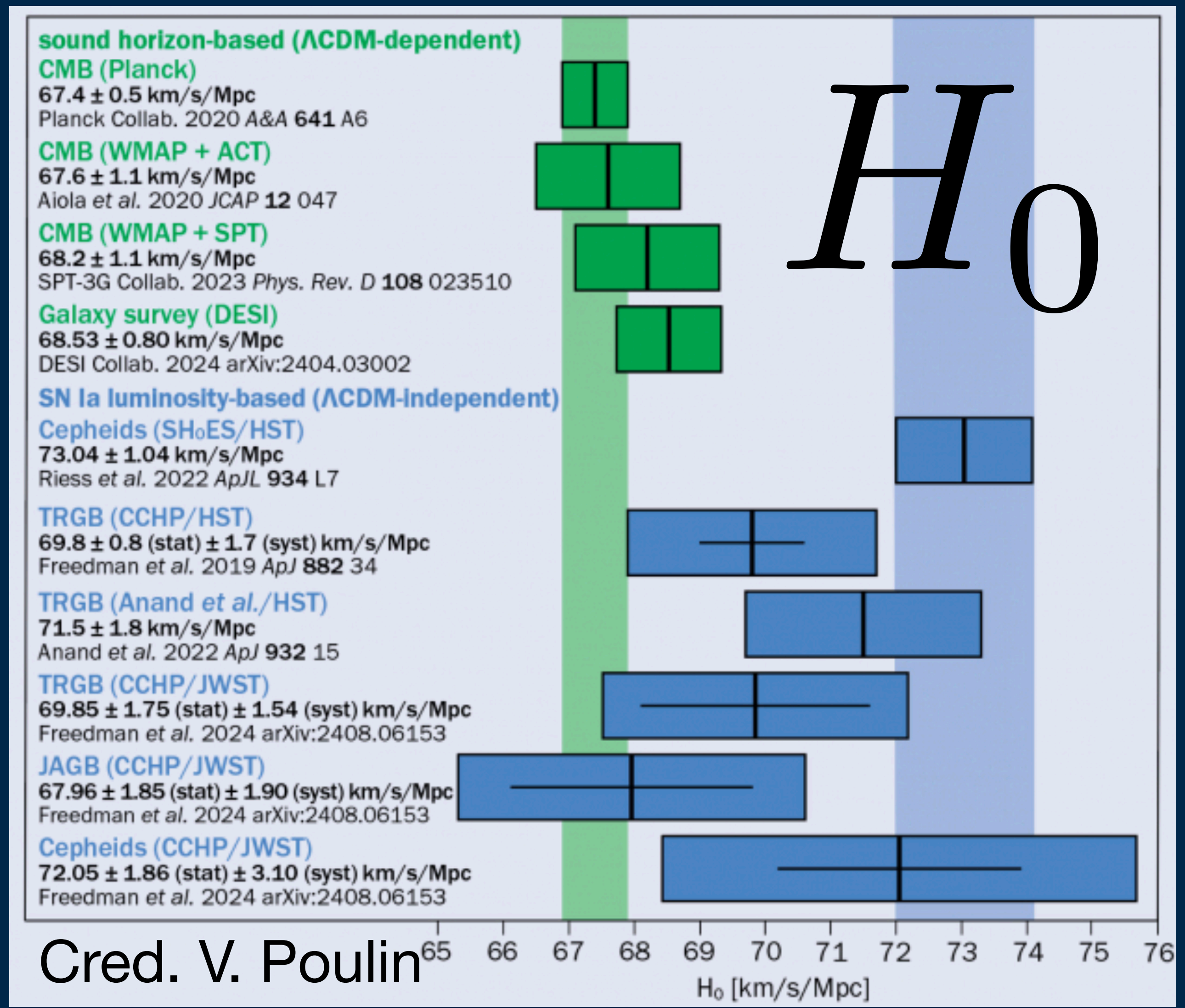


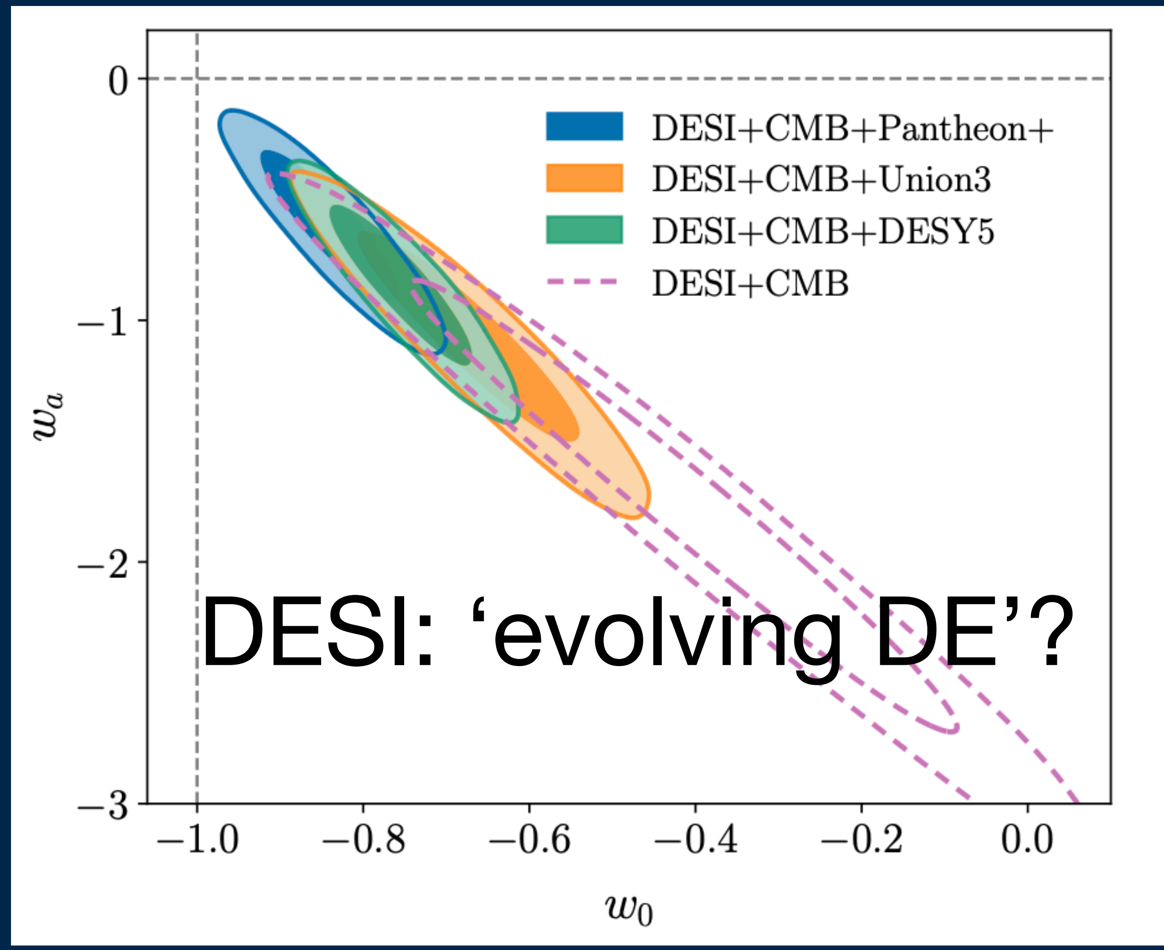


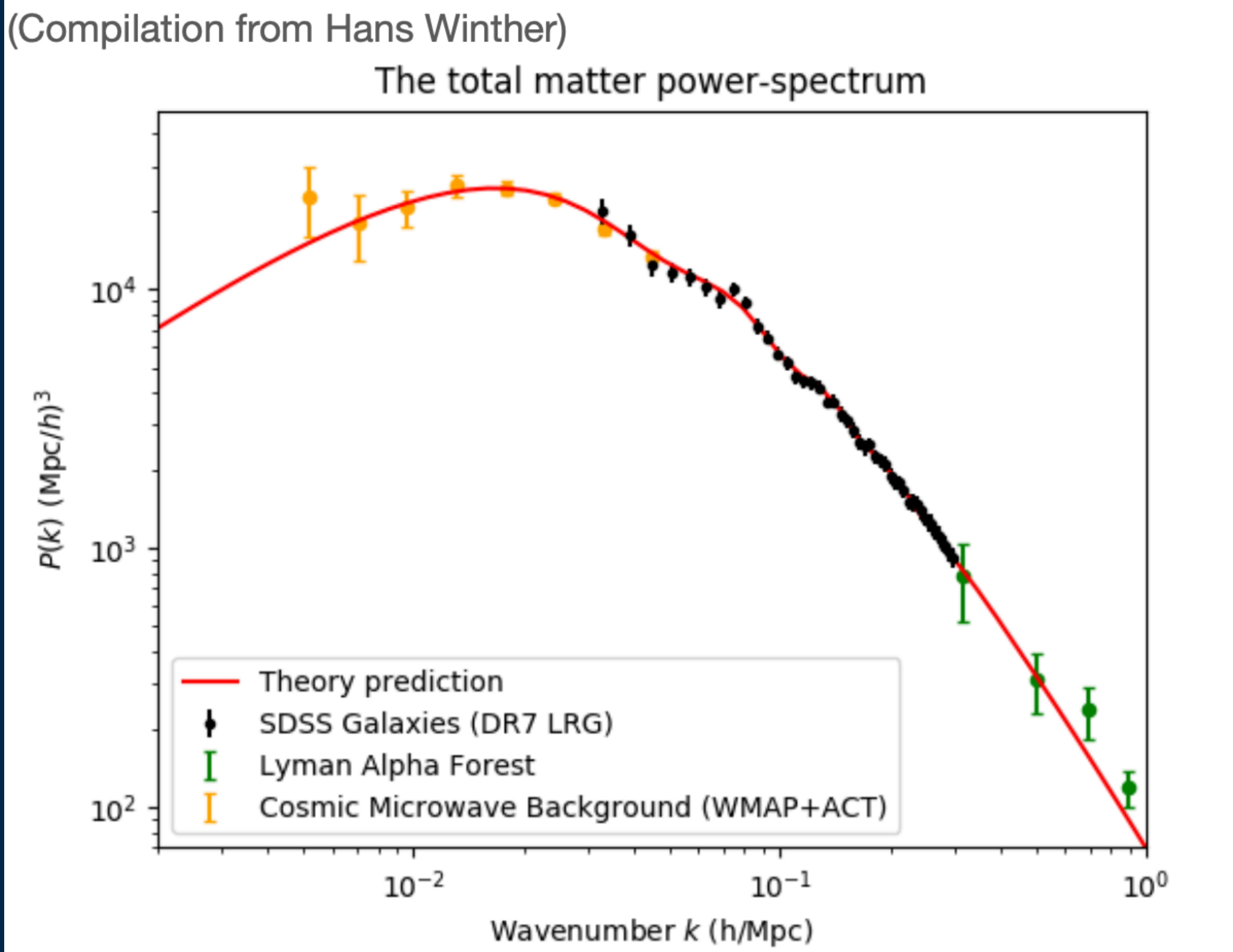
Cold dark matter + cosmological constant: ΛCDM model

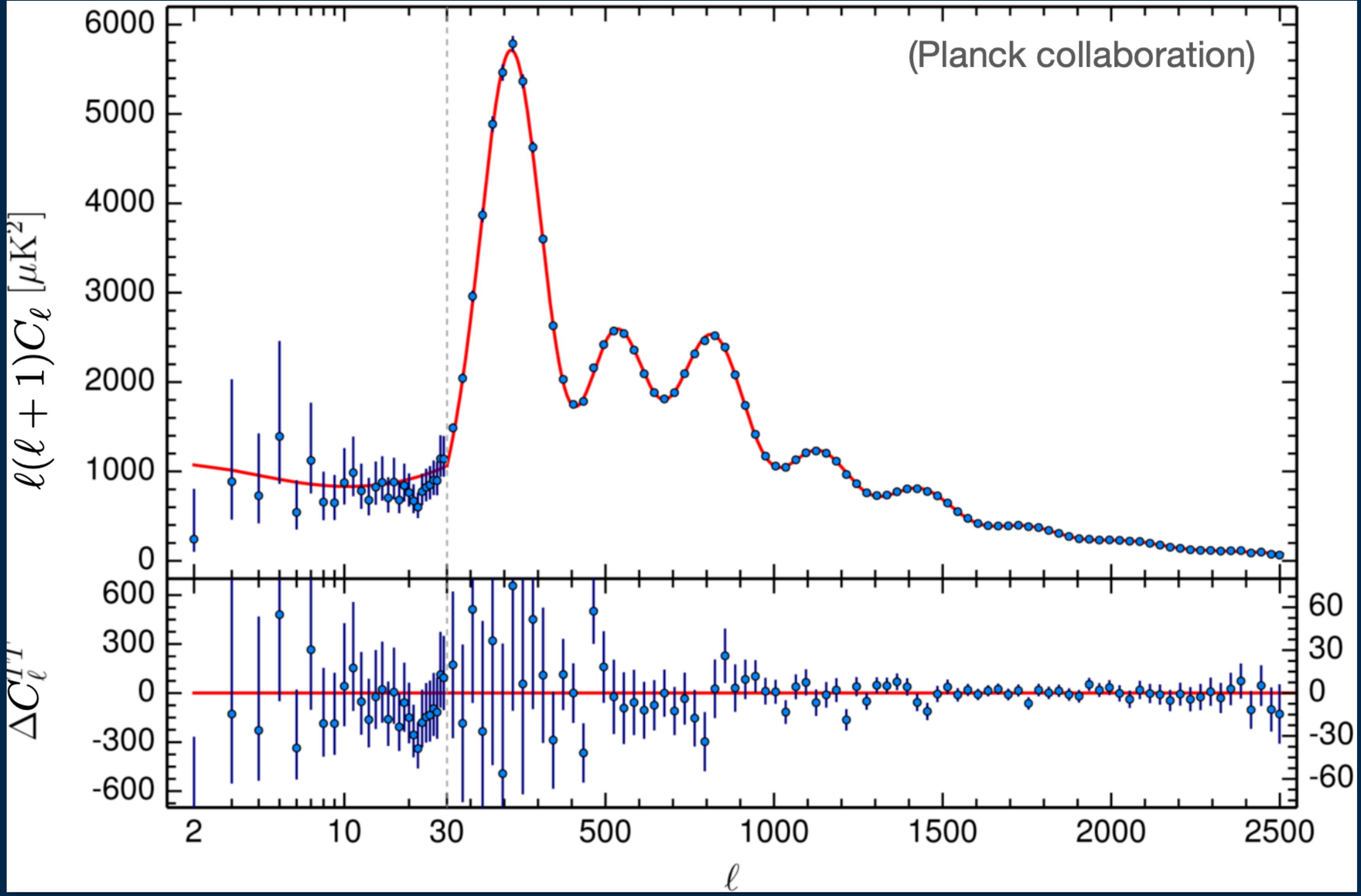
Superb fit on scales $\gtrsim Mpc$

But **Large-scale tensions**









Cold dark matter + cosmological constant: ΛCDM model

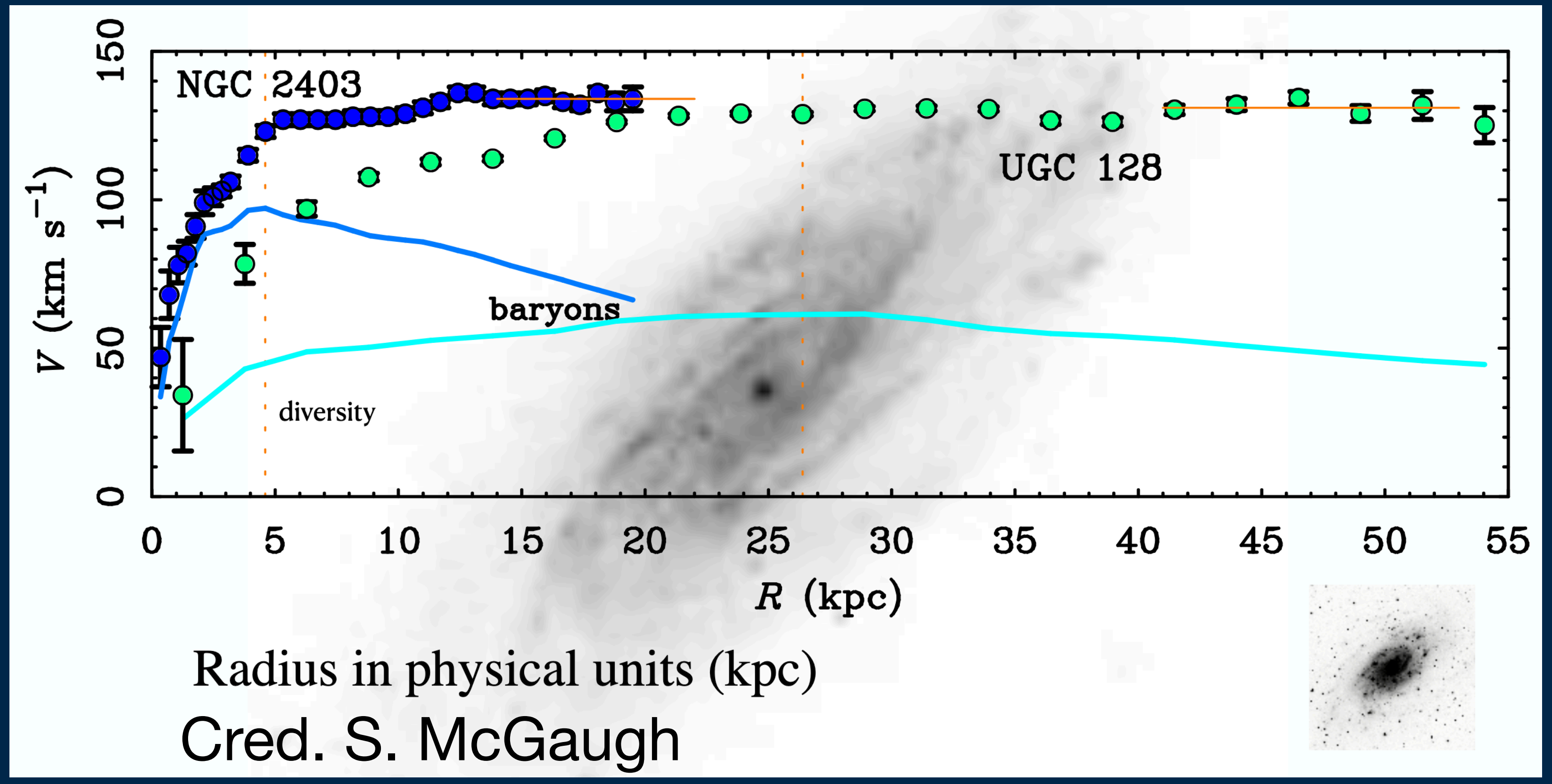
Superb fit on scales $\gtrsim Mpc$

But

Small-scale 'crisis'

See e.g. Bullock & Boylan-Kolchin (2017)

- Missing satellites
- Too big to fail
- Diversity of spiral rotation curves



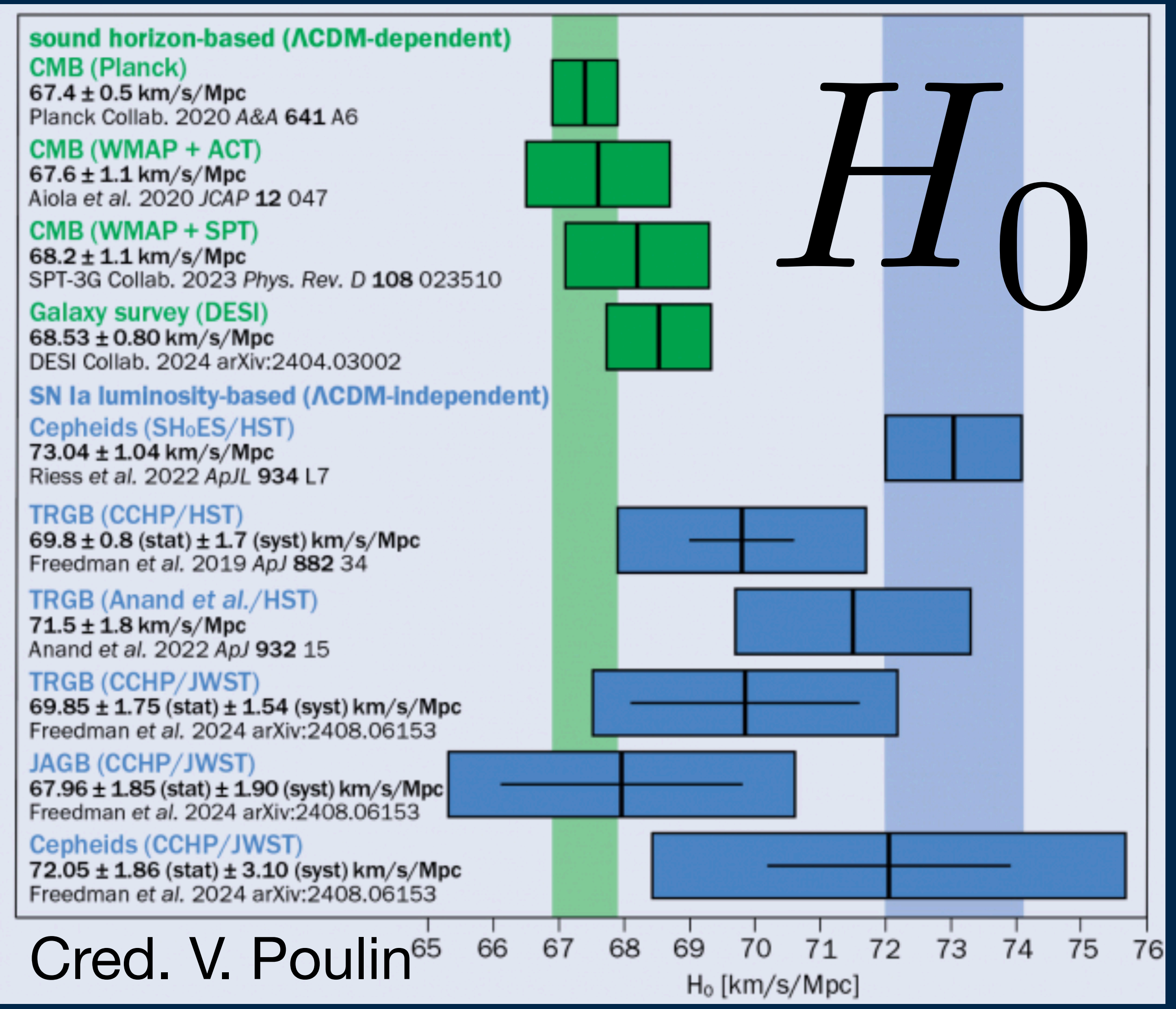
Very different mass distributions

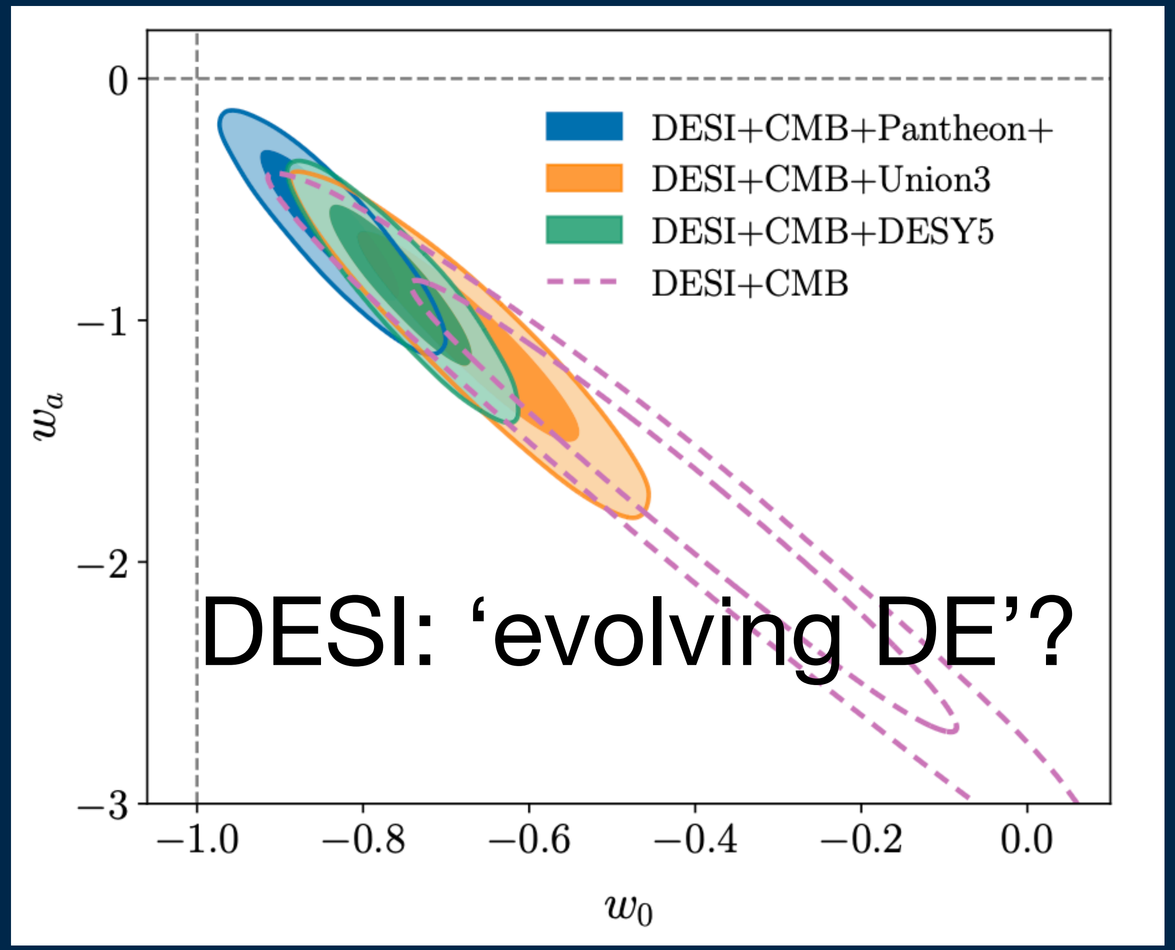
Order of mag. Difference in surface brightness

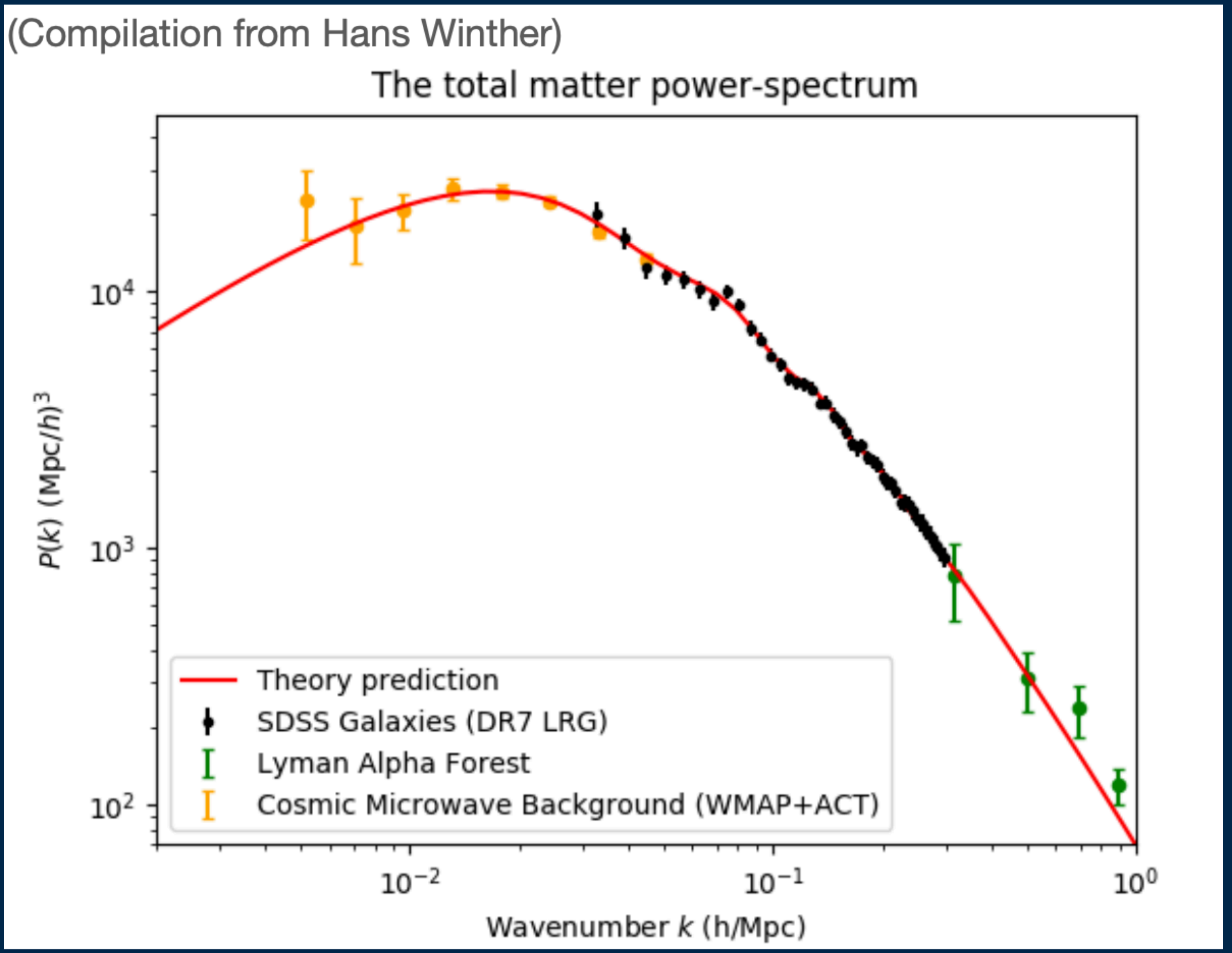
Same flat rotation speed

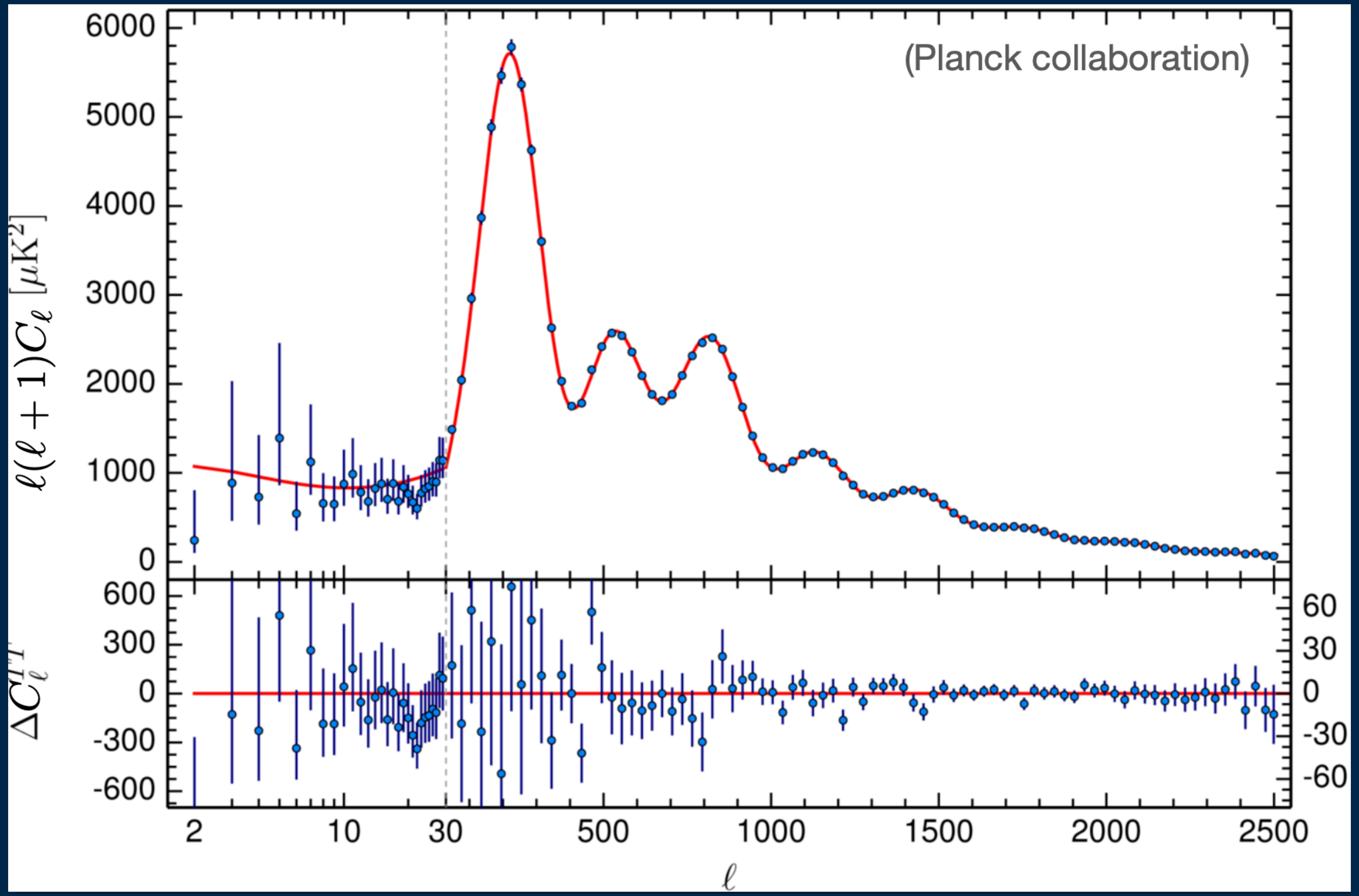
Same mass

Large-scale tensions





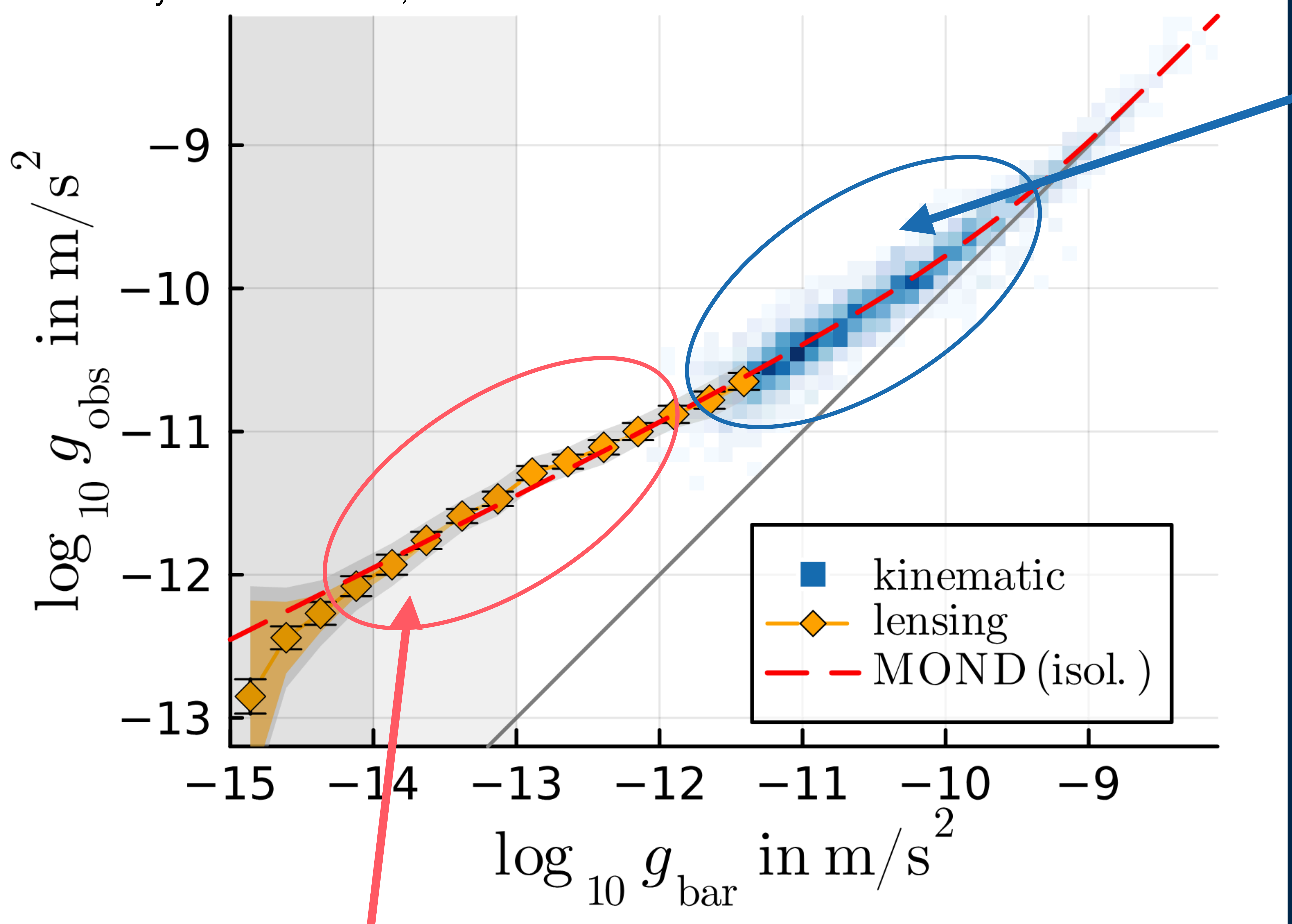




RAR: Radial acceleration relation

S. McGaugh, F. Lelli & J. Schombert, PRL 117, 201101 (2016)

B Famaey & A. Durakovic, ArXiV: 2501.17006



T. Mistele et al., JCAP 04, 020 (2024)

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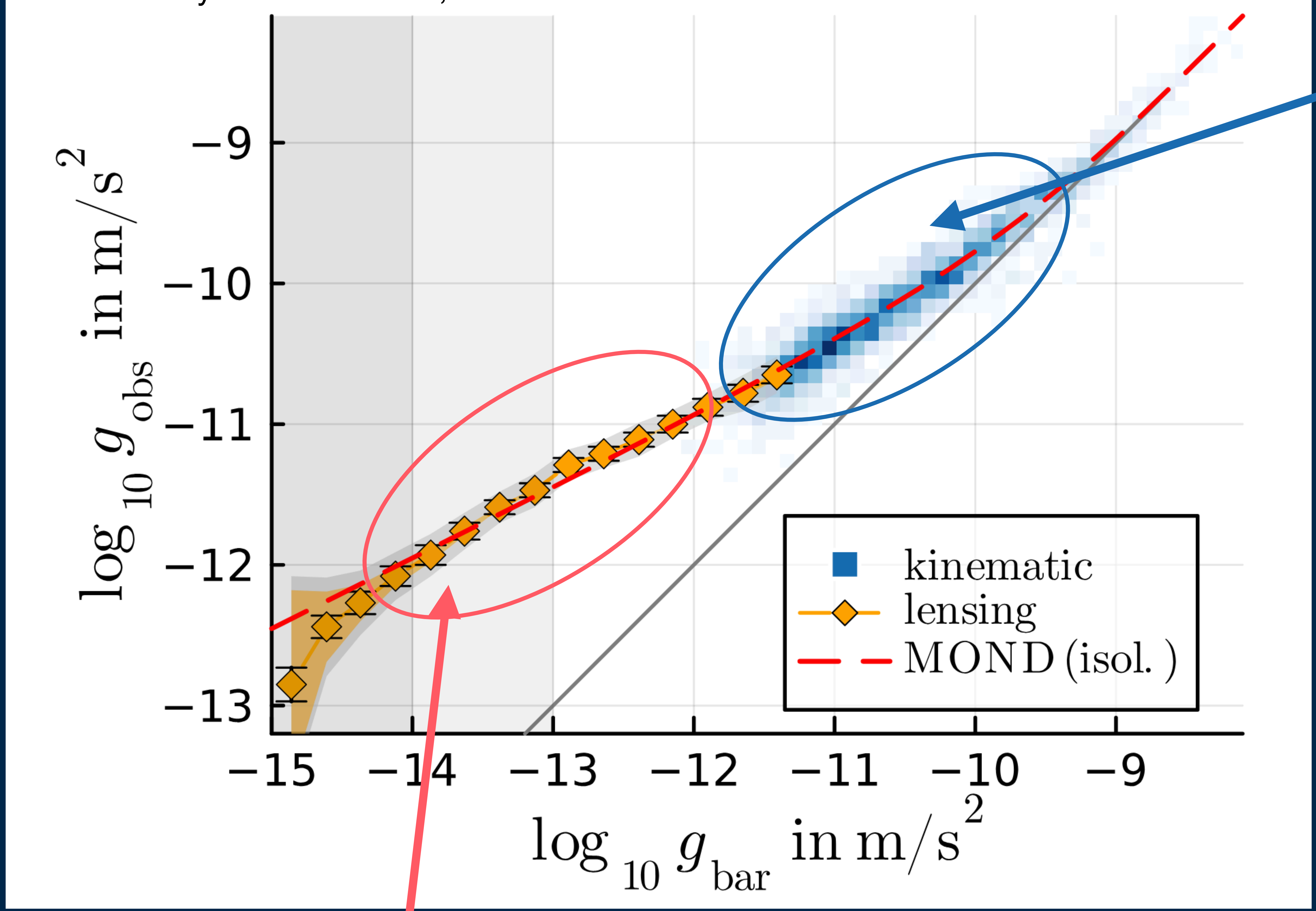

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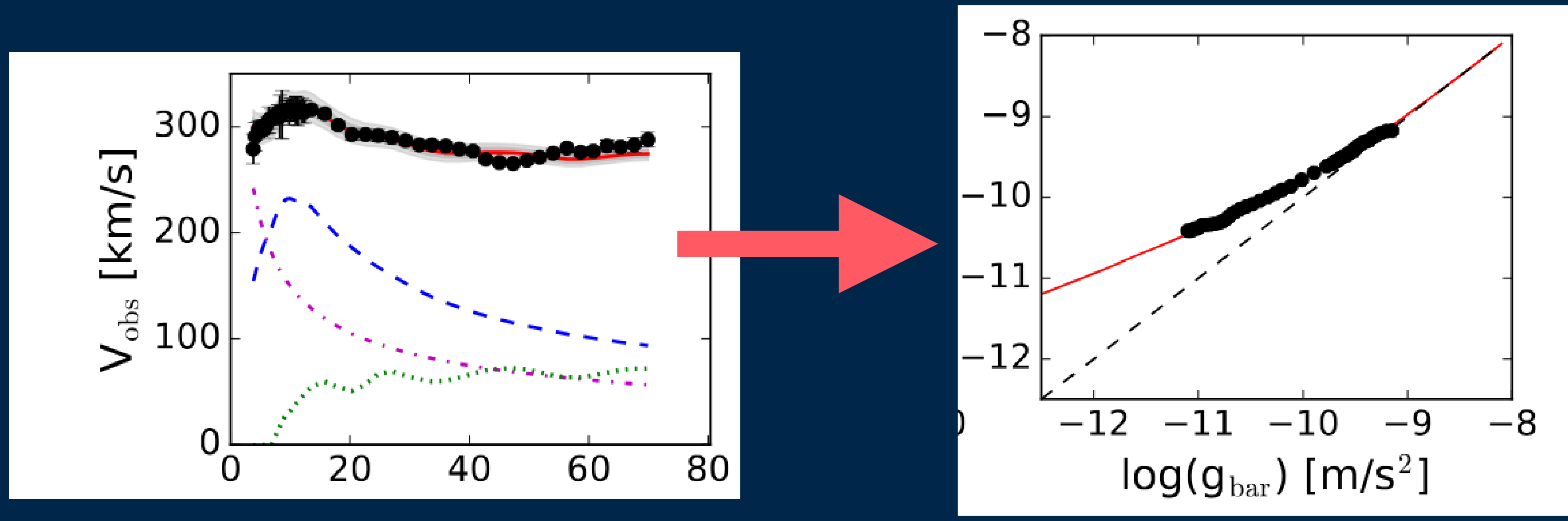
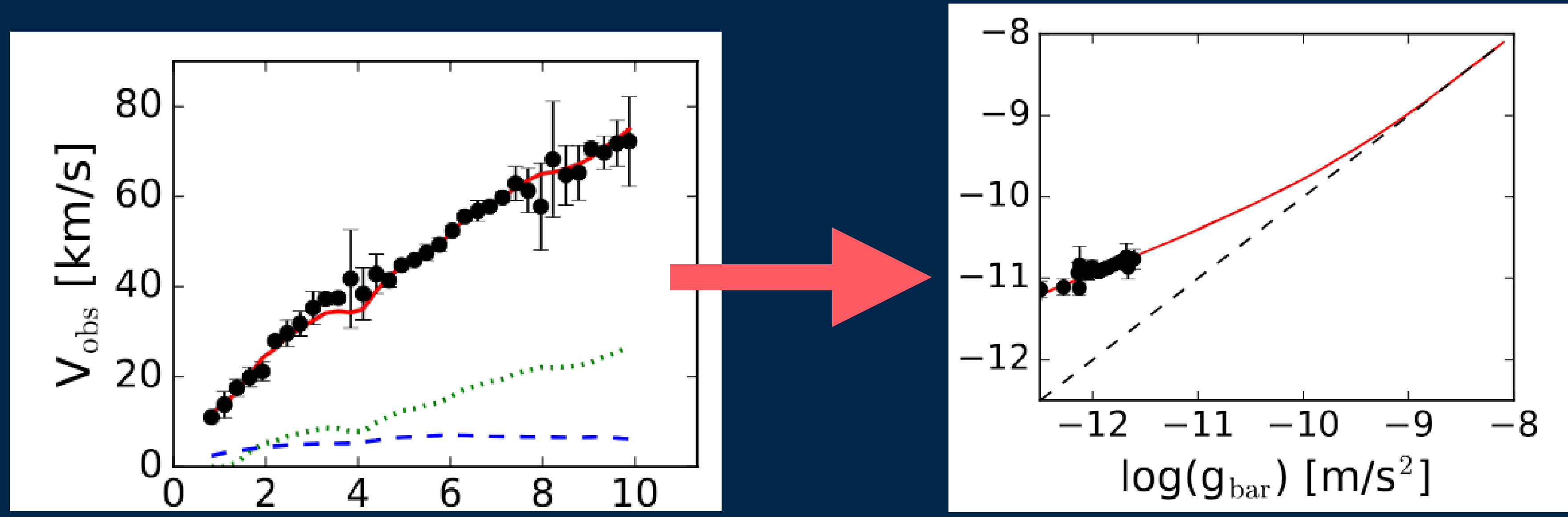


Li et al., A&A 615, A3 (2018)



T. Mistele et al., JCAP 04, 020 (2024)

The RAR is a fundamental relation

- Residuals do not correlate significantly with any other quantity
- Strongest dynamical-to-baryonic relation
- Sufficient to explain all other correlations involving radial dynamics
- No other relation possesses these properties

R. Stiskalek & H. Desmond, MNRAS 525, 6130 (2023)

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Can we understand this in terms of:

- Baryonic physics (e.g. feedback)

Navarro *et al.*, MNRAS 471, 1841 (2017)
Ludlow *et al.*, PRL 118, 161103 (2017)
Paranjape & Sheth, MNRAS 507, 632 (2021)
Challenging view: Desmond, MNRAS 464, 4160 (2017)

- DM physics (e.g. self-interaction, condensate)

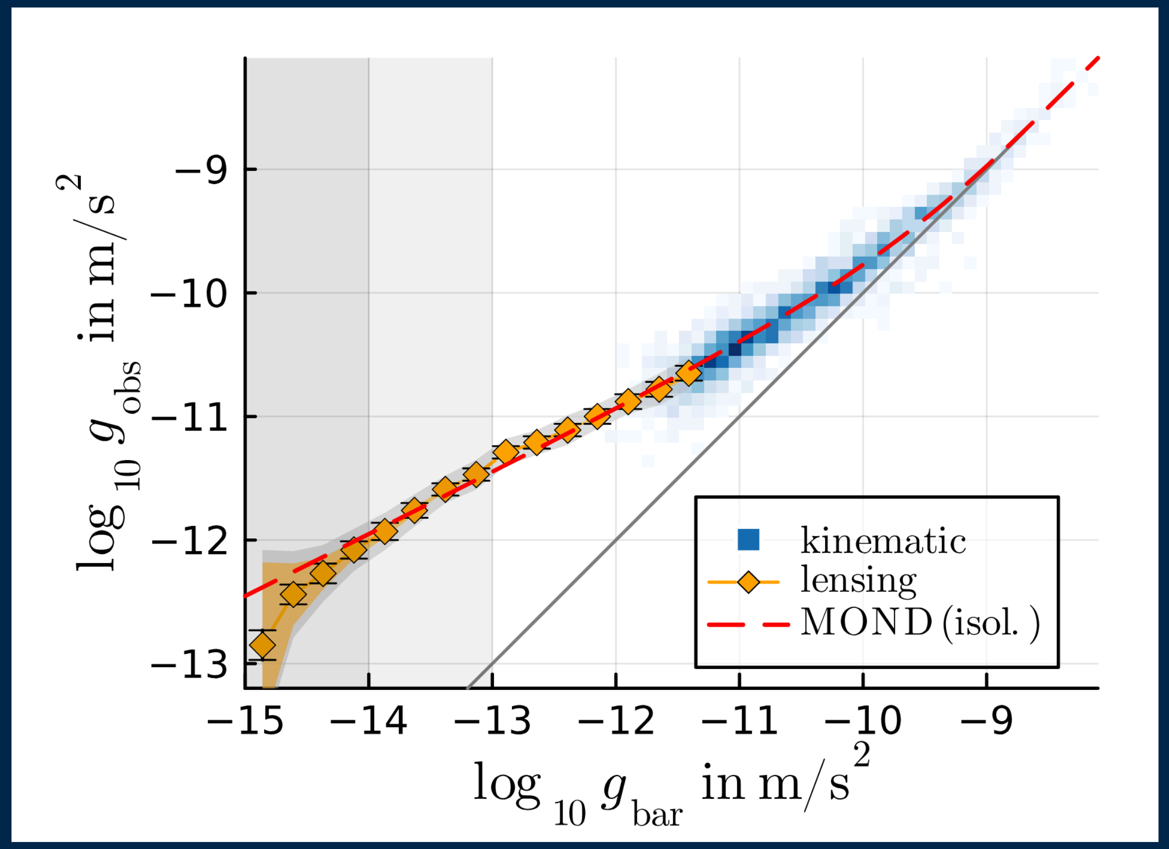
- Self-interacting DM

Spergel & Steinhardt, PRL 84, 3760 (2000)
Ren et al, PRX 9, 031020 (2019)

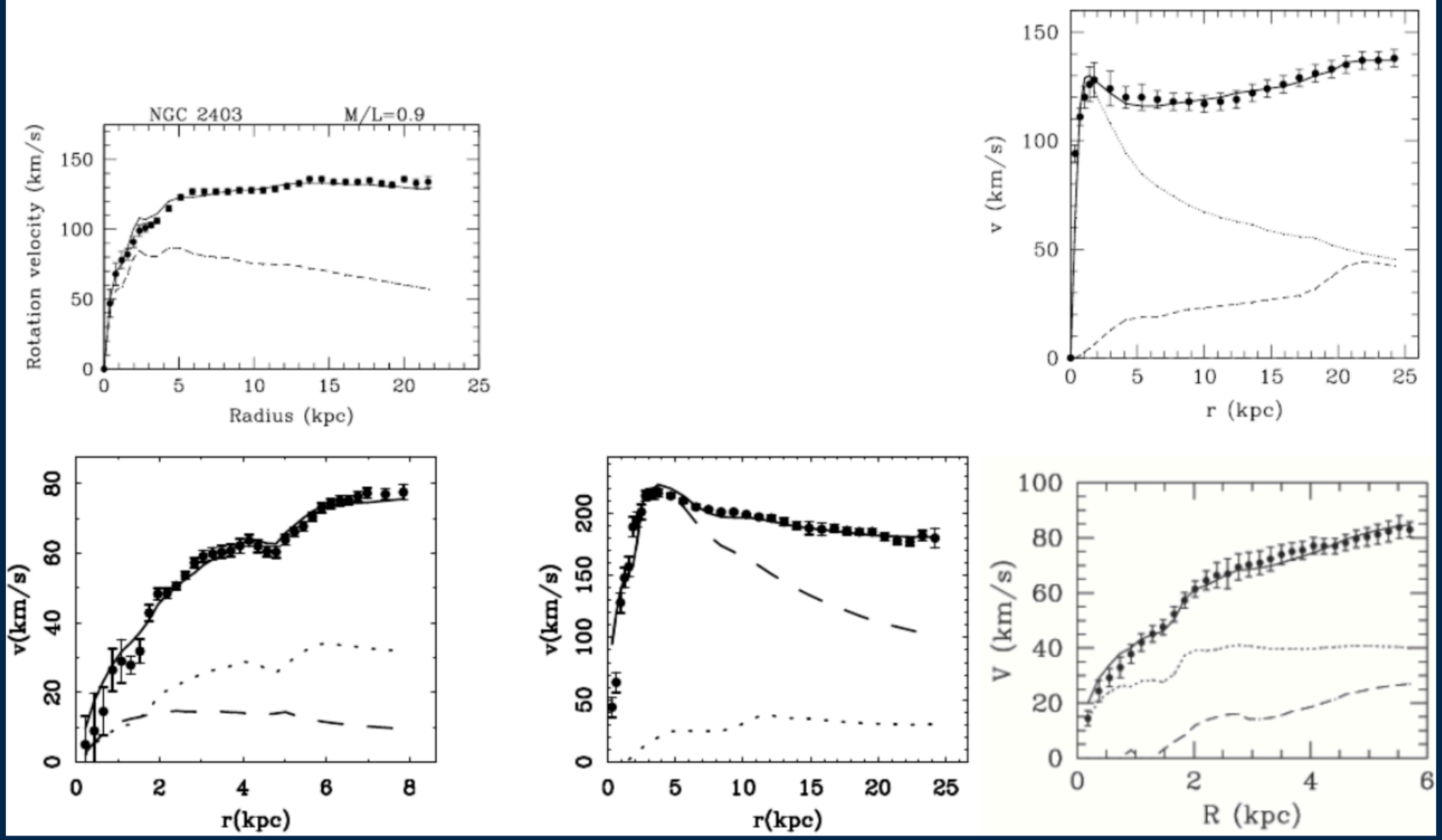
Superfluid dark matter (Khoury & Berezhiani, 2015)

- **Deviations from GR**

RAR: Radial acceleration relation



Dynamics follows baryons



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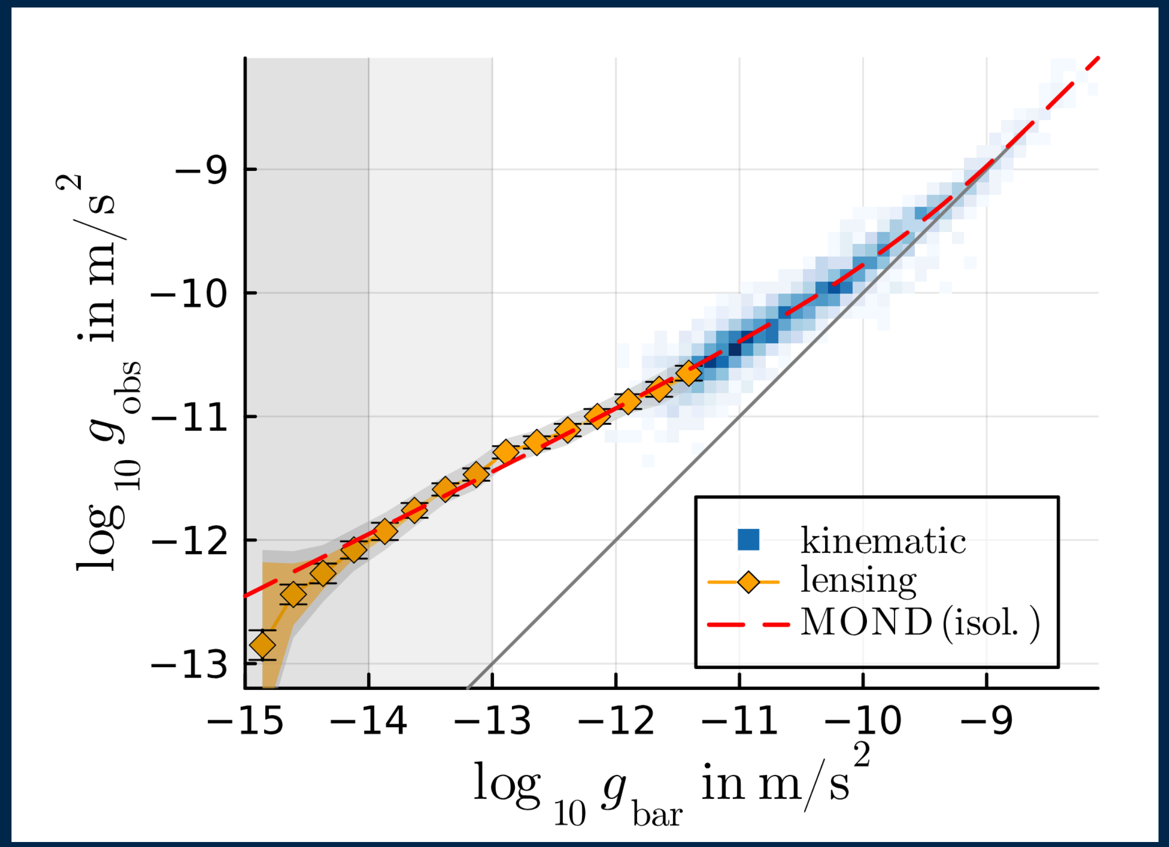

Modified Newtonian Dynamics

Milgrom (1983), Bekenstein & Milgrom (1984)

Deviation from Newton when

$$a < a_0 \sim 1.2 \times 10^{-10} m/s^2$$

Universal constant

$$\vec{\nabla} \cdot \left(\frac{|\vec{\nabla}\Phi|}{a_0} \vec{\nabla}\Phi \right) = 4\pi G_N \rho$$

Not valid for
- Gravitational Lensing
- CMB
- LSS

GR extension

MOND term $\vec{\nabla} \cdot \left(\frac{|\vec{\nabla}\Phi|}{a_0} \vec{\nabla}\Phi \right) = 4\pi G_N \rho$

Derivable from an action $\int d^3x \left(\frac{1}{4\pi G_N} \frac{2}{3a_0} |\vec{\nabla}\Phi|^3 + \Phi\rho \right)$

With $|\vec{\nabla}\Phi|^3 = \left(|\vec{\nabla}\Phi|^2 \right)^{3/2}$

All models for which a MOND limit exists have this term built-in
(either for the grav. potential or, more usually, for some other field)

Restoring Newton: 'interpolation function'

MOND $\vec{\nabla}\cdot\left(\frac{|\vec{\nabla}\Phi|}{a_0}\vec{\nabla}\Phi\right) = 4\pi G_N\rho$

Newton (Poisson) $\vec{\nabla}^2\Phi = 4\pi G_N\rho$

$$x \equiv \frac{|\vec{\nabla}\Phi|}{a_0}$$

$$\vec{\nabla}\cdot\left(f(x)\vec{\nabla}\Phi\right) = 4\pi G_N\rho$$

$$\begin{cases} x \ll 1 & f \to 1 \\ x \gg 1 & f \to x \end{cases}$$

e.g. $f = \frac{x}{1+x}$

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$x \ll 1 \quad f \to 1$

$x \gg 1 \quad f \to x$

e.g. $f = \frac{x}{1+x}$

This can only be an effective description at best

We expect new dof to be there → Φ not enough

$f(x)$ →
- Where is it coming from
- Non-Fourier expandable $|\vec{\nabla}\Phi| \equiv \sqrt{|\vec{\nabla}\Phi|^2}$
- Non-analytic

New scalar φ [Bekenstein & Milgrom 1984] -- generalized Brans-Dicke theory (subset of Horndeski)

Suppose that

Always Poisson $$\vec{\nabla}^2 \tilde{\Phi} = 4\pi G_N \frac{\lambda_s}{1+\lambda_s}\rho$$

Always MOND $$\vec{\nabla} \cdot \left(\frac{|\vec{\nabla}\varphi|}{a_0} \vec{\nabla}\varphi \right) = 4\pi G_N \rho$$

Matter follows $$\vec{\nabla}\Phi = \vec{\nabla}\tilde{\Phi} + \vec{\nabla}\varphi$$

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Spherical source:

$$|\vec{\nabla}\tilde{\Phi}| = \frac{\lambda_s}{1+\lambda_s} \frac{G_N M}{r^2}$$

$$|\vec{\nabla}\varphi| = \frac{\sqrt{a_0 G_N M}}{r}$$

$$|\vec{\nabla}\Phi| \sim \frac{G_N M}{r^2} + \frac{\sqrt{a_0 G_N M}}{r}$$

$\lambda_s \to \infty$

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MOND returns to Newton trivially

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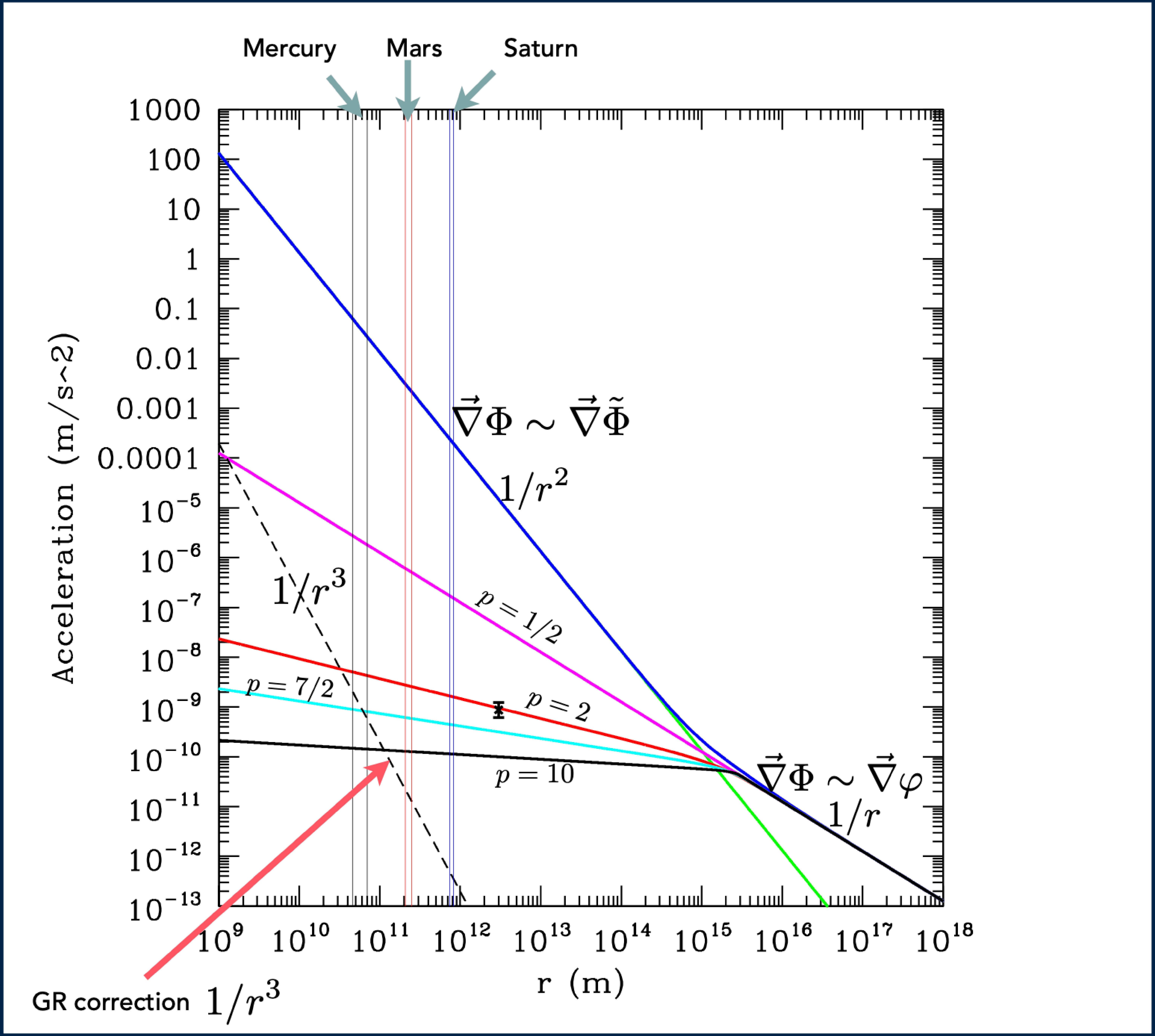
But this doesn't work: GR

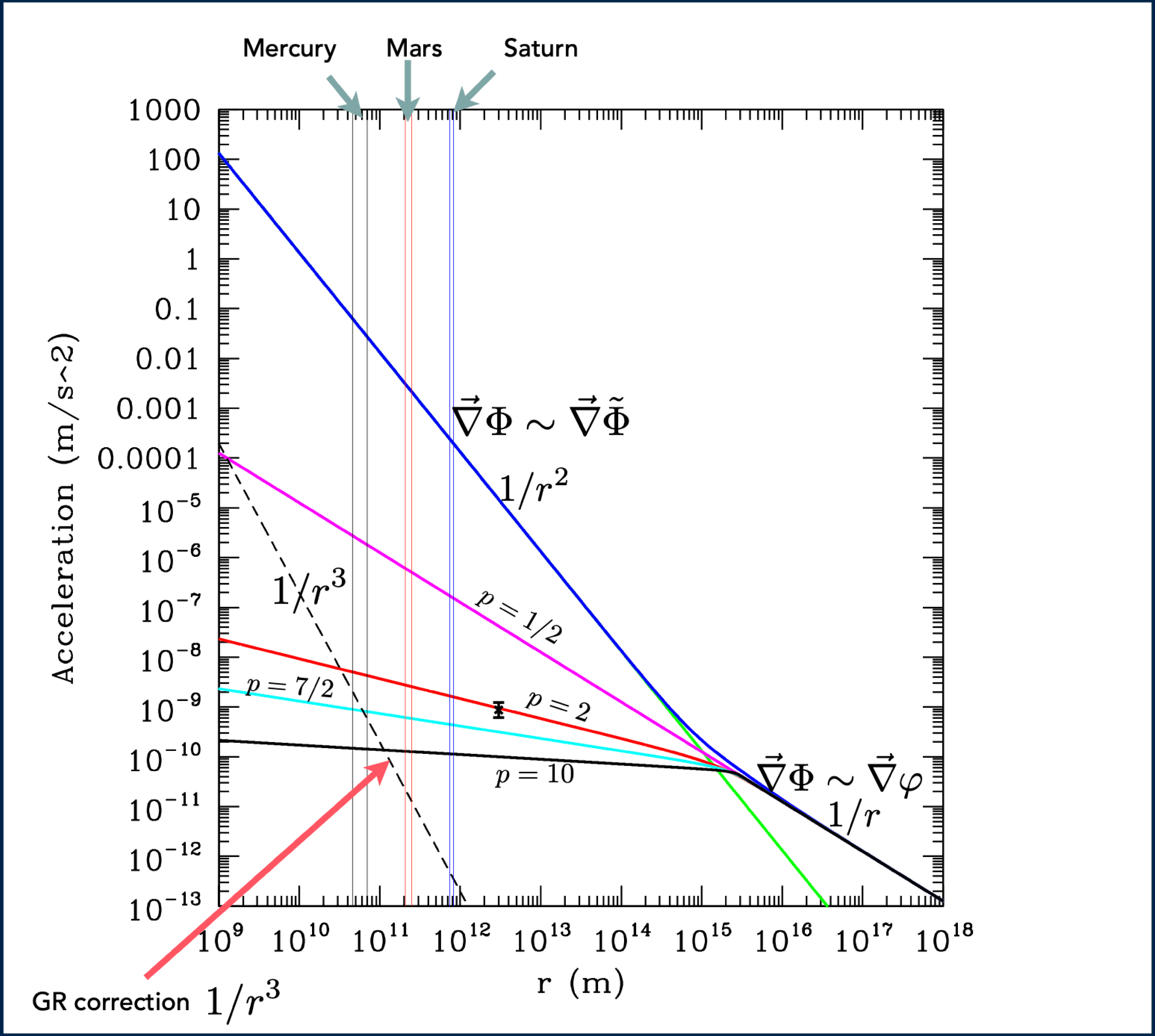
Transition at the MOND radius: $r_M = \sqrt{\frac{G_N M}{a_0}}$



$$\Phi = \tilde{\Phi} + \varphi$$

Screening

$$\mathcal{L} \sim \mathcal{J}(\mathcal{Y}) \sim \frac{|\vec{\nabla}\varphi|^3}{a_0} + \beta_p \frac{|\vec{\nabla}\varphi|^{2(p+1)}}{a_0^{2p}}$$



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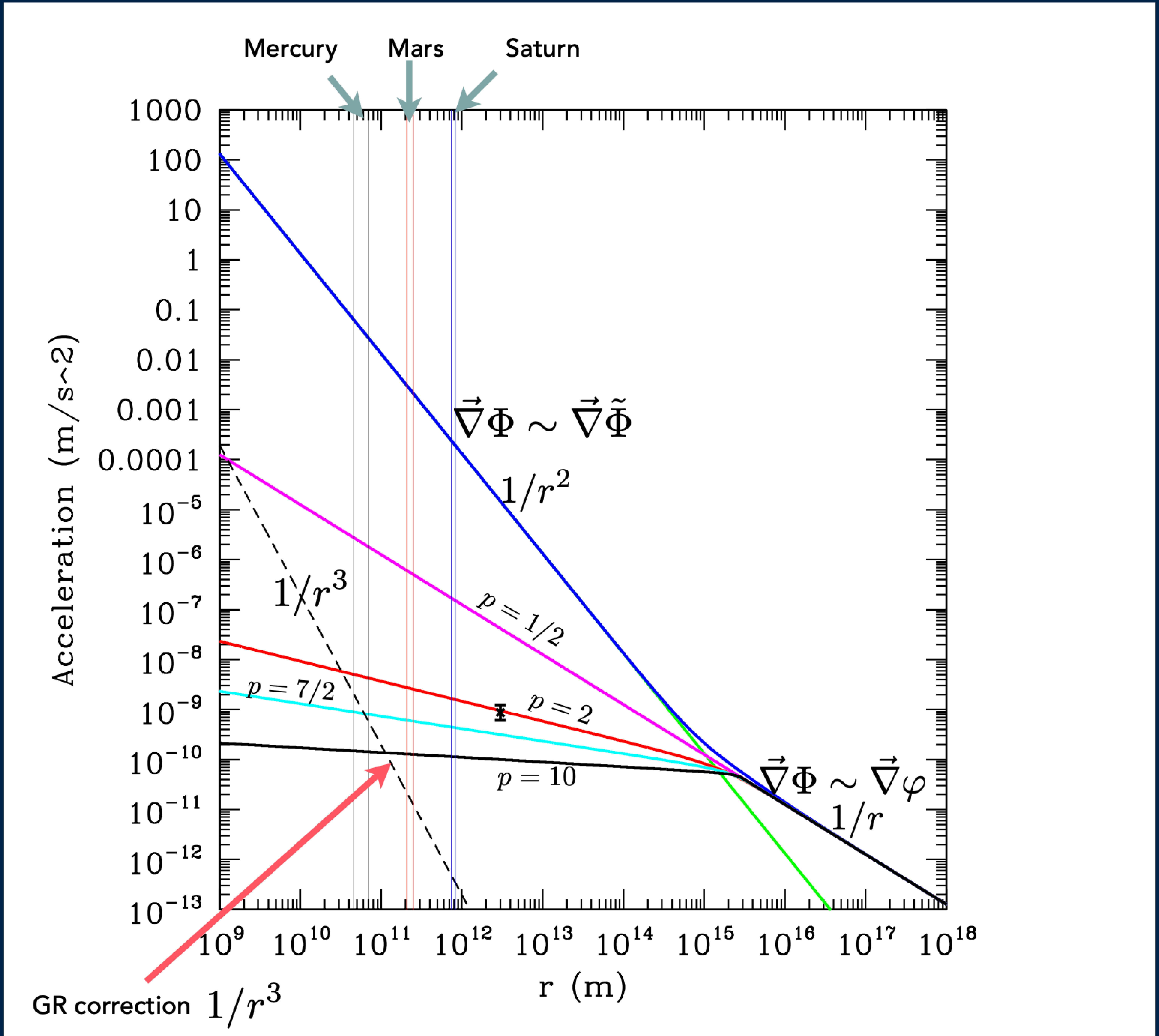

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$$f \equiv \frac{d\mathcal{J}}{d\mathcal{Y}}$$

Tracking

$$\vec\nabla \cdot \left[f\left(\frac{|\vec\nabla\varphi|}{a_0} \right) \vec\nabla\varphi \right] = 4\pi G\rho$$

Interpolation function: $f\left(\frac{|\vec\nabla\varphi|}{a_0} \right)$

- $\frac{|\vec\nabla\varphi|}{a_0} \gg 1$ → Const.
- $\frac{|\vec\nabla\varphi|}{a_0} \ll 1$ → $\frac{|\vec\nabla\varphi|}{a_0}$
- → ?

Lensing : GR extension

$$ds^2 = a^2\left[-(1+2\Psi)d\tau^2 + (1-2\Phi)d\vec{x}\cdot d\vec{x}\right]$$

NR gravity $\vec{\nabla}\cdot\left[f\left(\frac{|\vec{\nabla}\Psi|}{a_0}\right)\vec{\nabla}\Psi\right] = 4\pi G_N\rho$ → If consistent with dynamics

Then

Assumption: $\Phi = \Psi$ → this would give the right lensing as if DM is present

Bekenstein-Milgrom 1984

$\Phi = -\Psi$

doesn't work

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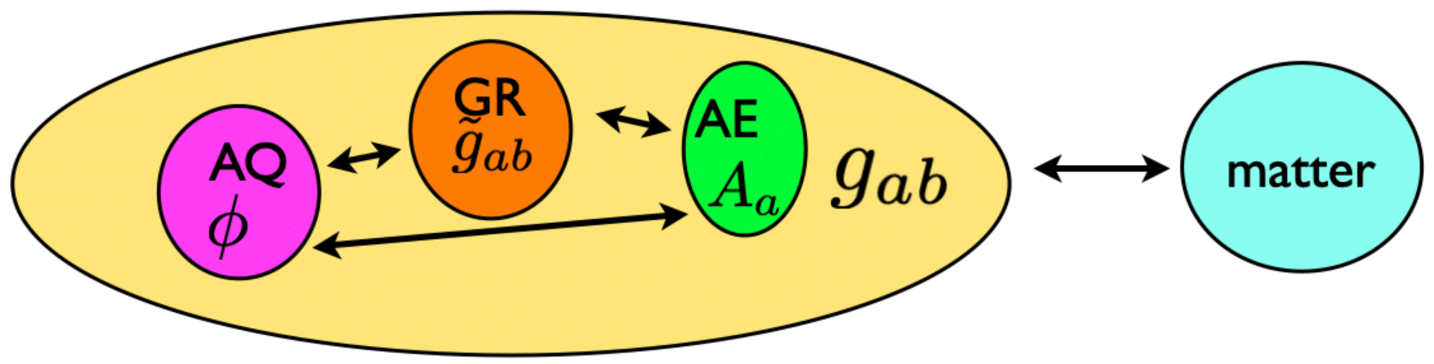
few years & theories later →

Tensor-Vector-Scalar (TeVeS) theory



Bekenstein (2004)

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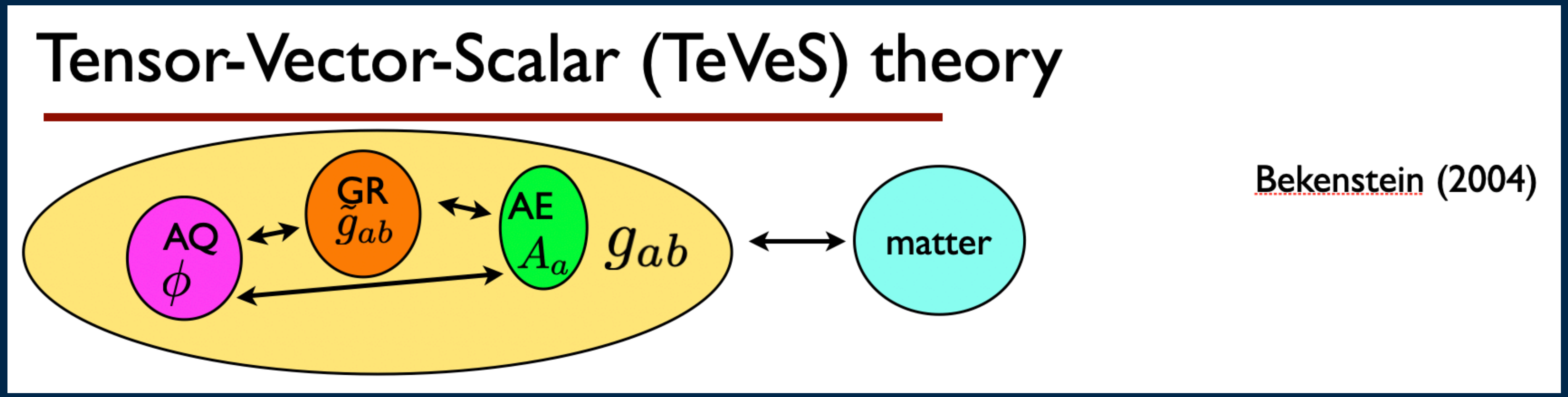

Ruled out by CMB (WMAP 7) CS et al, PRL 96, 011301 (2006)

Ruled out by GW LIGO + EM counterparts CS & Zlonsik, PRD 96, 011301 (2006)

TeVeS theory

(Sanders (1997), Bekenstein (2004) : two metrics)

Tensor mode speed $\neq 1$

Disagreement with CMB

One metric: $g_{\mu\nu}$

ΛCDM

Scherrer (2004)
Arkani-Hamed et al (2004)

New scalar dof. ϕ

Bekenstein & Milgrom (1984)

MOND

Sanders (1997), Bekenstein (2004)
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Aether:
New time-like vector dof. A_μ

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Dirac's new theory of electrons (1963)

Lorentz violation

Einstein-Aether theory
Jacobson. & Mattingly (2002)

Bumblebee field
Kostelecky & Samuel, PRD 40, 1886 (1989)

C.S. & Zlosnik, PRL 127, 161302 (2021)

•Aether Scalar Tensor (AeST)

MOND

On galactic scales

ΛCDM

On cosmological scales

One metric: $g_{\mu\nu}$

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Ingredients:

$g_{\mu\nu}$ ϕ A_μ

Shift symmetry: $\phi \to \phi + \mathrm{const}$

$$S = \int d^4x \frac{\sqrt{-g}}{16\pi\tilde{G}} \left\{ R - 2\Lambda - \frac{K_B}{2} F^{\mu\nu} F_{\mu\nu} + (2 - K_B)\left(2J^\mu \nabla_\mu \phi - \mathcal{Y}\right) - \mathcal{F}(\mathcal{Y}, \mathcal{Q}) - \lambda(A^\mu A_\mu + 1) \right\} + S_m[g]$$

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$F_{\mu\nu} \equiv \nabla_\mu A_\nu - \nabla_\nu A_\mu$

$J_\mu \equiv A^\nu \nabla_\nu A_\mu$

C.S. & Zlosnik, PRL 127, 161302 (2021)

Ingredients:

$$g_{\mu\nu} \quad \phi \quad A_\mu$$

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Matter couples only to $g_{\mu\nu}$ (EEP but not SEP obeyed)

$$F_{\mu\nu} \equiv \nabla_\mu A_\nu - \nabla_\nu A_\mu$$

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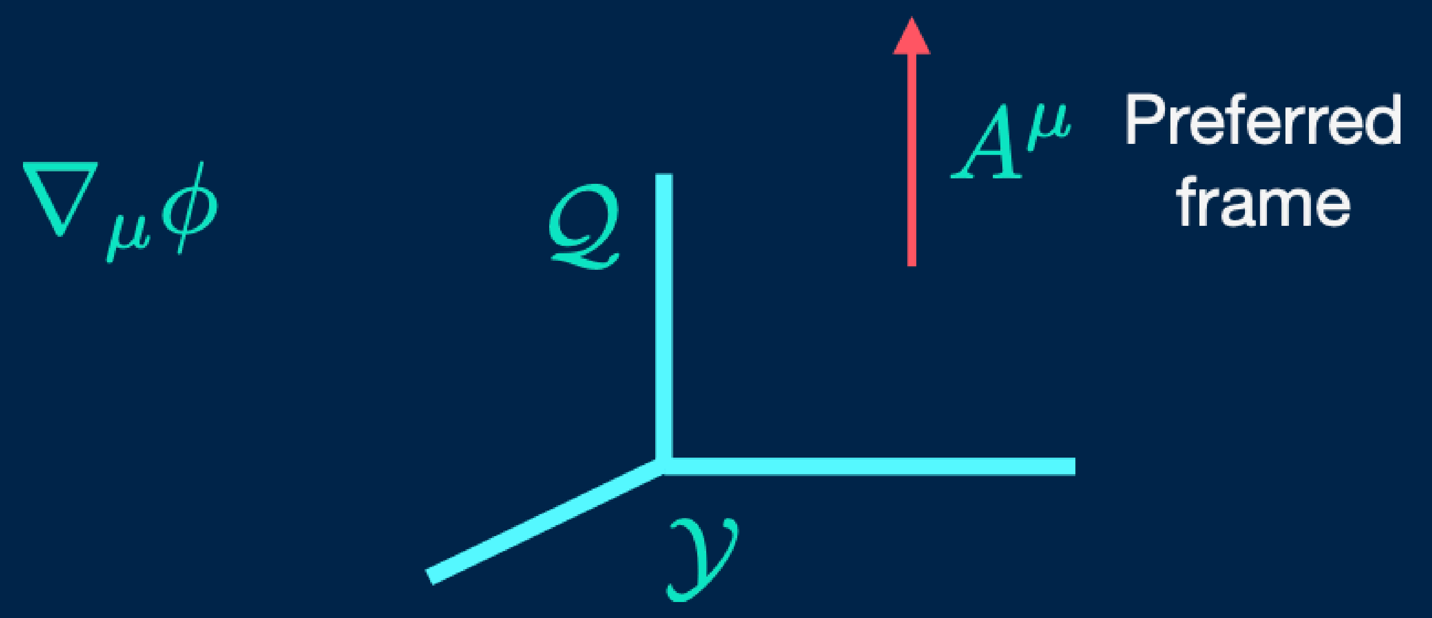

$$\begin{cases} \mathcal{Y} \equiv \left(g^{\mu\nu} + A^\mu A^\nu\right) \nabla_\mu \phi \nabla_\nu \phi \\ \mathcal{Q} \equiv A^\mu \nabla_\mu \phi \end{cases}$$

AeST: FLRW cosmology

C.S. & Zlosnik, PRL 127, 161302 (2021)

FLRW: no spatial gradients

$$\phi = \bar{\phi}(t)$$
$$A^0 \to 1$$
$$ds^2 = -dt^2 + a^2 d\vec{x}^2$$

$\rightarrow$

$$\mathcal{Y} = 0$$
$$\mathcal{Q} = \mathcal{Q}(t)$$

$\rightarrow$

$$\mathcal{F}(\mathcal{Y}, \mathcal{Q}) \to -2\mathcal{K}(\mathcal{Q})$$

Shift-symmetric K-essence

Energy density
$$8\pi\tilde{G}\rho_\phi = \mathcal{Q}\frac{d\mathcal{K}}{d\mathcal{Q}} - \mathcal{K}$$

Design $\mathcal{K}(\mathcal{Q})$ To give ΛCDM evolution for FLRW

The Scherrer model

Shift-symmetric k-essence:

Scherrer, Phys.Rev.Lett. 93, 011301 (2004)

$$\mathcal{L} \sim K_2 \left(X + X_0\right)^2 \quad \text{with} \quad X \equiv g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi \to -\dot{\phi}^2$$

→ **Dust solutions on FLRW**

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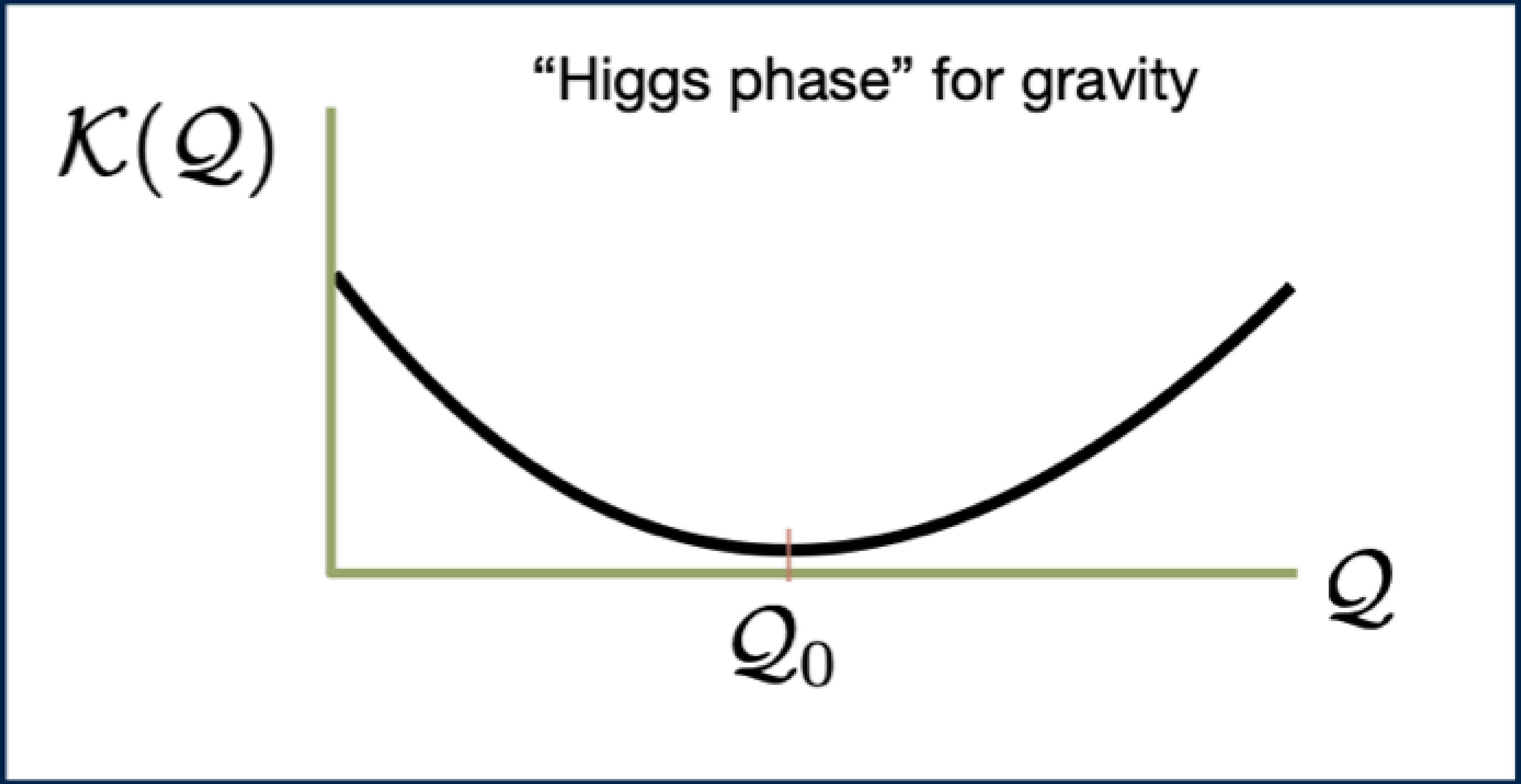
FLRW Limit of Ghost condensate

Arkani-Hamed et al., JHEP 05, 074 (2004)



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→ **Dust solutions on FLRW**

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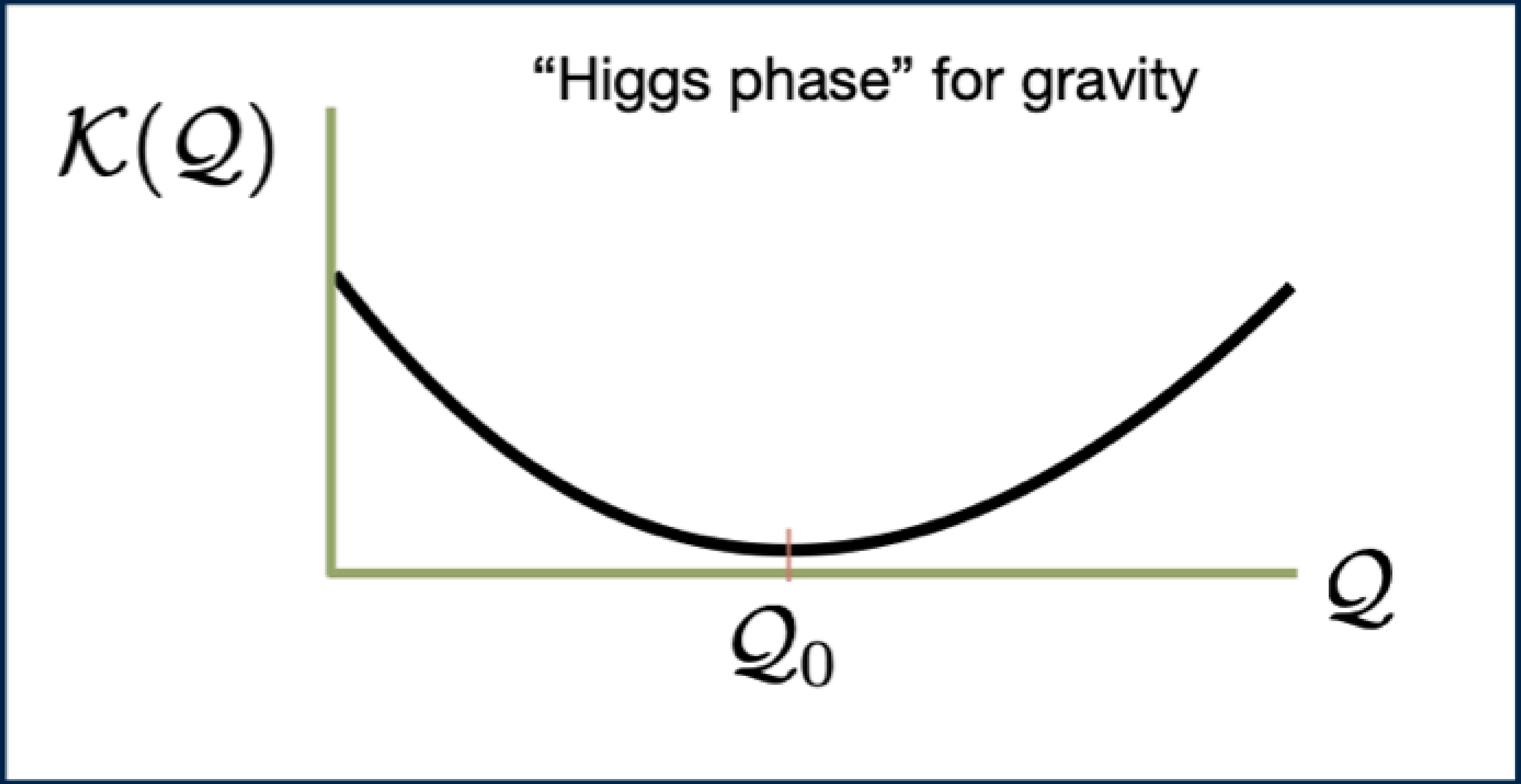
Adapt to AeST setting:

$$X \to \mathcal{Q} = \dot{\phi}$$

$$\mathcal{K}(\mathcal{Q}) = \mathcal{K}_2 \left(\mathcal{Q} - \mathcal{Q}_0\right)^2 + \ldots$$



FLRW in the Scherrer model

FLRW EOM: $\frac{d}{dt}\left(\frac{d\mathcal{K}}{d\mathcal{Q}}\right) = 0$

$$\frac{d\mathcal{K}}{d\mathcal{Q}} = I_0$$

Now Taylor expand: $\frac{d\mathcal{K}}{d\mathcal{Q}} = 2\mathcal{K}_2\left(\mathcal{Q} - \mathcal{Q}_0\right) + \ldots$

Initial condition

$$\mathcal{Q} = \dot{\phi} = \mathcal{Q}_0 + \frac{I_0/(2\mathcal{K}_2)}{a^3} + \ldots$$

At least two parameters: $\mathcal{Q}_0$ $\mathcal{K}_2$

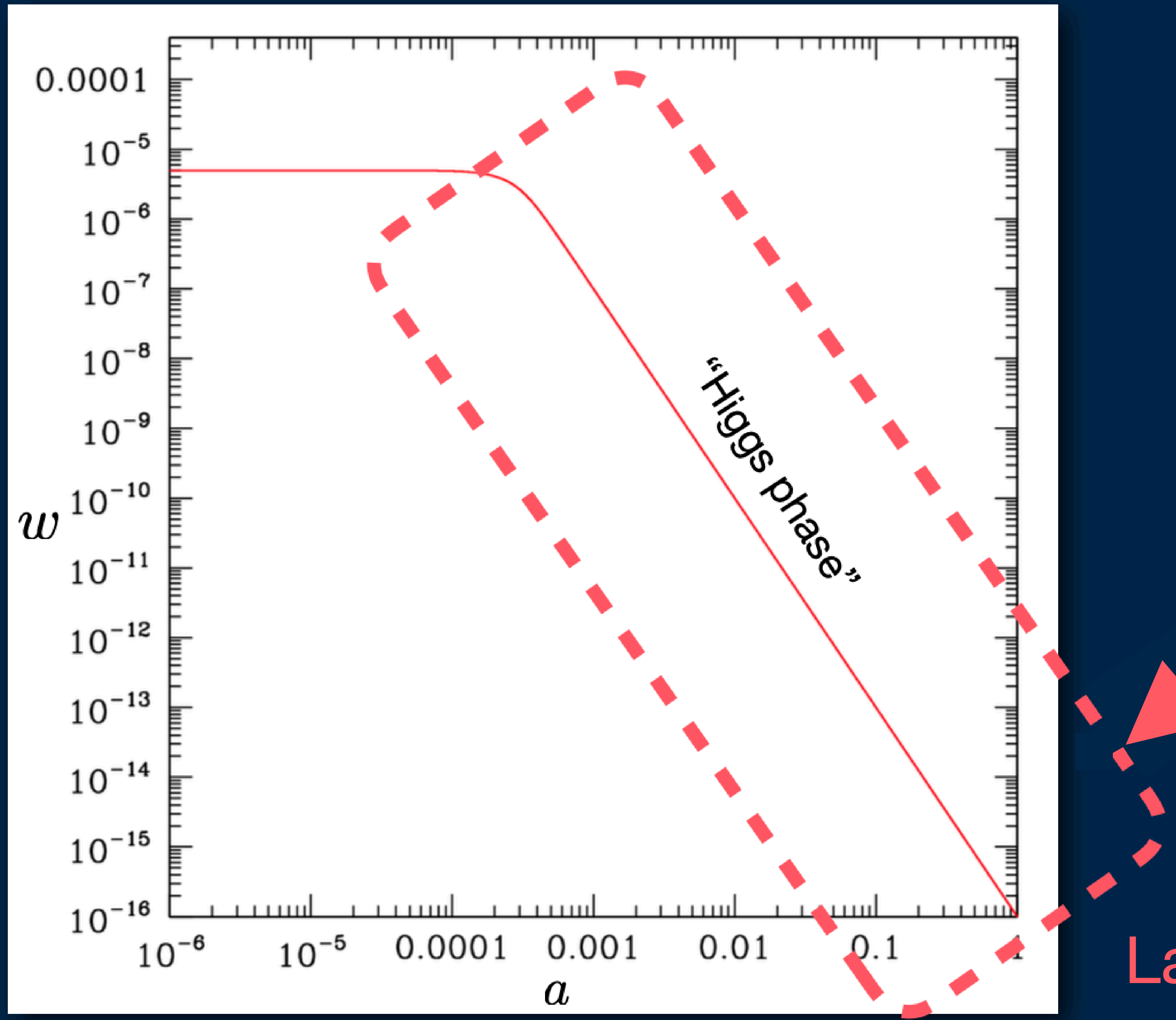
One initial condition: I_0

FLRW in the Scherrer model

$$\mathcal{Q} = \dot{\phi} = \mathcal{Q}_0 + \frac{I_0/(2\mathcal{K}_2)}{a^3} + \dots$$

Now Taylor expand the stress-energy tensor:

Equation of state w(t)



Late region: universal

Higgs phase: effective dust

$$\rho = \frac{\mathcal{Q}_0 I_0}{a^3} + \dots$$ Density

$$w = \frac{w_0}{a^3} + \dots$$ Equation of state

$$c^2_{\rm ad} = \frac{w_0}{a^3} + \dots$$ Adiabatic sound speed

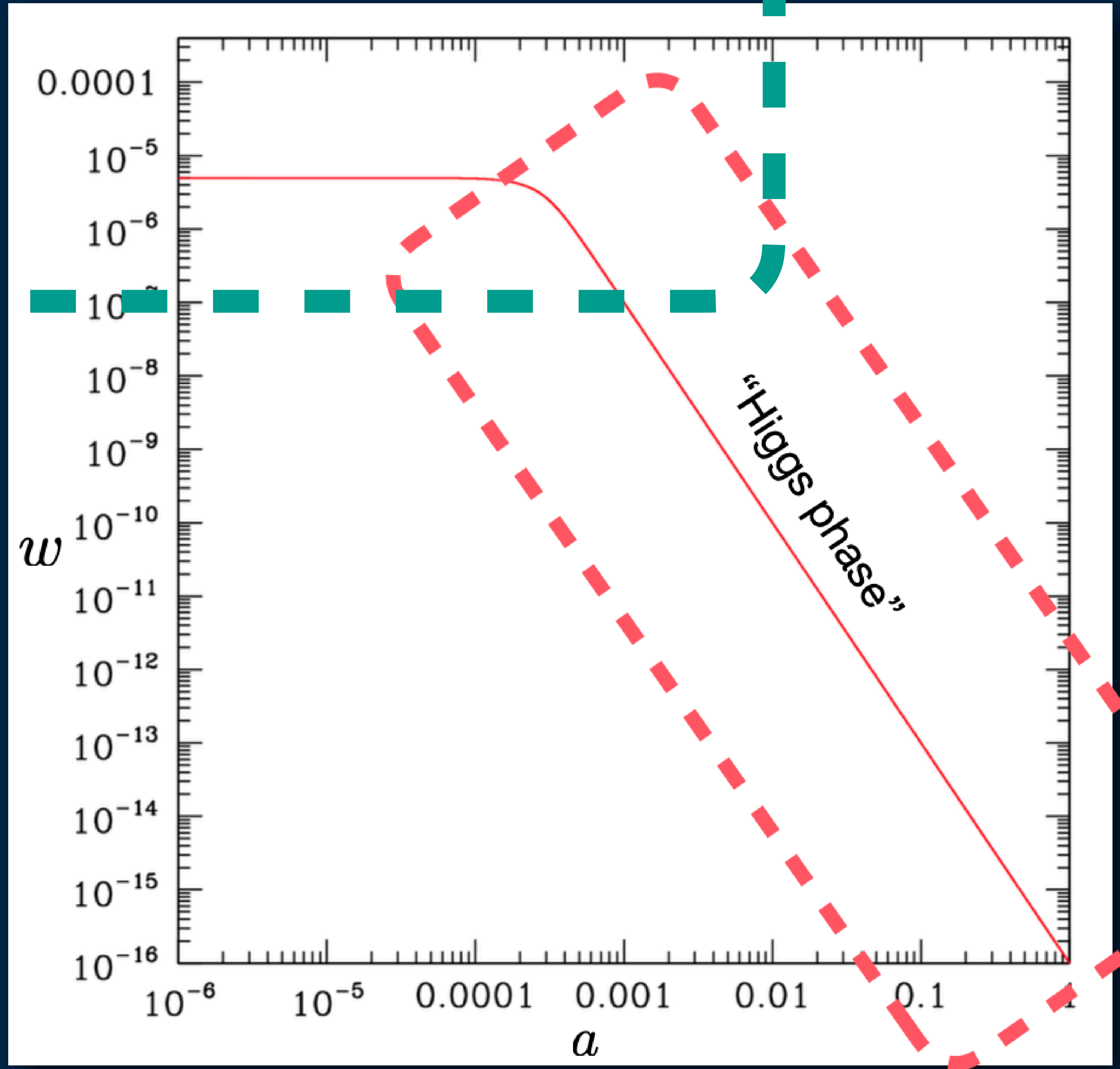
FLRW in the Scherrer model

$$\mathcal{Q} = \dot{\phi} = \mathcal{Q}_0 + \frac{I_0/(2\mathcal{K}_2)}{a^3} + \dots$$

Now Taylor expand the stress-energy tensor:

Early region: depends on form of $\mathcal{K}(\mathcal{Q})$

Equation of state w(t)



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$$w = \frac{w_0}{a^3} + \dots$$ Equation of state

$$c_{\rm ad}^2 = \frac{w_0}{a^3} + \dots$$ Adiabatic sound speed

Quasistatic weak-field limit

$$ds^2 = -(1+2\Psi)\,dt^2 + (1-2\Phi)\,d\vec{x}^2$$

$$\phi = Q_0 t + \varphi(\vec{x})$$

$$A^0 = 1 - \Psi$$

$$A_i = \vec{\nabla}_i \alpha + (\vec{\nabla} \times \beta)_i$$

suppressed

Symmetry: $\dot{\xi}_T = 0$

$$A_i \to A_i - \vec{\nabla}_i \xi_T$$

$$\varphi \to \varphi + Q_0 \xi_T$$

Ignoring curl, set $A_i = 0$

Quasistatic weak-field limit

$$ds^2 = -(1+2\Psi)\,dt^2 + (1-2\Phi)\,d\vec{x}^2$$

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→

$$\vec{\nabla}\phi = \vec{\nabla}\varphi$$

$$\mathcal{Y} \rightarrow |\vec{\nabla}\varphi|^2$$

$$\mathcal{Q} \rightarrow \mathcal{Q}_0$$

→

$$\mathcal{F}(\mathcal{Y}, \mathcal{Q}) \rightarrow \mathcal{J}(\mathcal{Y})$$

Quasistatic weak-field limit

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→ $\vec{\nabla}\phi = \vec{\nabla}\varphi$

$$\mathcal{Y} \to |\vec{\nabla}\varphi|^2$$

$$\mathcal{Q} \to \mathcal{Q}_0$$

→ $\mathcal{F}(\mathcal{Y}, \mathcal{Q}) \to \mathcal{J}(\mathcal{Y})$

Field equations → $\Psi = \Phi$ (lensing works)

Quasistatic Field equations (not MOND)

$$\vec{\nabla}^2\tilde{\Phi} + \mu^2\Phi = \frac{8\pi\tilde{G}}{2-K_B}\rho$$

$$\vec{\nabla}^2\tilde{\Phi} = \vec{\nabla}\cdot\left(\frac{d\mathcal{J}}{d\mathcal{Y}}\vec{\nabla}\varphi\right)$$

Baryon density

$$\mu^2 = \frac{2\mathcal{K}_2\mathcal{Q}_0^2}{2-K_B}$$

Acceleration

$$\vec{\nabla}\Phi = \vec{\nabla}\tilde{\Phi} + \vec{\nabla}\varphi$$

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Baryon density

$$\mu^2 = \frac{2\mathcal{K}_2 Q_0^2}{2-K_B}$$

Acceleration

$$\vec{\nabla}\Phi = \vec{\nabla}\tilde{\Phi} + \vec{\nabla}\varphi$$

$$\mu \to 0$$

$$\tilde{\Phi} \approx \frac{G_N M}{r}$$

$$\vec{\nabla}\cdot\left(\frac{d\mathcal{J}}{d\mathcal{Y}}\vec{\nabla}\varphi\right) \approx \frac{8\pi\tilde{G}}{2-K_B}\rho$$

MOND

Quasistatic Field equations (not MOND)

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$$\tilde{\Phi} \approx \frac{G_N M}{r}$$

$$\vec{\nabla}\cdot\left(\frac{d\mathcal{J}}{d\mathcal{Y}}\vec{\nabla}\varphi\right) \approx \frac{8\pi\tilde{G}}{2-K_B}\rho$$

MOND

Parameter

$$\mathcal{J}(\mathcal{Y}) \to \lambda_s\mathcal{Y} = \lambda_s|\vec{\nabla}\phi|^2 \qquad |\vec{\nabla}\phi| \gg a_0$$

$$\mathcal{J}(\mathcal{Y}) \to \frac{\lambda_s}{1+\lambda_s}\frac{\mathcal{Y}^{3/2}}{a_0} = \frac{\lambda_s}{1+\lambda_s}\frac{|\vec{\nabla}\varphi|^3}{a_0} \qquad |\vec{\nabla}\phi| \ll a_0$$

Parameter

Example: $\mathcal{J} = \lambda_s\left\{\mathcal{Y} - 2a_0(1+\lambda_s)\sqrt{\mathcal{Y}} + 2(1+\lambda_s)^2 a_0^2 \ln\left[1 + \frac{\sqrt{\mathcal{Y}}}{(1+\lambda_s)a_0}\right]\right\}$

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$$\vec{\nabla}^2\tilde{\Phi} = \vec{\nabla}\cdot\left(\frac{d\mathcal{J}}{d\mathcal{Y}}\vec{\nabla}\varphi\right)$$

Baryon density

$$\mu^2 = \frac{2\mathcal{K}_2 Q_0^2}{2-K_B}$$

Acceleration

$$\vec{\nabla}\Phi = \vec{\nabla}\tilde{\Phi} + \vec{\nabla}\varphi$$

Observed Newton's constant

$$G_N = \frac{1+\frac{1}{\lambda_s}}{1-\frac{K_B}{2}}\tilde{G}$$

$\mu \to 0$

$$\tilde{\Phi} \approx \frac{G_N M}{r}$$

$$\vec{\nabla}\cdot\left(\frac{d\mathcal{J}}{d\mathcal{Y}}\vec{\nabla}\varphi\right) \approx \frac{8\pi\tilde{G}}{2-K_B}\rho$$

MOND

Parameter

$$\mathcal{J}(\mathcal{Y}) \to \lambda_s\mathcal{Y} = \lambda_s|\vec{\nabla}\phi|^2 \qquad |\vec{\nabla}\phi| \gg a_0$$

$$\mathcal{J}(\mathcal{Y}) \to \frac{\lambda_s}{1+\lambda_s}\frac{\mathcal{Y}^{3/2}}{a_0} = \frac{\lambda_s}{1+\lambda_s}\frac{|\vec{\nabla}\varphi|^3}{a_0} \qquad |\vec{\nabla}\phi| \ll a_0$$

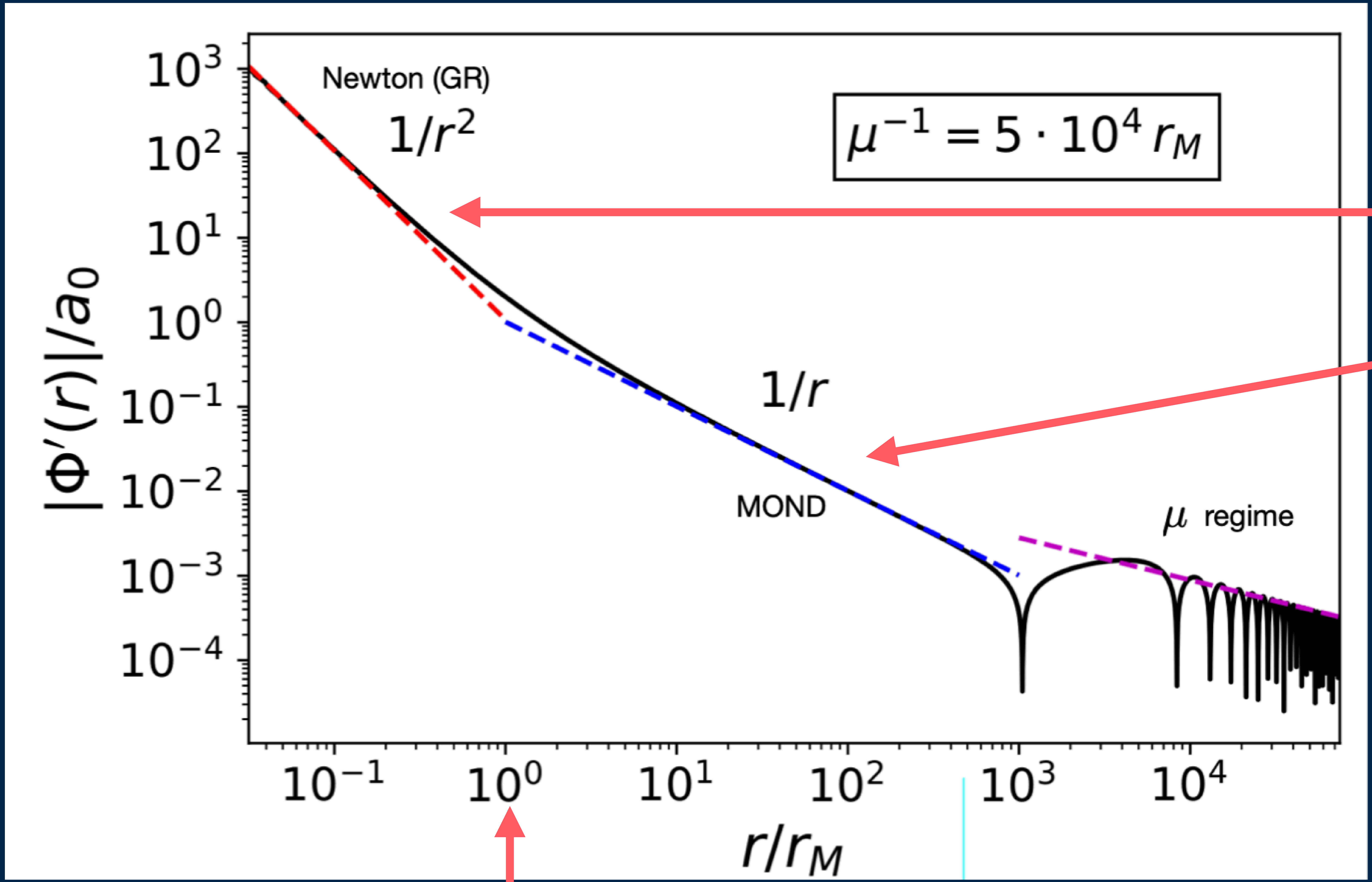
Parameter

Example: $\mathcal{J} = \lambda_s\left\{\mathcal{Y} - 2a_0(1+\lambda_s)\sqrt{\mathcal{Y}} + 2(1+\lambda_s)^2 a_0^2 \ln\left[1 + \frac{\sqrt{\mathcal{Y}}}{(1+\lambda_s)a_0}\right]\right\}$

AeST: Quasistatic limit

Verwayen, Boehm, C.S., MNRAS 531, 272 (2024)

Durakovic & C.S., JCAP 04 (2024) 040



$\mathcal{F}(\mathcal{Y}, \mathcal{Q}_0) \to \mathcal{Y}$

Inherited from TeVeS

$\mathcal{F}(\mathcal{Y}, \mathcal{Q}_0) \to \mathcal{Y}^{3/2}/a_0$

$\Phi = \Psi$ Lensing

$r_M \sim \sqrt{\frac{GM}{a_0}}$

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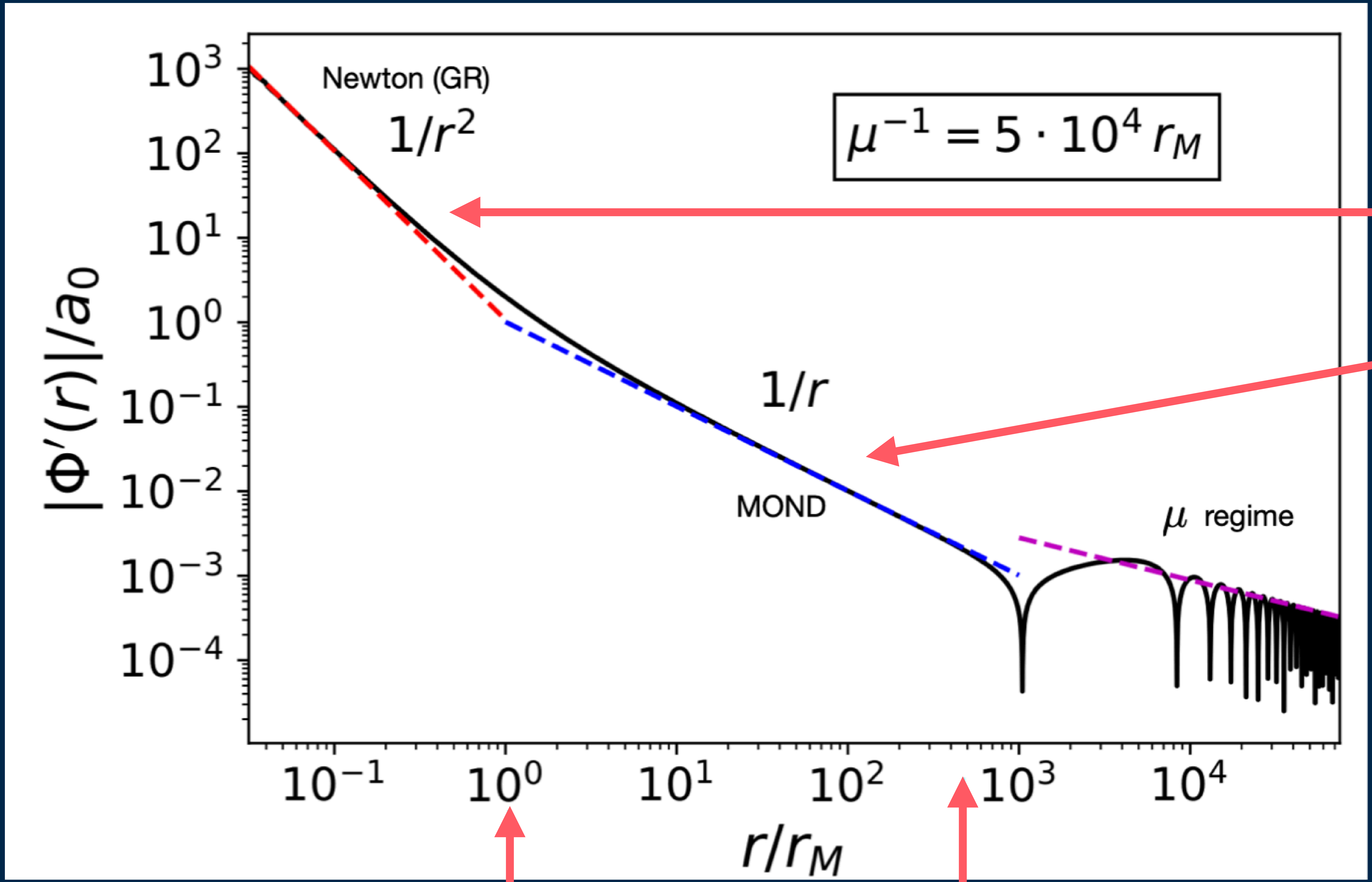

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$\mathcal{F}(\mathcal{Y}, \mathcal{Q}_0) \to \mathcal{Y}^{3/2}/a_0$

$\Phi = \Psi$ Lensing

$r_M \sim \sqrt{\frac{GM}{a_0}}$

$r_C \sim \left(\frac{r_M}{\mu^2}\right)^{1/3}$ New scale $\mu^{-1} \gtrsim Mpc$

(Observationally)

$(\mu \lesssim 6 \times 10^{-30} eV)$

Back to cosmology

MOND compatibility

$$\mu^2 = \frac{2\mathcal{K}_2}{2 - K_B} Q_0^2$$

$$\mu^{-1} \gtrsim Mpc$$

Higgs phase:

$$w \approx \frac{w_0}{a^3} + \dots$$

$$w_0 = \frac{3H_0^2\Omega_Q}{4Q_0^2\mathcal{K}_2} = \frac{3H_0^2\Omega_Q}{2(2 - K_B)\mu^2}$$

$$w_0 \gtrsim 10^{-8}$$

$$w_{rec} \sim O(1)$$

Data: $w_{rec} \lesssim 10^{-4}$

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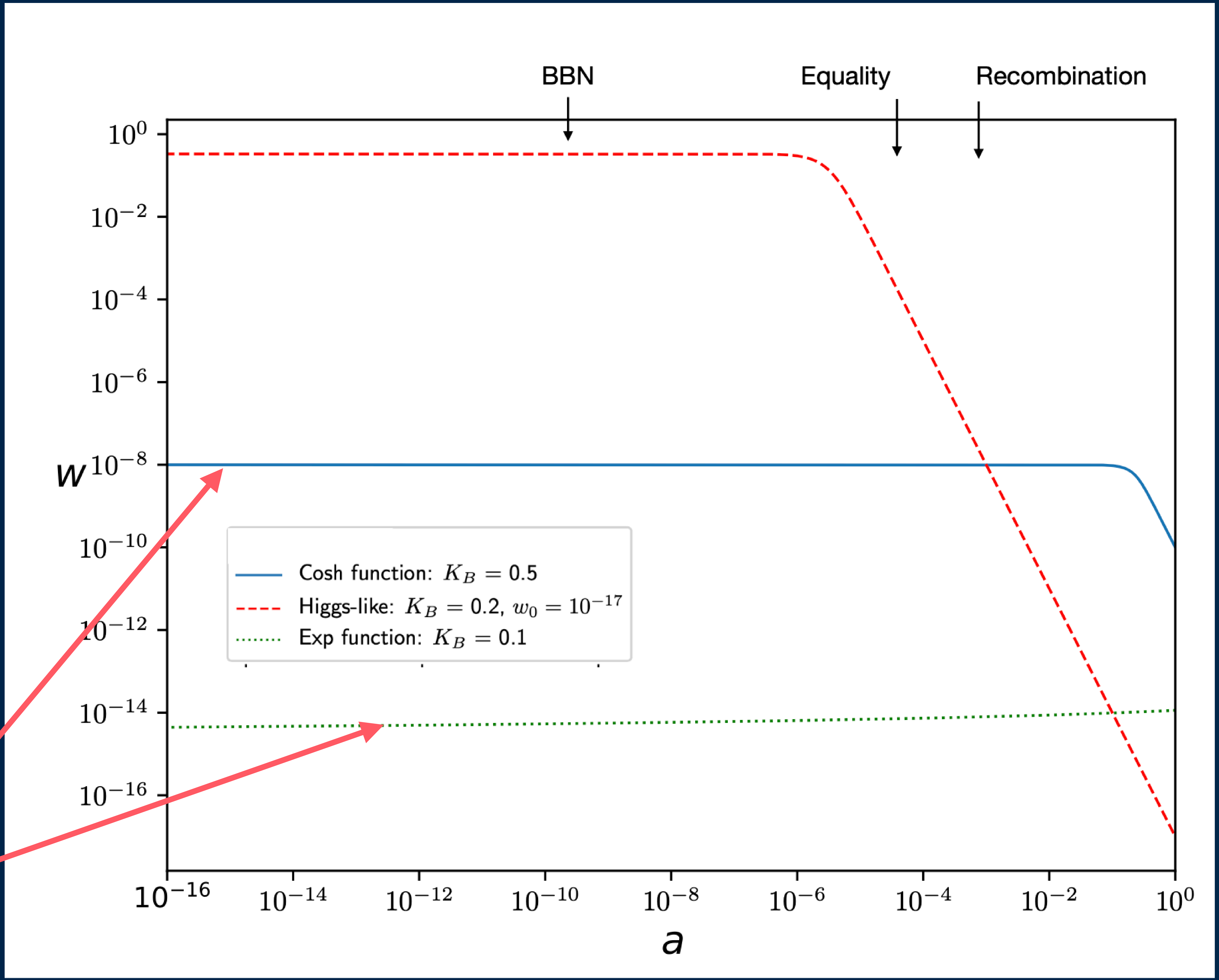
FLRW “Equation of state” w = Pressure/ density



$$w \ll 1$$

$$c_{\rm ad}^2 \equiv w - \frac{1}{3(1+w)}\frac{dw}{d\ln a} \ll 1$$

Back to cosmology

FLRW "Equation of state" w = Pressure/ density

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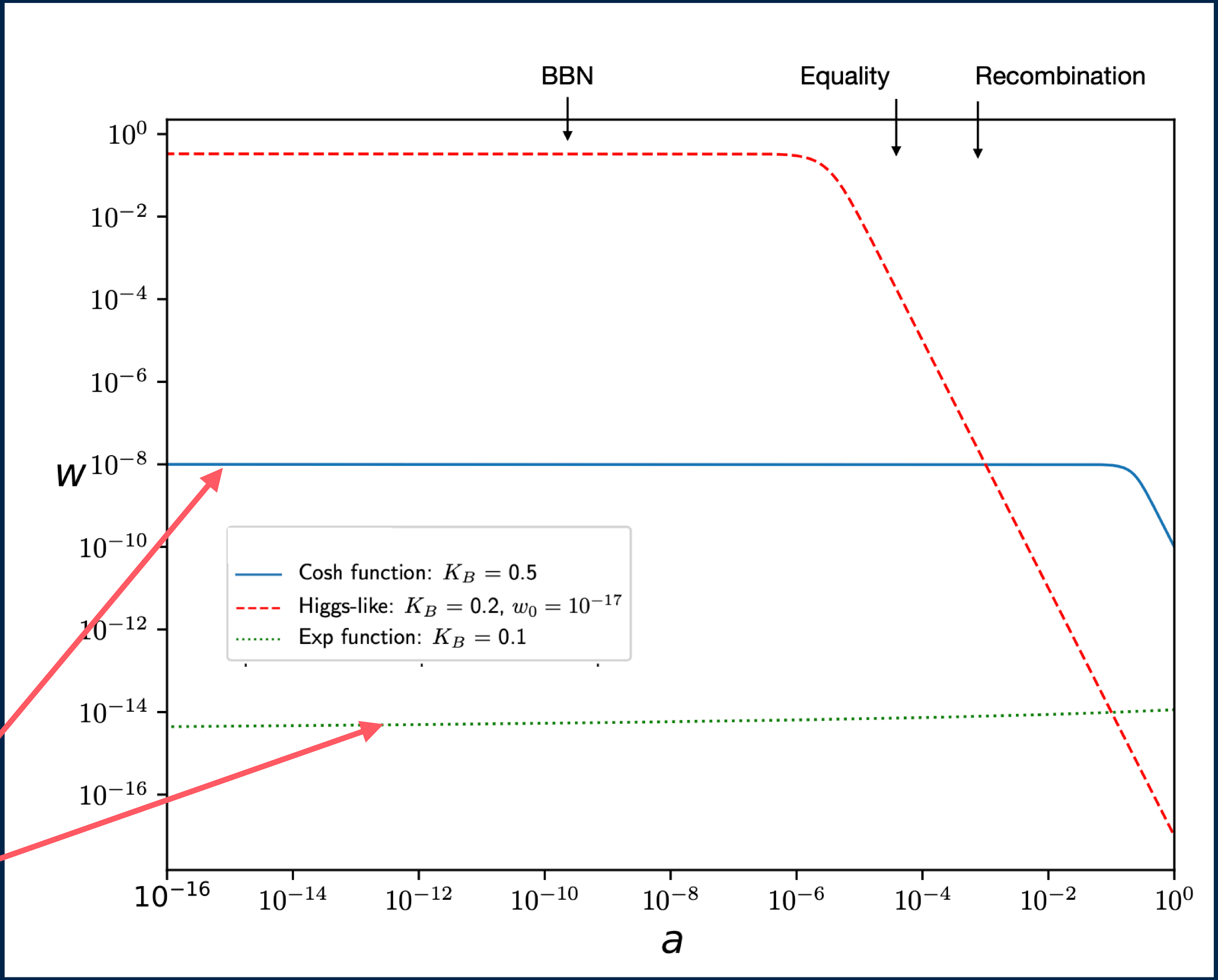


✗	Higgs-like	$\mathcal{K} \sim (\mathcal{Q}^2 - \mathcal{Q}_0^2)^2$
✓	Cosh	$\mathcal{K} \sim \cosh\left(\frac{\mathcal{Q}-\mathcal{Q}_0}{\mathcal{Z}_0}\right)$
✓	Exp	$\mathcal{K} \sim e^{\left(\frac{\mathcal{Q}-\mathcal{Q}_0}{\mathcal{Z}_0}\right)^2}$

$$w \ll 1$$

$$c_{\rm ad}^2 \equiv w - \frac{1}{3(1+w)}\frac{dw}{d\ln a} \ll 1$$

linear fluctuations around FLRW

$$ds^2 = -\left(1+2\Psi\right)dt^2 + a^2\left(1-2\Phi\right)d\vec{x}^2$$

$$\phi = \bar{\phi} + \varphi$$

$$A_i = \vec{\nabla}_i \alpha$$

$$E = \dot{\alpha} + \Psi$$

$$\chi = \varphi + \dot{\bar{\phi}}\alpha$$

$$\gamma = \dot{\varphi} - \dot{\bar{\phi}}\Psi$$

Density contrast $\delta = \frac{1+w}{\dot{\bar{\phi}} c_{\rm ad}^2}\gamma + \frac{1}{8\pi G a^2 \bar{\rho}}\vec{\nabla}^2\left[K_B E + (2-K_B)\chi\right]$

Velocity divergence $\theta = \frac{\varphi}{\dot{\bar{\phi}}}$

Pressure contrast $\Pi = c_{\rm ad}^2\delta - \frac{c_{\rm ad}^2}{8\pi G a^2 \bar{\rho}}\vec{\nabla}^2\left[K_B E + (2-K_B)\chi\right]$

Fluid-like

$$\dot{\delta} = 3H\left(w\delta - \Pi\right) + (1+w)\left(3\dot{\Phi} - \frac{k^2}{a^2}\theta\right)$$

$$\dot{\theta} = 3c_{\rm ad}^2 H\theta + \frac{\Pi}{1+w} + \Psi$$

Field

$$K_B\left(\dot{E} + HE\right) = \frac{d\mathcal{K}}{d\mathcal{Q}}\chi - (2-K_B)\left[\frac{\dot{\bar{\phi}}}{1+w}\Pi + \left(H + \dot{\bar{\phi}}\right)\chi - 3c_{\rm ad}^2 H\dot{\bar{\phi}}\alpha\right]$$

$$\dot{\alpha} = E - \Psi$$

$$w \to 0$$

$$c_{\rm ad} \to 0$$

CDM-like

$$\dot{\delta} \approx 3\dot{\Phi} - \frac{k^2}{a^2}\theta$$

$$\dot{\theta} \approx \Psi$$

Field (decoupled)

$$K_B\left(\dot{E} + HE\right) \approx \left[\frac{3H_0^2\Omega_{0\mathcal{Q}}}{a^3} - (2-K_B)H\mathcal{Q}_0\right](\theta + \alpha)$$

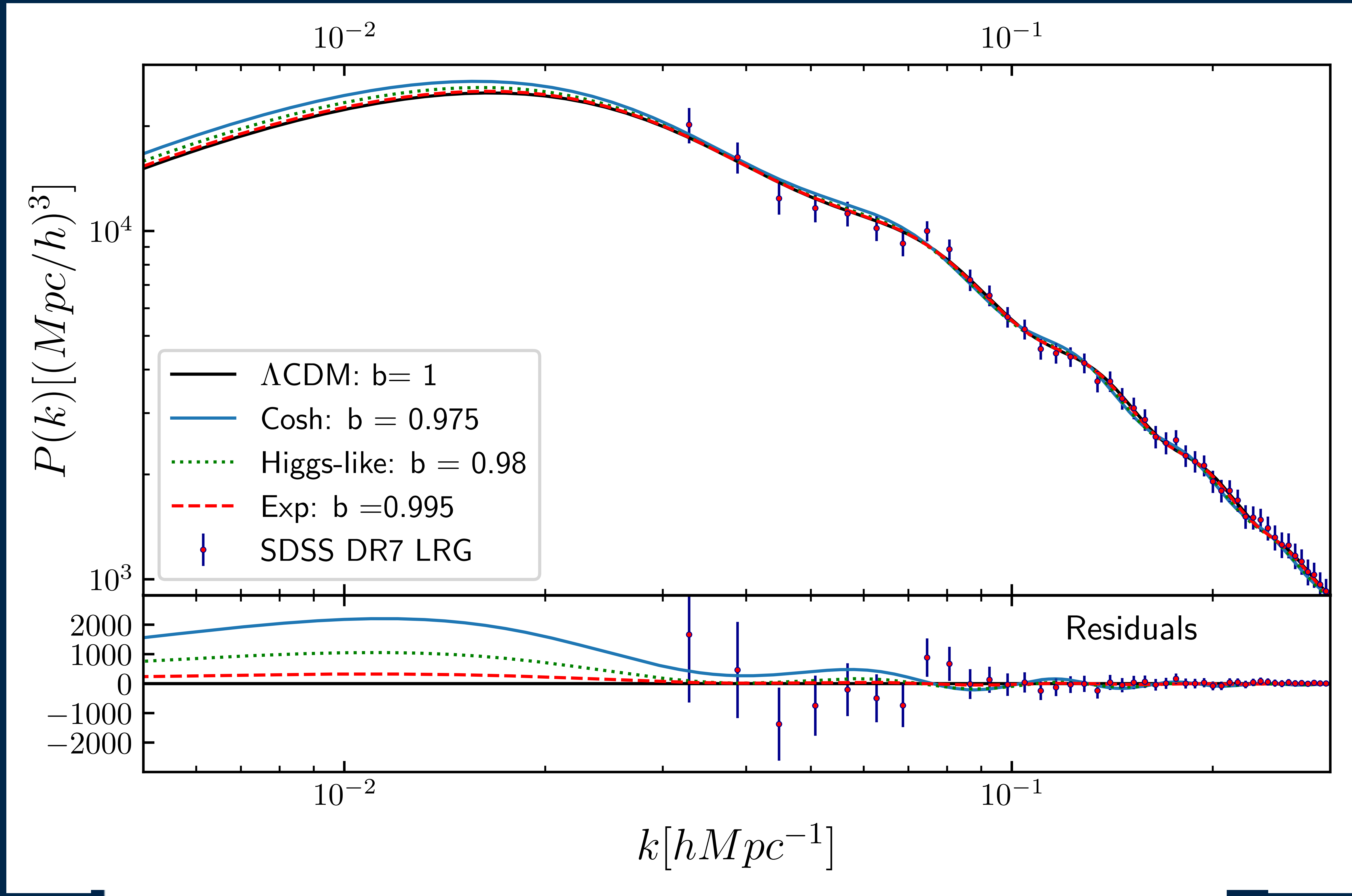
Outcome

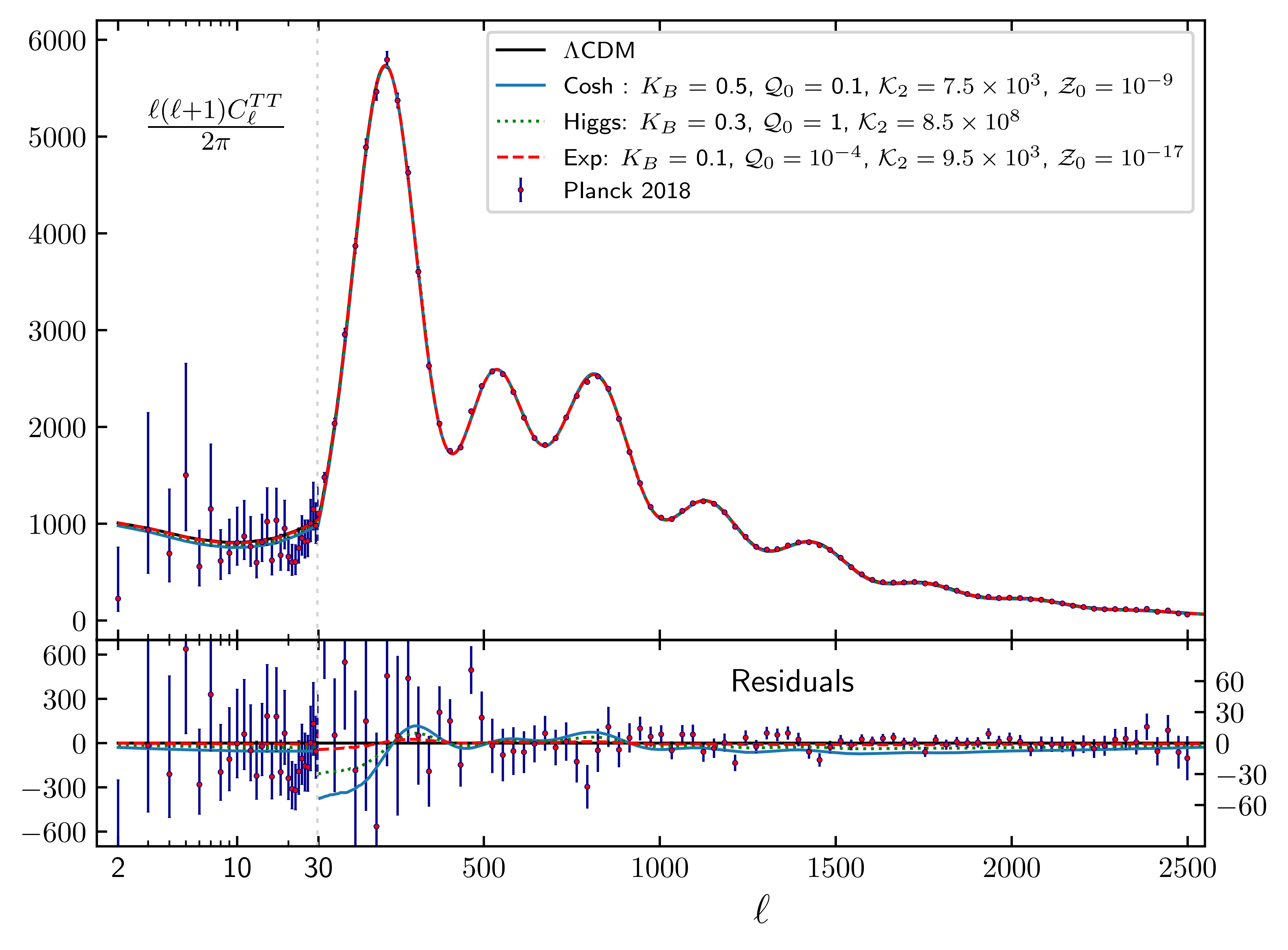
AeST parameters:

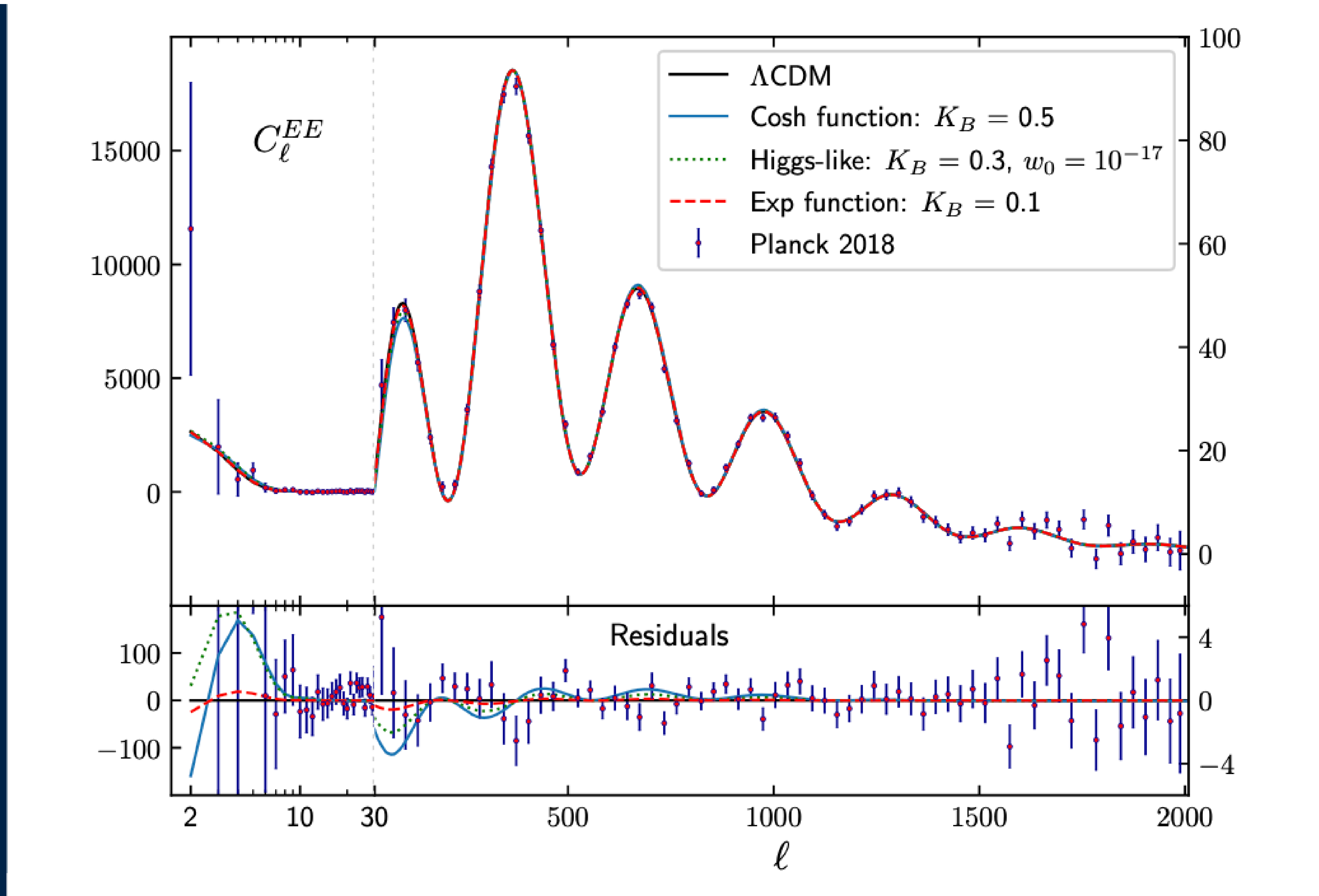
$K_{\rm B}$ $\mathcal{Q}_0$ $\mathcal{K}_2$

Initial condition: $I_0 \to \rho_{0c}$

(MCMC pending)







Khronon model

Ingredients

Blanchet & Marsat (2012)

C.S. & Blanchet, JCAP 11 (2024) 040

$$g_{\mu\nu} \quad \tau$$

$$S = \frac{c^3}{16\pi G}\int d^4x\,\sqrt{-g}\left[R - 2\mathcal{J}(\mathcal{Y}) + 2\mathcal{K}(\mathcal{Q})\right] + S_m\left[\Psi, g\right]$$

Derived unit-timelike vector $n_\mu = -\frac{c}{\mathcal{Q}}\nabla_\mu \tau$

$$\mathcal{Q} \equiv c\sqrt{-g^{\mu\nu}\nabla_\mu\tau\nabla_\nu\tau}$$

Acceleration $A_\mu = c^2 n^\nu \nabla_\nu n_\mu$

$$\mathcal{Y} \equiv \frac{A_\mu A^\mu}{c^4}$$

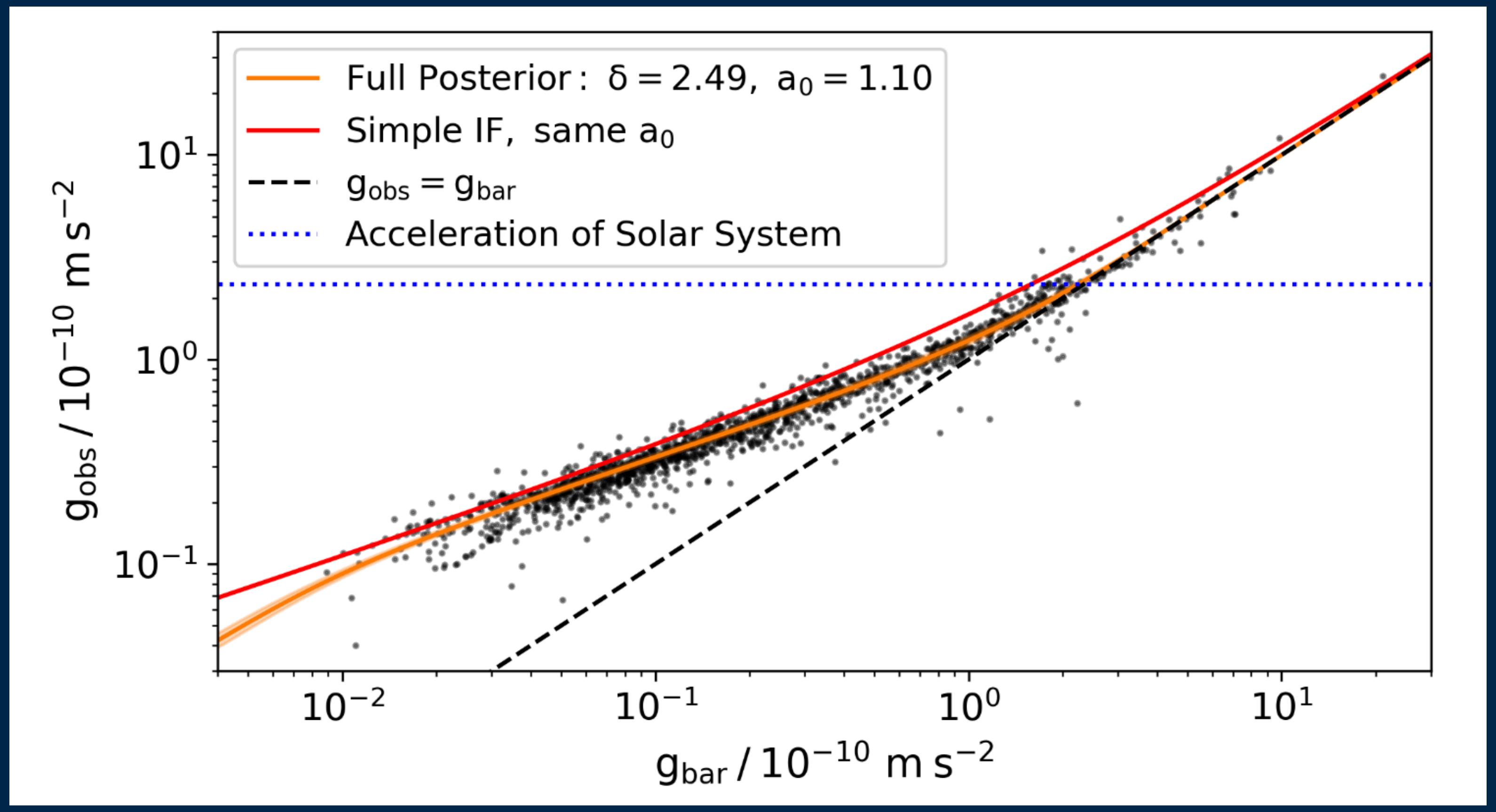
Essentially a Scalar-Tensor theory

Low energy limit of Khronometric Horava-Lifshitz theory (with two functions inserted…)

Quasistatic & Cosmological behaviour similarly to AeST

MOND and solar system

D. Vokrouhlicky, D. Nesvorny & S.Tremaine (2024)



H. Desmond, A. Hees & B. Famaey (2024)

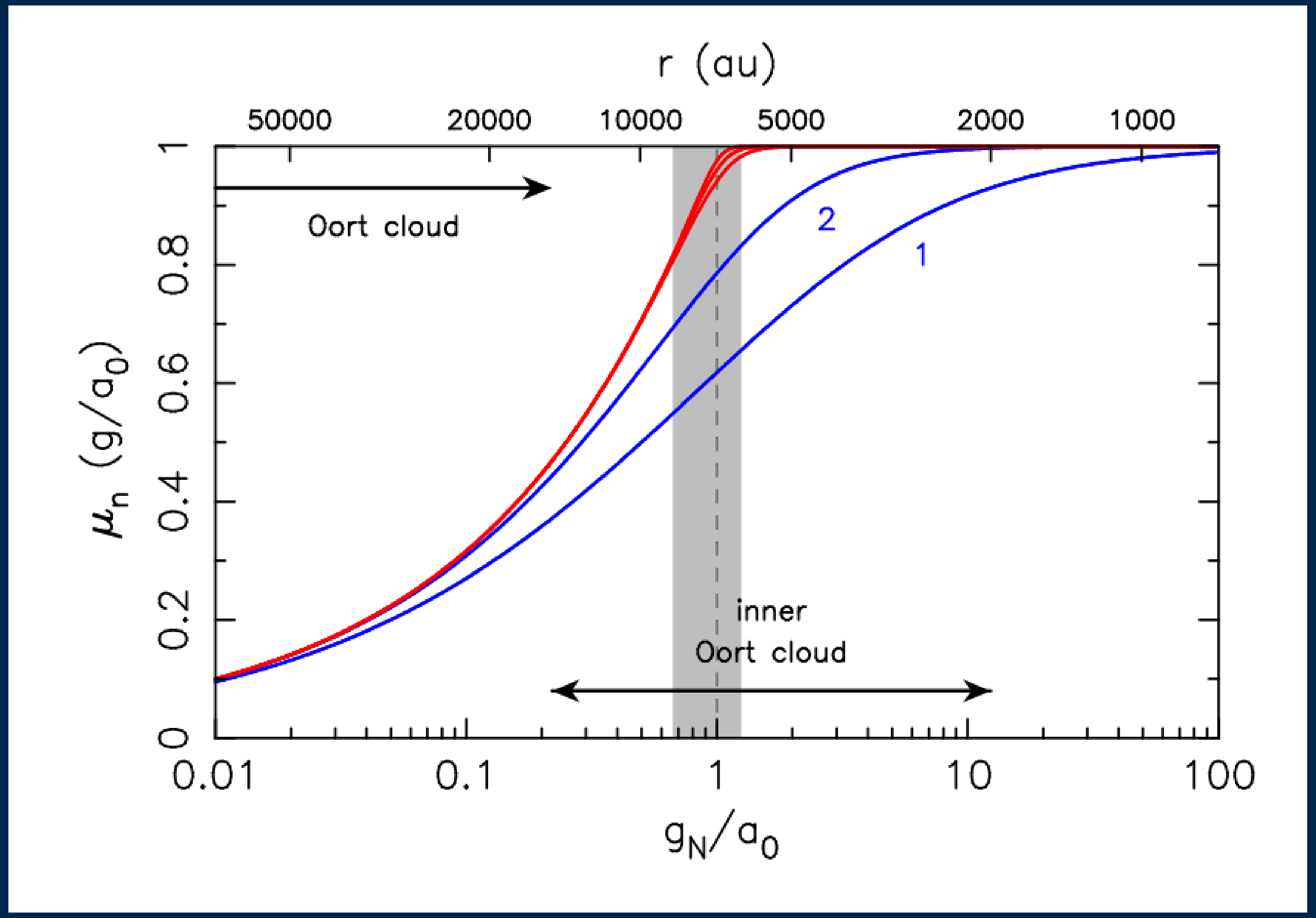
Solar system quadrupole moment

$$\delta\Phi = -\frac{Q_2}{2} x^i x^j \left(\hat{e}_i \hat{e}_j - \frac{1}{3}\delta_{ij} \right)$$

$$Q_2 = (3 \pm 3) \times 10^{-27} s^{-2}$$

Measured by Cassini (Saturn's orbit)

Solar system very GR like:
inconsistent with MOND



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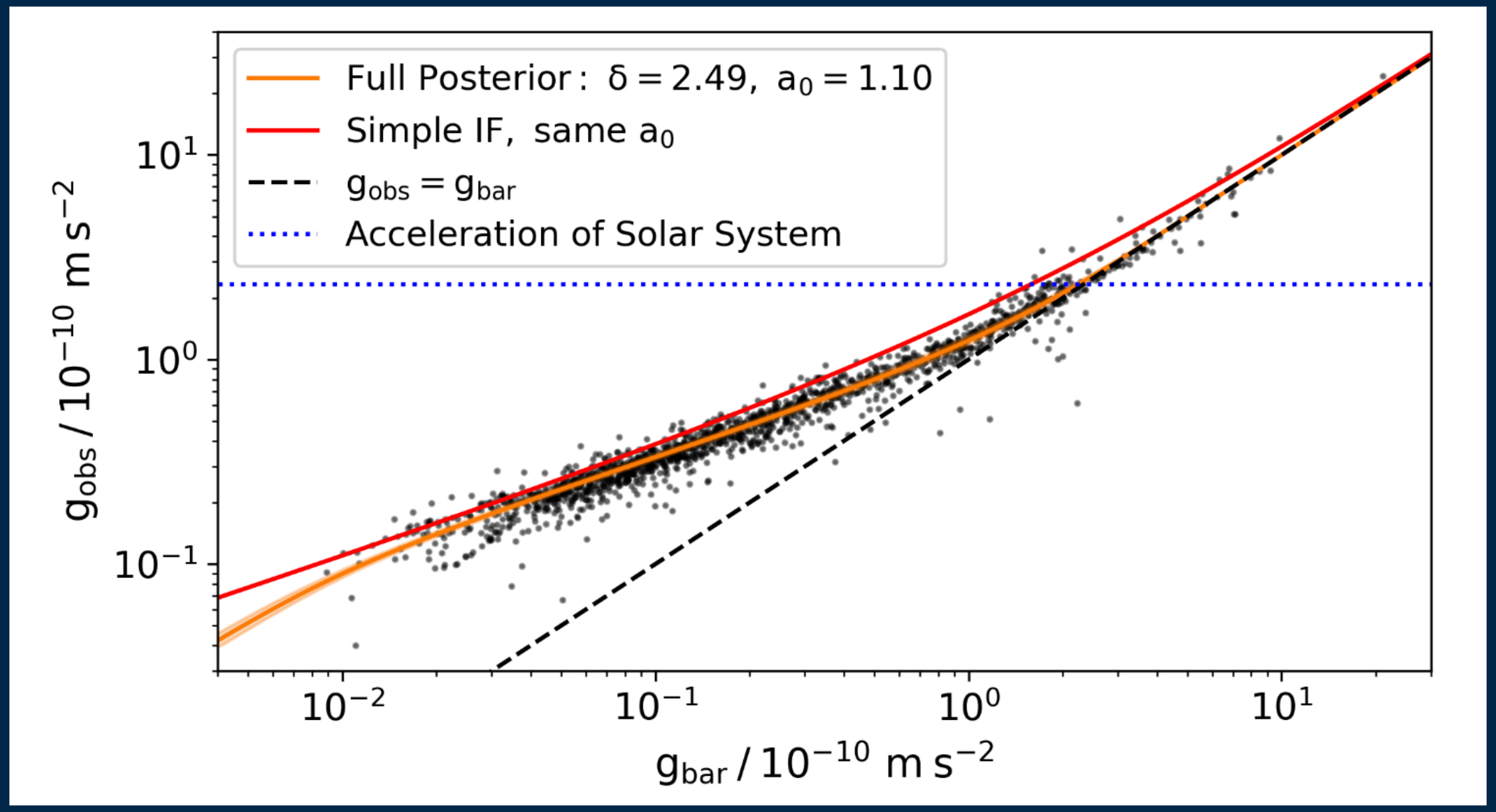

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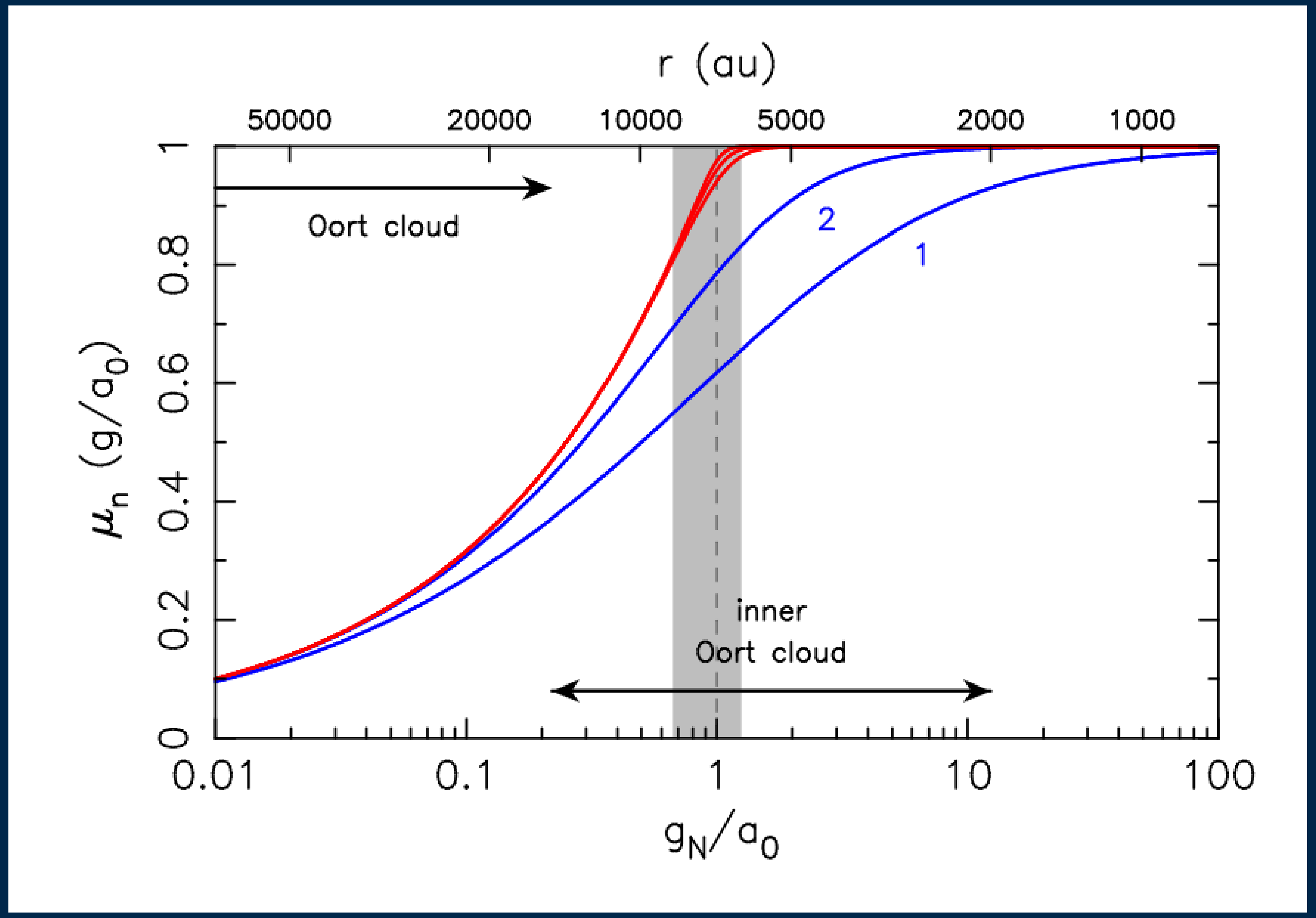

Wide-binary constraints from GAIA data

Banik et al., MNRAS 527, 4573 (2024

Rule out MOND at 16σ

Chae, ApJ 960, 114 (2024)

Confirms MOND at 5σ

Hernandez, MNRAS 525, 1401 (2023)

Confirms MOND at ?? σ

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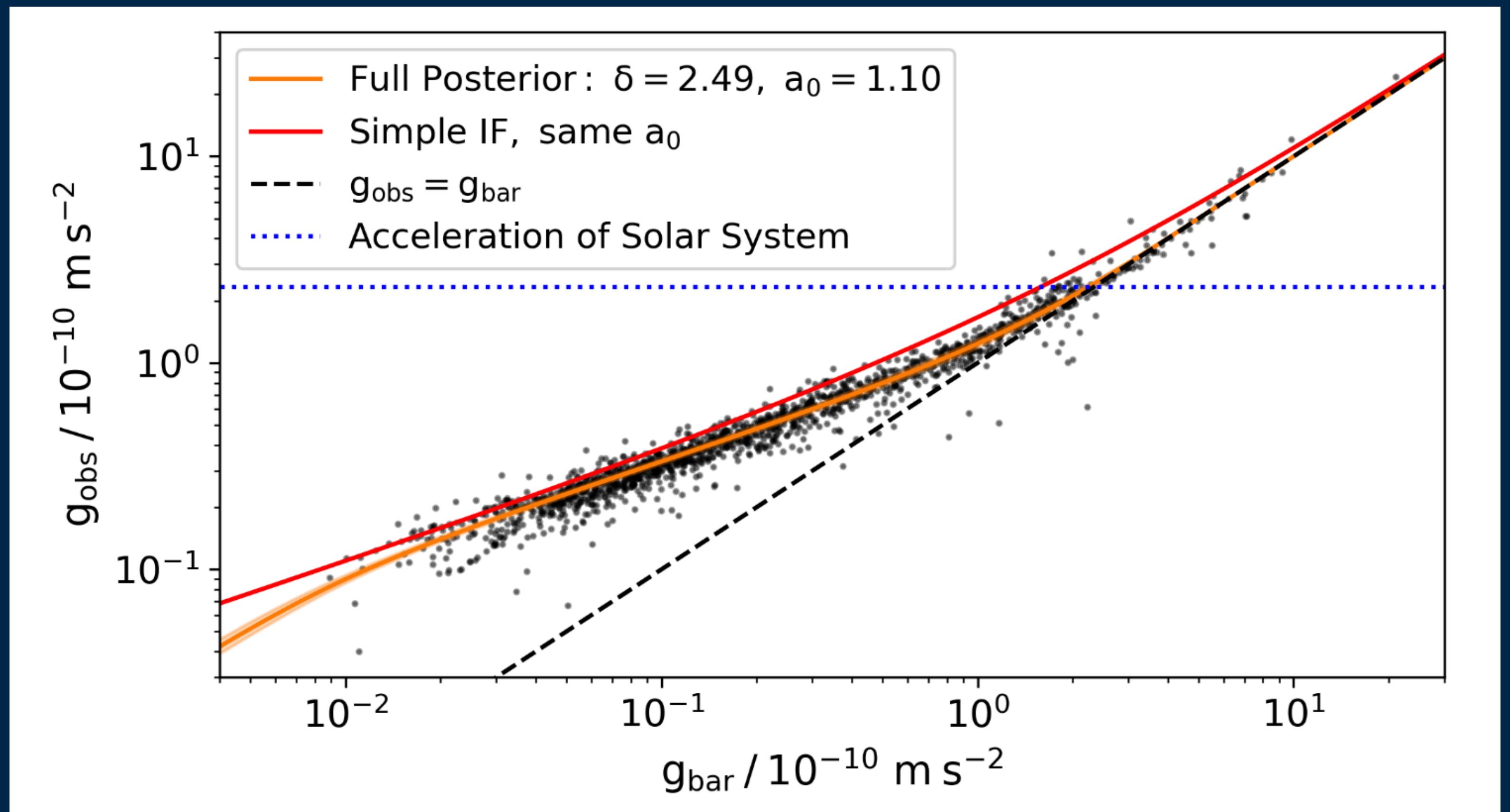

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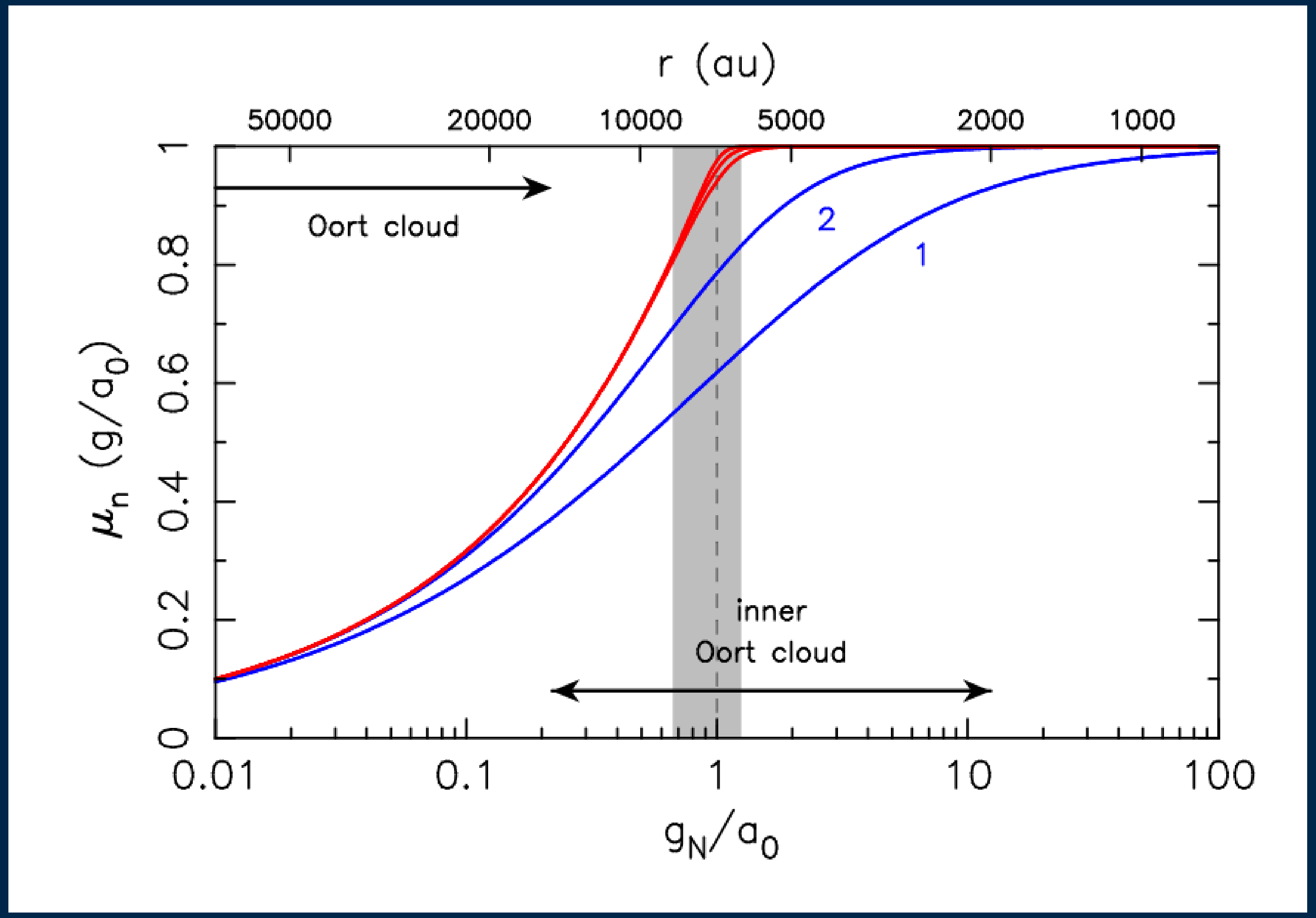

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Note of caution

New RAR in preparation with Varasteanu, Jarvis, et al. (Yasin, Desmond, private comm.)

Several bands in photometry, almost no scatter, consistent with SS & Wide binaries

see also
Vărăşteanu et al,
Mon.Not.Roy.Astron.Soc. 541, 2366 (2025)

Hamiltonian formulation

M. Bataki, C.S. & T. Zlosnik, PRD 110, 044015 (2024)

6 dof (fully non-linear)

4 First class constraints

4 Second class constraints (Momenta of auxiliary fields + functional freedom)

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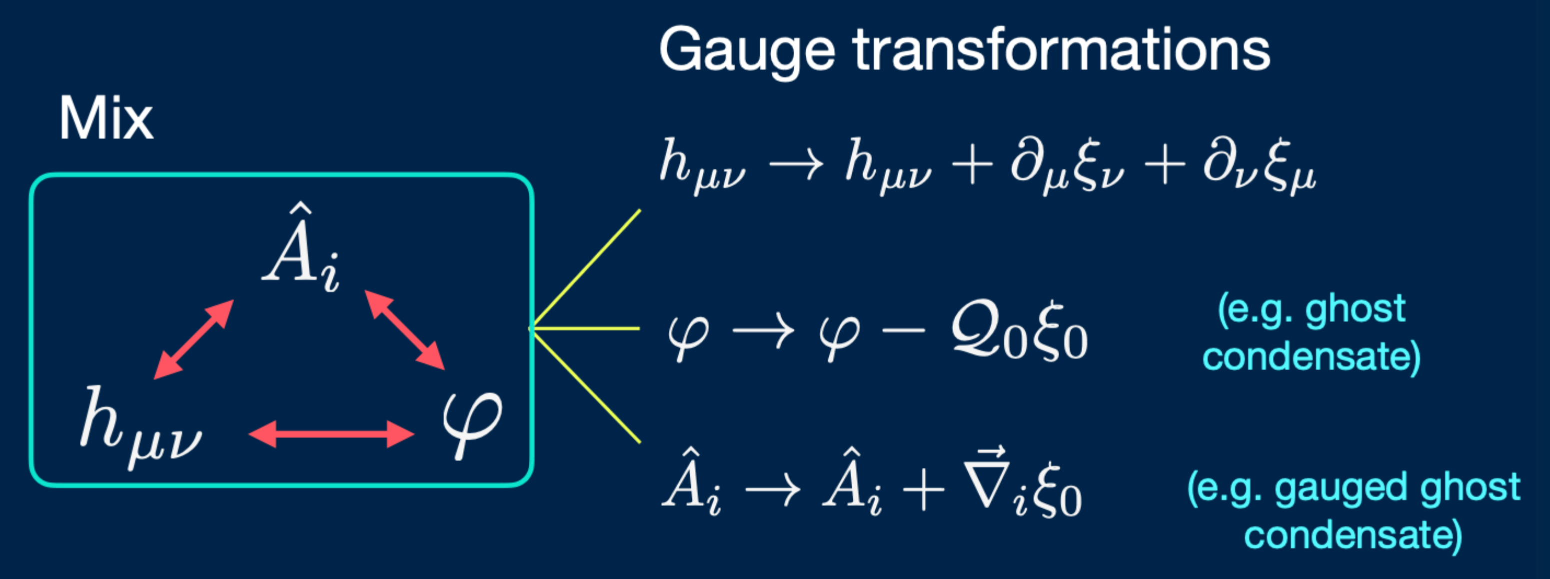

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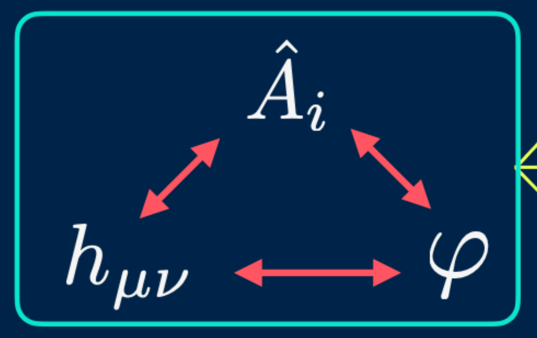

Gauge transformations

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$\varphi \to \varphi - \mathcal{Q}_0 \xi_0$ (e.g. ghost condensate)

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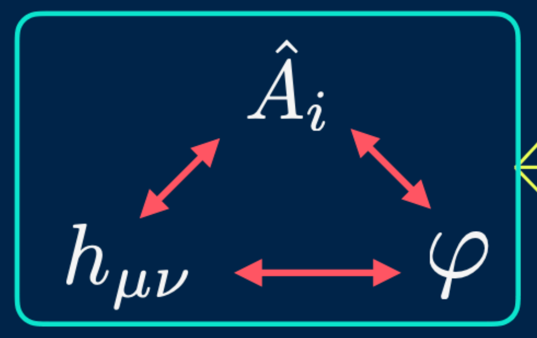

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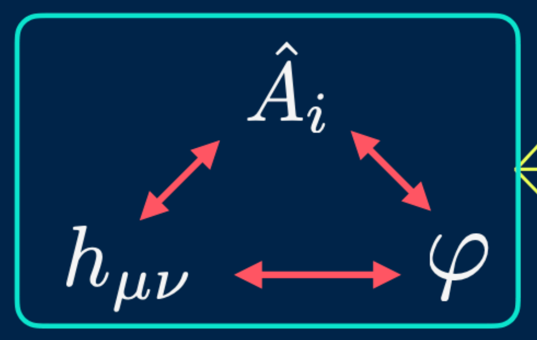

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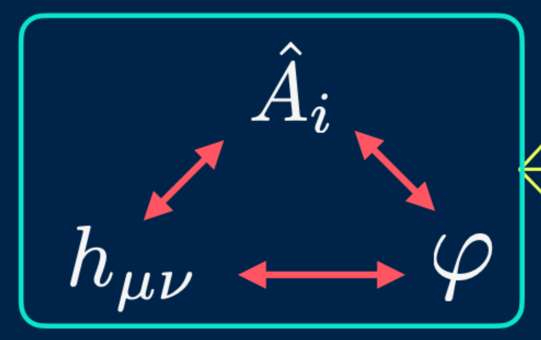

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$\mathcal{K}_2 > 0$

$0 < K_B < 2$

$\lambda_s > 0$

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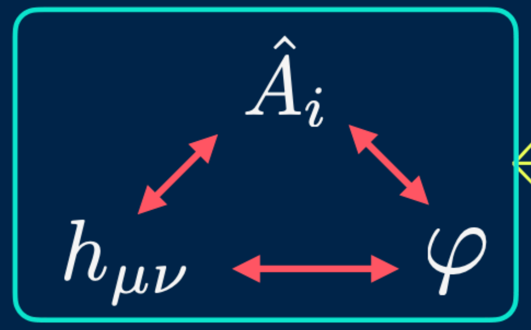

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$\omega^2 = 0$

- Positive Hamiltonian $k > \mu($
- Negative Hamiltonian $k < \mu$

$\mathcal{K}_2 > 0$

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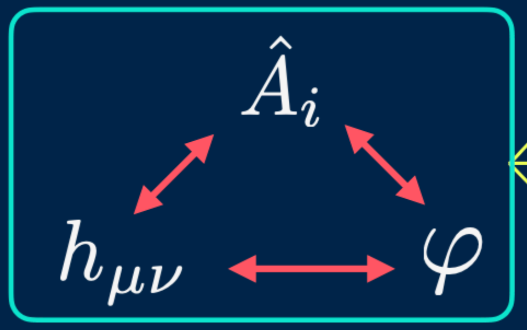

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- Negative Hamiltonian $k < \mu$

$\mathcal{K}_2 > 0$

$0 < K_B < 2$

$\lambda_s > 0$

De Sitter: M. Bataki & C.S., in preparation : Cures Minkowski (linear) instability

Static, spherically symmetric vacuum solutions in AeST

C.S. & David Vokrouhlický, JCAP 03, 035 (2025)

— PhD student —

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Strong-field regime

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Strong-field regime

$$ds^2 = -e^{2\Phi}dt^2 + e^{2\Psi}dr^2 + r^2 d\Omega^2$$

$$\underline{A} = \sqrt{1 + A^2 e^{-2\Psi}}\,dt + A\,dr$$

$$\phi = \mathcal{Q}_0\,(qt + R)$$

Static, spherically symmetric vacuum solutions in AeST

C.S. & David Vokrouhlický, JCAP 03, 035 (2025)

— PhD student —

$$r \ll r_M \ll \mu^{-1} \qquad \mathcal{F} = (2 - K_B)\left[\lambda_s \mathcal{Y} - \mu^2 \left(\mathcal{Q}/\mathcal{Q}_0 - 1\right)^2\right] + \dots$$

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0: ~~Joins to cosmology~~ (Unphysical)

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Two separate cases:

- $A(r) \neq 0$
- $A = 0$

$A(r) \neq 0$

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C.S. & D. Vokrouhlický, JCAP 03, 035 (2025)

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C.S. & D. Vokrouhlický, JCAP 03, 035 (2025)

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Reissner-Nordstrom soln (unique)

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C.S. & D. Vokrouhlický, JCAP 03, 035 (2025)

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Conserved current is J^μ

Yang et al., arXiv: 2504.20144

$$\tilde{n} \equiv \frac{2 + K_B\lambda_s}{2(1+\lambda_s)}$$

$$0 < \tilde{n} < 1$$

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Vector

$$A = \pm e^{-2\Phi} \sqrt{\frac{2(|q_A| + G_{\rm N}M)}{r} + \frac{(1-\tilde{n})q_A^2}{r^2}}$$

Scalar

$$\phi = -\mathcal{Q}_0 \int^r dr' \frac{1}{1 + \frac{|q_A|}{r'}} \left[\frac{1}{(1+\lambda_s)\mathcal{Q}_0} \frac{|q_A|}{r'^2} \mp A(r') \right]$$

All scalar combinations are regular at both horizons; soln can be extended through

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C.S. & D. Vokrouhlický, JCAP 03, 035 (2025)

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---

$q = 0$ RN still unique soln, A(r) same, $\phi(r)$ Simpler

Scalar charge: $\varphi_0 = \pm \mathcal{Q}_0 \sqrt{\frac{2 + K_B\lambda_s}{2 - K_B}} \sqrt{q_A^2 - \frac{q_{BH}^2}{\tilde{n}}}$

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C.S. & D. Vokrouhlický, JCAP 03, 035 (2025)

L. Yang et al., arXiv: 2504.20144 (2025)

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New coordinate $$r(u) = \frac{r_0}{|u|} \frac{|u - u_1|^{\frac{1+n}{2n}}}{|u - u_2|^{\frac{1-n}{2n}}}$$

$$n \equiv \sqrt{1 - \tilde{n}}$$

$$0 < n < 1$$

$$u_2 = -\frac{1}{1-n} \quad < \quad u_1 = -\frac{1}{1+n} \quad < 0$$

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u — $-\infty$ — u_2 — u_1 — 0 — ∞ — u

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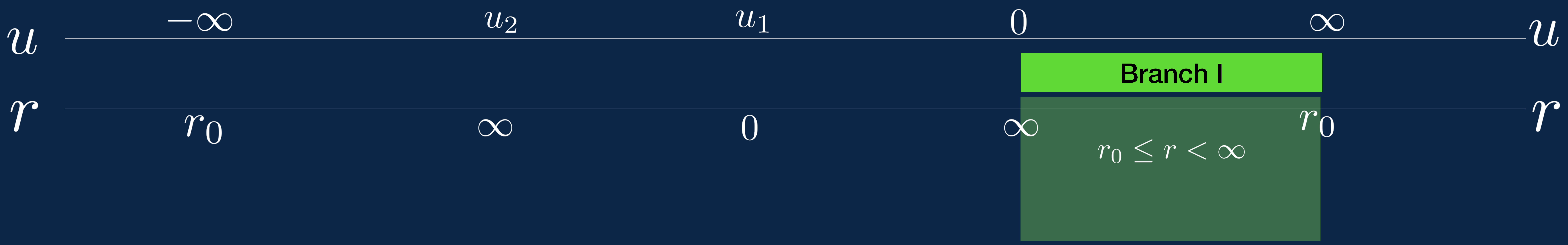

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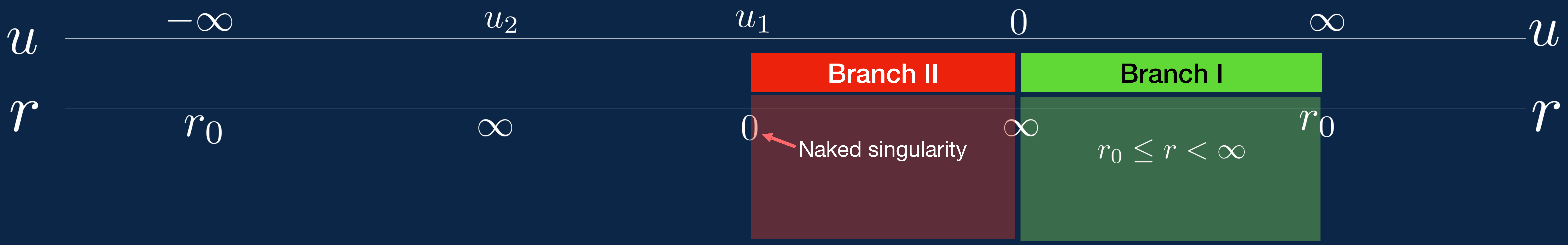

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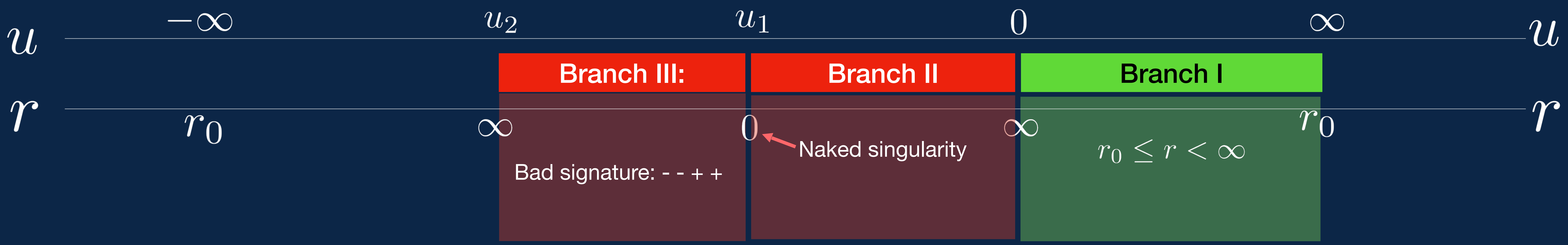

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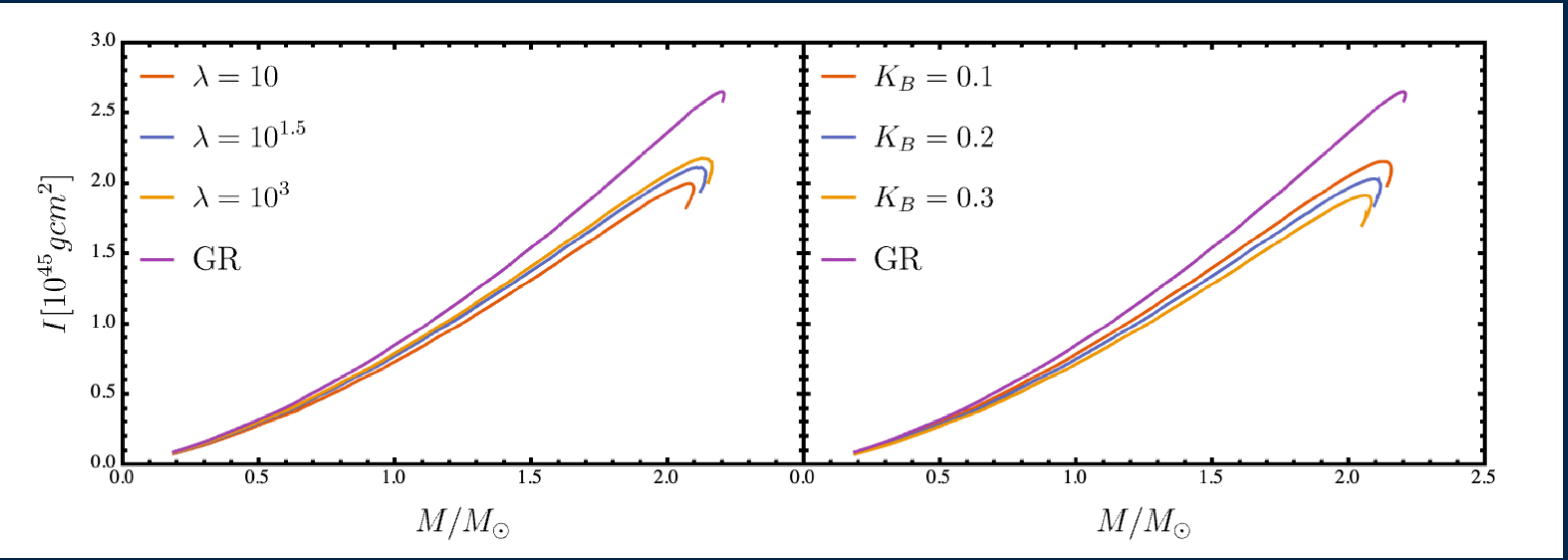
Eling-Jacobson wormhole

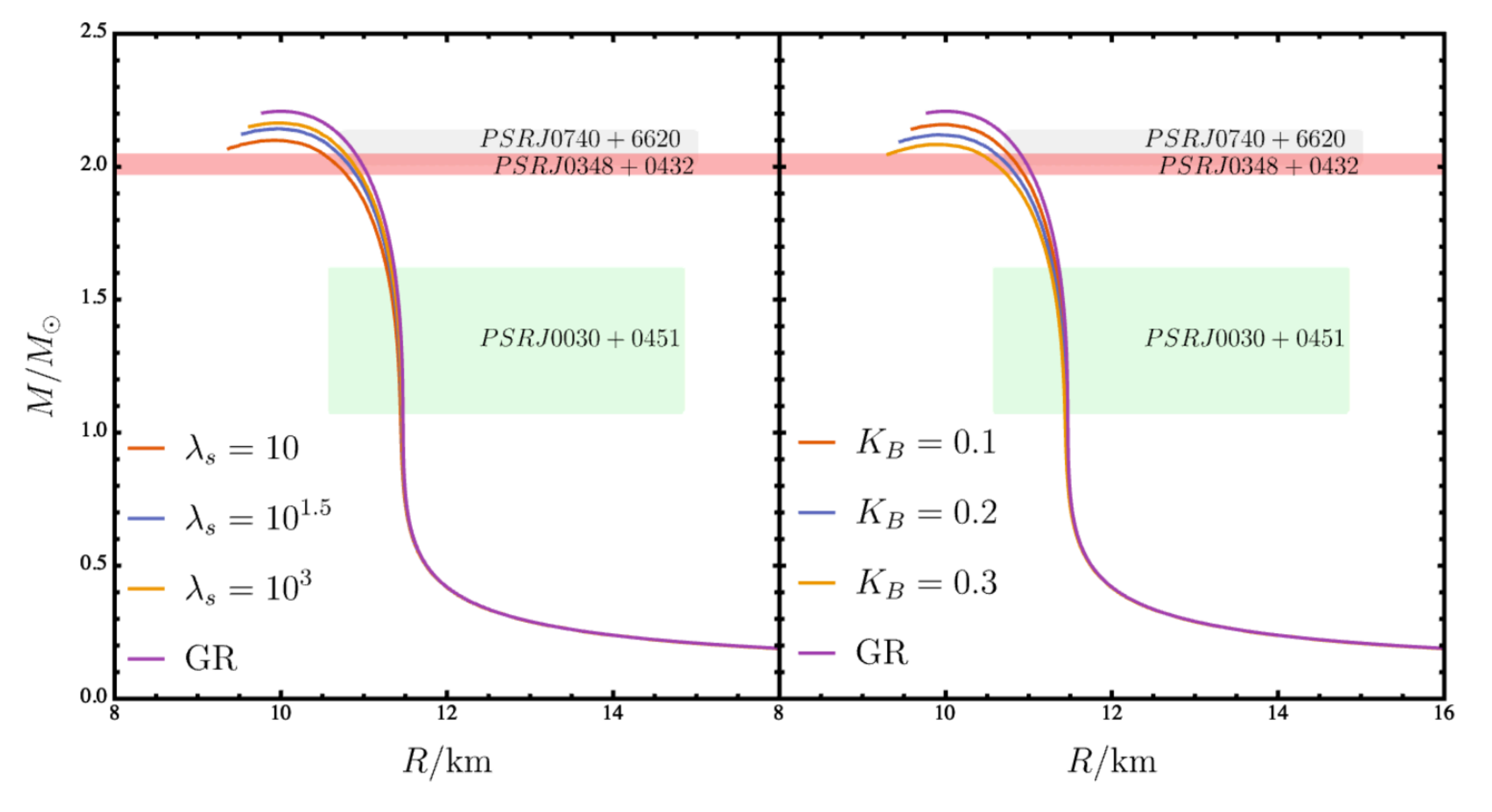
C. Eling & T. Jacobson, Class.Quant.Grav. 23, 5625 (2006)

Neutron stars

Spherically symmetric — Reyes & Sakstein, PRD110, 084019 (2024)

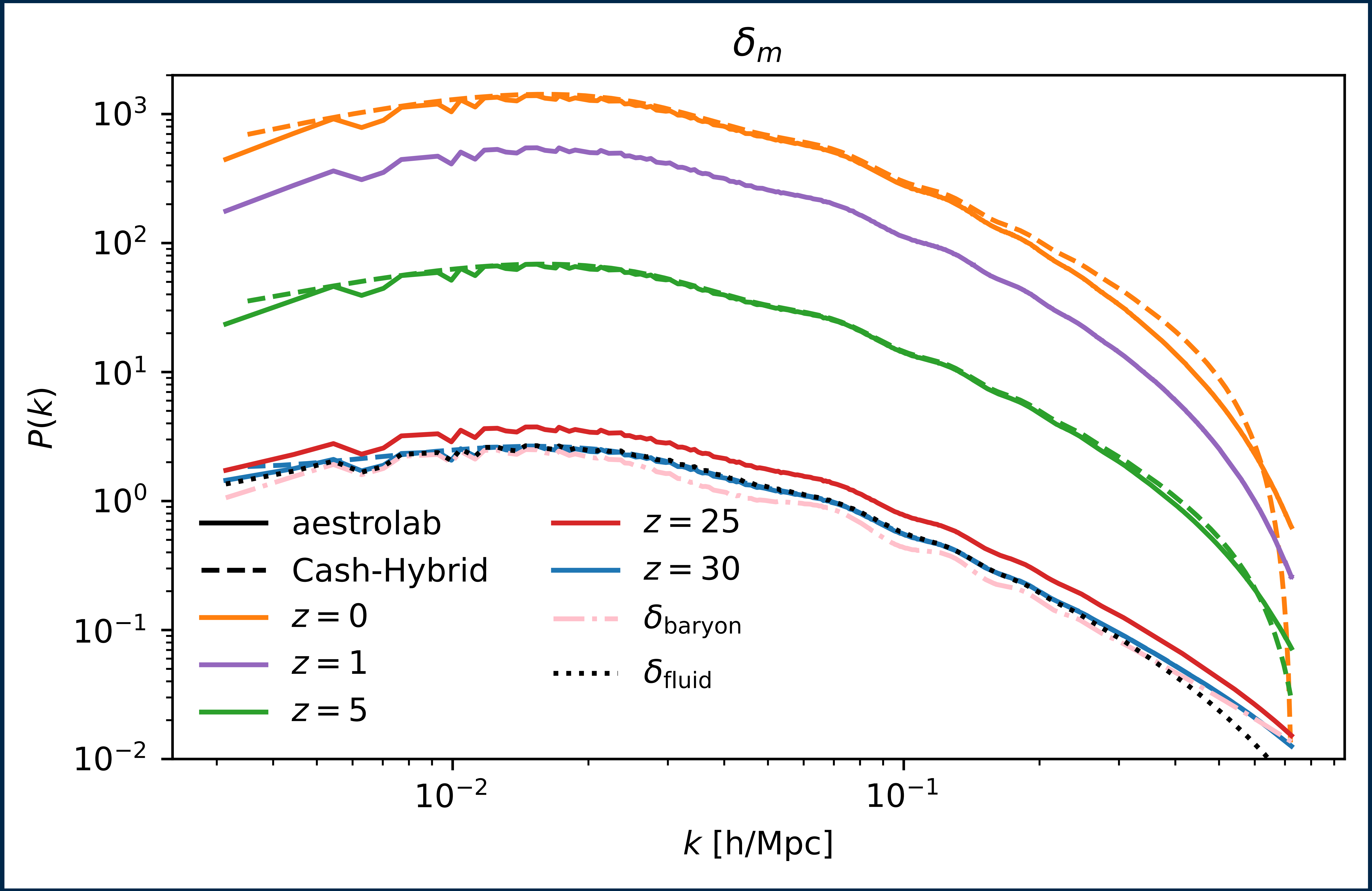
Slowly rotating — Reyes & Sakstein, arXiv: 2505.03527





interesting to check I-Love-Q relations

N-body simulations (with O. Christiansen, C. Boehm & D. Mota)



In progress

Linear theory Milestone:

matched lattice-based with Fourier-mode transfer functions from Boltzman code

Next stage:

Include the 'MOND' non-linear terms

Theory

Dependence on $\mathcal{F}(\mathcal{Q}, \mathcal{Y})$ has issues

quasistatic:

Function → Non-analytic

$$\mathcal{J}(\mathcal{Y}) = \begin{cases} \sim \mathcal{Y} & \mathcal{Y} \gg 1 \\ \sim \mathcal{Y}^{3/2} & \mathcal{Y} \ll 1 \end{cases}$$

MOND term $\mathcal{Y}^{3/2} = |\vec{\nabla}\varphi|^3$ → Non-Fourier expandable

All models for which a MOND limit exists have this term built-in (either for the potential or, more usually, for some other field)

Cosmology

Higgs phase incompatible with MOND → $\mathcal{K}(\mathcal{Q})$

Different field content but keep essential features → Is there an EFT?

AeST / Khronon

MOND (galaxies) ~CDM (linear cosmology)

In progress

De Sitter stability (to be submitted with M. Bataki)

Cosmological simulations (with Christiansen, Boehm & Mota)

Should be done

PPN parameters

Gravitational waves (additional polarisations)

Outlook

Consistent phenomenological models for testing this paradigm further

Better model is necessary (and possible)

EXTRA

AeST $g_{\mu\nu}$ ϕ A_μ

NEW DOF

Khronon $g_{\mu\nu}$ $\mathcal{T}$

AeST $g_{\mu\nu}$ ϕ A_μ

NEW DOF

Dark matter??

Khronon $g_{\mu\nu}$ $\mathcal{T}$

New dof

$\varphi^{(I)}$ $\alpha_\mu^{(I)}$

$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

Minkowski

$h_{\mu\nu}$

$\varphi^{(I)}$ $\alpha_\mu^{(I)}$

Matter

e.g. $\mathcal{L} \sim (\partial h)^2 + (\partial \varphi)^2$

Transformations

$h_{\mu\nu} \leftrightarrow \alpha_\mu^{(I)}$

$\varphi^{(I)}$

Gravity

e.g. $\mathcal{L} \sim (\partial h)^2 + (\partial \varphi)^2 + (\partial h)(\partial \varphi)$

AeST: New d.o.f.

Breaking of diff-invariance: preferred frame

$$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$$

$$A_0 = -1 + \frac{1}{2}h^{00}$$

$$A_i = \hat{A}_i$$

$$\dot{\phi} = \mathcal{Q}_0 + \dot{\varphi}$$

$$\vec{\nabla}_i \phi = \vec{\nabla}_i \varphi$$

Background

Fluctuations

Similar (but not identical to) for Khronon model.
(C.S. & Blanchet)

AeST: New d.o.f.

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Fluctuations

Mix

$\hat{A}_i$, $h_{\mu\nu}$, φ

Gauge transformations

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu\xi_\nu + \partial_\nu\xi_\mu$$

$$\varphi \to \varphi - \mathcal{Q}_0\xi_0$$ (e.g. ghost condensate)

$$\hat{A}_i \to \hat{A}_i + \vec{\nabla}_i\xi_0$$ (e.g. gauged ghost condensate)

Mix metric perturbation with new fields

N.B.

Dark fields (e.g. DM)

$$\chi \to \chi$$

No mixing with $h_{\mu\nu}$

Similar (but not identical to) for Khronon model. (C.S. & Blanchet)