General features of energy extraction from black holes through charged particle production

Filip Heida

Centro de Astrofísica e Gravitação, Departamento de Física, Instituto Superior Técnico, Universidade de Lisboa, Portugal CEICO, Institute of Physics of the Czech Academy of Sciences, Prague, Czech Republic

> based on work in progress expanding and reinventing the results of PhysRevD.105.024014













Penrose process and its improvements

- Penrose process requires high relative velocity of the fragments: J. Bardeen, W. Press, S. Teukolsky, Astrophys. J. **178**, 347 (1972).
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- BSW effect for extremal Kerr seems ideal collisions with arbitrarily high centre-of-mass energy with particles coming from rest at infinity: M. Bañados, J. Silk, S. M. West, PRL 103, 111102 (2009).
- However, strict upper bounds on the extracted energy were found: T. Harada, H. Nemoto, U. Miyamoto, PRD 86, 024027 (2012). J. D. Schnittman, PRL 113, 261102 (2014).

Back to electrovacuum

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Outline of the setup

• First-order EOM for a test particle with mass m in an axially symmetric, stationary spacetime can be written using function X

$$\frac{\mathrm{d}\,t}{\mathrm{d}\lambda} \equiv p^t = \frac{X}{N^2} \qquad \qquad \frac{\mathrm{d}\,r}{\mathrm{d}\lambda} \equiv p^r = \frac{\sigma}{N\sqrt{g_{rr}}}\sqrt{X^2 - N^2\left(m^2 + \frac{p_\varphi^2}{g_{\varphi\varphi}}\right)}$$

- Function X has the physical meaning of "locally measured energy redshifted to spatial infinity": $NE_{LNRF} = X = -p_t - \omega p_{\omega}$
- For Schwarzschild, X is just the conserved energy, i.e. $X \equiv E$; Penrose process is possible only when X has spatial dependence
- $p^r \in \mathbb{R} \Longrightarrow$ function X cannot change sign (assuming $g_{\varphi\varphi} > 0$)

$$|X| \geqslant N\sqrt{m^2 + \frac{p_{\varphi}^2}{g_{\varphi\varphi}}} > 0$$

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• Hence, there must be a turning point between two points with different signs of function X (we selected X > 0 to preserve causality)

- Let us consider a model process in which a collision of two uncharged particles gives rise to two oppositely charged particles
- For uncharged particles, $-p_t \equiv E$ and $p_{\varphi} \equiv L$ are conserved; function X is given by $X = E - \omega L$; only ω spatially dependent
- Provided the asymptotics is healthy (no Melvin!) $|\omega_H| \geqslant |\omega| > 0$
- Considering only particles that can get close to the black hole leads to
- Marginally bound particles $(E \approx m)$ with I (defining L = Im) in a
- Photons (m = 0) with impact parameter b (defining L = bE) in a
- Function X is additive and conserved at the instant of collision:
- Bound on the total value: $X_3^C + X_4^C = X_0 = X_1^C + X_2^C \le f_{in}(E_1, E_2)$
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- Marginally bound particles $(E \approx m)$ with I (defining L = lm) in a certain range $I_{min} < I < I_{max}$ won't get reflected; hence $X \le f_{mb}(m)$
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- For charged particles, $-p_t$ and p_{φ} have spatial dependence, whereas $-\Pi_t = -p_t - qA_{\varphi} = E$ and $\Pi_{\varphi} = p_{\varphi} + qA_{\varphi} = L$ are conserved
- Spatial dependence of X gets a contribution proportional to q: $X = X_{\mathsf{C}} + \Delta X = X_{\mathsf{C}} - \Delta \omega p_{\omega}^{\mathsf{C}} - q \xi$
- For realistic particles, $|q| \gg m$ in geometric units (an electron has $q \approx -2 \cdot 10^{21} m$)
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- N.B. For uncharged initial particles we have bounds on $X_{\mathbb{C}}$ (and $p_{in}^{\mathbb{C}}$)
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- Didn't assume extremality anywhere, generic conclusion
- Limit of infinitesimal coordinate distance from the horizon not suitable for this kind of process (but fine in the uncharged case!)

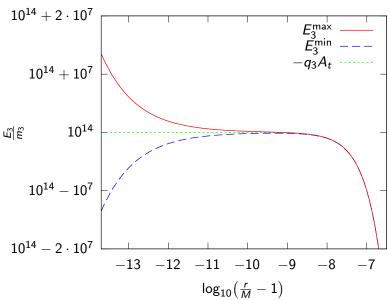
A more detailed look

- Unless unreasonably close to the horizon, production of oppositely charged particles will extract energy. But how much?
- Classical works parametrised the outcome of an event using relative three-velocity, Lorentz factor and the scattering angle
- \bullet An alternative parametrisation using function X avoids the three-velocity: O. B. Zaslavskii, PRD 108, 084022 (2023)
- Can be generalised to the electrovacuum case
- There is a range for $-p_t$, so corresponding range for E_3 is centered (roughly) around $-q_3A_t$; a large value assuming $|Qq|\gg Mm$
- $E_{\rm cm}$ controls width of the range: $E_3^{\rm max} E_3^{\rm min} \approx \sqrt{g_{tt}} E_{\rm cm}$
- For the BSW effect (in the extremal case) $E_{cm}^2 \sim (r_{\rm C} r_{\rm H})^{-1}$
- In the subextremal case, E_{cm} is bounded
- In both cases the width is negligible compared to the mean!
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- N.B. No exotic particles in subextremal case due to $m_3 + m_4 < E_{cm}$ Filip Hejda (CENTRA/CEICO)

extremal Kerr-Newman $Q=0.5\cdot 10^{-7}M$, $q_3=2\cdot 10^{21}m_3$



Conclusions

- The simple model of pair creation close to an electrovacuum black holes shows that it can lead to ejection of highly energetic particles
- Key ingredients are the constraints on the momentum of the centre-of-mass frame, can also work for other (quantum) models
- The general argument holds regardless of extremality; previously
- Thus, the BSW effect does not seem to be an important ingredient,
- Conventional wisdom is that black holes should be largely neutral due
- Stay tuned for a more detailed treatment of the subextremal case

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