A multi-parameter expansion for Extreme Mass Ratio Inspirals in astrophysical environments

arXiv:2507.04471 [gr-qc]

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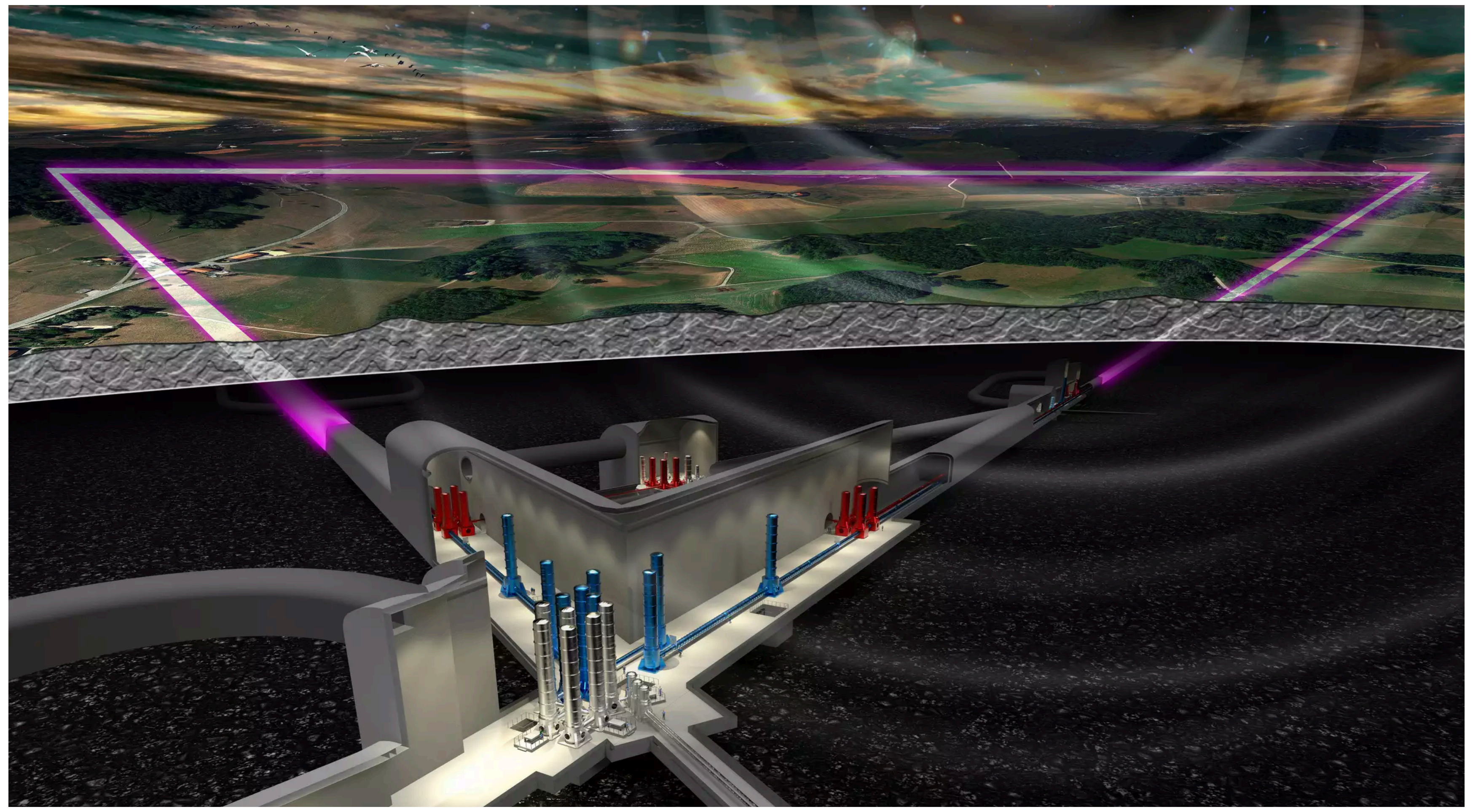
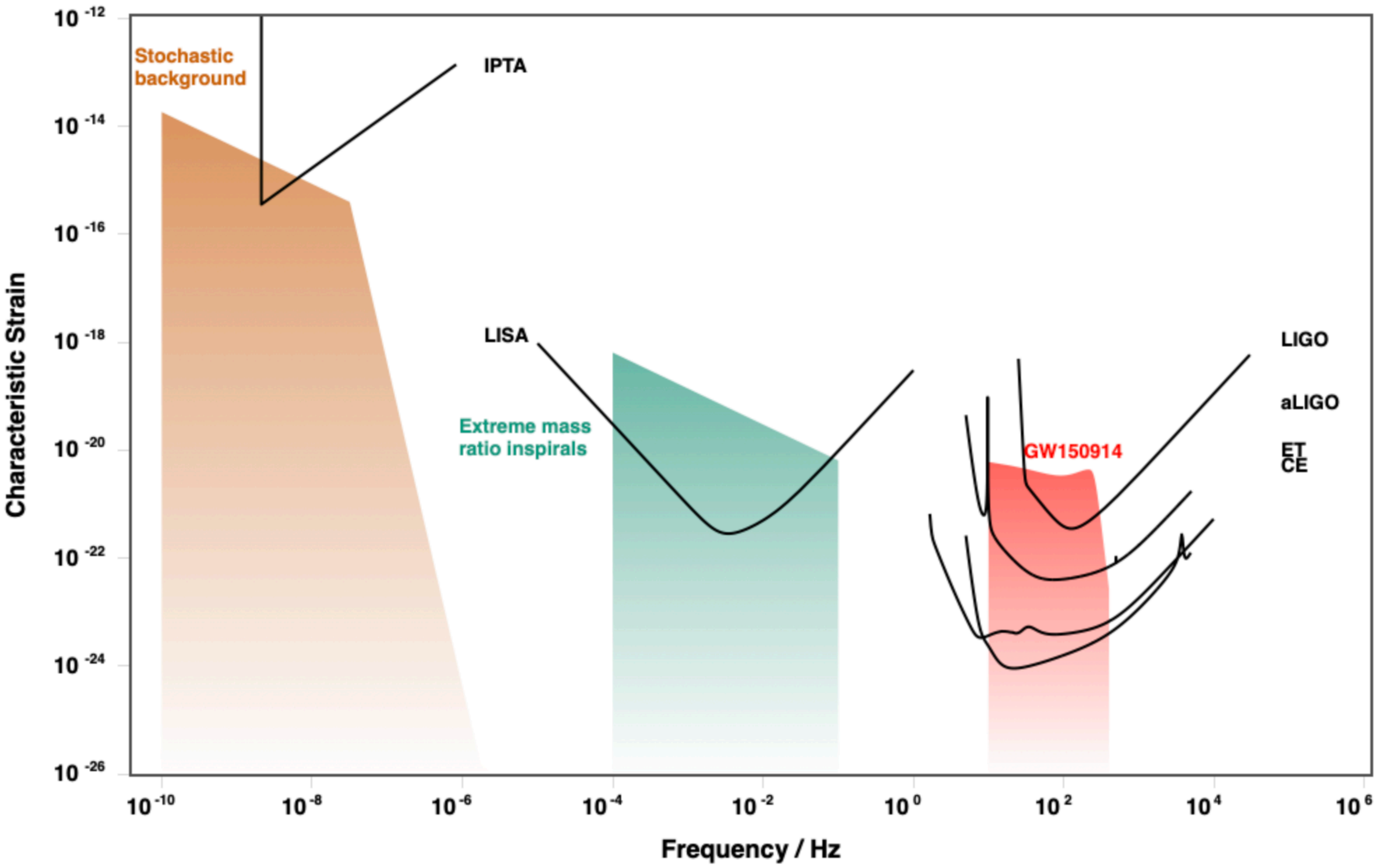
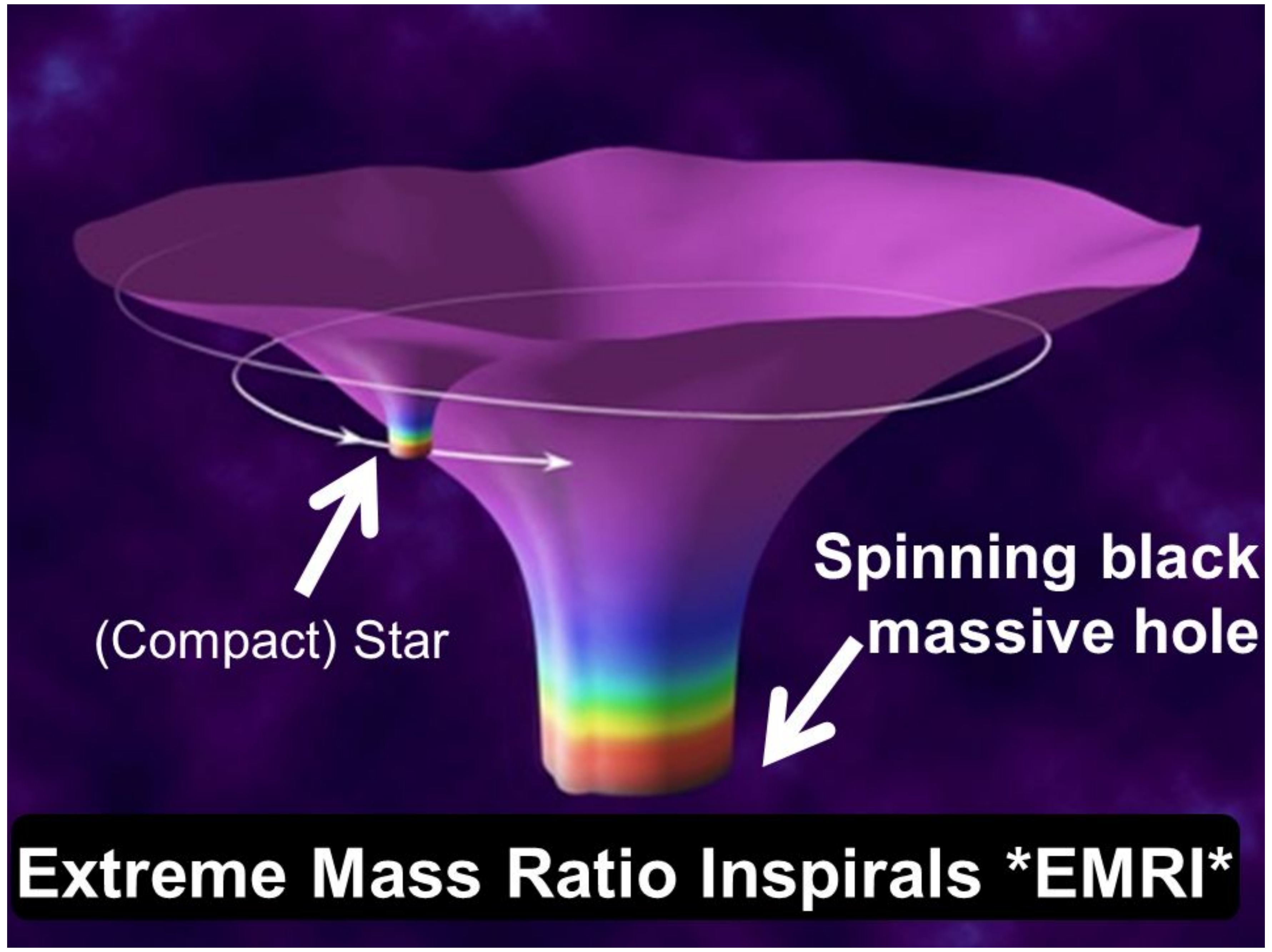
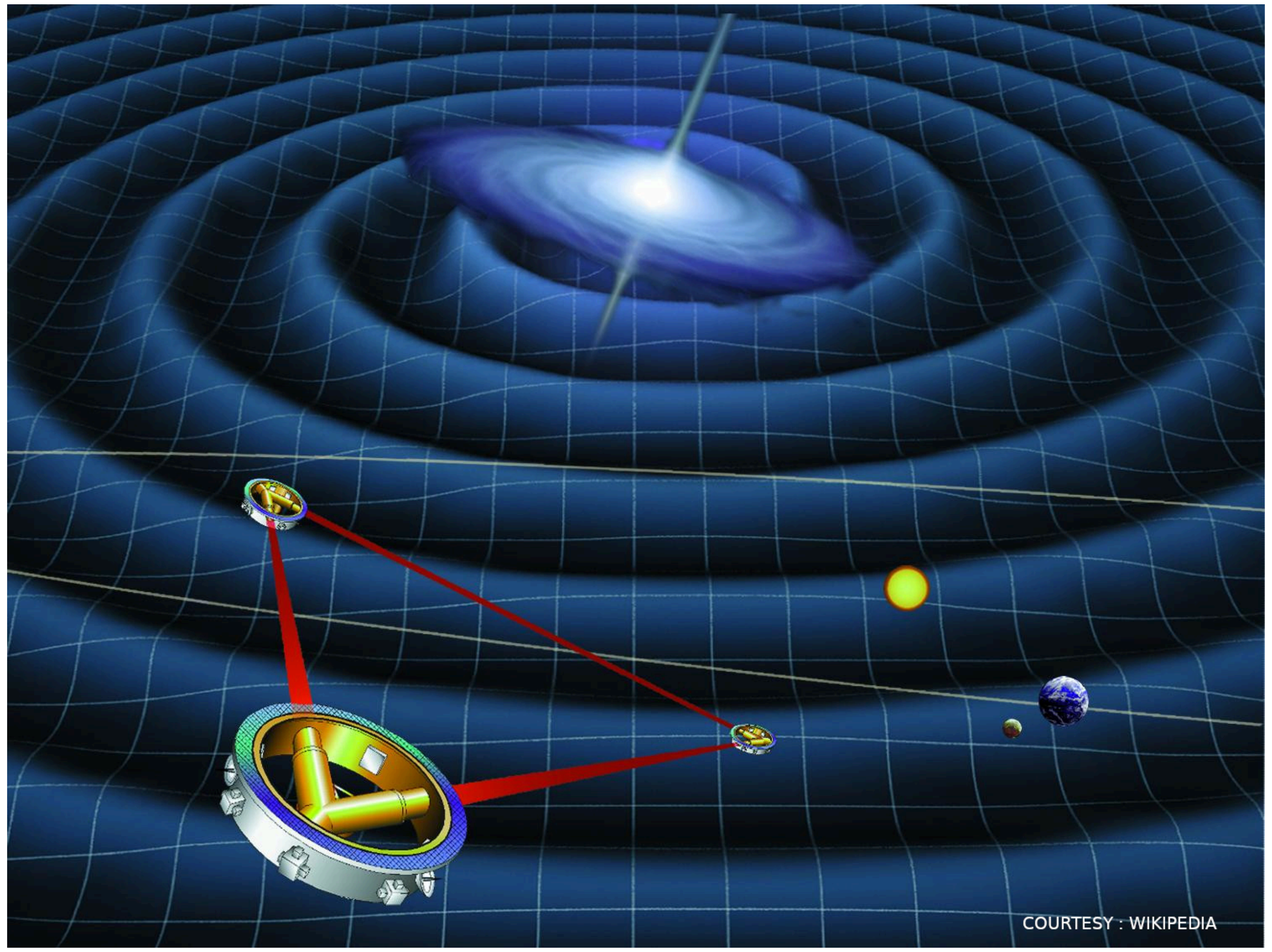
- Current GBDs are being upgraded.
- New detectors Cosmic Explorer, Einstein telescope, and space based LISA is also coming.
- These will be more sensitive detectors.
- This opportunity can be used to test GR.
- Also the nature of the compact objects.
- Exotic compact object (ECO), quantum effects near BH.



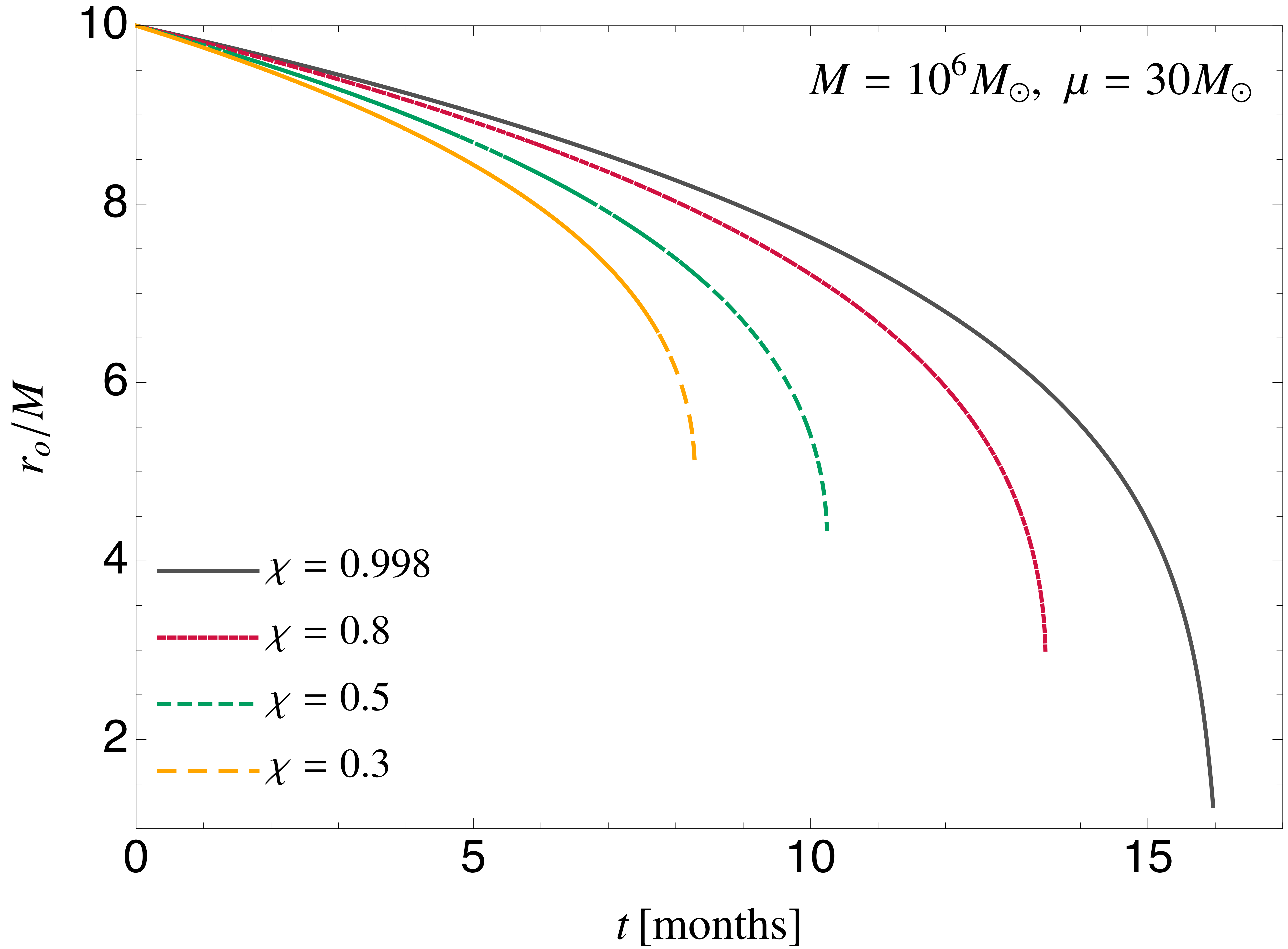


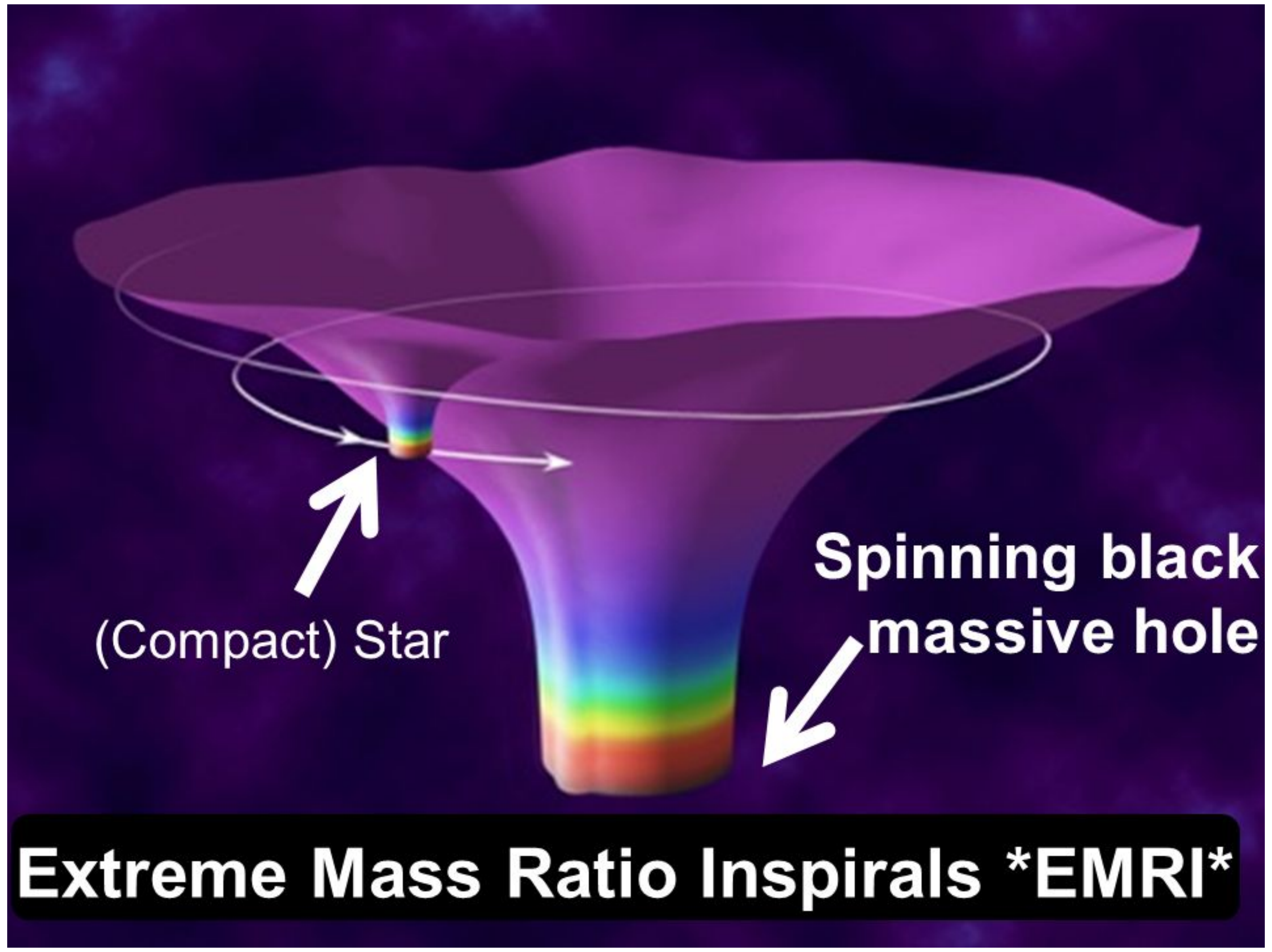
- Center of a galaxy can host SMBH of mass $M \sim 10^6 - 10^7 M_\odot$.
- Stellar mass stars, BHs get captured in inspiral around such SMBHs.
- Mass ratio $\leq 10^{-4}$.
- Frequency of EMRI $\frac{c^3}{50MG} \leq f \leq \frac{c^3}{MG}$.
- For $M \sim 10^6 M_\odot$, $.004Hz \leq f \leq .2Hz$.
- Perfect for LISA$(10^{-4} - .1)Hz$





- We will focus on EMRI first, where a stellar mass $\sim 10 - 100 M_\odot$ Inspirals around SMBH of $\sim 10^5 - 10^7 M_\odot$, observable in LISA.
- Hence we calculate perturbation around BH by a small particle.





- ψ_4 is the perturbation quantity satisfying Teukolsky equation.
- From ψ_4 GW waveform, energy fluxes at infinity and also the flux at horizon can be calculated.

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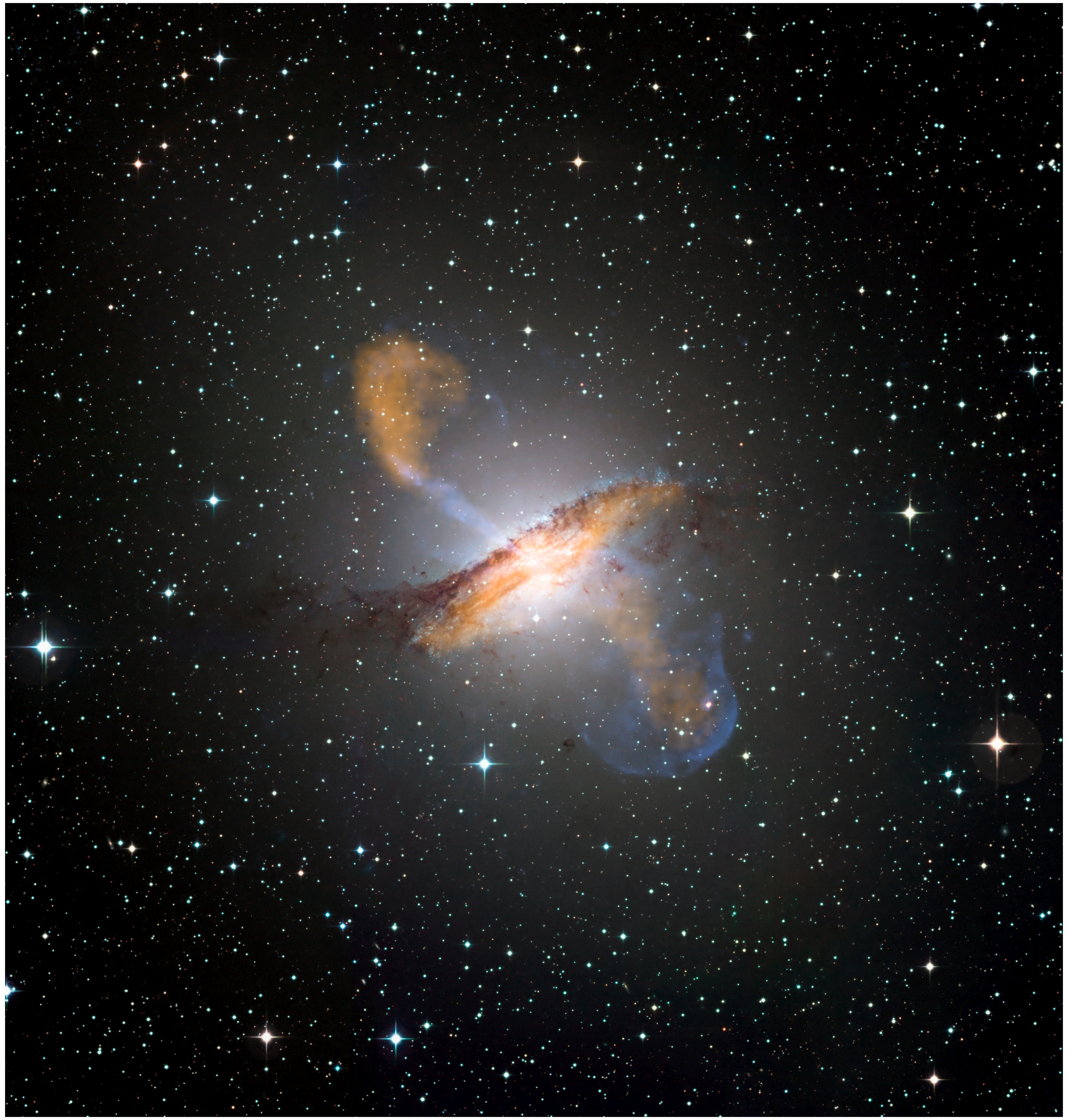

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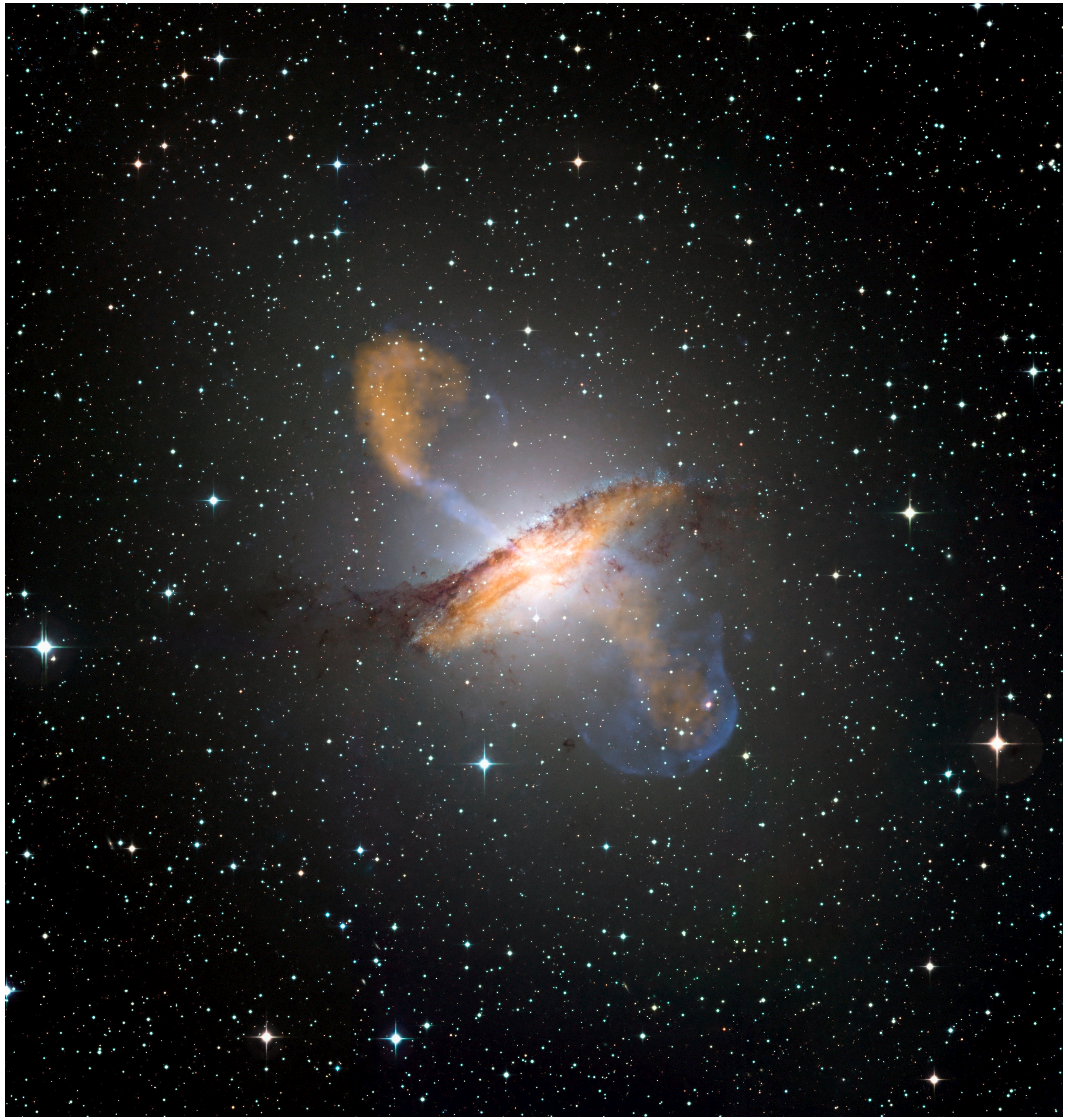
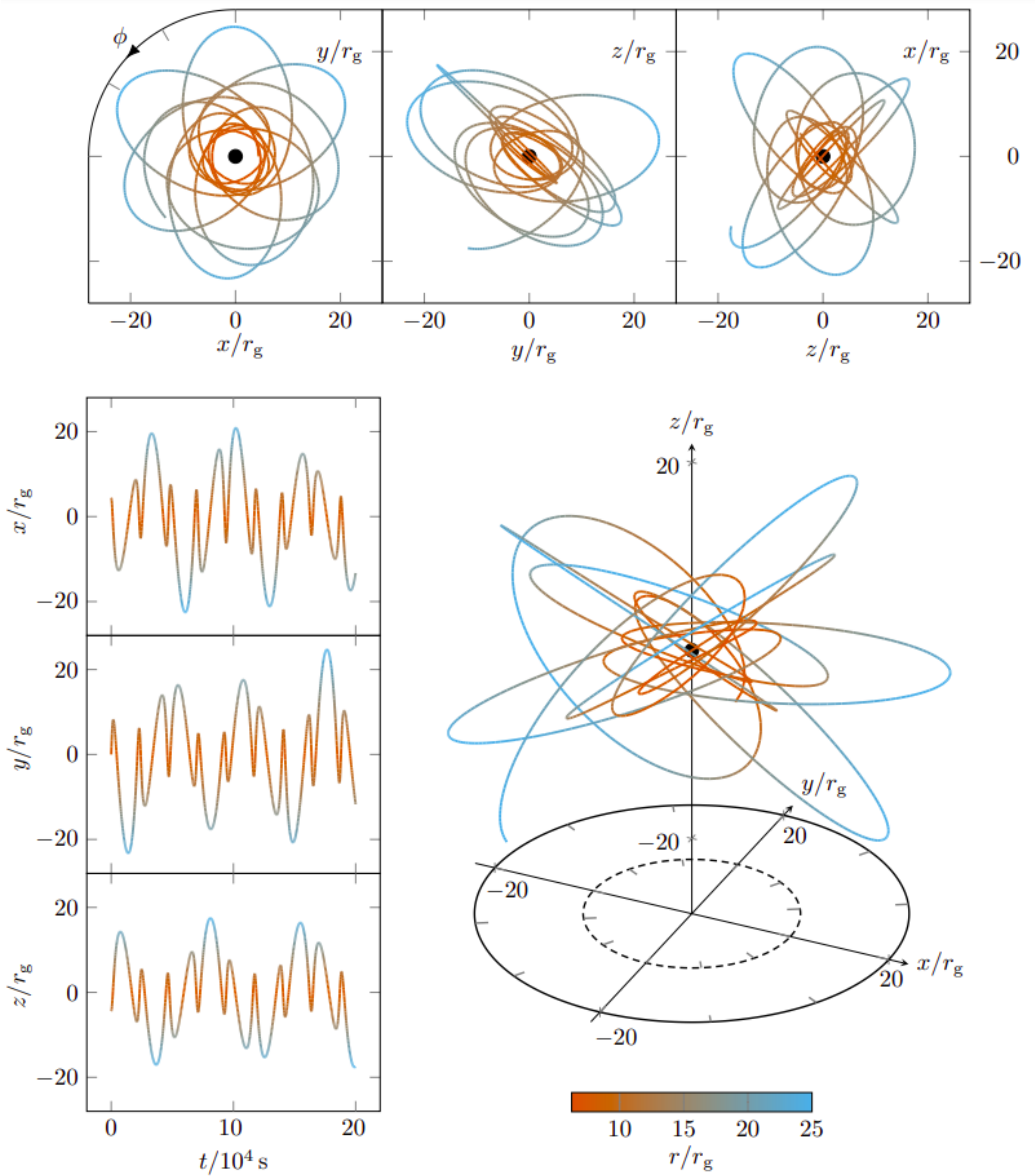


- Do they impact EMRI? How to quantify it?

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

- Presence of matters and fields introduces non-zero energy momentum tensor (EMT).
- EMT changes the geometry.
- This modifies the geodesics.
- As a result, GW emission changes.
- This requires a systematic computation of environmental effects (EEs).

- In non rotating case some progress was done. Cardoso + 2021, 2022, Rahman 2023, Speeney 2024
- Computed flux with DM distributions in circular, eccentric orbit.



- Mihaylov and Gair [2017], Berry+ [2019]

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- In perturbation, $\delta g_{\mu\nu}$ induces due to point particle.
- $\delta g_{\mu\nu} =$ Axial + Polar, due to Parity $\theta \to \pi - \theta, \phi \to \phi + \pi$.
- Axial case reduces to, $\left(d_x^2 - V(x) + \omega^2\right)\psi = S_{PP}$
- $V = V_{BH} + something.$

- For BH even polar case reduces to $\left(d_x^2 - V_P(x) + \omega^2\right)\Psi = \bar{S}_{PP}$
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- Results in solving several coupled equation.

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- We need a simpler approach for polar.
- Which can be added to existing vacuum result.

Multi parameter expansion

- We use 2 perturbative parameters. Brito+ PRD 108 (2023) 8, 084019
- Mass ratio $= q = m_2/m_1 \ll 1$, environment coupling $= \epsilon$.

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- Mass ratio $= q = m_2/m_1 \ll 1$, environment coupling $= \epsilon$.
- $T_{\mu\nu} = T^e_{\mu\nu} + T^p_{\mu\nu}$
- $g_{\mu\nu} = g^{(0,0)}_{\mu\nu} + \epsilon g^{(0,1)}_{\mu\nu} + q g^{(1,0)}_{\mu\nu} + q\epsilon g^{(1,1)}_{\mu\nu}$
- $T^e_{\mu\nu} = \epsilon T^{e\ (0,1)}_{\mu\nu} + q\epsilon T^{e\ (1,1)}_{\mu\nu}$
- $T^p_{\mu\nu} = q T^{p\ (1,0)}_{\mu\nu} + q\epsilon T^{p\ (1,1)}_{\mu\nu}$, with$(i,j) \equiv \mathcal{O}(q^i, \epsilon^j)$.
- (0,0) is Vacuum. (0,1) metric changed due to static environment. (1,0) is Vacumm EMRI. (1,1) is EMRI-EE coupling.

$\mathcal{O}(0,1)$

- $T^{e\mu}{}_{\nu}{}^{(0,1)} = \mathrm{diag}(-\rho^{(0,1)}, p_r^{(0,1)}, p_t^{(0,1)}, p_t^{(0,1)})\,,$
- $g_{\mu\nu}^{(0,1)} = \mathrm{diag}\left(-fH, \frac{2}{rf^2}m\,, r^2, r^2\sin^2\theta\right),$
- $m' = 4\pi r^2\rho$
- $\frac{r^2f^2}{2}H' = m + 4\pi r^3 f p_r$
- $p_r' = \frac{2}{r}p_t + \frac{(3M-2r)}{r^2f}p_r - \frac{M}{r^2f}\rho$

$\mathcal{O}(1,0)+\mathcal{O}(1,1)$

- $\delta g_{\mu\nu} = g^{(1,0)}_{\mu\nu} + g^{(1,1)}_{\mu\nu}\,,\ \ \delta g_{\mu\nu}(x^\alpha) = g^{A}_{\mu\nu}(x^\alpha) + g^{P}_{\mu\nu}(x^\alpha),$
- $\delta g^{A}_{\mu\nu} \sim i\, h_{1,\ell m}(t,r)\ \mathbf{c}_{\ell m}(\theta,\phi) - \ h_{0,\ell m}(t,r)\ \mathbf{c}^{0}{}_{\ell m}(\theta,\phi)$
- $\delta g^{P}_{\mu\nu} \sim -\, g_{tt}\, H_{0,\ell m}(t,r)\ -i\sqrt{2}\ H_{1,\ell m}(t,r)\ + g_{rr}\ H_{2,\ell m}(t,r)\ +\ K_{\ell m}(t,r)$
- $\rho = \rho(r) + \ \rho^{(1,1)}(t,r,\theta,\phi)$
- $\rho^{(1,1)} \sim \rho^{(1,1)}_{\ell m}(t,r) Y_{\ell m}(\theta,\phi)$
- $p^{(1,1)}_{r,\ell m} = c^2_{r,\ell m}(r)\ \rho^{(1,1)}_{\ell m}$, and same for tangential pressure. SD PRD 109 (2024) 10, 104042

Velocity perturbation

- $u^{t(1,0)} \sim H^{(1,0)}_{0,\ell m}(t,r)$
- $u^{r(1,0)} \sim W^{(1,0)}_{\ell m}(t,r)$,
- $u^{\theta(1,0)} \sim \left[V^{(1,0)}_{\ell m}(t,r)\ \partial_\theta - \ U^{(1,0)}_{\ell m}(t,r)\ \csc\theta\partial_\phi\right] Y_{\ell m}(\theta,\phi)$
- $u^{\phi(1,0)}$ depends on same functions.

How to choose r_* : example

- $-\partial_t^2\Phi + \mathscr{F}\partial_r\left(\mathscr{F}\partial_r\Phi\right) - \mathscr{F}V\Phi = 0$
- where $\mathscr{F} = \sqrt{AB} = \left(1 - \frac{r_h}{r}\right)Z(r).$
- $\phi = \sqrt{Z}\Phi, \qquad Z(r) = [1 + \delta Z(r)]$ Cardoso+ 2019, Hatsuda+ 2024
- $-(1+2\delta Z)\frac{\partial^2\phi}{\partial t^2} + f\frac{d}{dr}\left(f\frac{d\phi}{dr}\right) - f\tilde{V}\phi = 0$

Axial

- $\bar{\phi}_{\ell m} \sim - h_{1,\ell m}$
- $\delta Z(r) = \dfrac{H(r)}{2} - \dfrac{m(r)}{rf}$
- $[\partial^2_{r_\star} - \partial^2_t - V^A]\ \phi^{(1,0)}_{\ell m}(t,r) = S^{A(1,0)}_{\ell m}(t,r),$
- $[\partial^2_{r_\star} - \partial^2_t - V^A]\ \phi^{(1,1)}_{\ell m}(t,r) = S^{A(1,1)}_{\ell m}(t,r),$
- For frequency domain, $\partial_t \to - i\omega$
- Flux $\sim \mathscr{F}_V + \sim \phi^{(1,0)}_{\ell m} \phi^{(1,1)*}_{\ell m} + \ldots$

Polar

- $\partial_t K_{\ell m}(t,r) = \alpha\, \bar{\chi}_{\ell m}(t,r) + \beta\, \bar{R}_{\ell m}(t,r)$
- $H_{1,\ell m}(t,r) = \gamma\, \bar{\chi}_{\ell m}(t,r) + \delta\, \bar{R}_{\ell m}(t,r)$

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- Scale by δZ.
- $\partial^2_{r_\star} \chi^{(1,0)}_{\ell m} + (V^P - \partial^2_t)\, \chi^{(1,0)}_{\ell m} = S^{P(1,0)}_{\ell m}$
- $\partial^2_{r_\star} \chi^{(1,1)}_{\ell m} + (V^P - \partial^2_t)\, \chi^{(1,1)}_{\ell m} = S^{P(1,1)}_{\ell m} - (z_1 + z_2 f \partial_r)\, V^{(1,0)}_{\ell m} - (z_3 + z_4 f \partial_r)\, W^{(1,0)}_{\ell m}$

Matter perturbation

- $\kappa_t \partial_t V^{(1,0)}_{\ell m} + 4\pi c^2_{t,\ell m} \rho^{(1,1)}_{\ell m} = S^V_{\ell m},$
- $\kappa_r \partial_t W^{(1,0)}_{\ell m} + (w_1 + w_2 f \partial_r) \rho^{(1,1)}_{\ell m} = S^W_{\ell m},$
- $(\partial^2_{r_\star} - c^{-2}_{r,\ell m} \partial^2_t + V^\rho + \gamma_1 \partial_{r_\star}) \rho^{(1,1)}_{\ell m} = S^\rho_{\ell m}$
- For frequency domain, $\partial_t \to -i\omega$
- Flux $\sim \mathcal{F}_V + \sim \chi^{(1,0)}_{\ell m} \chi^{(1,1)*}_{\ell m} + \dots$

Take home

- We consider generic matter configuration.
- Multiparameter approach provides wave equation for Polar sector too.
- 2- sets of equations with same operator but different sources in each sector.
- Flux modifies.