Some Constraints on $f(Q)$ Gravity

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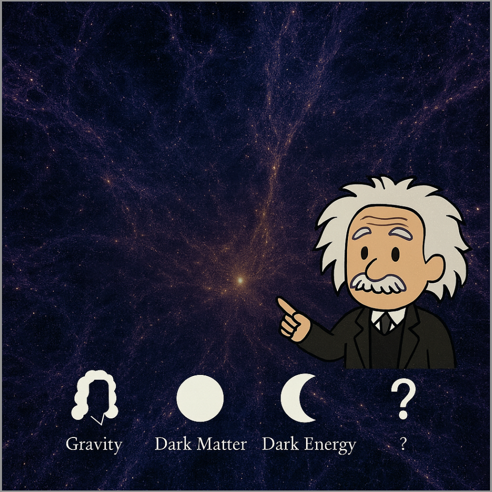
The Ghosts in Our Cosmic History



Cosmic Web + Unseen Ingredients (AI illustration)

Cosmology at a Crossroads

- **Success:** ΛCDM explains CMB, BAO, SNe with remarkable precision.
- **Tensions: cracks emerging across independent datasets:**
  - H_0 tension: early-universe $H_0 \sim 67$ vs late-universe $H_0 \sim 73$ km/s/Mpc, discrepancy $\sim$5–6σ
  - S_8 tension: CMB vs weak lensing $\sim$ 2–3σ
  - Curvature hints: Planck data sometimes favor $\Omega_K < 0$ at $\gtrsim 3\sigma$, but BAO+SNe restore flatness
  - Lensing anomalies: CMB lensing amplitude slightly higher than ΛCDM expectations
- **The Challenge:** cracks in the model, or hidden systematics?

The Ghosts in Cosmology

- **Invisible Sector:** $\sim$95% of the Universe remains unseen—dark matter, dark energy, and puzzling anomalies.
- **Frontier:** Systematics or new physics—early dark energy, interacting DM/DE, modified gravity, extra neutrinos.
- **Theme:** Cosmological tensions as opportunities to uncover deeper laws of nature.

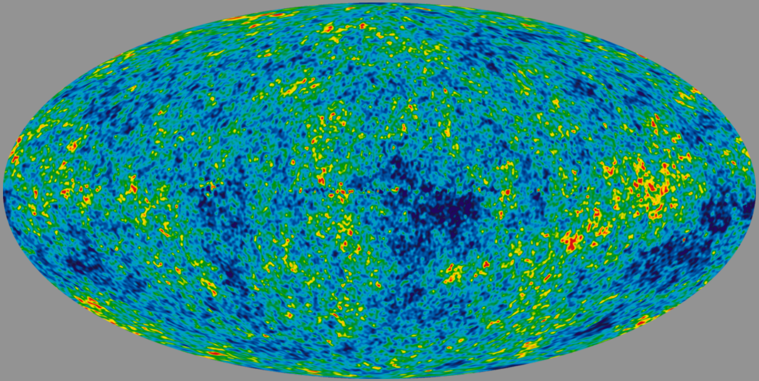
Talk Outline

1. The Dark Side of the Universe
2. Cosmological Tensions
3. Matters of Gravity
4. Some Constraints on $f(Q)$ Gravity

The Dark Side of the Universe

The Current Cosmological Paradigm

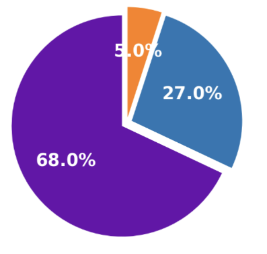
- Based on the current cosmological paradigm, the universe is a 4-dimensional, maximally symmetric (FLRW), spacetime that started off at the Big Bang and has since been expanding (for the last ~ 13.8 billion years)
  - Homogeneous: all regions of space look alike, no preferred positions
  - Isotropic: no preferred directions
  - Perfect-fluid assumptions

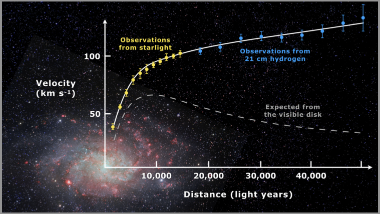


The baby universe @ $\sim 380{,}000$ years old. Today, $T \sim 2.726K$, $\frac{\delta T}{T} \sim 10^{-5}$.

Challenges to the Cosmological Paradigm. . .

- The isotropy and homogeneity assumptions are too simplistic, valid only on very large scales, i.e. on scales bigger than galaxy clusters
- Recent cosmological observations have shown that the universe is currently (i.e., since $\sim 5 - 6$ billion years ago, or $z \sim 0.5$ - 1) undergoing accelerated expansion
- Not conclusively known what caused this accelerated expansion, the prevailing argument being that *dark energy* caused it, often considered to be sourced by Λ
- Rotational curves of galaxies: due to *dark matter*?





The current cosmic acceleration is attributed to dark energy, whereas the discrepancy between the predicted and observed rotation curves of galaxies is attributed to dark matter [Credit: Wiki Commons]

Challenges to the Cosmological Paradigm...

Some serious problems (or *tensions*) with the dark sector:

- Natures unknown: no direct prediction, nor detection so far
- Cosmological Constant Problem [1](vacuum catastrophe): measured energy density of the vacuum over 120 orders of magnitude less than the theoretical prediction
  - Worst prediction in the history of physics (and of science in general)
  - Casts doubt on dark energy being a cosmological constant
- Cosmic Coincidence Problem [2]: dark matter and dark energy densities have the same order of magnitude at the present moment of cosmic history, while differing with many orders of magnitude in the past and the predicted future
  - The initial conditions of dark matter and dark energy should be fine-tuned to about 95 orders of magnitude to produce a universe where the two densities nearly coincide today, approximately 14 billion years later[3]
- Way too many models of dark energy and dark matter!

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[1]Weinberg, S. Rev. Mod. Phys, 61 (1), 1 (1989)

[2]Velten, H. E. et al., Eur. Phys. J. C, 74 (11), 1 (2014)

[3]Zlatev, I., Wang, L., & Steinhardt, P. J. , Physi. Rev. Lett., 82(5), 896 (1999).

Cosmological Tensions

Cosmological Tensions

Latest tensions vis-á-vis precise theoretical predictions and observational measurements:

- H_0 CMB vs local measurements, $\sim 5\sigma$ discrepancy
  - Planck, ΛCDM model
    $$H_0 \approx 67\ km/s/Mpc$$
  - Estimate using SNIa, Cepheid measurements
    $$H_0 \approx 73\ km/s/Mpc$$
  - Late Universe probes tend to give a higher value for H_0 compared to early Universe probes within the ΛCDM framework.
  - This tension persists across independent datasets, suggesting it's not just a statistical fluke.
- S_8 vs cosmic shear data, more than 3σ discrepancy between Planck data and local measurements of
  $$S_8 = \sigma_8 \sqrt{\Omega_m/0.3}$$
  σ_8 measures the amplitude of the linear power spectrum on the $8h^{-1}$Mpc scale
  - σ_8 measures amplitude of the matter fluctuations on a $8h^{-1}$Mpc scales
    - RMS fluctuation of matter density field in spheres of $8h^{-1}$ Mpc
  - Measurements of S_8 from late Universe probes like weak lensing and galaxy surveys tend to be lower than those inferred from early Universe (CMB) data within ΛCDM.
- Ω_K, zero or not zero? ΛCDM assumes flat universe, but Planck temperature and polarisation power spectra give an above 3σ deviation:
  $$\Omega_K \approx -0.044^{+0.018}_{-0.015}$$
- Other challenges and anomalies exist, such anisotropic anomalies in the CMB, and hints of dynamical dark energy

Possible Explanations for Tensions

Three possibilities [4]

- Systematic errors
- New physics beyond ΛCDM
- Modified gravity, dark energy models, interactions

The CosmoVerse White Paper: Addressing observational tensions in cosmology with systematics and fundamental physics

The CosmoVerse Network

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arXiv:2504.01669v1 [astro-ph.CO] 2 Apr 2025

[4] The CosmoVerse White Paper (arXiv: 2504.01669) by the CosmoVerse Network: E Di Valentino et al, *Physics of the Dark Universe* 49, 101965 (2025)

Proposed Solutions - Early Universe (before recombination) Modifications

- **Early Dark Energy (EDE)**: This scenario proposes an additional, short-lived component of dark energy in the early Universe
  - EDE can reduce the size of the sound horizon at recombination, allowing for a higher inferred H_0 from CMB data, potentially bridging the Hubble tension
  - However, EDE models face challenges with fitting other cosmological data, such as the amplitude of matter fluctuations S_8
- **Extra Relativistic Species (ERS)**: Introducing additional relativistic particles in the early Universe, parameterized by an increase in the effective number of neutrinos (N_{eff}), can also affect the expansion rate and sound horizon, potentially increasing the inferred H_0
  - These ERS could be sterile neutrinos or other forms of dark radiation
  - Current cosmological data place constraints on the allowed amount of extra relativistic species, limiting their ability to fully resolve the Hubble tension

Proposed Solutions - Late Universe (after recombination) Modifications

- **Interacting Dark Energy (IDE)**: non-gravitational interactions b/n dark matter and dark energy
  - Such interactions can modify the late-time expansion history and the growth of structure, potentially alleviating both the H_0 and S_8 tensions.
  - Some IDE models can even feature phantom dark energy behaviour, which is known to potentially mitigate the H_0 tension
- **Modified Gravity (MG)**: Instead of introducing new components, MG theories alter the laws of gravity on cosmological scales
  - These modifications can affect both the expansion history and structure formation, offering potential ways to resolve the tensions
  - MG can be applied at both early and late times

Other Potential Avenues

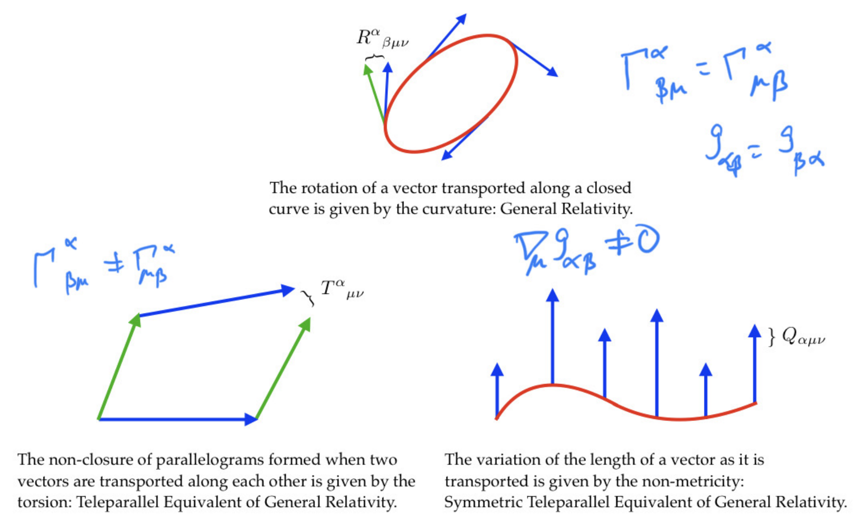
- Matter Sector Solutions: Investigating alternative properties of dark matter, such as interacting or decaying dark matter, or warm dark matter
- Local Physics Solutions: Considering the possibility that the Hubble tension arises from new physics operating specifically in our local cosmic neighbourhood, affecting distance measurements (e.g., modifications to the gravitational constant or interstellar medium)
- Revisiting Fundamental Assumptions: Questioning fundamental assumptions like the cosmological principle (homogeneity and isotropy of the Universe on large scales), e.g., existence of a large local void
- Quantum Gravity Phenomenology: Exploring potential effects from quantum gravity that might manifest on cosmological scales
- Varying Fundamental Constants: Investigating if fundamental constants might have varied over cosmic time, affecting cosmological observations

Summary of fundamental physics solutions proposed to solve the cosmological tensions [Credit: The CosmoVerse White Paper (arXiv: 2504.01669) by the CosmoVerse Network]

Matters of Gravity

The geometrical "trinity" of gravity

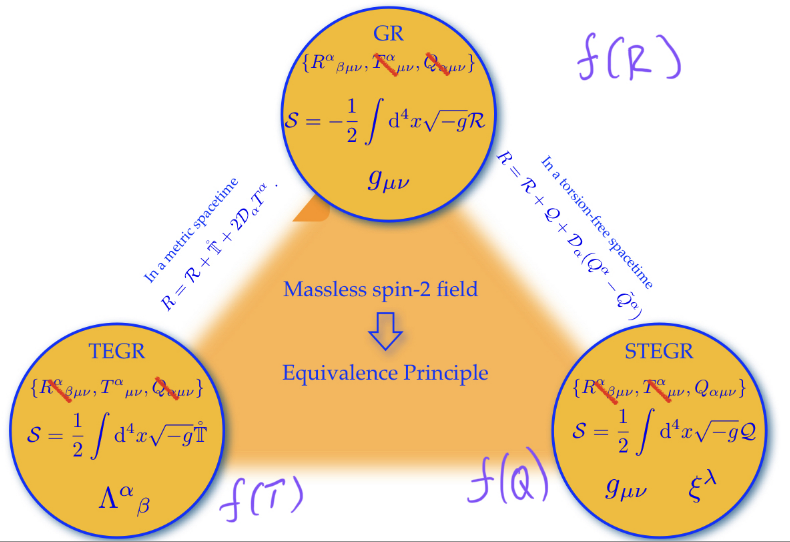
- Three different geometrical representations of spacetime curvature possible



The geometrical meaning of curvature, torsion and non-metricity. [Credit: Jimenez et al, arXiv 1903.06830]

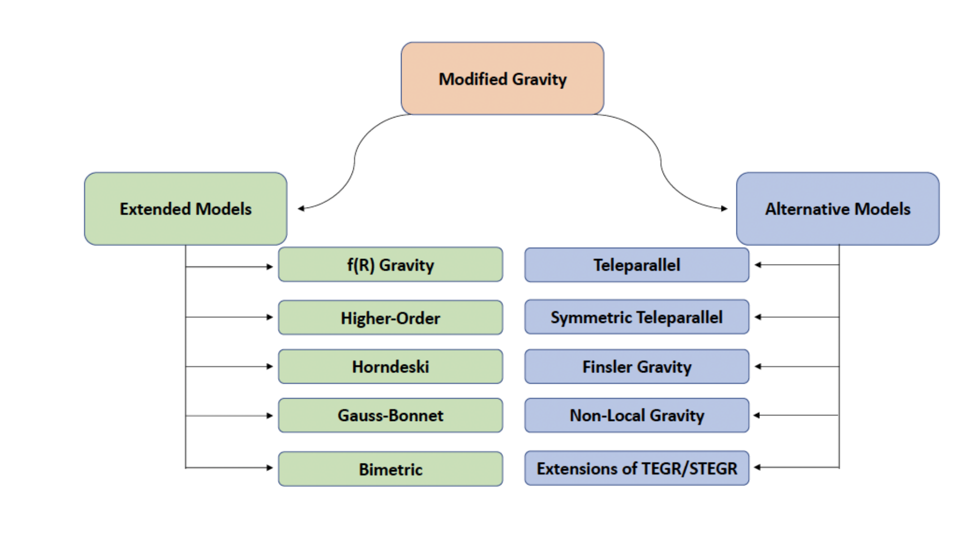
The geometrical "trinity" of gravity...

- Three possible gravitational interpretations



Three alternative gravitational descriptions. [Credit: Jimenez et al, arXiv 1903.06830]

Modifying Gravity. . .



Roadmap to messing with gravity [Credit: The CosmoVerse White Paper (arXiv: 2504.01669) by the CosmoVerse Network]

Some Constraints on $f(Q)$ Gravity

General Motivation and Aim

- The ΛCDM model explains many cosmological observations, but persistent tensions (H_0, S_8) and the unknown dark sector motivate exploration of modified gravity.
- Among these, $f(Q)$ gravity generalizes the symmetric teleparallel equivalent of GR, characterized by the non-metricity scalar Q, with potential to reproduce cosmic acceleration, providing an alternative to curvature- and torsion-based frameworks.
- Viscosity in the cosmic fluid has also been proposed to explain late-time acceleration and affect clustering.
- Combining both?: testing three paradigmatic $f(Q)$ models (power-law, exponential, logarithmic) with and without viscous fluids against multiple cosmological datasets (CC, BAO, Pantheon+SH0ES, f, $f\sigma_8$).

Aims

Goal 1: Test $f(Q)$ gravity with joint expansion + growth data (OHD, Pantheon+ SNe, RSD $f\sigma_8$, VIPERS/SDSS f and σ_8)
Goal 2: Assess whether bulk viscosity improves or worsens the observational viability of $f(Q)$ models compared to ΛCDM

Akaike (AIC) and Bayesian (BIC) information criteria used to compare $f(Q)$ gravity with ΛCDM.

- **Definitions:**
  - $\text{AIC} = \chi^2 + 2K$, $\text{BIC} = \chi^2 + K \ln N$
  - K: number of parameters, N: number of data points
- **Interpretation of ΔAIC:**
  - $\Delta\text{AIC} \leq 2 \Rightarrow$ substantial support
  - $4 \leq \Delta\text{AIC} \leq 7 \Rightarrow$ less support
  - $\Delta\text{AIC} \geq 10 \Rightarrow$ no support
- **Interpretation of ΔBIC:**
  - $0 \leq \Delta\text{BIC} \leq 2 \Rightarrow$ negligible evidence
  - $2 \leq \Delta\text{BIC} \leq 6 \Rightarrow$ positive evidence
  - $6 \leq \Delta\text{BIC} \leq 10 \Rightarrow$ strong evidence
  - $\Delta\text{BIC} > 10 \Rightarrow$ extremely strong evidence

Some Studies on $f(Q)$ Gravity: Structure Growth in Ccosmology [5]

- Investigated expansion history and growth of large-scale structures in the power-law i.e., $f(Q) = Q + \alpha Q^n$
- Employed MCMC simulations and Bayesian selection criteria to determine statistical significance: using Hubble data (OHD) and the Pantheon+ SNe sample to constrain parameters Ω_m, H_0, and the exponent n.
- Competitive viability against the ΛCDM model when considering both background expansion history and the growth of large-scale structures
- While statistical analyses provide varying levels of support depending on the specific datasets and criteria used (AIC vs. BIC), the model is not ruled out by the current data

Highlight: Background Expansion

- Consistent with ΛCDM at background level, with $H_0 \simeq 72.9^{+2.2}_{-1.7}$ km/s/Mpc, $\Omega_m \simeq 0.30$. Preferred n values are small, slightly negative, indicating only mild deviations from GR.
- $f(Q)$ fits late-time H_0 values better than early-time (Planck) estimates, showing observational competitiveness at the background level.

[5] Sahlu, S.; de la Cruz-Dombriz, Á.; **AA**, *Mon. Not. R. Astron. Soc.*, *530*, 3973-3988 (2025).

Constraints on $f(Q)$ vs ΛCDM

Dataset	Parameters	ΛCDM	$f(Q)$ gravity
OHD	n	–	$0.010^{+0.328}_{-0.381}$
	Ω_m	$0.287^{+0.039}_{-0.035}$	$0.297^{+0.040}_{-0.036}$
	H_0	$69.72^{+1.92}_{-1.70}$	$69.26^{+3.20}_{-2.70}$
SNIa	n	–	$-0.080^{+0.324}_{-0.283}$
	Ω_m	$0.336^{+0.048}_{-0.045}$	$0.332^{+0.054}_{-0.042}$
	H_0	$73.42^{+1.26}_{-1.52}$	$73.26^{+1.60}_{-2.11}$
OHD+SNIa	n	–	$-0.085^{+0.300}_{-0.190}$
	Ω_m	$0.323^{+0.089}_{-0.041}$	$0.304^{+0.029}_{-0.027}$
	H_0	$72.56^{+0.60}_{-0.60}$	$72.85^{+2.22}_{-1.66}$

Best-fit cosmological parameters at 1σ.

Data	$\mathcal{L}$	χ^2	$\chi^2_{\rm red}$	AIC	ΔAIC	BIC	ΔBIC
ΛCDM							
OHD	-14.87	29.74	0.74	33.74	–	37.26	–
SNIa	-761.56	1523.12	0.90	1527.12	–	1536.99	–
OHD+SNIa	-781.82	1563.64	0.90	1567.64	–	1578.57	–
$f(Q)$ model							
OHD	-14.60	29.20	0.78	35.20	1.46	40.48	3.21
SNIa	-761.93	1522.86	0.86	1528.86	1.74	1544.20	7.20
OHD+SNIa	-781.03	1562.06	0.95	1569.06	1.42	1584.45	5.88

Model selection (AIC, BIC) for ΛCDM vs $f(Q)$.

Perturbations and Growth

The full system of perturbation equations was derived for the study of structure growth beyond the background, numerical solutions were compared with the commonly used quasi-static approximation, which simplifies the equations at sub-horizon scales:

$$(1+z)^2\mathcal{D}''_m = (1+z)\left[1-(1+z)\frac{H'}{H}+\frac{H'}{H}(1-\Omega_m)\,nE^{2n-2}\right]\mathcal{D}'_m + \frac{3\Omega_m}{2E^2}(1+z)^3\,\mathcal{D}_m$$

Normalised (at $z_{in} \approx 1089$) density contrast, and growth rate:

$$\delta(z) \equiv \frac{\mathcal{D}_m(z)}{\mathcal{D}_m(z_{in})}$$

$$f \equiv \frac{\mathrm{d}\ln\delta_m}{\mathrm{d}\ln a} = -(1+z)\frac{\delta'_m(z)}{\delta_m(z)}$$

Highlight

Quasi-static approximation closely matches full solutions (94–99% accuracy) for best-fit n values, making it a reliable tool for $f(Q)$ growth studies.

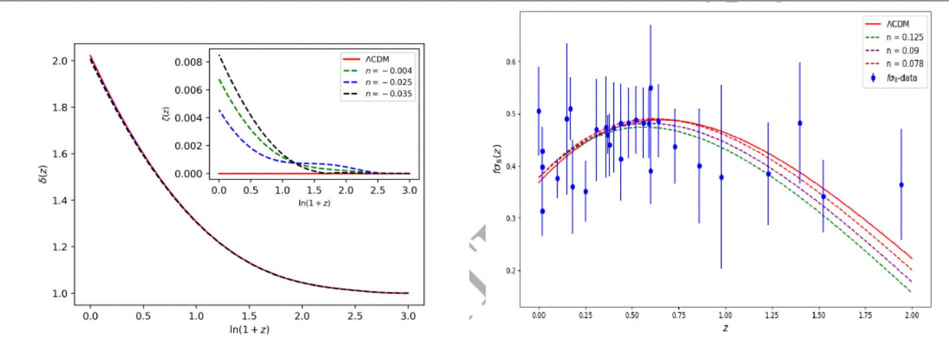
The analysis incorporated $f\sigma_8$ data (30 points), as well as direct f and σ_8 measurements from VIPERS and SDSS, together with OHD and SNIa.

Structure Growth Fits

Joint fits yielded $\Omega_m \simeq 0.327$, $\sigma_8 \simeq 0.826$, and $n \simeq -0.025$, showing that small deviations from GR remain allowed. $f(Q)$ tends to predict higher H_0 values, closer to local measurements, while σ_8 remains consistent with high-redshift probes.

Highlight

$f(Q)$ models reproduce both expansion and growth data, remaining competitive with ΛCDM across multiple datasets.



Some Studies on $f(Q)$ Gravity: Viscous-fluid Models [6]

- Insights into the viability of $f(Q)$ gravity models with and without bulk viscosity, with fluid description

$$T_{\mu\nu} = (\rho + \bar{p})u_\mu u_\nu + \bar{p}g_{\mu\nu} \; ,$$

such that $\bar{p} = p_m + \zeta(\rho)H$, with $\zeta(\rho) = \zeta\rho^\delta$ as most common parametrisation:

$$\dot{\rho}_m + 3H\left(\rho_m - \zeta\rho_m\right) = 0$$

$$\dot{\rho}_{de} + 3H\left(\rho_{de} + p_{de}\right) = 0$$

- Considered power-law, exponential, and logarithmic $f(Q)$ models:

$$f_1 := Q + \alpha Q^n \; , \qquad f_2 := Q + \beta Q_0\left(1 - \mathrm{e}^{-p\sqrt{\frac{Q}{Q_0}}}\right) \; , \qquad f_3 := Q + \epsilon \ln\left(\Gamma \frac{Q}{Q_0}\right)$$

[6] Sahlu, S.; Hough, R.;**AA**, de la Cruz-Dombriz Á, *Eur. Phys. J. C*, **85** 746 (2025)

Non-viscous scenarios

Data	CC	BAO	$PantheonP$ + $SH0ES$	$CC + BAO$	$f\sigma_8$	f	$f + f\sigma_8$	$PantheonP$ + $SH0ES + f$	$PantheonP$ + $SH0ES + f\sigma_8$
	f_1CDM								
H_0	$67.320^{+2.953}_{-2.759}$	$69.179^{+7.200}_{-6.426}$	$73.534^{+1.012}_{-1.003}$	$68.184^{+1.812}_{-1.830}$	–	–	–	$73.548^{+1.031}_{-1.000}$	$73.534^{+1.026}_{-1.005}$
Ω_m	$0.297^{+0.056}_{-0.055}$	$0.288^{+0.016}_{-0.017}$	$0.284^{+0.036}_{-0.041}$	$0.289^{+0.016}_{-0.016}$	$0.261^{+0.067}_{-0.072}$	$0.266^{+0.051}_{-0.061}$	$0.263^{+0.054}_{-0.064}$	$0.292^{+0.024}_{-0.032}$	$0.280^{+0.035}_{-0.038}$
n	$0.246^{+0.170}_{-0.166}$	$0.168^{+0.164}_{-0.117}$	$0.245^{+0.169}_{-0.164}$	$0.164^{+0.161}_{-0.114}$	$0.263^{+0.161}_{-0.174}$	$0.183^{+0.175}_{-0.129}$	$0.202^{+0.161}_{-0.137}$	$0.200^{+0.144}_{-0.121}$	$0.264^{+0.154}_{-0.167}$
r_d	–	$145.163^{+14.845}_{-13.659}$	–	$147.178^{+3.534}_{-3.352}$	–	–	–	–	–
M_{abs}	–	–	$-19.243^{+0.029}_{-0.029}$	–	–	–	–	$-19.244^{+0.030}_{-0.029}$	$-19.243^{+0.029}_{-0.029}$
σ_8	–	–	–		$0.810^{+0.045}_{-0.038}$	–	$0.807^{+0.026}_{-0.023}$	–	$0.803^{+0.044}_{-0.036}$
s_8	–	–	–	–	$0.752^{+0.099}_{-0.099}$	–	$0.754^{+0.081}_{-0.081}$	–	$0.776^{+0.064}_{-0.063}$
γ	–	–	–	–	$0.552^{+0.098}_{-0.100}$	$0.575^{+0.087}_{-0.106}$	$0.563^{+0.094}_{-0.102}$	$0.619^{+0.054}_{-0.076}$	$0.580^{+0.063}_{-0.063}$
	f_2CDM								
H_0	$70.211^{+3.061}_{-3.103}$	$69.540^{+7.003}_{-6.540}$	$73.752^{+1.010}_{-1.002}$	$70.931^{+1.918}_{-1.876}$	–	–	–	$73.813^{+1.013}_{-1.008}$	$73.762^{+1.023}_{-1.002}$
Ω_m	$0.280^{+0.056}_{-0.049}$	$0.263^{+0.014}_{-0.012}$	$0.321^{+0.034}_{-0.024}$	$0.236^{+0.013}_{-0.012}$	$0.260^{+0.072}_{-0.068}$	$0.219^{+0.058}_{-0.051}$	$0.248^{+0.062}_{-0.051}$	$0.307^{+0.018}_{-0.017}$	$0.309^{+0.021}_{-0.018}$
p	$0.842^{+0.110}_{-0.153}$	$0.918^{+0.054}_{-0.063}$	$0.918^{+0.059}_{-0.098}$	$0.926^{+0.049}_{-0.061}$	$0.827^{+0.127}_{-0.405}$	$0.694^{+0.172}_{-0.154}$	$0.869^{+0.090}_{-0.120}$	$0.948^{+0.037}_{-0.057}$	$0.949^{+0.037}_{-0.065}$
r_d	–	$150.934^{+15.705}_{-13.859}$	–	$147.501^{+3.553}_{-3.385}$	–	–	–	–	–
M_{abs}	–	–	$-19.245^{+0.029}_{-0.030}$	–	–	–	–	$-19.245^{+0.029}_{-0.030}$	$-19.245^{+0.029}_{-0.030}$
σ_8	–	–	–	–	$0.773^{+0.046}_{-0.115}$	–	$0.798^{+0.025}_{-0.025}$	–	$0.790^{+0.025}_{-0.027}$
s_8	–	–	–	–	$0.7203^{+0.02}_{-0.01}$	–	$0.7245^{+0.113}_{-0.097}$	–	$0.802^{+0.037}_{-0.037}$
γ	–	–	–	–	$0.487^{+0.125}_{-0.067}$	$0.536^{+0.105}_{-0.088}$	$0.517^{+0.112}_{-0.084}$	$0.651^{+0.034}_{-0.049}$	$0.632^{+0.045}_{-0.059}$
	f_3CDM								
H_0	$70.708^{+2.633}_{-2.397}$	$69.403^{+7.032}_{-6.551}$	$73.768^{+1.008}_{-1.005}$	$71.179^{+1.784}_{-1.780}$	–	–	–	$73.920^{+1.020}_{-1.000}$	$73.809^{+1.015}_{-1.002}$
Ω_m	$0.352^{+0.033}_{-0.045}$	$0.310^{+0.015}_{-0.014}$	$0.388^{+0.009}_{-0.014}$	$0.313^{+0.015}_{-0.014}$	$0.278^{+0.070}_{-0.065}$	$0.245^{+0.051}_{-0.048}$	$0.258^{+0.058}_{-0.043}$	$0.359^{+0.016}_{-0.016}$	$0.384^{+0.011}_{-0.015}$
Γ	$3.457^{+1.269}_{-1.014}$	$2.704^{+0.826}_{-0.506}$	$2.459^{+0.531}_{-0.330}$	$2.661^{+0.768}_{-0.479}$	$3.887^{+1.500}_{-1.284}$	$3.751^{+1.825}_{-1.243}$	$3.009^{+1.156}_{-0.728}$	$2.208^{+0.306}_{-0.155}$	$2.410^{+0.498}_{-0.297}$
r_d	–	$153.498^{+14.856}_{-14.461}$	–	$147.304^{+3.506}_{-3.343}$	–	–	–	–	–
M_{abs}	–	–	$-19.246^{+0.029}_{-0.029}$	–	–	–	–	$-19.247^{+0.030}_{-0.029}$	$-19.246^{+0.029}_{-0.029}$
σ_8	–	–	-	–	$0.720^{+0.037}_{-0.042}$	–	$0.772^{+0.024}_{-0.023}$	–	$0.709^{+0.016}_{-0.016}$
s_8	–	–	-	–	$0.693^{+0.123}_{-0.121}$	–	$0.715^{+0.102}_{-0.083}$	–	$0.802^{+0.020}_{-0.020}$
γ	–	–	–	–	$0.536^{+0.105}_{-0.091}$	$0.518^{+0.117}_{-0.077}$	$0.463^{+0.088}_{-0.046}$	$0.685^{+0.011}_{-0.023}$	$0.648^{+0.036}_{-0.053}$
	ΛCDM								
H_0	$68.335^{+2.833}_{-2.542}$	$69.116^{+7.175}_{-6.318}$	$73.588^{+1.002}_{-1.001}$	$69.181^{+1.714}_{-1.697}$	–	–	–	$73.684^{+1.012}_{-1.002}$	$73.592^{+1.032}_{-0.991}$
Ω_m	$0.316^{+0.049}_{-0.050}$	$0.294^{+0.015}_{-0.014}$	$0.331^{+0.018}_{-0.018}$	$0.296^{+0.015}_{-0.014}$	$0.270^{+0.068}_{-0.071}$	$0.257^{+0.047}_{-0.059}$	$0.250^{+0.056}_{-0.058}$	$0.321^{+0.015}_{-0.015}$	$0.328^{+0.017}_{-0.017}$
r_d	–	$147.488^{+14.894}_{-13.814}$	–	$147.197^{+3.496}_{-3.386}$	–	–	–	–	–
M_{abs}	–	–	$-19.244^{+0.029}_{-0.029}$	–	–	–	–	$-19.244^{+0.029}_{-0.029}$	$-19.245^{+0.030}_{-0.029}$
σ_8	–	–	–	–	$0.776^{+0.033}_{-0.028}$	–	$0.792^{+0.023}_{-0.021}$	–	$0.760^{+0.020}_{-0.020}$
s_8	–	–	–	–	$0.736^{+0.031}_{-0.035}$	–	$0.737^{+0.025}_{-0.024}$	–	$0.795^{+0.029}_{-0.029}$
γ	–	–	–	–	$0.540^{+0.102}_{-0.095}$	$0.577^{+0.086}_{-0.107}$	$0.544^{+0.099}_{-0.094}$	$0.667^{+0.024}_{-0.039}$	$0.621^{+0.050}_{-0.059}$

Viscous scenarios

Data	CC	BAO	$PantheonP + SH0ES$	$CC + BAO$	$f\sigma_8$	f	$f + f\sigma_8$	$PantheonP + SH0ES + f$	$PantheonP + SH0ES + f\sigma_8$
	f_1CDM								
H_0	$67.907^{+2.550}_{-2.427}$	$68.677^{+7.498}_{-5.756}$	$73.467^{+1.020}_{-0.996}$	$67.662^{+1.854}_{-1.875}$	–	–	–	$73.508^{+1.029}_{-0.994}$	$73.472^{+1.016}_{-0.997}$
Ω_m	$0.335^{+0.045}_{-0.061}$	$0.321^{+0.041}_{-0.029}$	$0.332^{+0.043}_{-0.050}$	$0.321^{+0.039}_{-0.028}$	$0.294^{+0.068}_{-0.077}$	$0.289^{+0.013}_{-0.015}$	$0.243^{+0.052}_{-0.054}$	$0.303^{+0.024}_{-0.028}$	$0.326^{+0.045}_{-0.048}$
n	$0.261^{+0.167}_{-0.173}$	$0.135^{+0.160}_{-0.100}$	$0.256^{+0.160}_{-0.163}$	$0.139^{+0.155}_{-0.100}$	$0.241^{+0.164}_{-0.190}$	$0.236^{+0.172}_{-0.160}$	$0.284^{+0.147}_{-0.176}$	$0.311^{+0.122}_{-0.149}$	$0.276^{+0.147}_{-0.166}$
r_d	–	$145.178^{+13.205}_{-14.586}$	–	$147.171^{+3.542}_{-3.339}$	–	–	–	–	–
M_{abs}	–	–	$-19.244^{+0.030}_{-0.029}$	–	–	–	–	$-19.244^{+0.030}_{-0.029}$	$-19.243^{+0.029}_{-0.029}$
ζ	$0.054^{+0.060}_{-0.037}$	$0.024^{+0.027}_{-0.017}$	$0.080^{+0.066}_{-0.054}$	$0.024^{+0.026}_{-0.017}$	$0.133^{+0.105}_{-0.091}$	$0.151^{+0.095}_{-0.093}$	$0.085^{+0.083}_{-0.057}$	$0.059^{+0.055}_{-0.040}$	$0.077^{+0.067}_{-0.054}$
σ_8	–	–	–	–	$0.747^{+0.055}_{-0.048}$	–	$0.790^{+0.027}_{-0.027}$	–	$0.754^{+0.049}_{-0.041}$
S_8	–	–	–	–	$0.739^{+0.055}_{-0.055}$	–	$0.711^{+0.080}_{-0.080}$	–	$0.786^{+0.075}_{-0.072}$
γ	–	–	–	–	$0.527^{+0.096}_{-0.084}$	$0.573^{+0.087}_{-0.101}$	$0.524^{+0.109}_{-0.086}$	$0.640^{+0.042}_{-0.068}$	$0.596^{+0.059}_{-0.063}$
	f_2CDM								
H_0	$70.446^{+2.803}_{-2.779}$	$68.341^{+7.113}_{-5.651}$	$73.714^{+1.018}_{-1.007}$	$70.578^{+1.964}_{-1.952}$	–	–	–	$73.772^{+1.018}_{-1.011}$	$73.738^{+1.018}_{-1.013}$
Ω_m	$0.334^{+0.046}_{-0.060}$	$0.318^{+0.057}_{-0.057}$	$0.368^{+0.023}_{-0.034}$	$0.312^{+0.058}_{-0.052}$	$0.301^{+0.065}_{-0.074}$	$0.210^{+0.068}_{-0.056}$	$0.217^{+0.064}_{-0.043}$	$0.322^{+0.020}_{-0.019}$	$0.361^{+0.027}_{-0.035}$
p	$0.837^{+0.110}_{-0.136}$	$0.803^{+0.108}_{-0.086}$	$0.938^{+0.045}_{-0.073}$	$0.822^{+0.098}_{-0.088}$	$0.797^{+0.147}_{-0.346}$	$0.721^{+0.174}_{-0.181}$	$0.898^{+0.072}_{-0.113}$	$0.962^{+0.028}_{-0.049}$	$0.952^{+0.035}_{-0.058}$
r_d	–	$152.645^{+14.003}_{-14.383}$	–	$147.520^{+3.541}_{-3.388}$	–	–	–	–	–
M_{abs}	–	–	$-19.245^{+0.029}_{-0.029}$	–	–	–	–	$-19.246^{+0.029}_{-0.030}$	$-19.245^{+0.029}_{-0.030}$
ζ	$0.071^{+0.060}_{-0.047}$	$0.075^{+0.040}_{-0.049}$	$0.084^{+0.046}_{-0.049}$	$0.069^{+0.040}_{-0.044}$	$0.154^{+0.097}_{-0.100}$	$0.081^{+0.071}_{-0.063}$	$0.074^{+0.077}_{-0.050}$	$0.039^{+0.033}_{-0.026}$	$0.090^{+0.046}_{-0.052}$
σ_8	–	–	–	–	$0.698^{+0.063}_{-0.072}$	–	$0.781^{+0.027}_{-0.026}$	–	$0.731^{+0.037}_{-0.029}$
S_8	–	–	–	–	$0.699^{+0.078}_{-0.028}$	–	$0.653^{+0.108}_{-0.108}$	–	$0.801^{+0.050}_{-0.055}$
γ	–	–	–	–	$0.476^{+0.106}_{-0.057}$	$0.525^{+0.118}_{-0.091}$	$0.491^{+0.114}_{-0.067}$	$0.668^{+0.023}_{-0.040}$	$0.640^{+0.040}_{-0.056}$
	f_3CDM								
H_0	$71.132^{+2.407}_{-2.287}$	$69.116^{+7.277}_{-6.412}$	$73.824^{+1.027}_{-0.999}$	$70.413^{+1.829}_{-1.802}$	–	–	–	$73.926^{+1.003}_{-0.996}$	$73.815^{+1.015}_{-0.990}$
Ω_m	$0.366^{+0.025}_{-0.041}$	$0.365^{+0.025}_{-0.033}$	$0.391^{+0.007}_{-0.013}$	$0.367^{+0.023}_{-0.032}$	$0.309^{+0.059}_{-0.068}$	$0.192^{+0.051}_{-0.038}$	$0.190^{+0.042}_{-0.027}$	$0.362^{+0.016}_{-0.016}$	$0.389^{+0.008}_{-0.014}$
Γ	$3.247^{+1.204}_{-0.878}$	$2.946^{+0.935}_{-0.667}$	$2.360^{+0.468}_{-0.263}$	$2.870^{+0.867}_{-0.613}$	$3.882^{+1.795}_{-1.324}$	$3.754^{+1.629}_{-1.225}$	$2.969^{+1.129}_{-0.696}$	$2.192^{+0.286}_{-0.144}$	$2.323^{+0.437}_{-0.235}$
r_d	–	$150.509^{+15.405}_{-14.283}$	–	$147.335^{+3.499}_{-3.386}$	–	–	–	–	–
M_{abs}	–	–	$-19.246^{+0.030}_{-0.029}$	–	–	–	–	$-19.248^{+0.029}_{-0.029}$	$-19.247^{+0.029}_{-0.029}$
ζ	$0.036^{+0.043}_{-0.026}$	$0.041^{+0.019}_{-0.022}$	$0.018^{+0.022}_{-0.013}$	$0.042^{+0.018}_{-0.022}$	$0.142^{+0.102}_{-0.095}$	$0.104^{+0.111}_{-0.074}$	$0.049^{+0.073}_{-0.036}$	$0.008^{+0.013}_{-0.006}$	$0.021^{+0.024}_{-0.015}$
σ_8	–	–	–	–	$0.675^{+0.044}_{-0.041}$	–	$0.765^{+0.024}_{-0.024}$	–	$0.700^{+0.016}_{-0.017}$
S_8	–	–	–	–	$0.540^{+0.045}_{-0.042}$	–	$0.603^{+0.042}_{-0.028}$	–	$0.786^{+0.020}_{-0.023}$
γ	–	–	–	–	$0.499^{+0.105}_{-0.070}$	$0.471^{+0.082}_{-0.051}$	$0.444^{+0.067}_{-0.032}$	$0.686^{+0.011}_{-0.022}$	$0.642^{+0.039}_{-0.054}$
	ΛCDM								
H_0	$68.965^{+2.487}_{-2.287}$	$69.210^{+7.222}_{-6.487}$	$73.587^{+0.998}_{-0.997}$	$68.426^{+1.752}_{-1.760}$	–	–	–	$73.681^{+1.029}_{-1.010}$	$73.591^{+1.022}_{-1.011}$
Ω_m	$0.348^{+0.036}_{-0.052}$	$0.331^{+0.037}_{-0.029}$	$0.365^{+0.024}_{-0.027}$	$0.333^{+0.037}_{-0.029}$	$0.298^{+0.066}_{-0.074}$	$0.230^{+0.050}_{-0.050}$	$0.224^{-0.063}_{-0.045}$	$0.329^{+0.017}_{-0.017}$	$0.362^{+0.025}_{-0.027}$
r_d	–	$145.698^{+15.006}_{-13.783}$	–	$147.199^{+3.521}_{-3.383}$	–	–	–	–	–
M_{abs}	–	–	$-19.244^{+0.029}_{-0.029}$	–	–	–	–	$-19.245^{+0.030}_{-0.029}$	$-19.244^{+0.029}_{-0.030}$
ζ	$0.049^{+0.052}_{-0.034}$	$0.028^{+0.025}_{-0.019}$	$0.058^{+0.042}_{-0.038}$	$0.028^{+0.025}_{-0.020}$	$0.135^{+0.106}_{-0.093}$	$0.128^{+0.102}_{-0.084}$	$0.060^{+0.076}_{-0.043}$	$0.020^{+0.024}_{-0.014}$	$0.058^{+0.043}_{-0.039}$
σ_8	–	–	–	–	– $0.753^{+0.045}_{-0.041}$	–	$0.779^{+0.025}_{-0.024}$	–	$0.726^{+0.028}_{-0.026}$
S_8	–	–	–	–	$0.750^{+0.016}_{-0.017}$	–	$0.785^{+0.038}_{-0.036}$	–	$0.789^{+0.041}_{-0.041}$
γ	–	–	–	–	$0.548^{+0.098}_{-0.095}$	$0.511^{+0.094}_{-0.076}$	$0.496^{+0.109}_{-0.068}$	$0.673^{+0.020}_{-0.035}$	$0.627^{+0.047}_{-0.057}$

Methodology and Datasets

- Modified Friedmann equations in $f(Q)$ with effective pressure $p \to p - 3\zeta H$.
- Growth equations with quasi-static approximation checked for scalar perturbations

Models tested:

- $f_1(Q) = Q + \alpha Q^n$ (power-law)
- $f_2(Q) = Q + \beta Q_0(1 - e^{-p\sqrt{Q/Q_0}})$ (exponential)
- $f_3(Q) = Q + \epsilon \ln(\Gamma Q/Q_0)$ (logarithmic)

Highlight

Joint MCMC analysis performed: cosmic chronometers (CC), BAO from DESI, Pantheon+SH0ES SNe sample, growth datasets f (VIPERS, SDSS) and $f\sigma_8$ (66 points):
Ω_m, H_0, σ_8, n, p, Γ, and ζ

Best-fit cosmological parameters were obtained for all models with and without bulk viscosity:

- For f_1, bulk viscosity raises Ω_m and slightly lowers S_8, closer to DES weak-lensing results
- For f_2, viscosity leads to higher Ω_m but significantly lower S_8, hinting at possible relevance to the S_8 tension
- For f_3, bulk viscosity increases parameter spread; Ω_m ranges from 0.19 to 0.39 across datasets

Highlight

Viscosity shifts Ω_m upward and σ_8 downward, but the effects are model-dependent and dataset-sensitive

Growth and Perturbations

Perturbation equations:

$$\frac{\mathrm{d}^2\delta_m}{\mathrm{d}z^2} = \left(\frac{1}{1+z} - \frac{\mathrm{d}E}{E\mathrm{d}z}\left(1+\bar{f}_{1,2,3}\right)\right)\frac{\mathrm{d}\delta_m}{\mathrm{d}z} + \frac{\Omega_m}{2E^2}(1+z)^{1-3\zeta}\delta_m \ ,$$

rewrite, with $f(z) \approx \tilde{\Omega}_m^{\gamma}(z)$,

$$(1+z)f' = f^2 + \left[2 - (1+z)\frac{dE}{Edz}\bar{f}_{1,2,3}\right]f - \frac{\Omega_m}{2E^2}(1+z)^{1-3\zeta}$$

Using best-fit parameters, growth rate $f(z)$ and RSD $f\sigma_8(z)$ were computed:

- Without viscosity, $f(Q)$ models track ΛCDM closely at high redshift, with mild deviations at $z < 0.5$.
- With viscosity, clustering is damped, predicting less structure growth.
- $f\sigma_8$ predictions align well with observational data across all $f(Q)$ models, even in viscous cases

Highlight

Bulk viscosity consistently damps growth, reducing clustering amplitude, but does not improve statistical fits

Statistical Viability and Conclusions

AIC and BIC were used to compare models with ΛCDM:

- Non-viscous f_1 achieves moderate support across datasets (no outright rejection).
- Exponential (f_2) and logarithmic (f_3) models perform worse, with several outright rejections.
- Adding bulk viscosity increases ΔAIC, ΔBIC values, penalizing all models.

Takeaway

- While the exponential $f(Q)$ model without viscosity showed promise as an alternative to ΛCDM based on background expansion data, the inclusion of bulk viscosity generally did not improve the models' performance statistically
- Interestingly, all $f(Q)$ models predicted a faster structure growth rate than ΛCDM
- Among all cases, only non-viscous f_1 remains a statistically viable alternative to ΛCDM; viscosity is not favored

Non-viscous scenarios

Data	$\mathscr{L}(\hat{\theta}\|data)$	χ^2	χ^2_ν	AIC	$\|\Delta\mathrm{AIC}\|$	BIC	$\|\Delta\mathrm{BIC}\|$
ΛCDM							
CC	−7.259	14.519	0.500	18.520	–	21.388	–
BAO	−6.378	12.756	1.417	18.756	–	20.211	–
PantheonP + SH0ES	−726.322	1452.645	0.878	1458.646	–	1474.884	–
CC+ BAO	−13.723	27.447	0.686	33.447	–	38.731	–
f	−3.082	6.164	0.513	10.164	–	11.442	–
$f\sigma_8$	−15.990	31.9808	0.507	37.981	–	44.550	–
$f + f\sigma_8$	−19.365	38.730	0.503	44.731	–	51.877	–
PantheonP + SH0ES + f	−729.775	1459.551	0.875	1467.552	–	1489.237	–
PantheonP + SH0ES + $f\sigma_8$	−742.462	1484.925	0.864	1494.925	–	1522.184	–
$f_{1\mathrm{CDM}}$							
CC	−7.318	14.636	0.522	20.636	2.115	24.938	3.549
BAO	−6.896	13.793	1.724	21.794	3.038	23.733	3.522
PantheonP + SH0ES	−726.220	1452.440	0.878	1460.440	1.794	1482.091	7.207
CC + BAO	−14.169	28.338	0.726	36.339	2.891	43.383	4.653
f	−3.398	6.796	0.617	12.797	2.633	14.714	3.272
$f\sigma_8$	−15.874	31.749	0.5121	39.750	1.769	48.508	3.958
$f + f\sigma_8$	−19.399	38.799	0.510	46.799	2.068	56.328	4.451
PantheonP + SH0ES + f	−729.603	1459.206	0.875	1469.207	1.655	1496.312	7.076
PantheonP + SH0ES + $f\sigma_8$	−742.087	1484.174	0.864	1496.175	1.250	1528.886	6.701
$f_{2\mathrm{CDM}}$							
CC	−7.3017	14.603	0.521	20.603	2.083	24.905	3.668
BAO	−7.864	15.728	1.962	23.729	4.973	25.668	5.457
PantheonP + SH0ES	−727.731	1455.462	0.880	1463.463	4.817	1485.114	10.230
CC + BAO	−15.001	30.002	0.769	38.002	4.555	45.047	6.316
f	−2.743	5.487	0.498	11.487	1.323	13.404	1.962
$f\sigma_8$	−17.697	35.394	0.570	43.395	5.414	52.153	7.614
$f + f\sigma_8$	−19.625	39.250	0.516	47.250	2.519	56.778	4.902
PantheonP + SH0ES + f	−731.209	1462.418	0.877	1472.418	4.866	1499.524	10.287
PantheonP + SH0ES + $f\sigma_8$	−743.738	1487.476	0.866	1499.476	4.551	1532.187	10.003
$f_{3\mathrm{CDM}}$							
CC	−7.377	14.7539	0.5269	20.754	2.234	25.056	3.668
BAO	−7.454	14.908	1.863	22.909	4.152	24.848	4.637
PantheonP + SH0ES	−727.034	1454.068	0.879	1462.069	3.423	1483.720	8.836
CC + BAO	−15.033	30.066	0.770	38.067	4.620	45.112	6.381
f	−2.7182	5.436	0.4942	11.436	1.272	13.354	1.911
$f\sigma_8$	−16.495	32.989	0.532	40.990	3.009	49.749	5.199
$f + f\sigma_8$	−19.781	39.562	0.520	47.563	2.832	57.091	5.214
PantheonP + SH0ES + f	−734.908	1469.816	0.882	1479.817	12.265	1506.923	17.686
PantheonP + SH0ES + $f\sigma_8$	−744.131	1488.263	0.866	1500.264	5.339	1532.974	10.790

Viscous scenarios

Data	$\mathscr{L}(\hat{\theta}\|data)$	χ^2	χ^2_ν	AIC	$\|\Delta\text{AIC}\|$	BIC	$\|\Delta\text{BIC}\|$
ΛCDM							
CC	−7.418	14.837	0.529	20.837	2.318	25.139	3.752
BAO	−6.640	13.281	1.660	21.282	2.525	23.221	3.010
PantheonP + SH0ES	−726.312	1452.624	0.878	1460.624	1.979	1482.275	7.391
CC+ BAO	−13.971	27.943	0.716	35.94	2.496	42.988	4.257
f	−2.785	5.571	0.506	11.572	1.407	13.489	2.047
$f\sigma_8$	−15.984	31.969	0.515	39.970	1.989	48.728	4.179
$f + f\sigma_8$	−19.490	38.981	0.512	46.982	2.251	56.5107	4.633
PantheonP + SH0ES + f	−730.028	1460.056	0.876	1470.0561	2.504	1497.162	7.926
PantheonP + SH0ES + $f\sigma_8$	−742.355	1484.710	0.864	1496.710	1.785	1529.421	7.237
$f_{1\text{CDM}}$							
CC	−7.501	15.003	0.555	23.004	4.484	28.740	7.352
BAO	−7.326	14.652	2.0932	24.652	5.896	27.077	6.866
PantheonP + SH0ES	−726.558	1453.116	0.879	1463.116	4.471	1490.180	15.296
CC + BAO	−14.553	29.107	0.766	39.108	5.660	47.914	9.183
f	−2.834	5.668	0.566	13.669	3.504	16.225	4.783
$f\sigma_8$	−15.943	31.887	0.522	41.887	3.906	52.835	8.286
$f + f\sigma_8$	−19.357	38.714	0.516	48.714	3.983	60.624	8.747
PantheonP + SH0ES + f	−729.688	1459.376	0.876	1471.377	3.825	1503.904	14.667
PantheonP + SH0ES + $f\sigma_8$	−742.391	1484.783	0.865	1498.783	3.858	1536.946	14.762
$f_{2\text{CDM}}$							
CC	−7.2881	14.576	0.539	22.576	4.056	28.312	6.924
BAO	−8.069	16.138	2.305	26.138	7.382	28.563	8.352
PantheonP + SH0ES	−727.226	1454.452	0.880	1464.452	5.807	1491.516	16.632
CC + BAO	−15.248	30.497	0.802	40.498	7.050	49.304	10.573
f	−2.710	5.421	0.542	13.421	3.257	15.977	4.535
$f\sigma_8$	−16.551	33.103	0.542	43.103	5.122	54.052	9.502
$f + f\sigma_8$	−19.437	38.874	0.518	48.874	4.143	60.784	8.908
PantheonP + SH0ES + f	−730.650	1461.300	0.877	1473.301	5.749	1505.828	16.591
PantheonP + SH0ES + $f\sigma_8$	−743.065	1486.130	0.866	1500.130	5.205	1538.293	16.109
$f_{3\text{CDM}}$							
CC	−7.531	15.061	0.557	23.062	4.542	28.798	7.410
BAO	−7.400	14.800	2.114	24.801	6.044	27.225	7.014
PantheonP + SH0ES	−727.172	1454.344	0.880	1464.345	5.699	1491.4081	16.525
CC + BAO	−14.562	29.125	0.766	39.126	5.678	47.932	9.201
f	−2.817	5.634	0.563	13.63	3.470	16.191	4.748
$f\sigma_8$	−16.210	32.421	0.531	42.421	4.440	53.369	8.820
$f + f\sigma_8$	−20.023	40.046	0.534	50.047	5.316	61.957	10.080
PantheonP + SH0ES + f	−735.435	1470.871	0.883	1482.871	15.320	1515.398	26.162
PantheonP + SH0ES + $f\sigma_8$	−744.146	1488.292	0.8673	1502.293	7.367	1540.455	18.271

Summary & Outlook

" Not only is the universe stranger than we think, it is stranger than we can think."

– Werner Heisenberg

- The existence and nature of dark matter and dark energy are key unsolved mysteries, and significant tensions in cosmology, such as the Hubble constant (H_0) and S_8 discrepancies, remain unresolved
- At the background level, $f(Q)$ models fit H_0 and Ω_m well. Perturbation analysis shows quasi-static approximation is accurate enough for practical growth studies. Joint constraints indicate that $f(Q)$ is not ruled out and can mimic ΛCDM, while offering slight improvements in matching local H_0 values.

Takeaway

- $f(Q)$ gravity is observationally viable and remains a competitive alternative to ΛCDM, pending future high-precision growth data.
- Among all cases considered here, only non-viscous f_1 remains a statistically viable alternative to ΛCDM; viscosity is not favored

Outlook

- Modified gravity and phenomenological models offer promising paths beyond the ΛCDM impasse
- More work ahead: deeper theoretical insights and observational data constraints

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