

A New model of spontaneous scalarization induced by curvature and matter

Based on : *Phys.Rev.D* 112 (2025) 4, 044015
arXiv : 2506.12137 With S. Mignemi

PhD in physics

Université de Genève
Département de physique théorique
Cosmology Group
Gravitational Wave Science Center

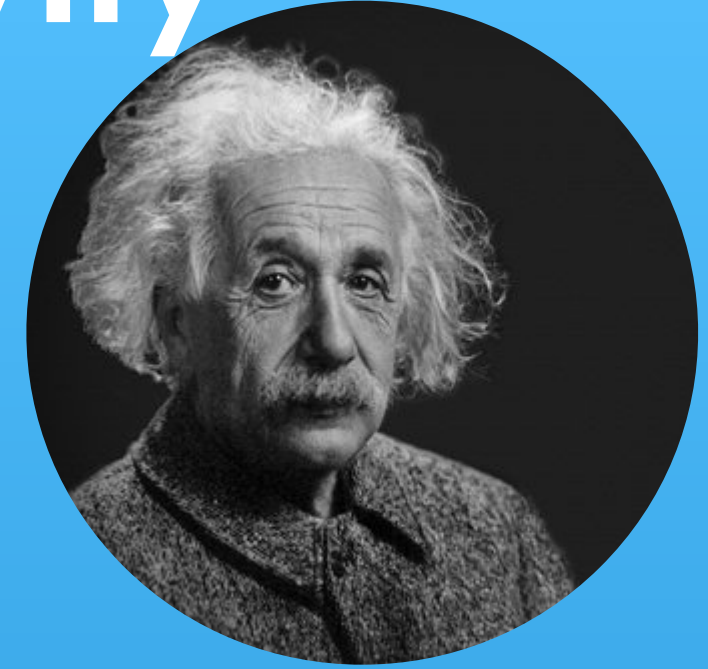


PhD in Mathematics

Università degli Studi di Cagliari
Dipartimento di Matematica e Informatica
INFN Theory of Cagliari

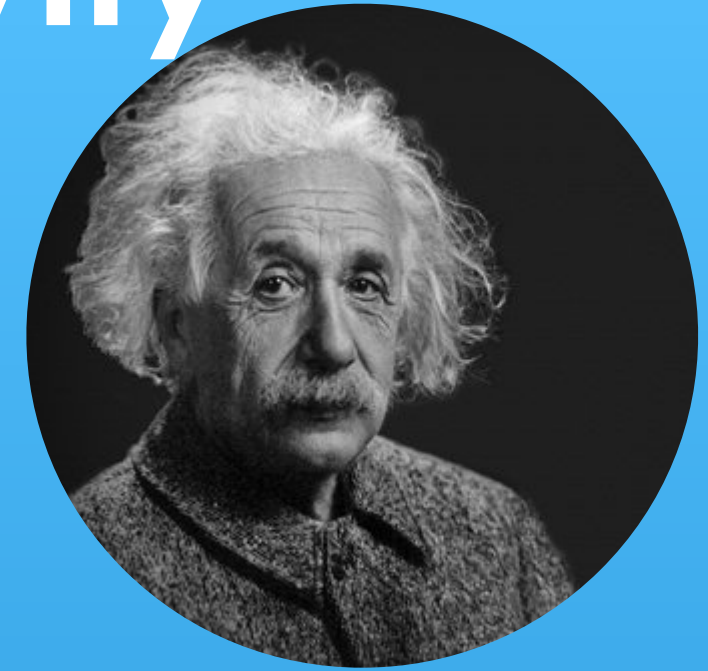


General relativity & Modified gravity

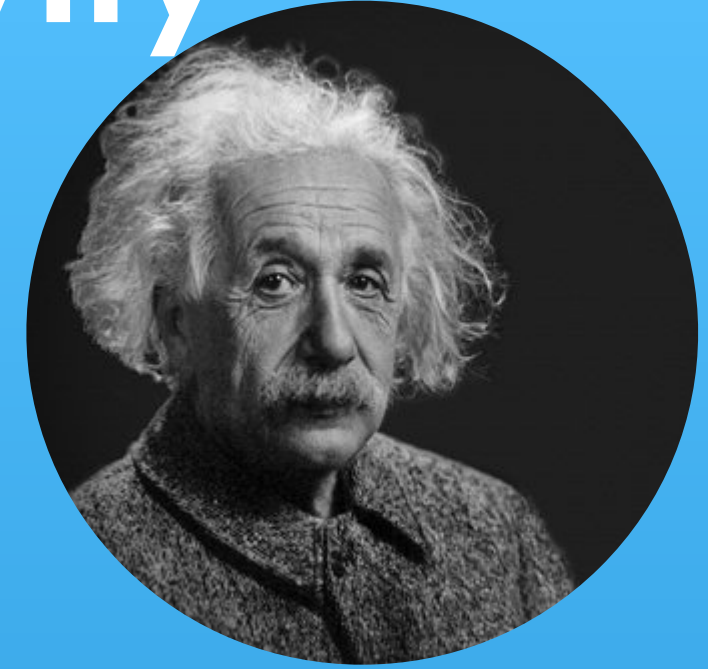


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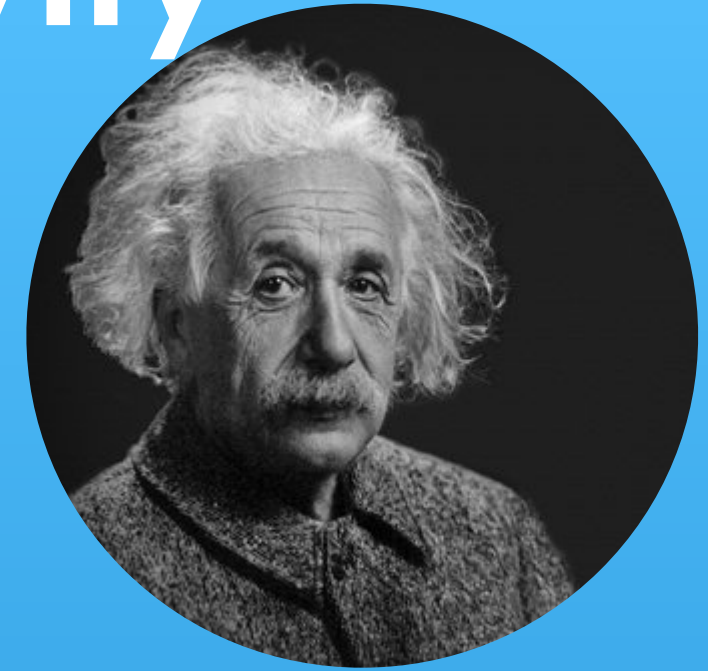


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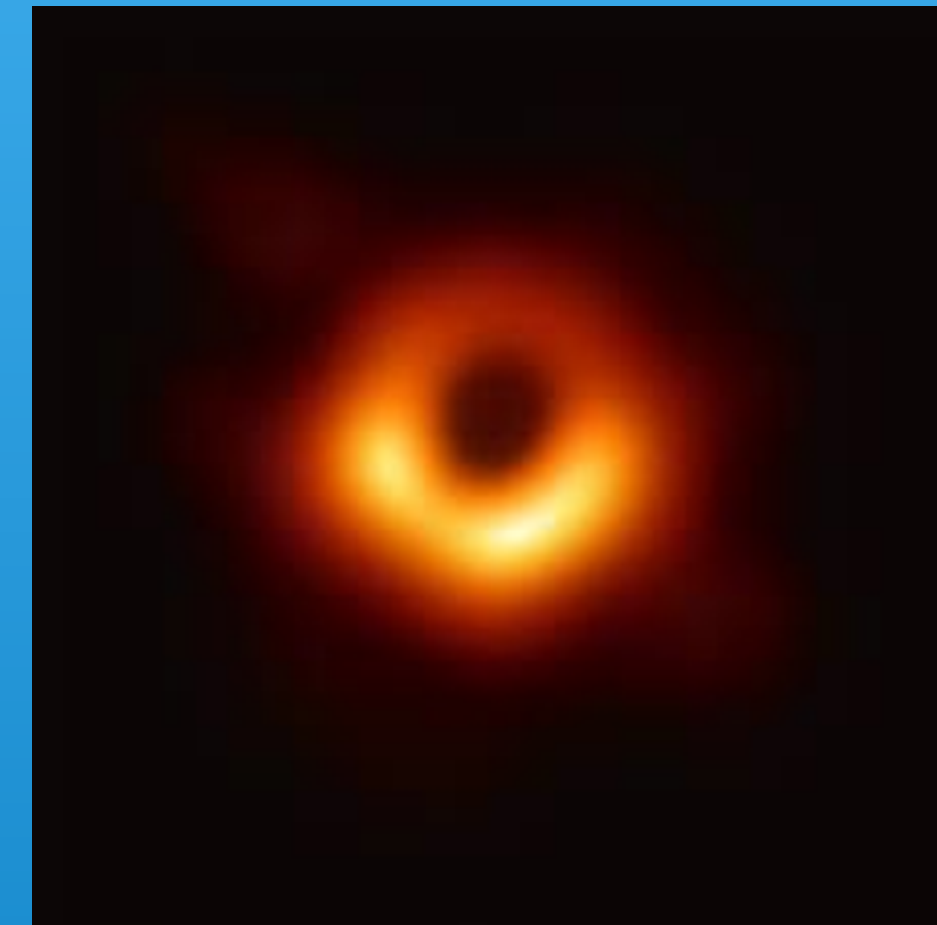
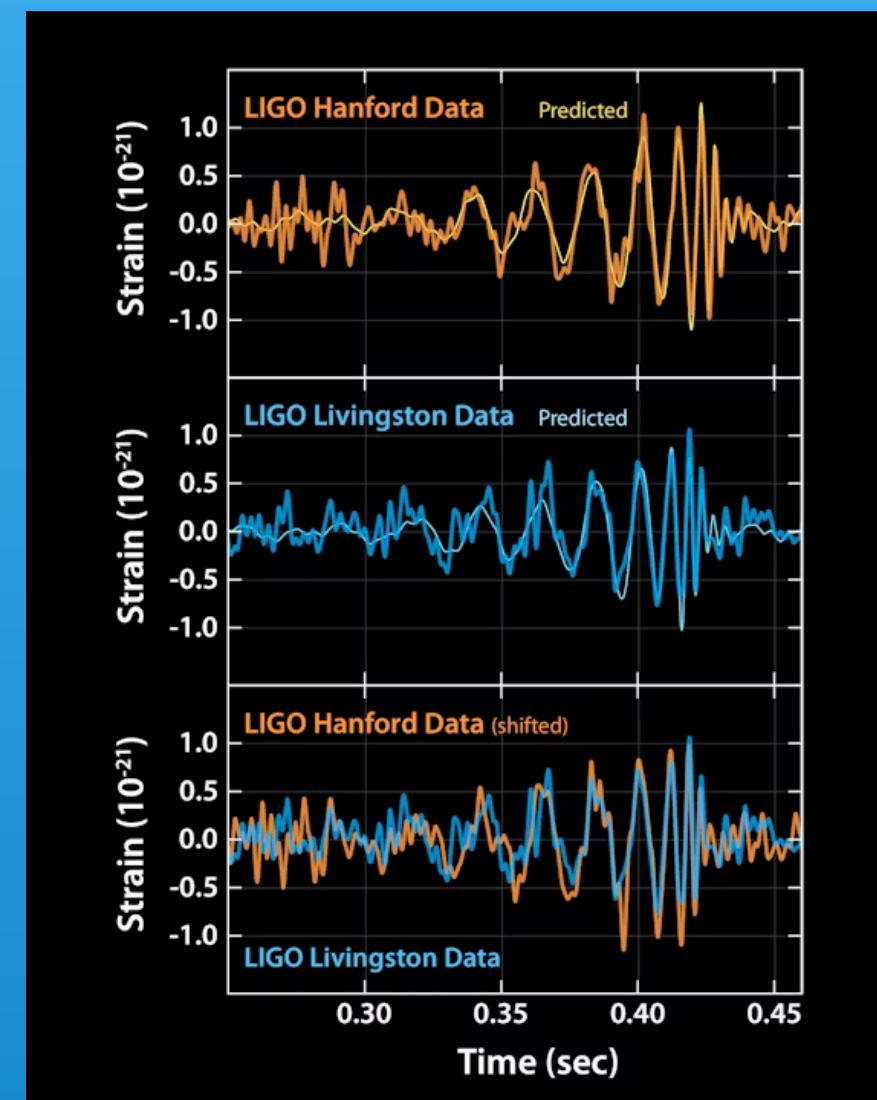


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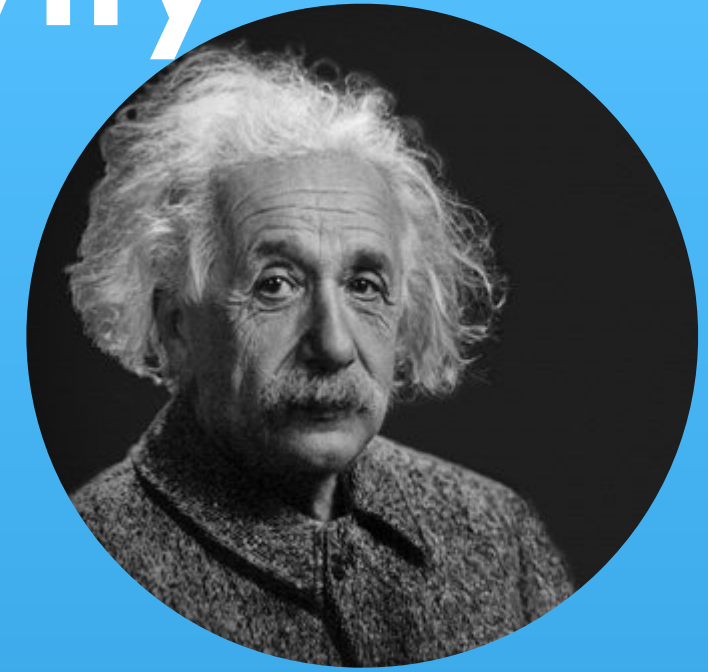
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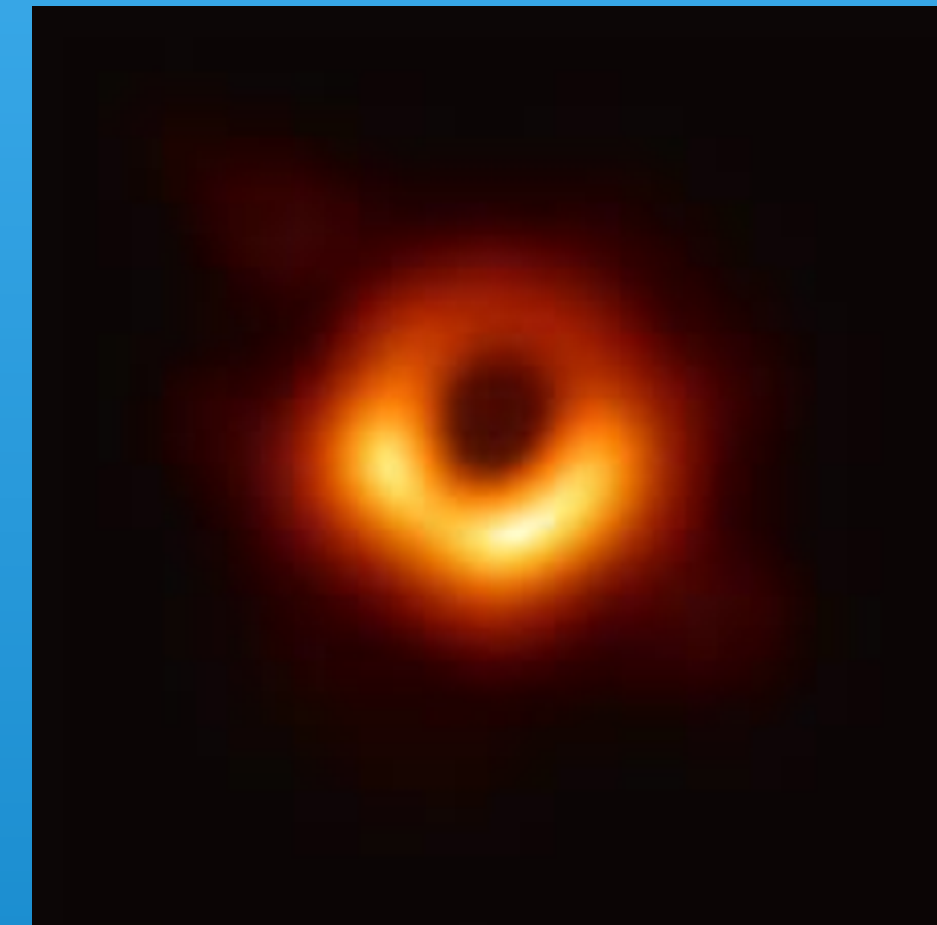
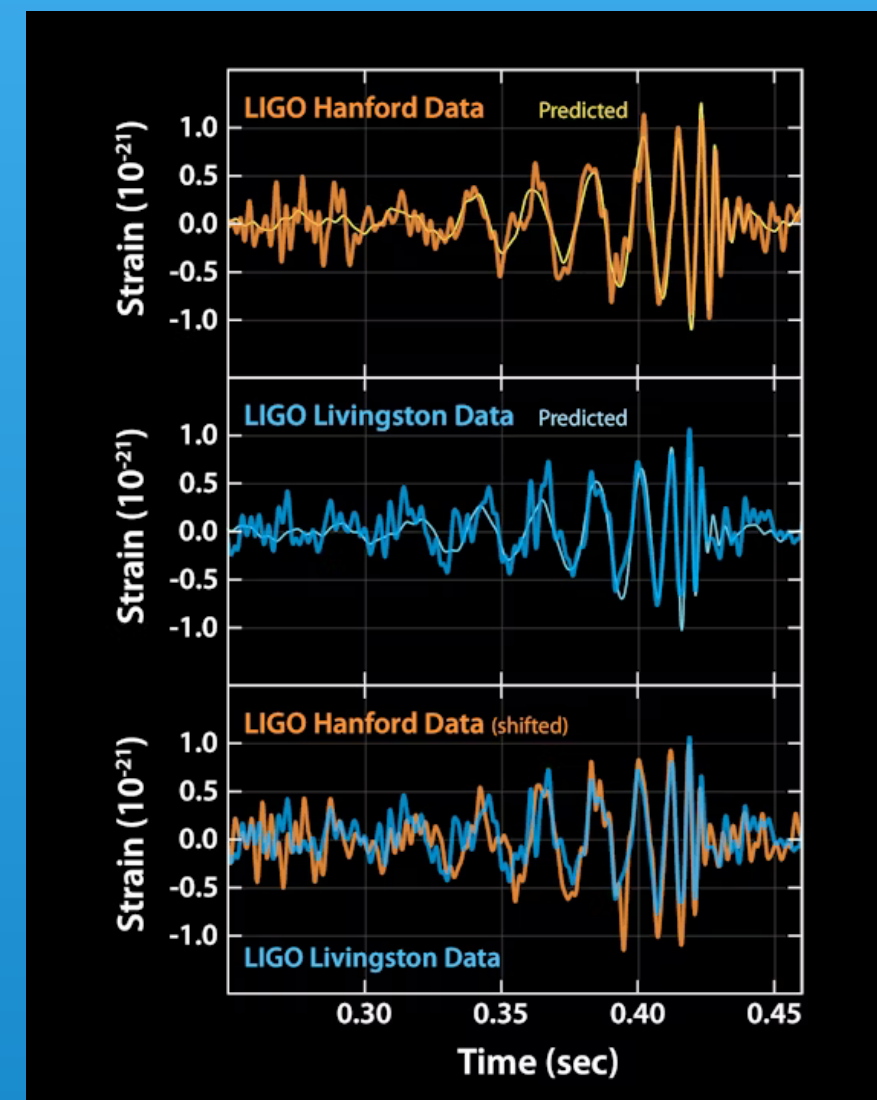


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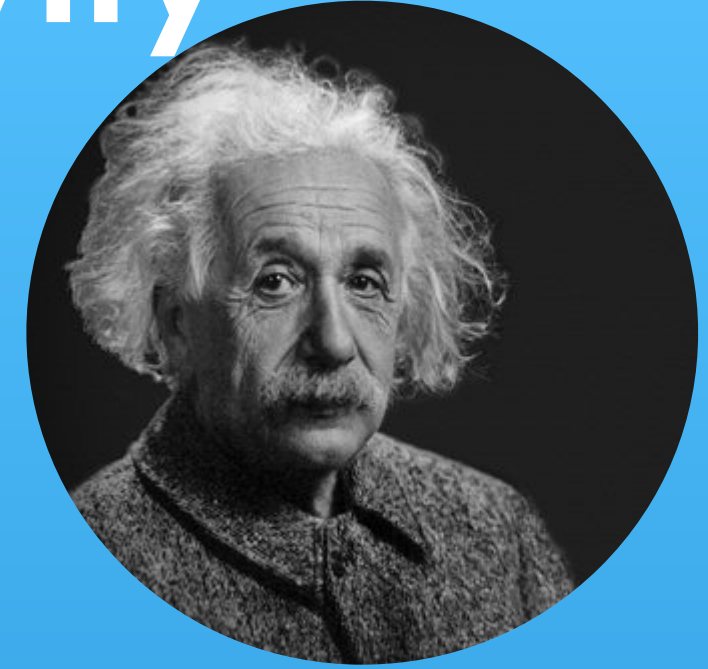


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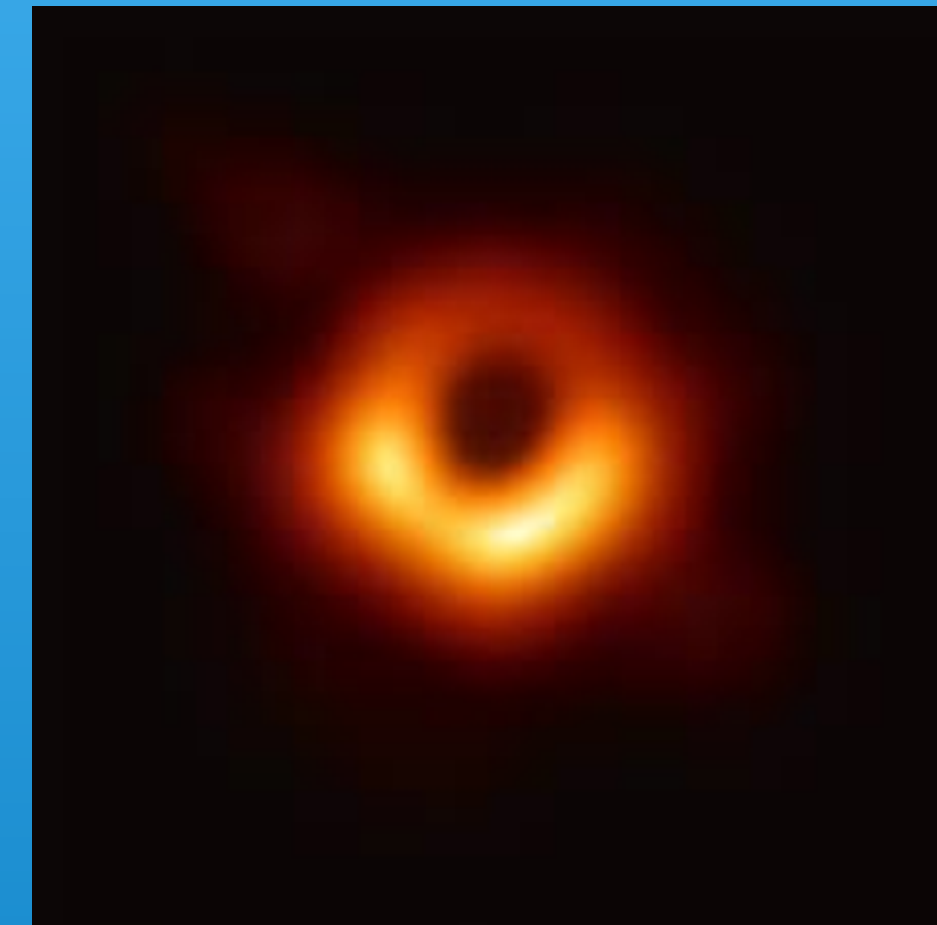
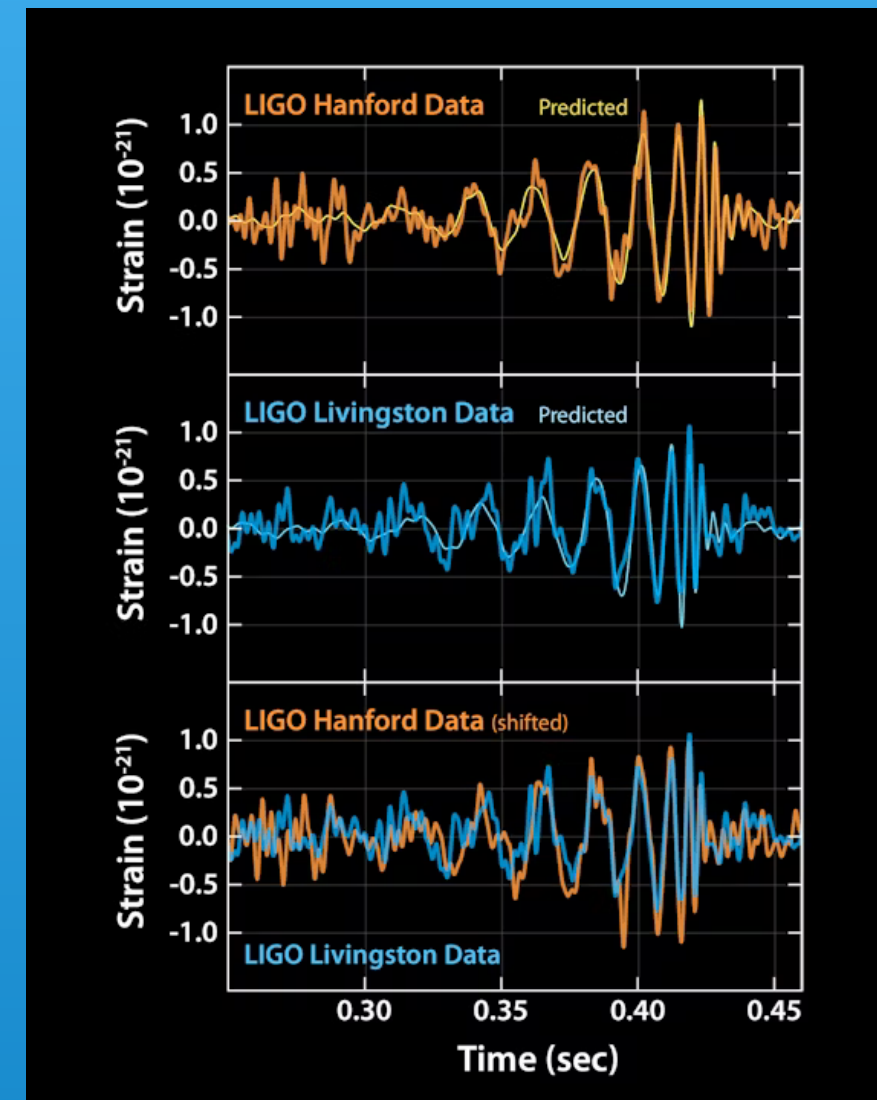


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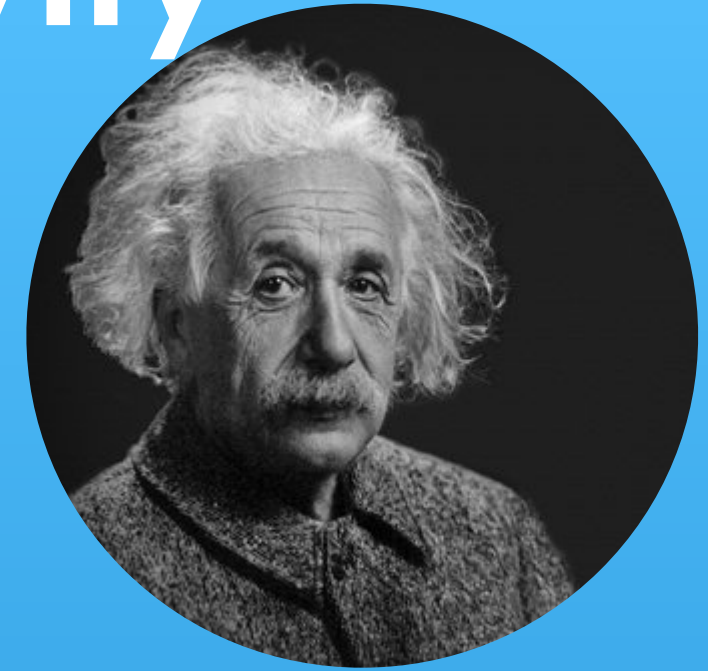
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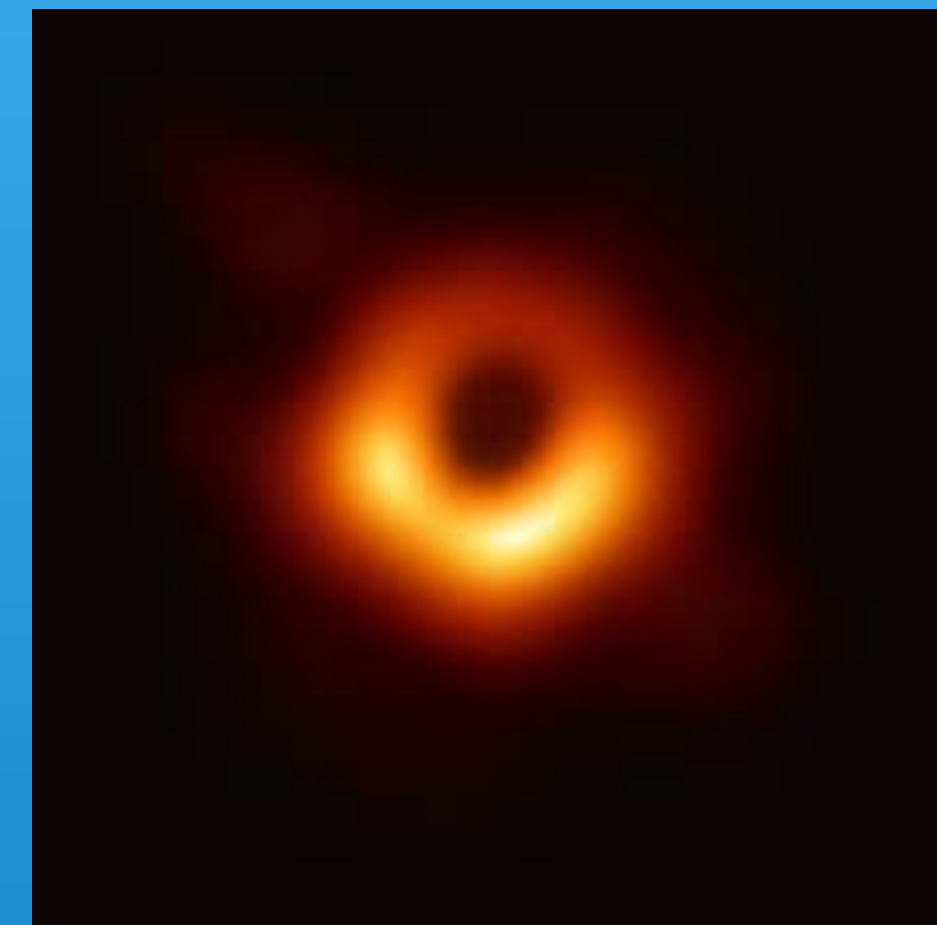
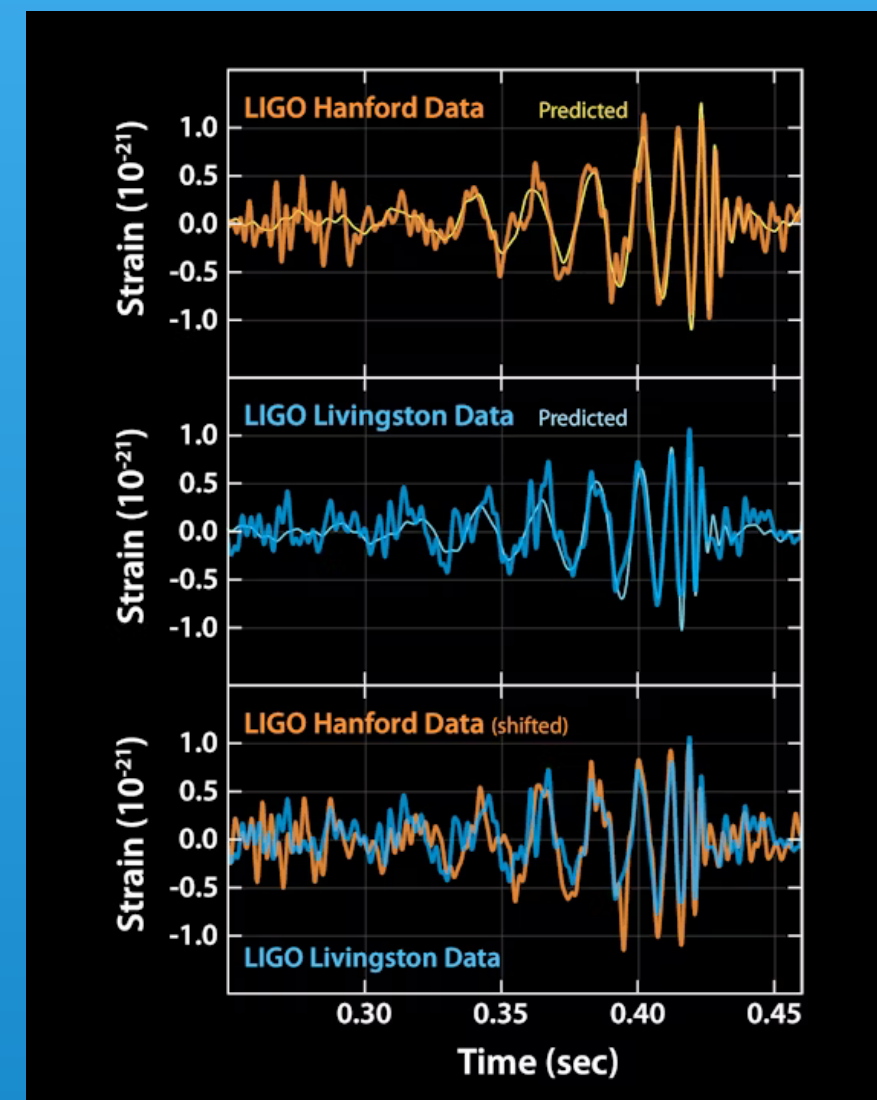
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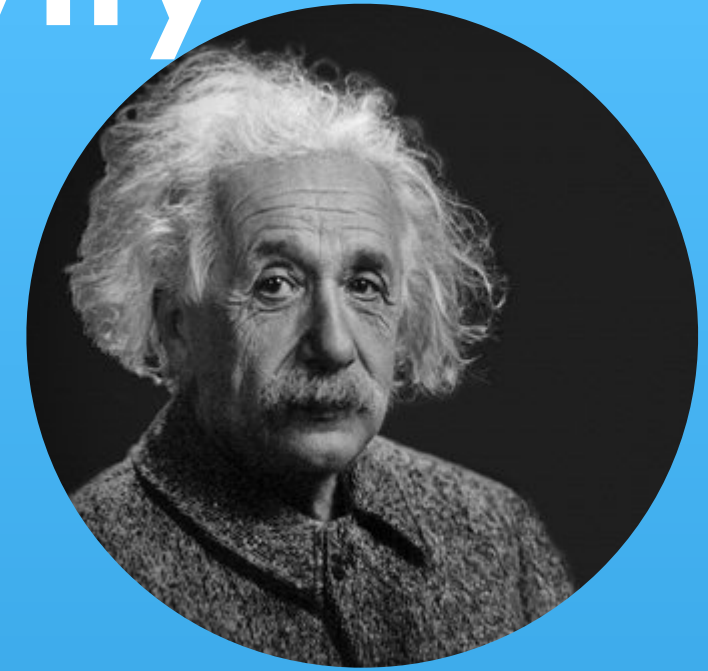
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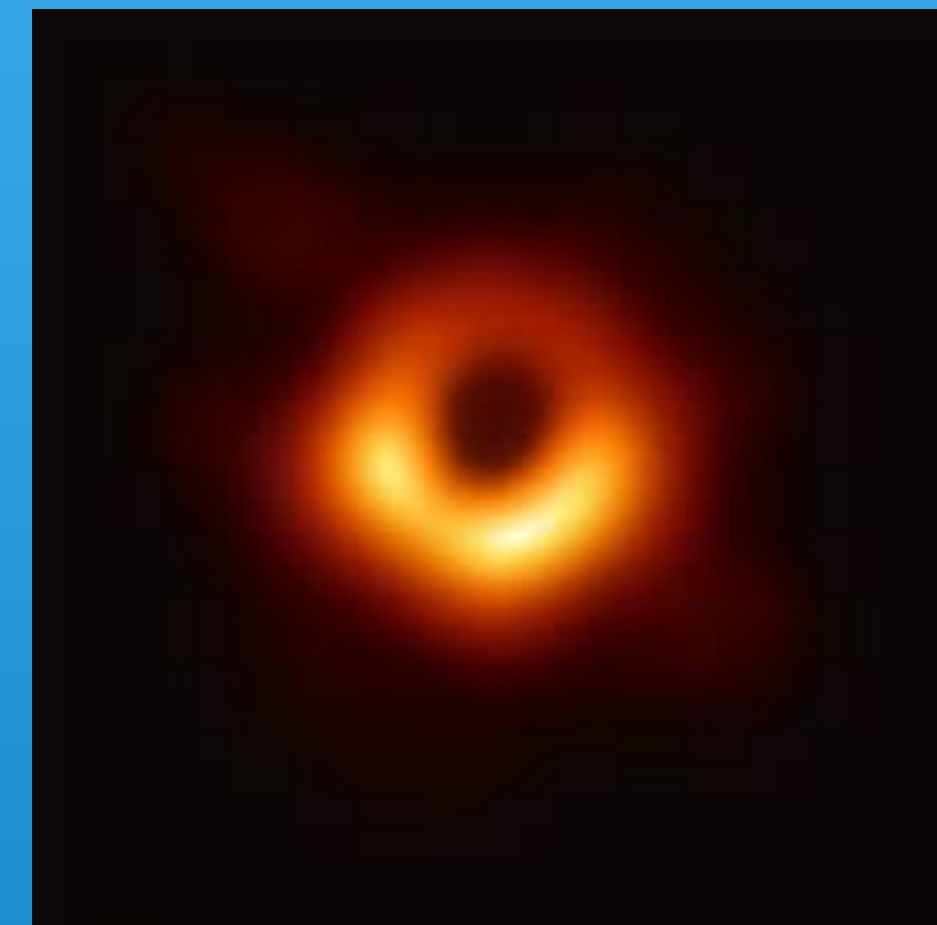
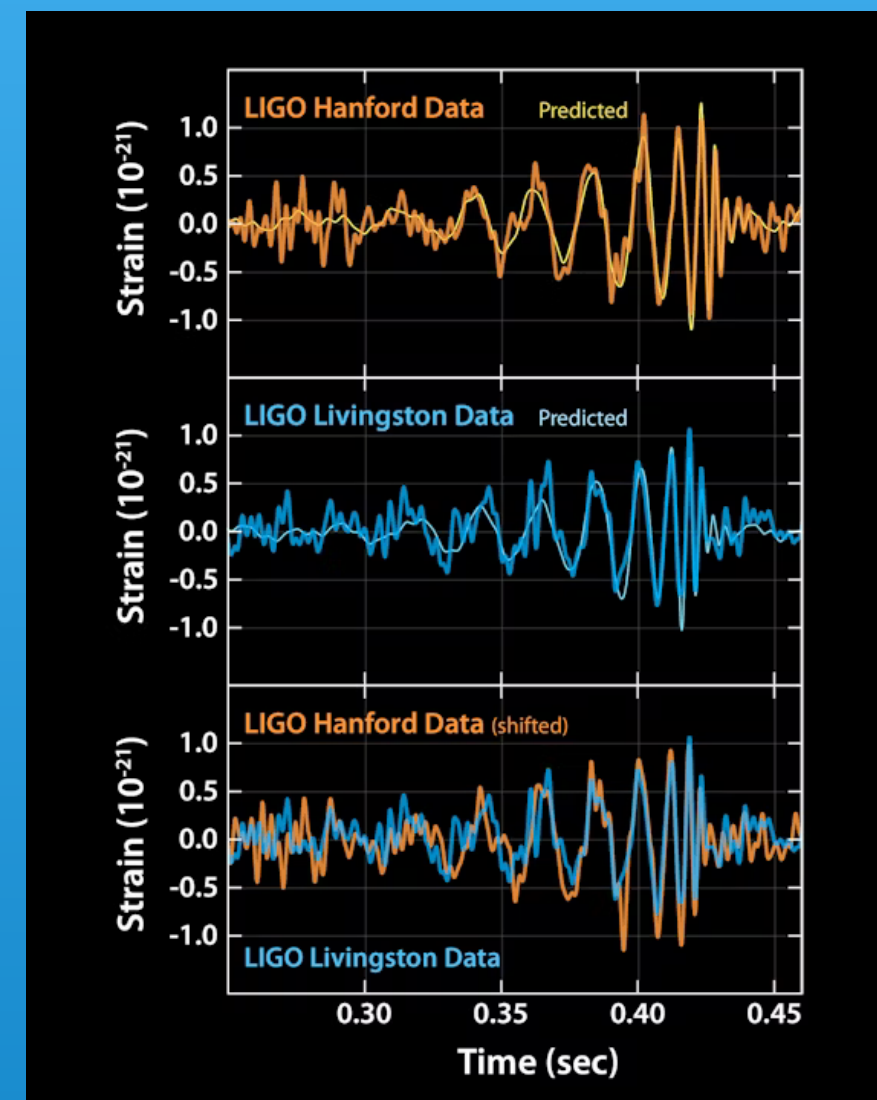
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No-hair conjecture

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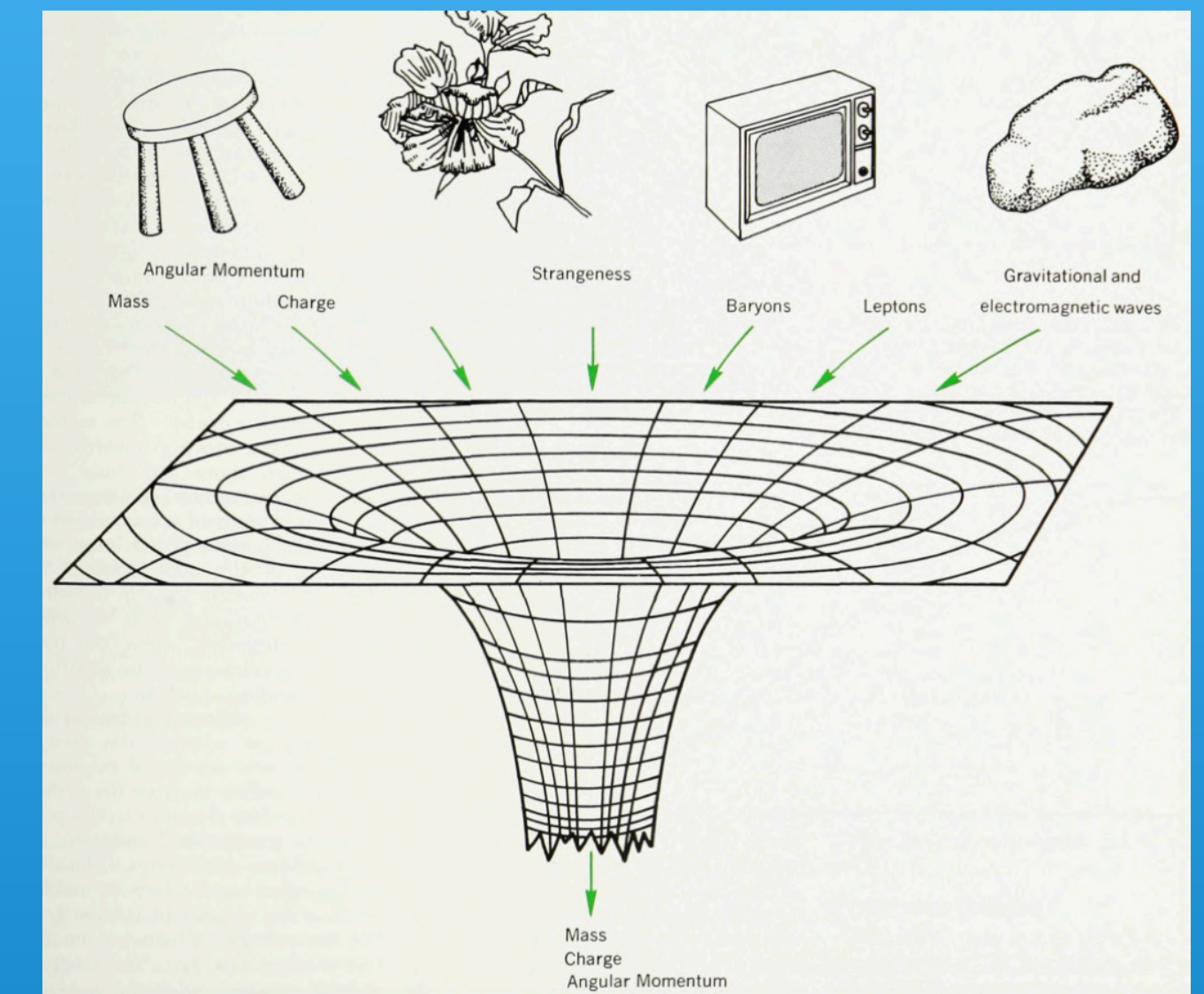
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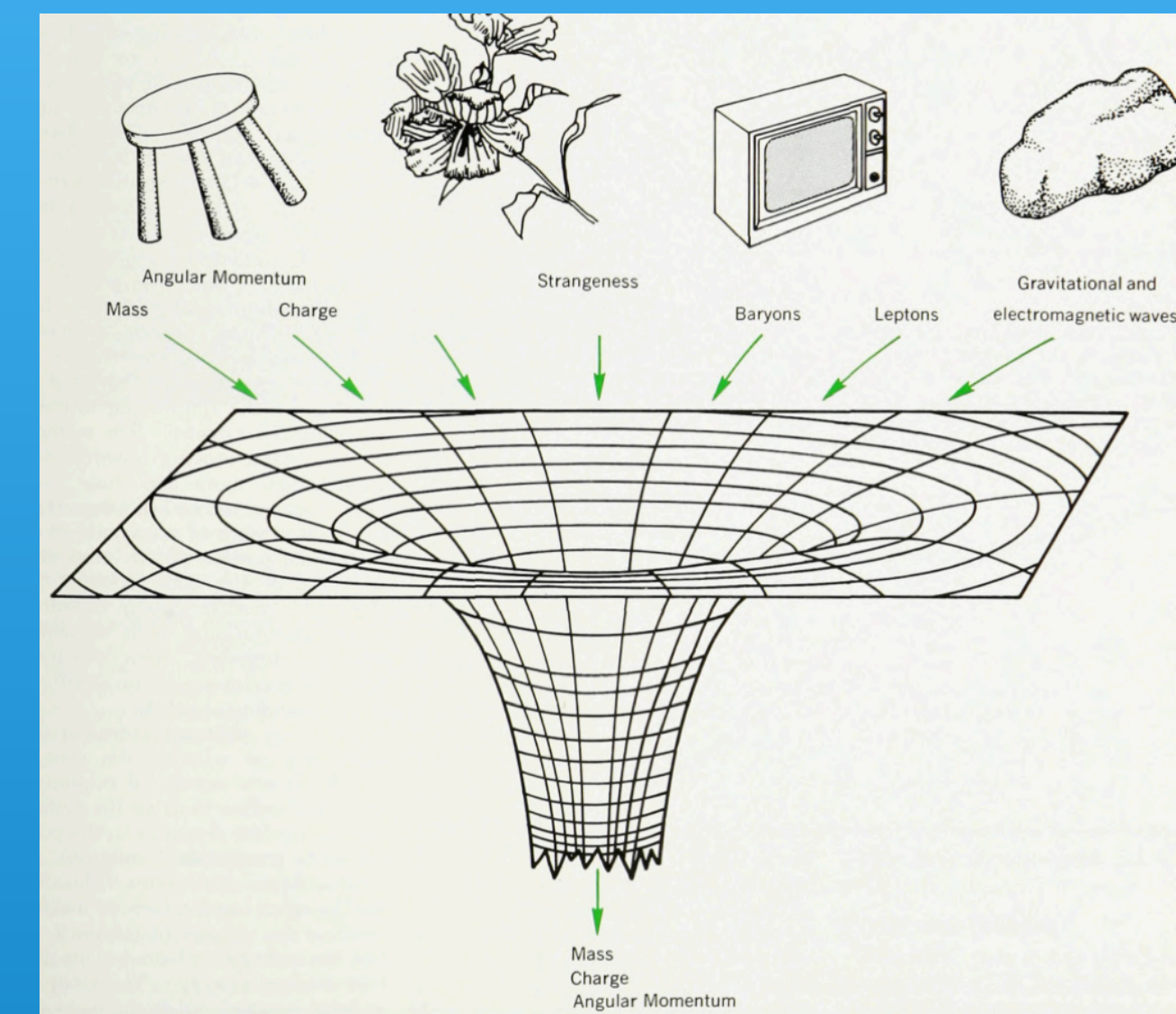


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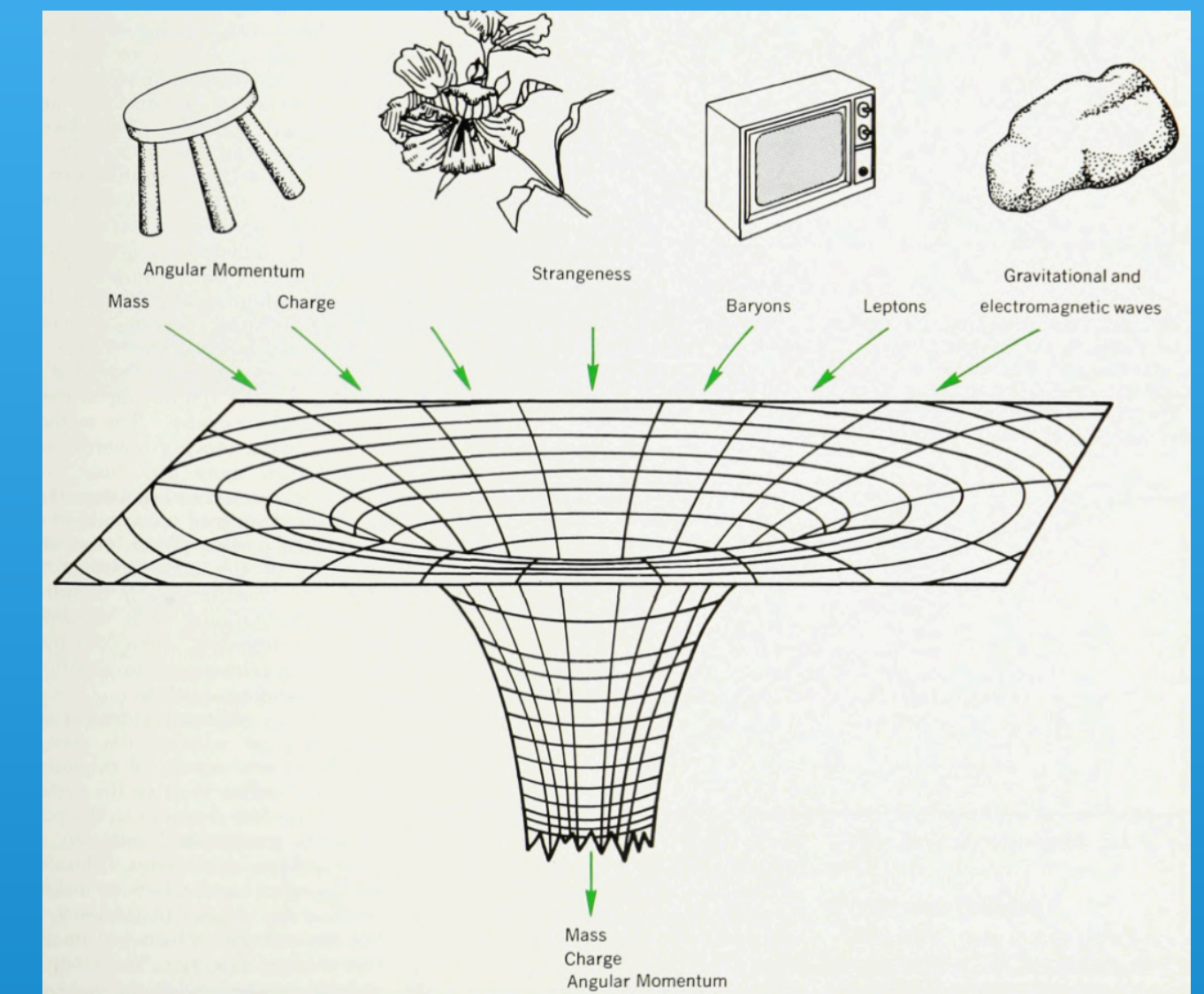
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Hair : is a metaphor of the lack of individuality, or an additional global charge



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Hairy Black Holes: Deviations from the No Hair Theorem, where black holes possess additional charges or "hair."

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This may implies in most of the cases to modify gravity or add some extra matter fields

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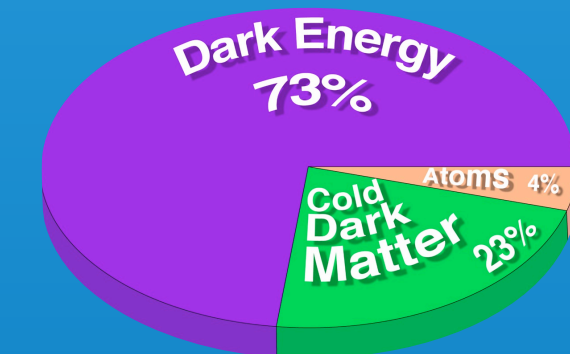
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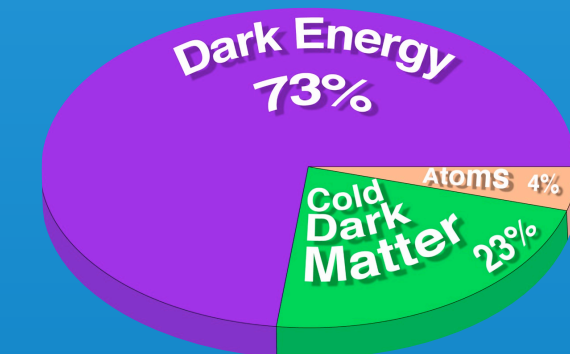


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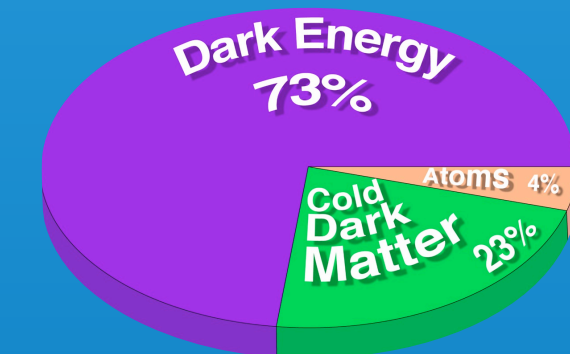


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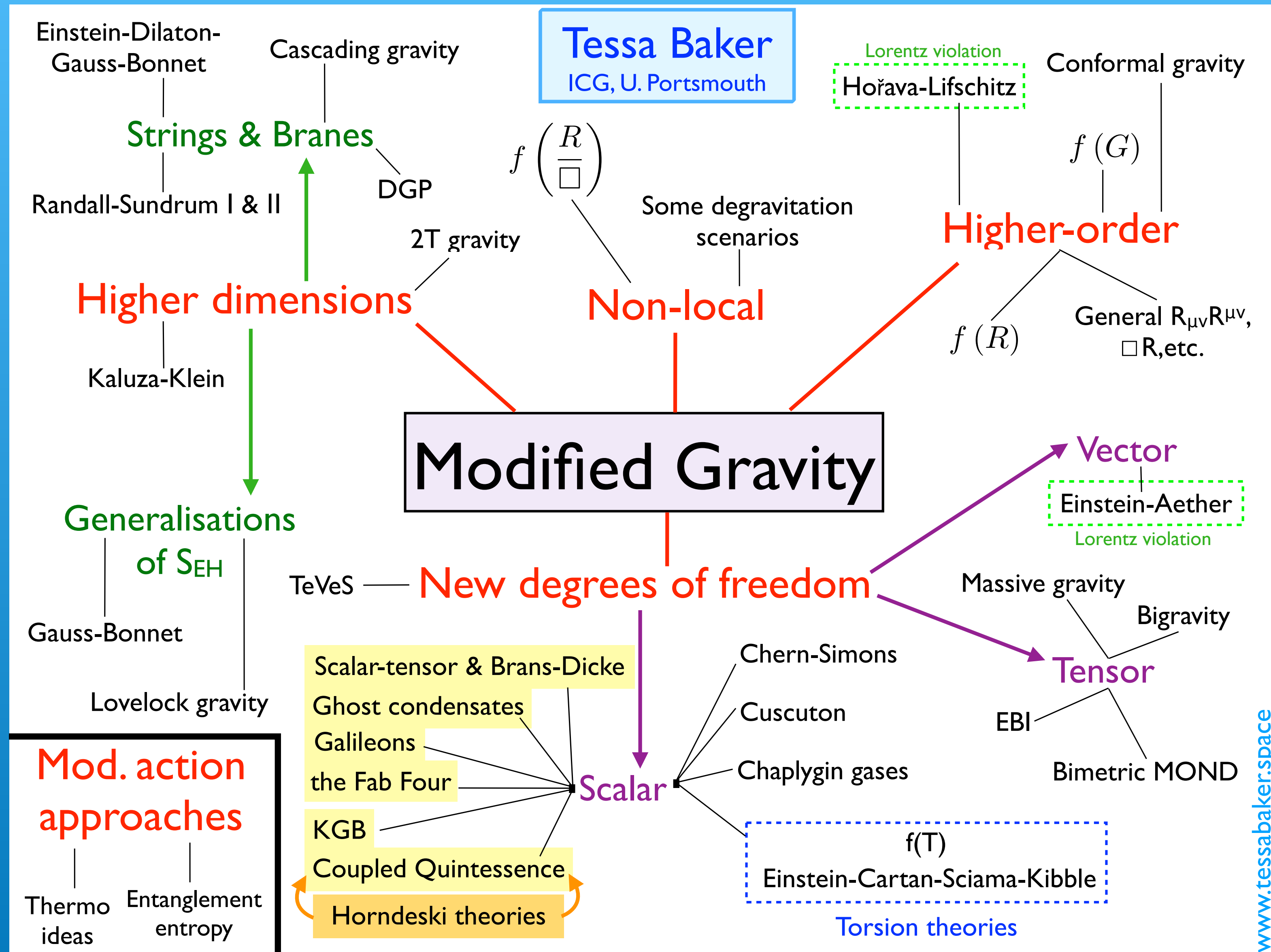
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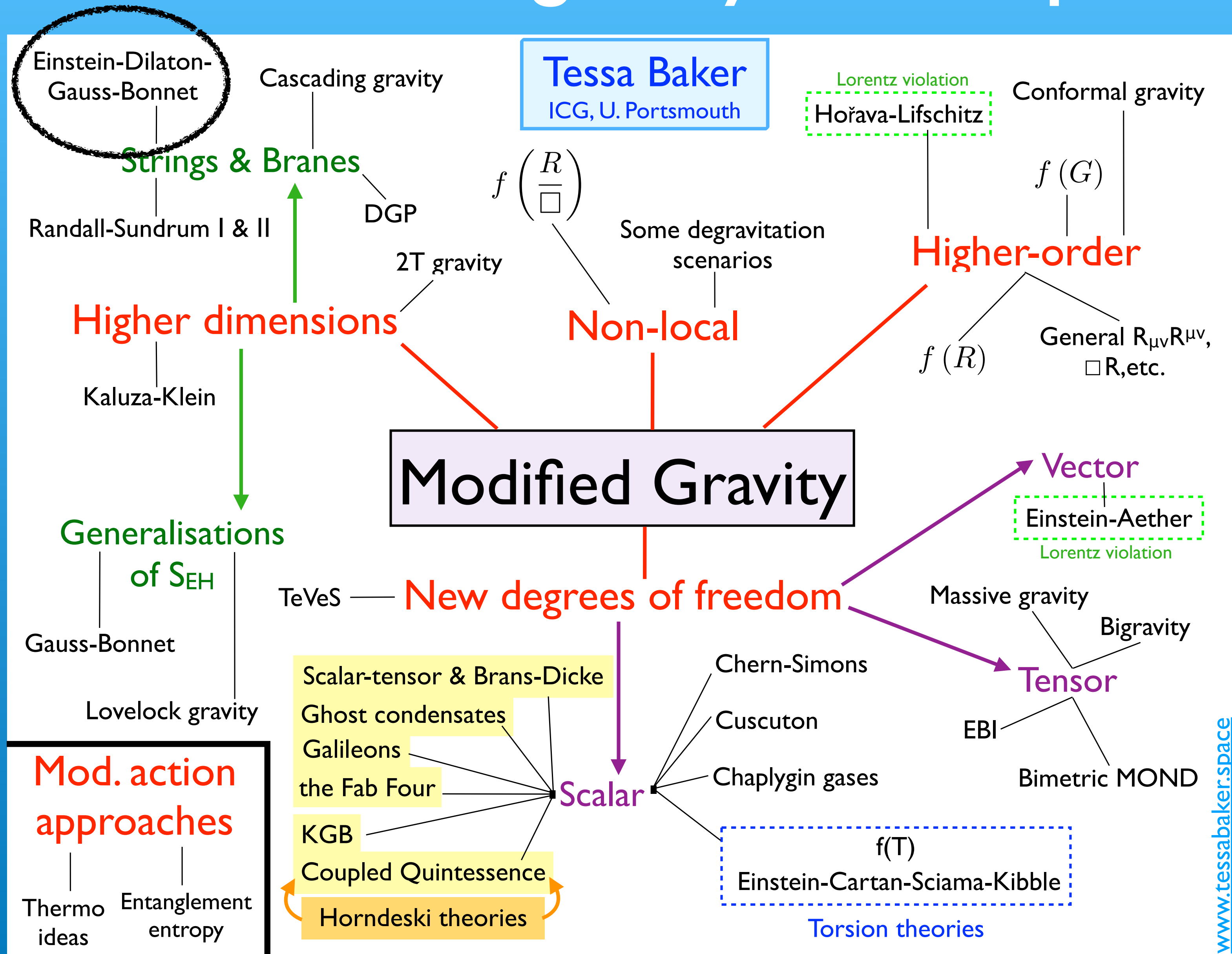
- Renormalizability , singularities, quantum gravity
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- Testing gravity in scales (not yet tested before)
- A better way to understand something is to break/ modify it (challenge GR)



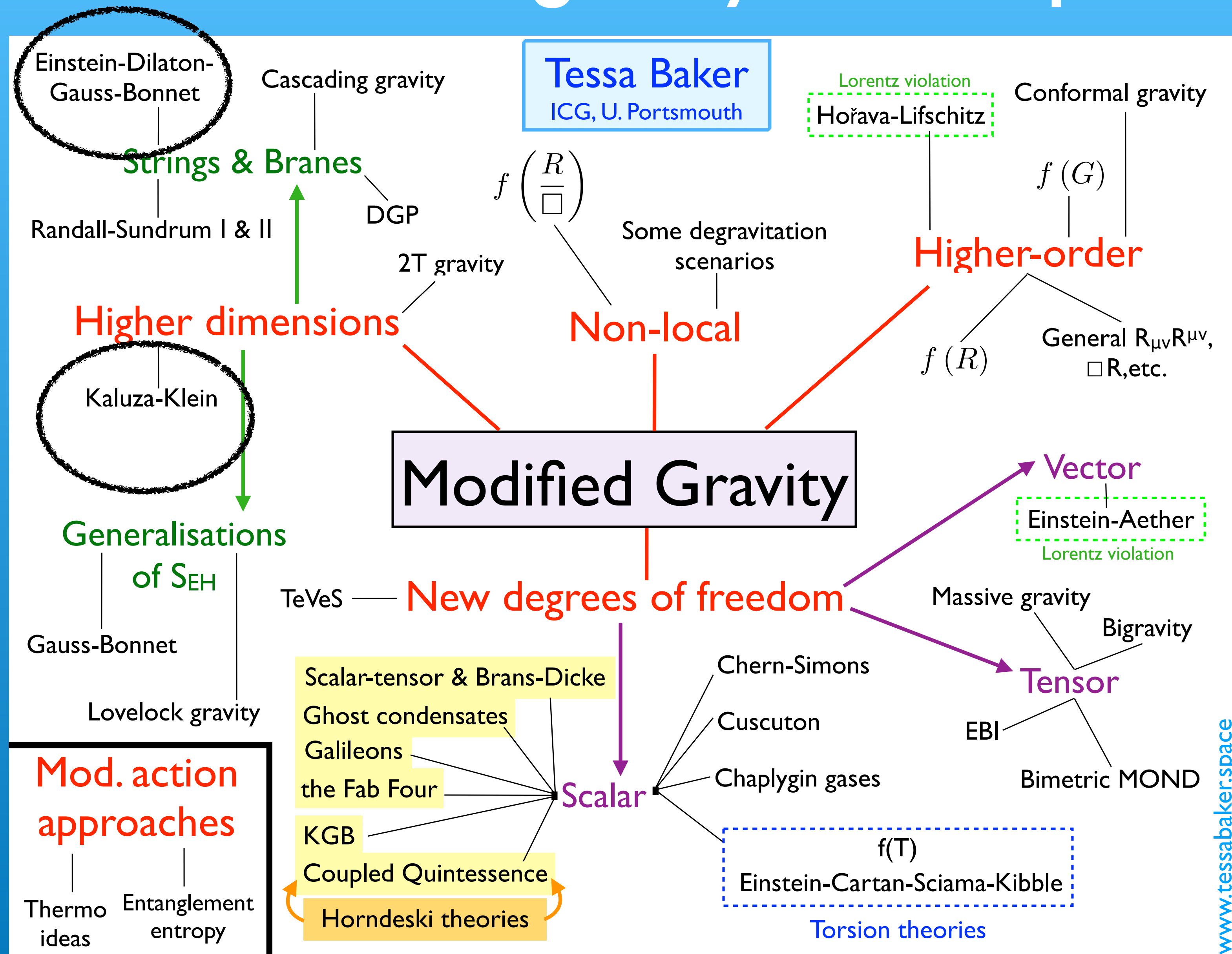
Modified gravity roadmap



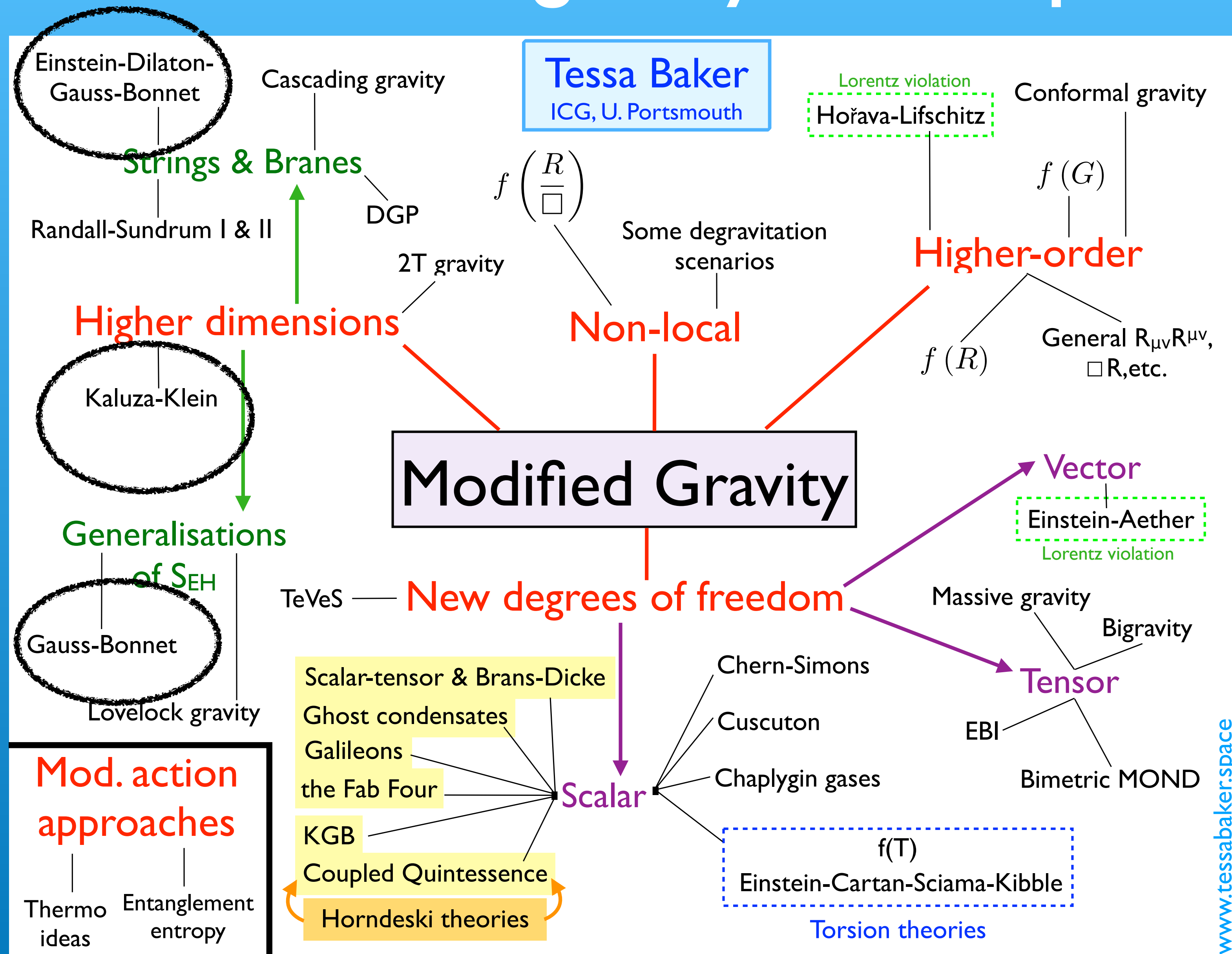
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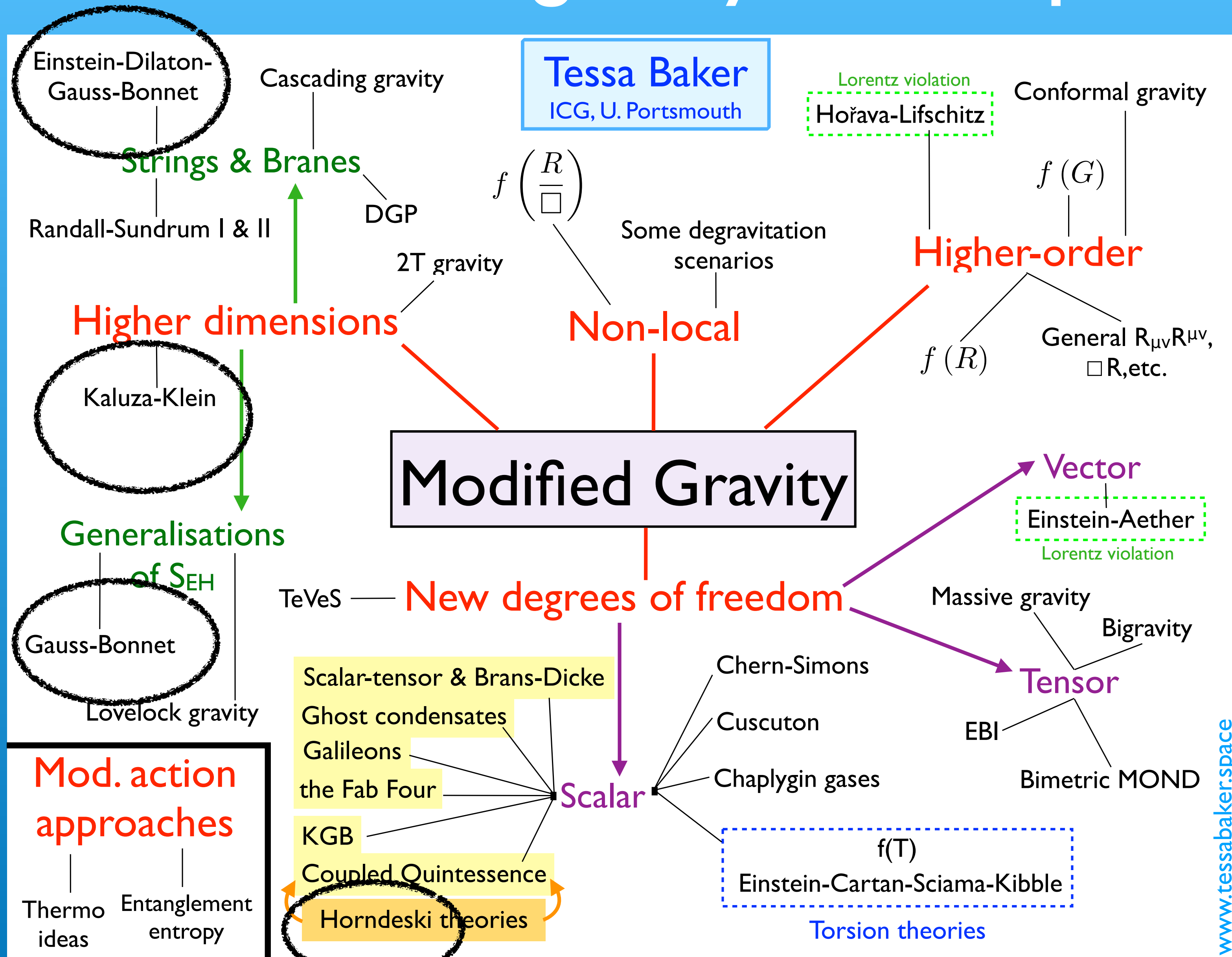
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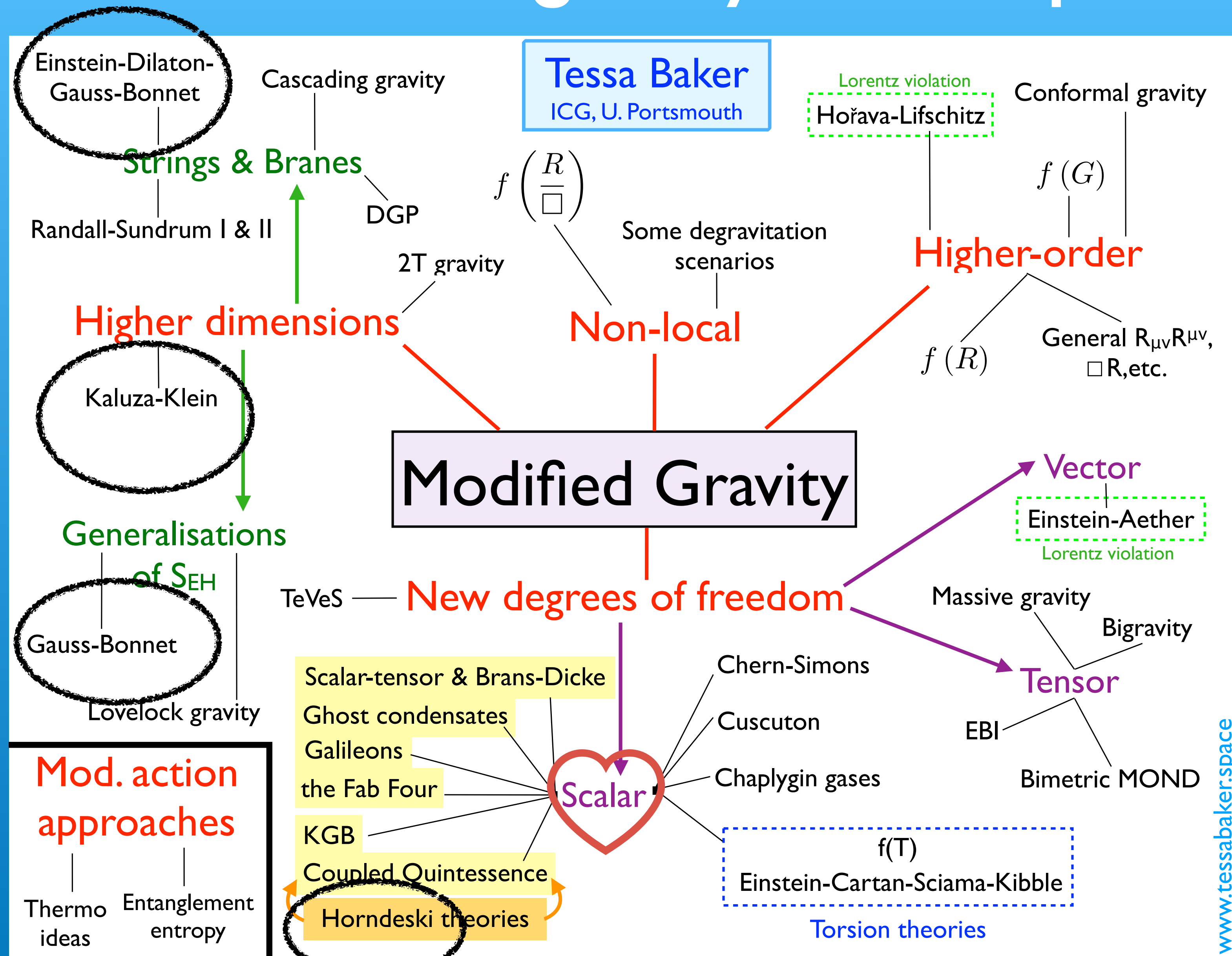
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Guide to modified gravity theory

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Yunes et al : arXiv : 2408.05240

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- ✱ If Non GR Black holes exist, they should have a dynamical formation mechanism

Modified gravity : Agnostic or specific

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Testing approach

How do we test deviations from vacuum GR?

Agnostic

- flexible & immediately applicable
- applicable to the coalescence phases
- useful for non-vacuum/beyond-GR/ECOs
- mostly pN based for the inspiral and QNM-based for the ringdown
- many parameters to add
- degeneracies for non-vacuum/beyond-GR/ECOs
- physical mapping requires specific models (aka false d.o.f. and *what if* we detect something)
- no parametrization for SF (but 1 case)

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- IMR waveforms
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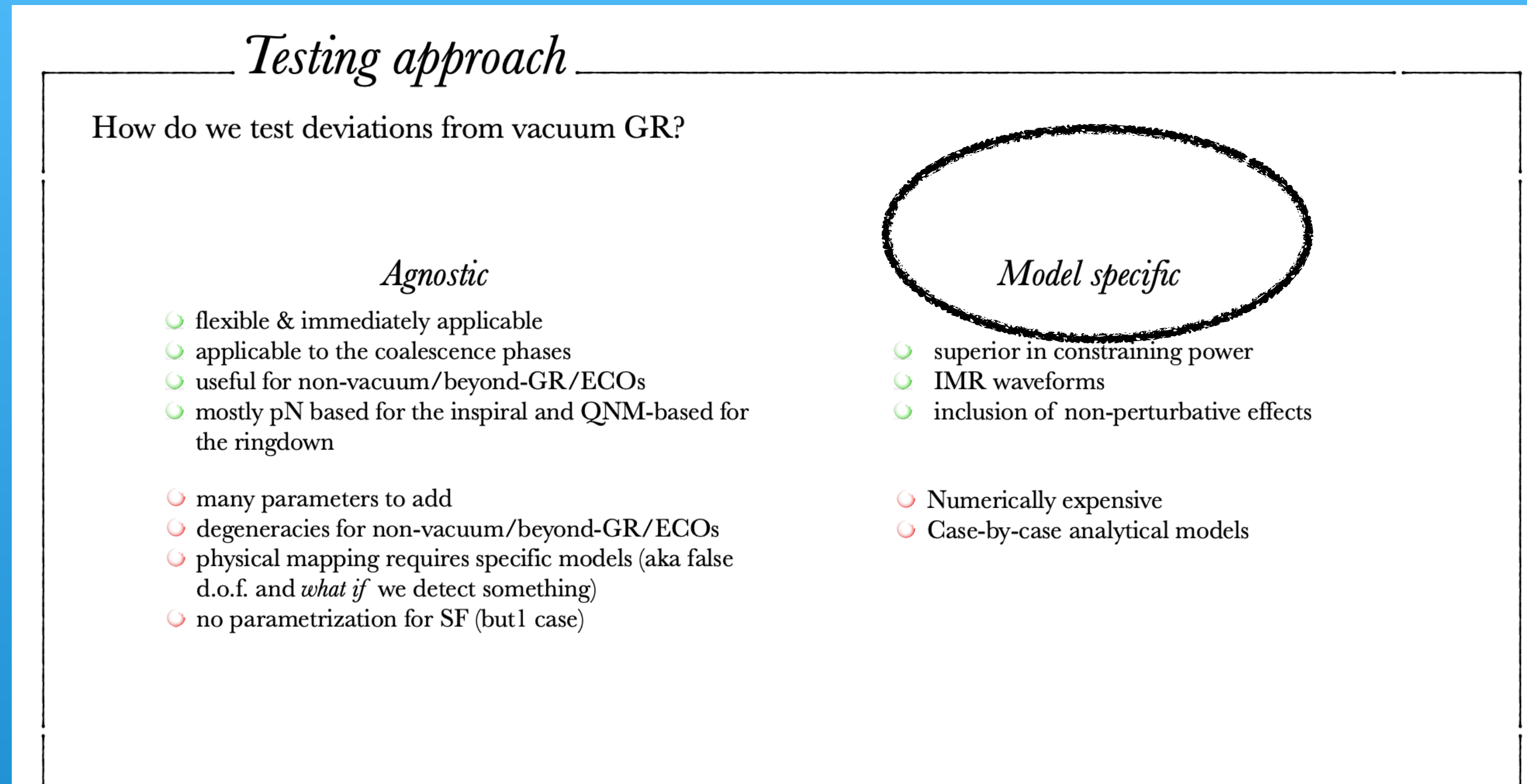
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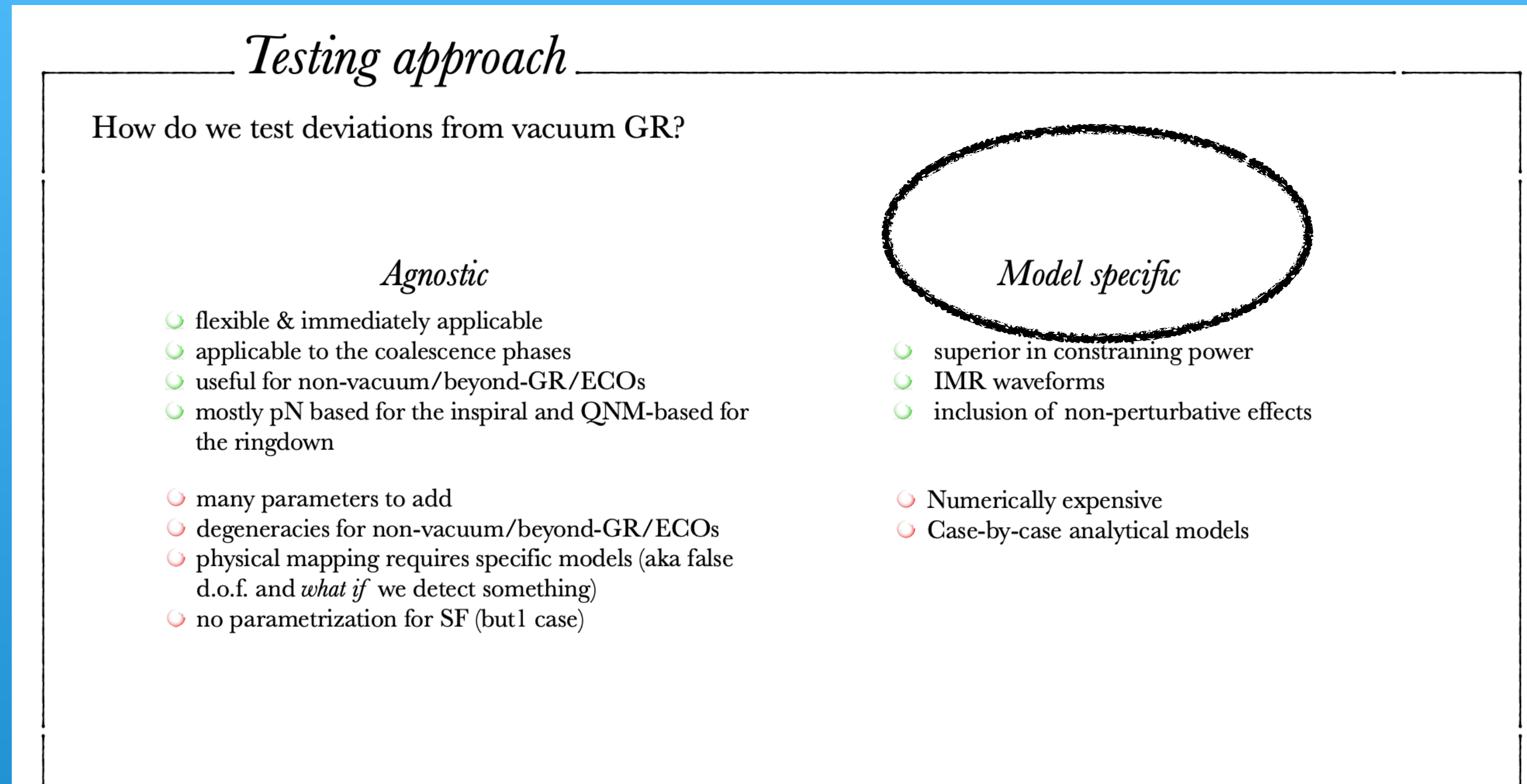
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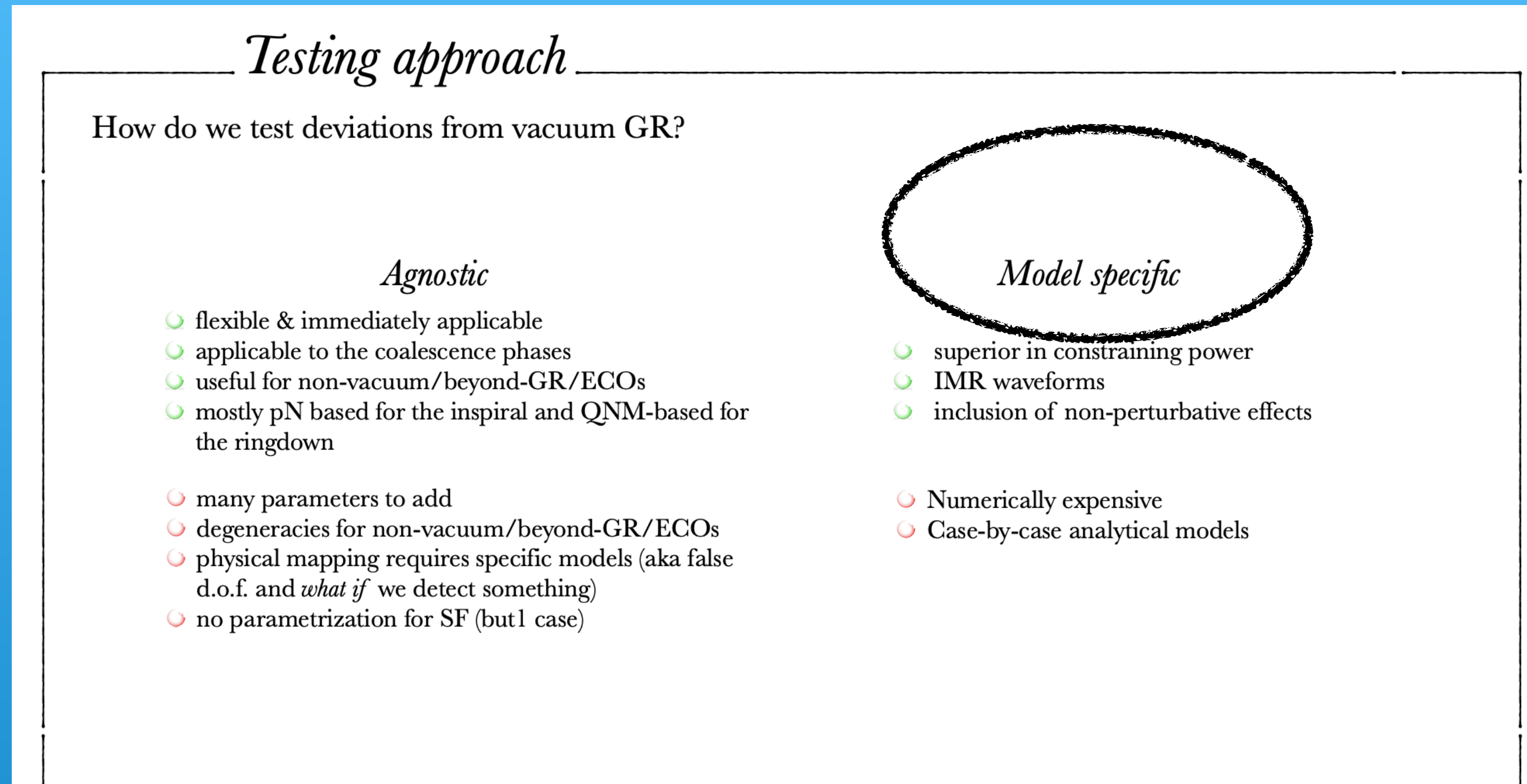
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- ✗ Agnostic : does not allow you to map to concrete models

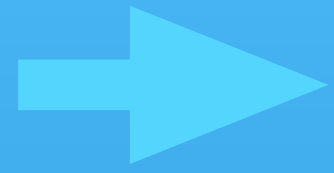
Modified gravity / New physics

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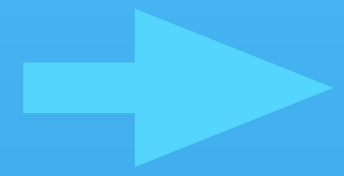
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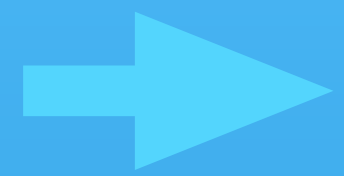
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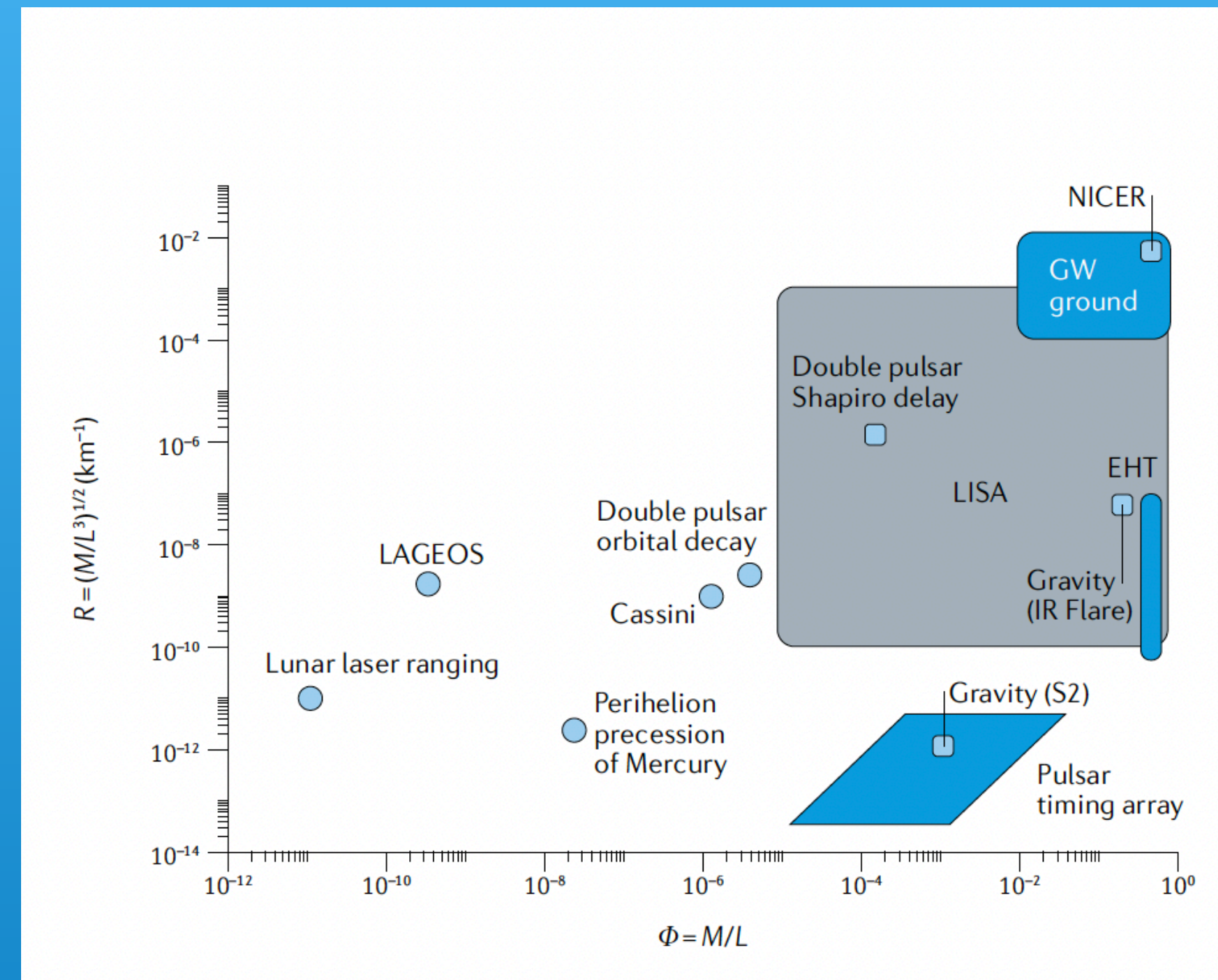
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Modified gravity / New physics

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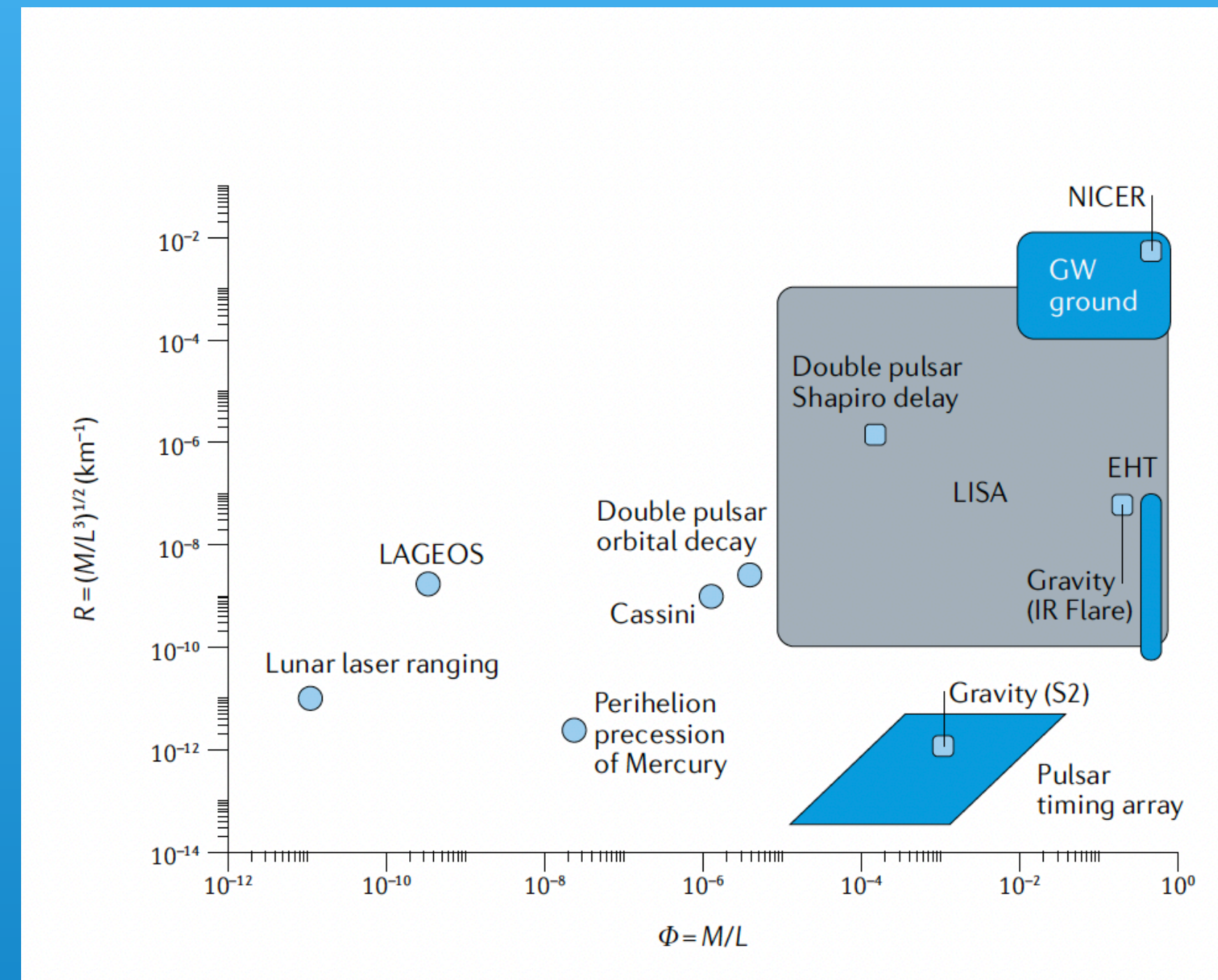
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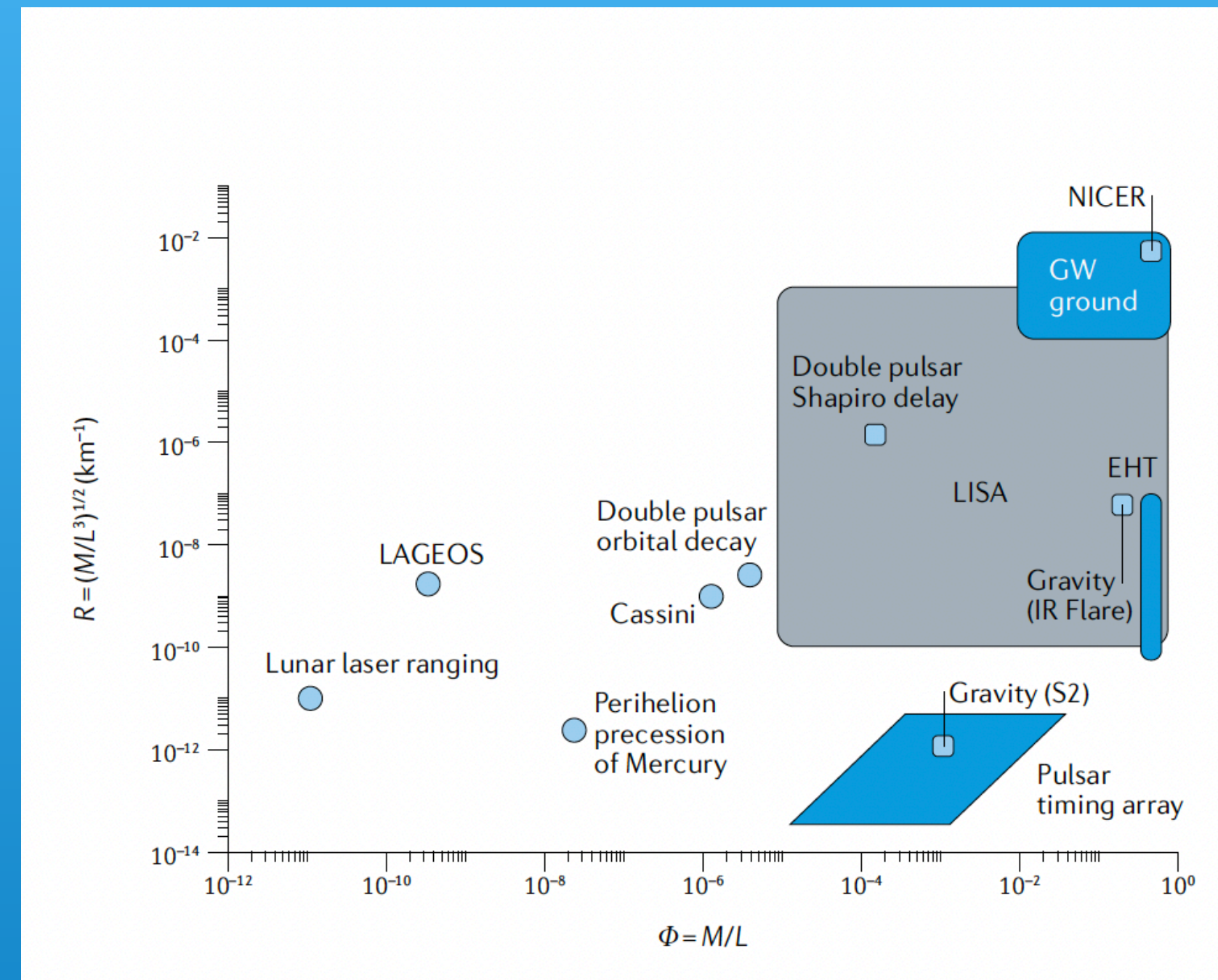


Bailes et al : Nature 2021

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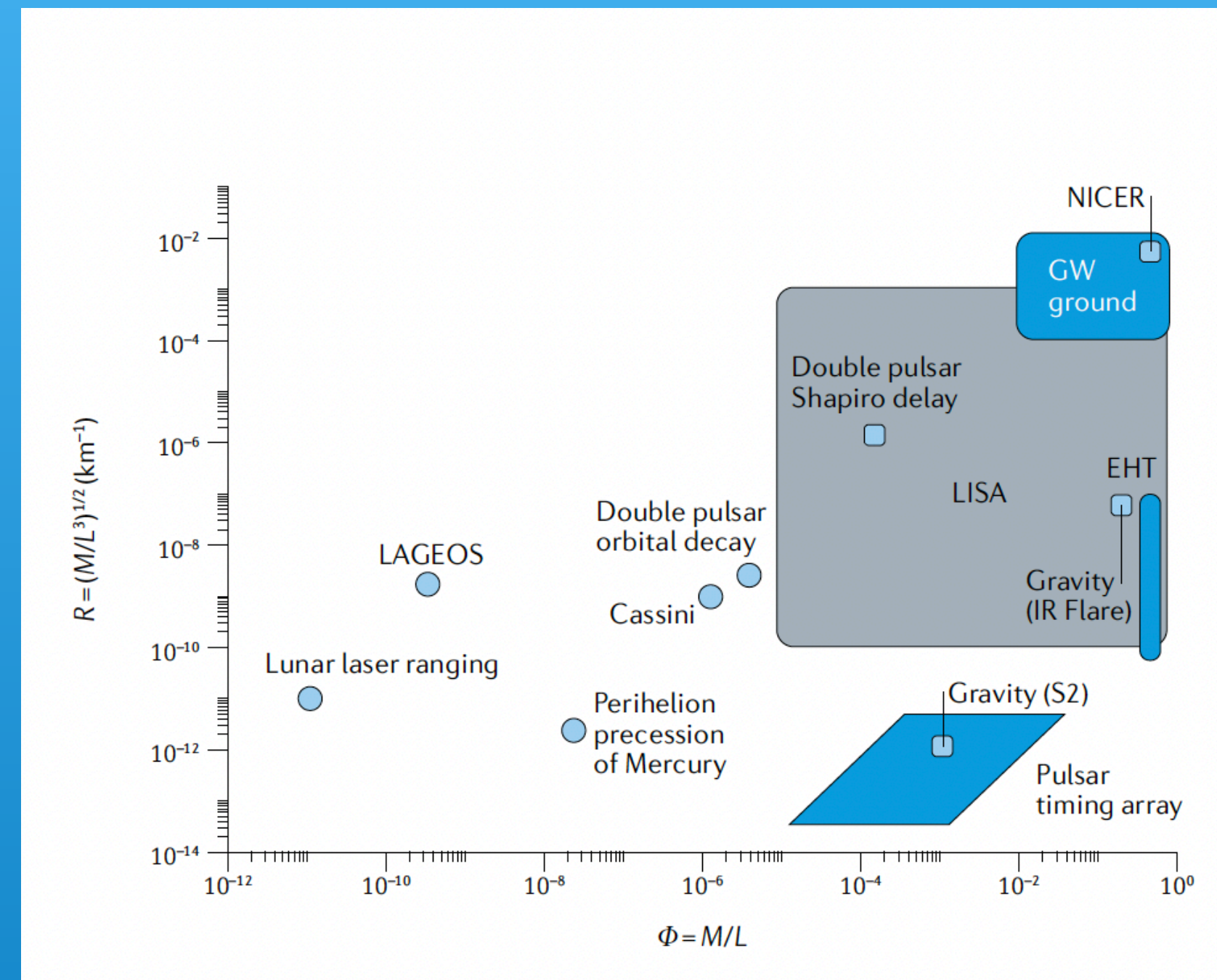
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Future detectors (ET, LISA, Cosmic Explorer)
allow gravity to be tested in new regimes
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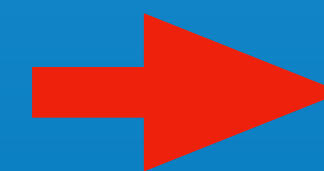
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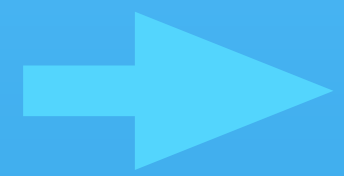
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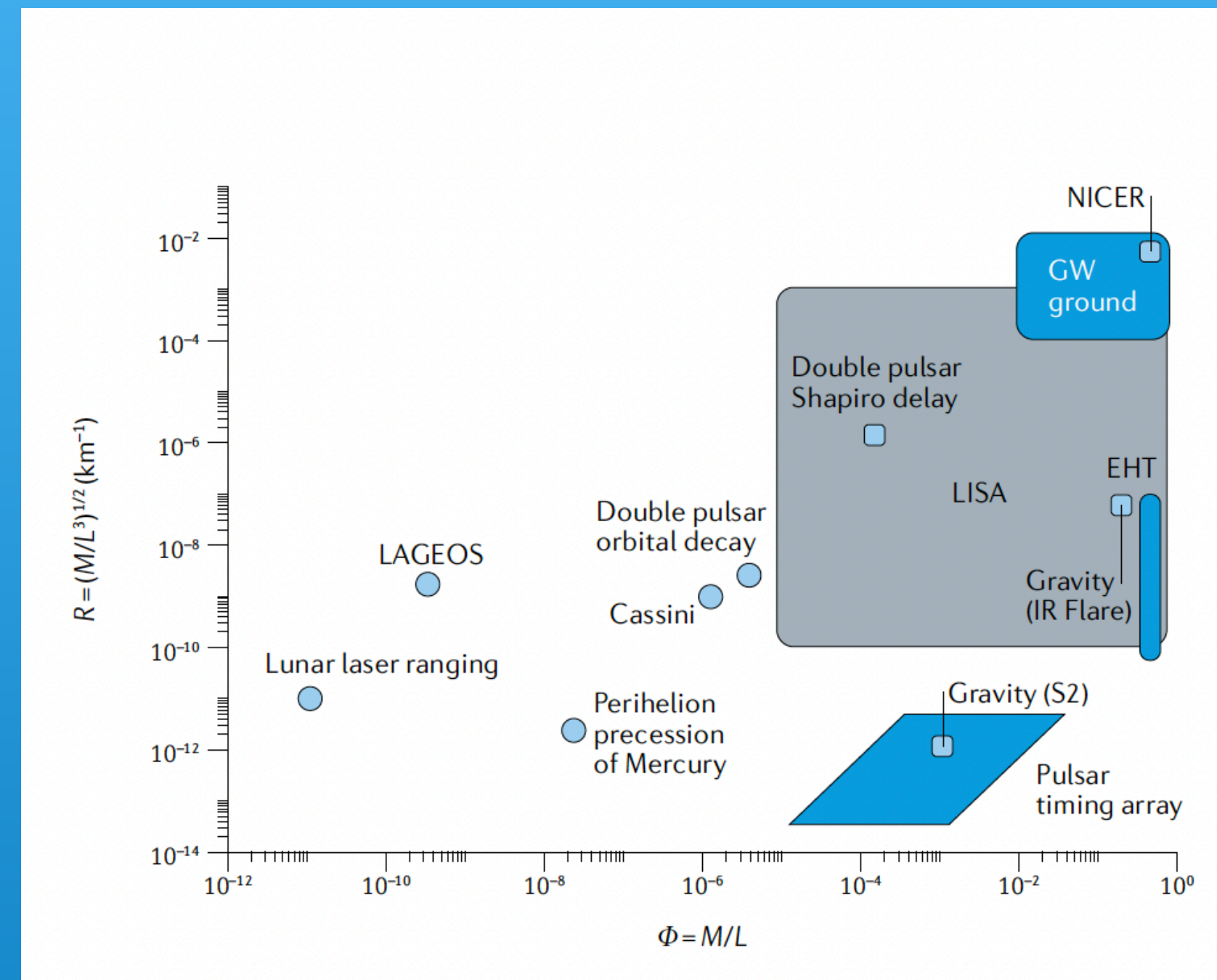


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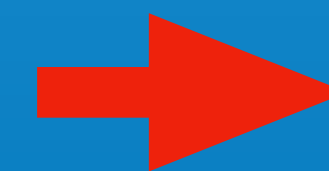


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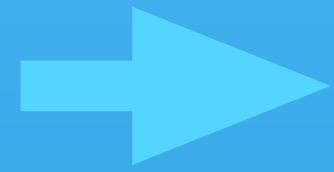
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Possibility of new physics :
New fields , new degrees of freedom

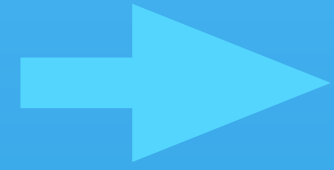
Modified gravity : New physics

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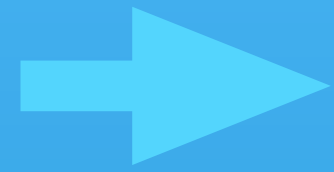
Modified gravity : New physics

Does this new fields modify GR black holes ? Do they become hairy ?



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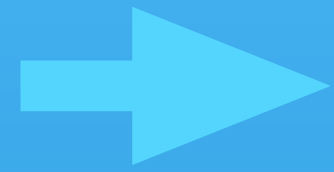
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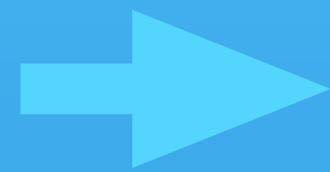


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Modified gravity : New physics

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Do they leave some observational signatures in any detector ?

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Is there any mechanism to allow you getting this new physics ?

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arXiv:gr-qc 0402007

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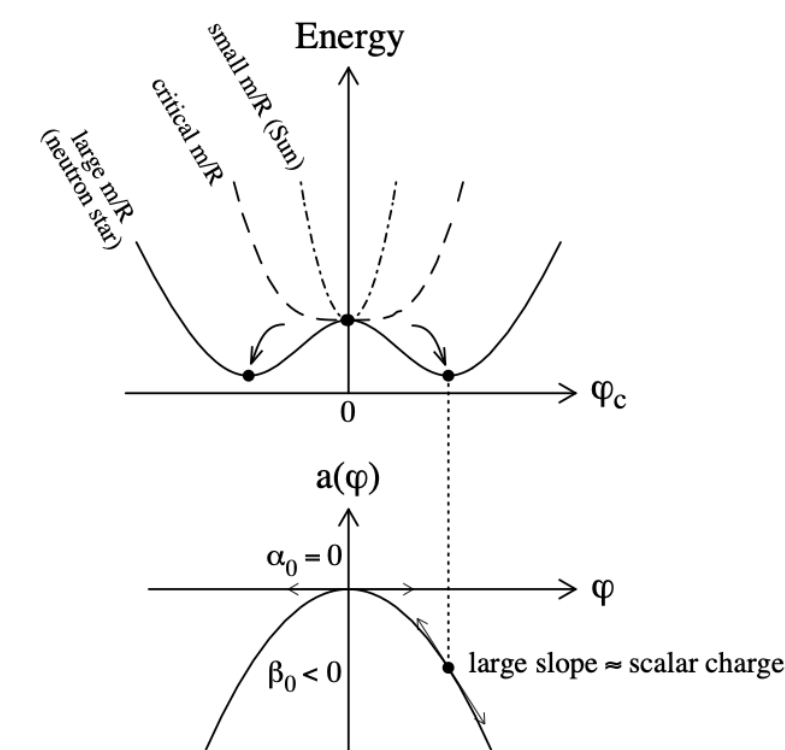


Figure 2. Heuristic argument to explain the phenomenon of “spontaneous scalarization”. When $\beta_0 < 0$ and the compactness Gm/Rc^2 of a body is large enough, it is energetically favorable to create a local scalar field different from the background value. The body becomes thus strongly coupled to the scalar field.

Spontaneous Scalarization of compact objects

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How it works concretely ?

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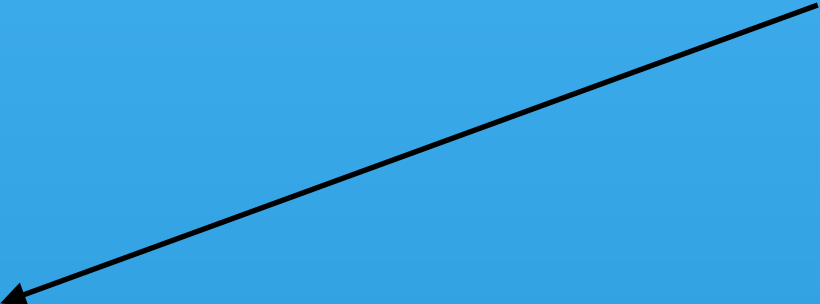
$$S_{scalarization} = \frac{1}{16\pi G_*} \int d^4x \sqrt{-g} [R - 2 \nabla_\mu \phi \nabla^\mu \phi] + \text{Source of Scalarization}$$

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$$\mathcal{J} = -F_{\mu\nu}F^{\mu\nu}$$

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function for scalarization :

$$f(0) = 0$$

Scalar field equation

$$\square \phi = -\frac{\dot{f}(\phi)\mathcal{J}}{4}$$

Scalar field linearised/perturbed equation

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Scalarization models of black holes

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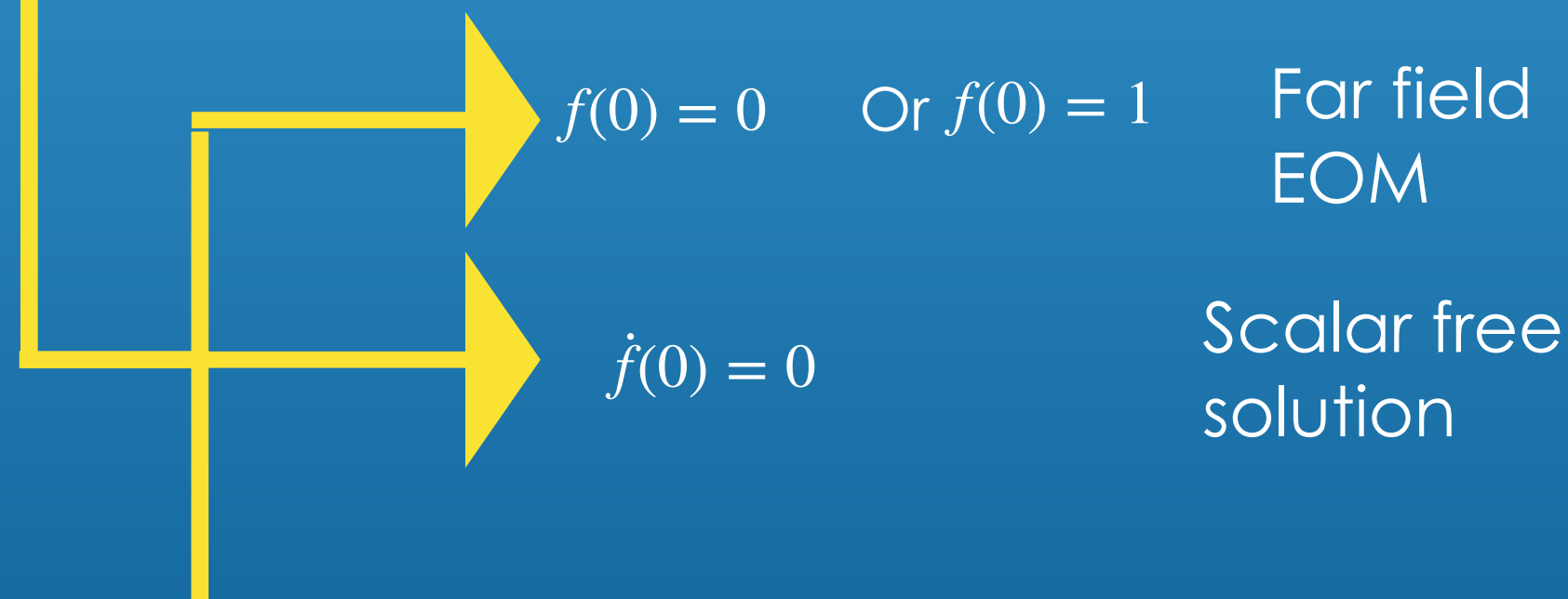
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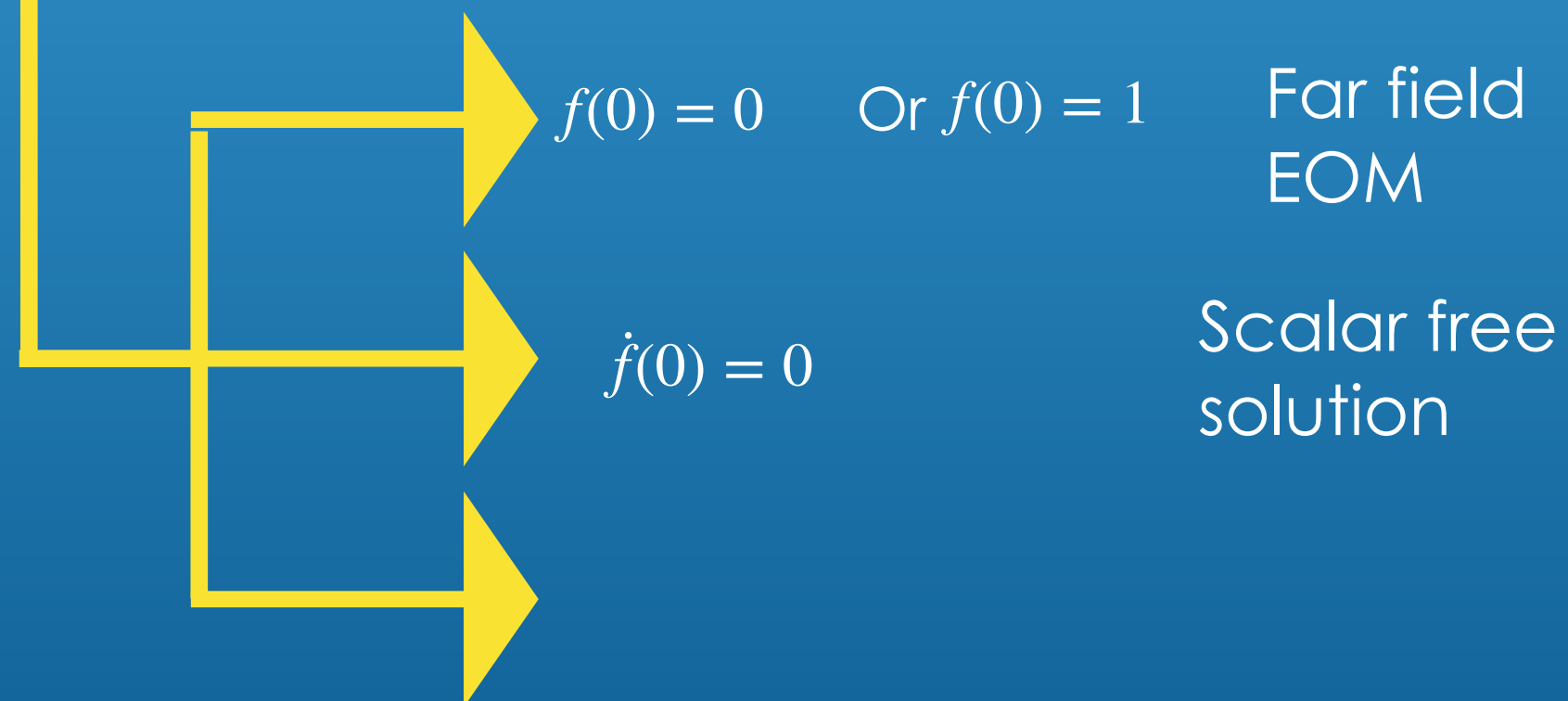
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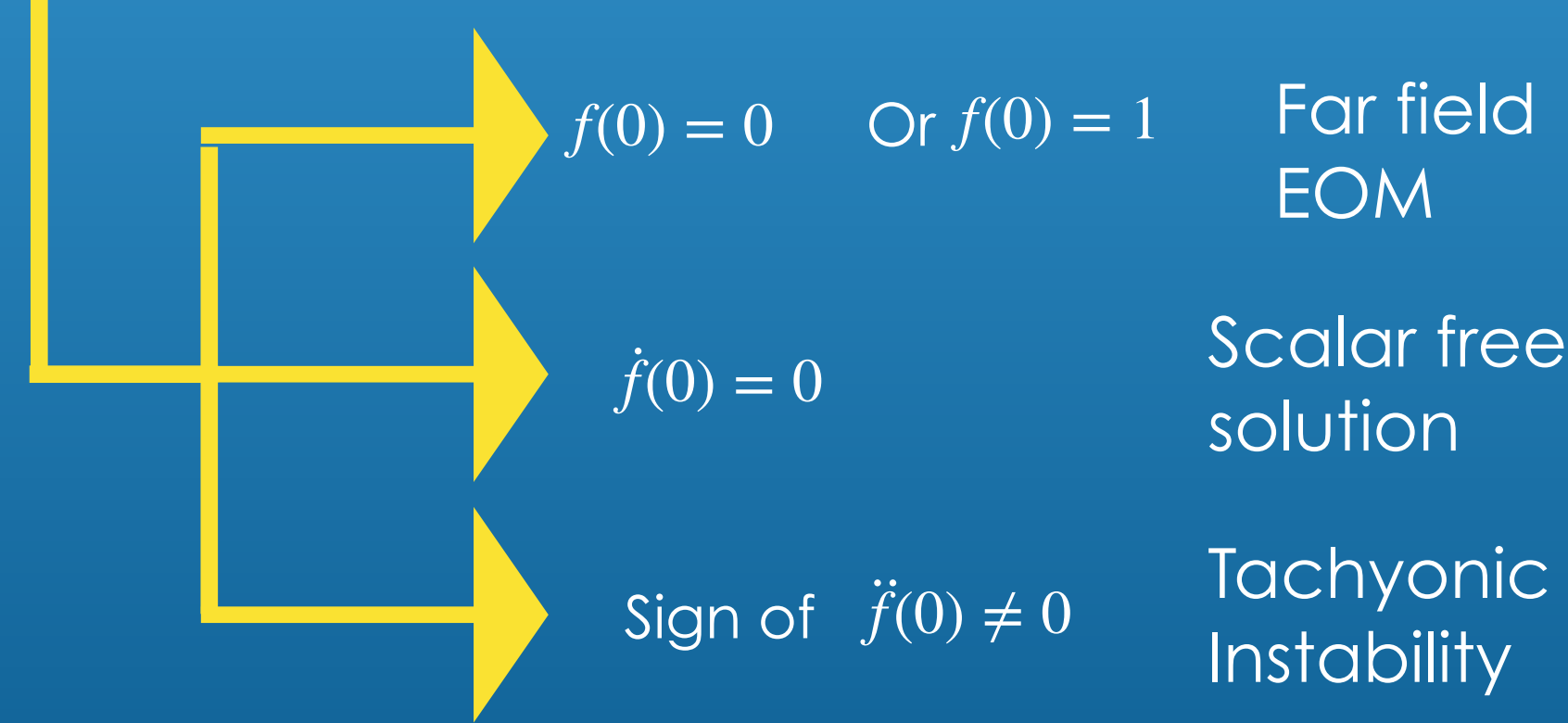
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Tachyonic
Instability

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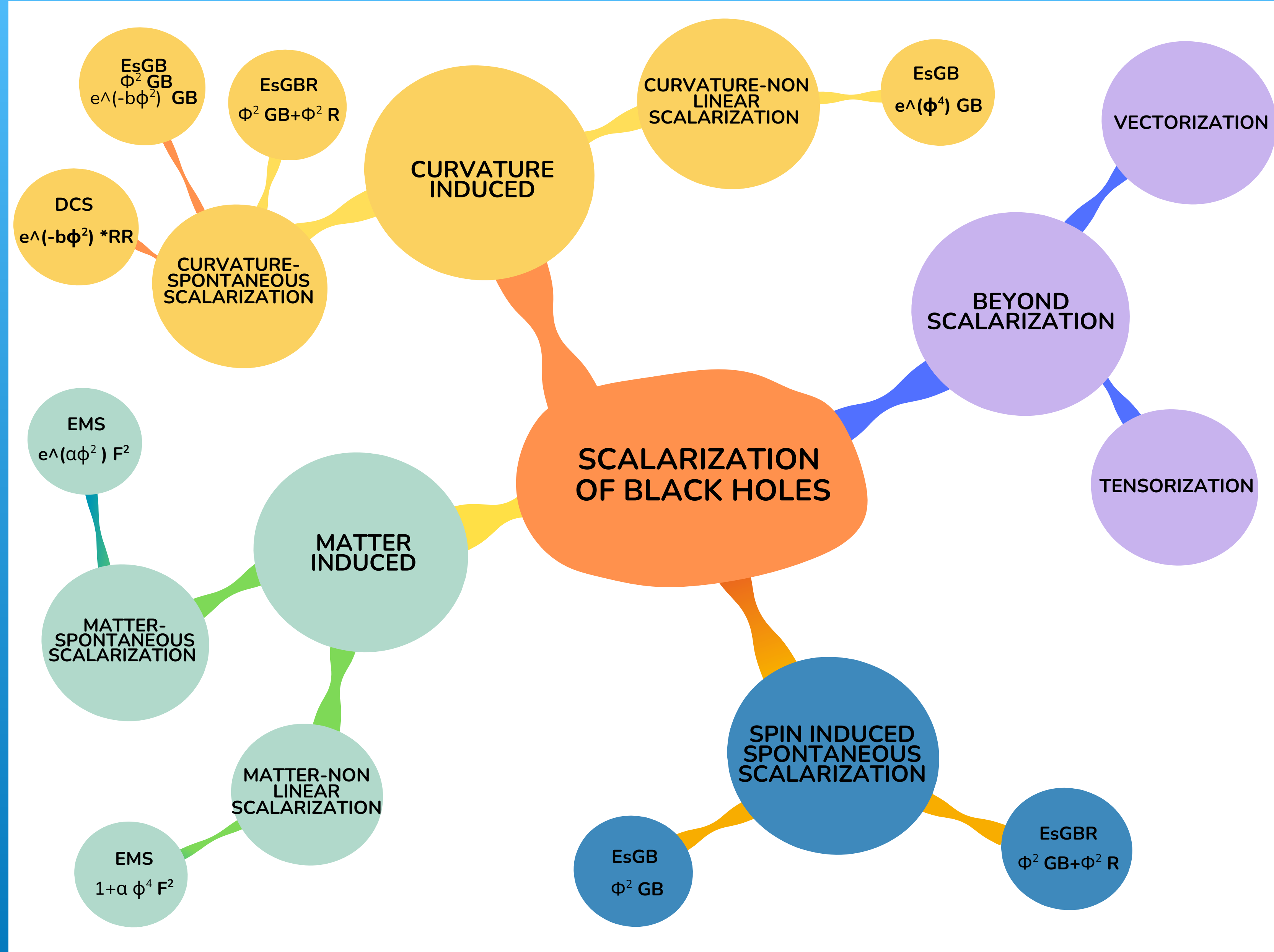
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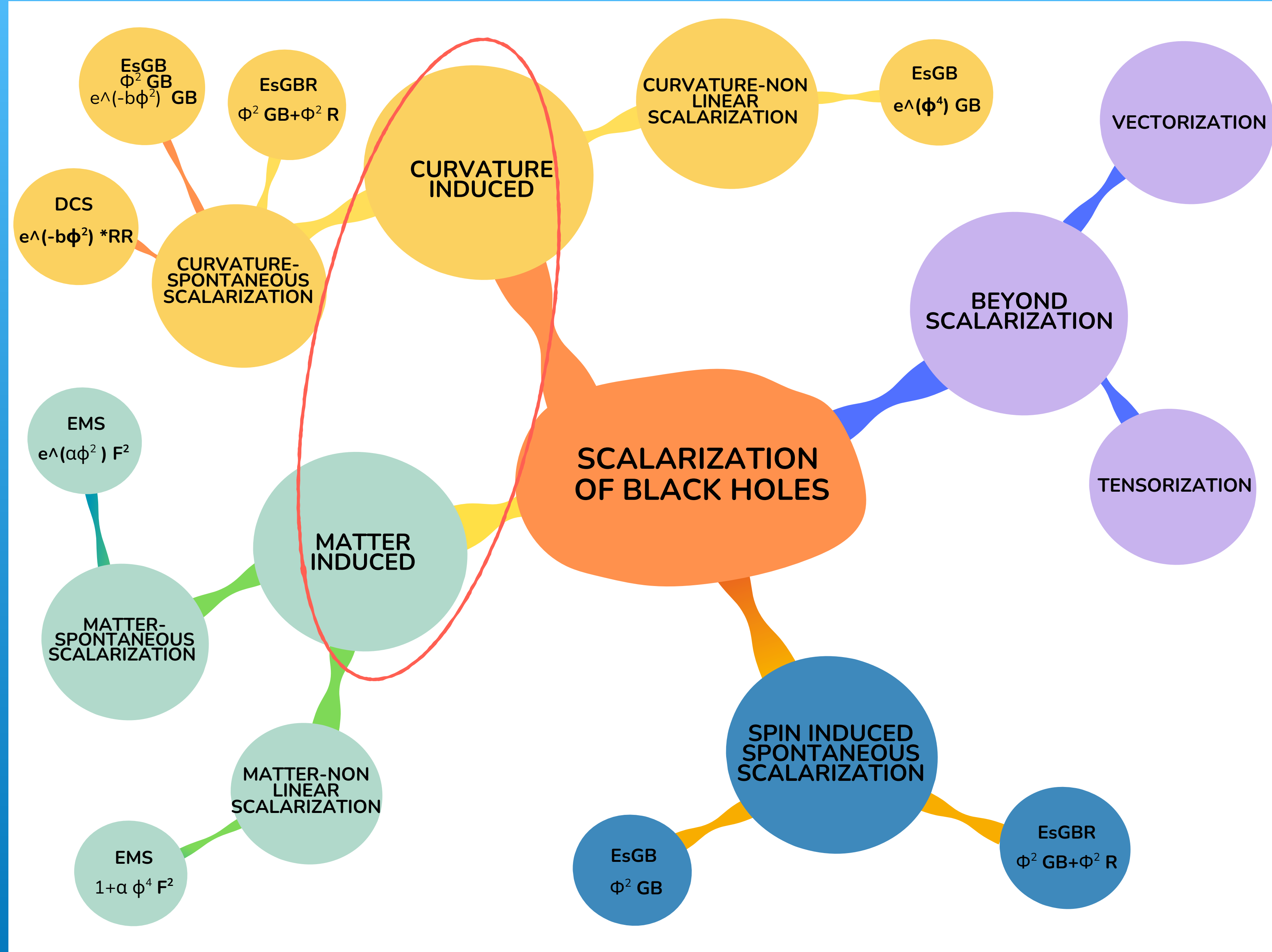
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Curvature induced scalarization

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Silva et al 2017
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Curvature-Matter induced scalarization

Scalarization induced by Curvature and matter

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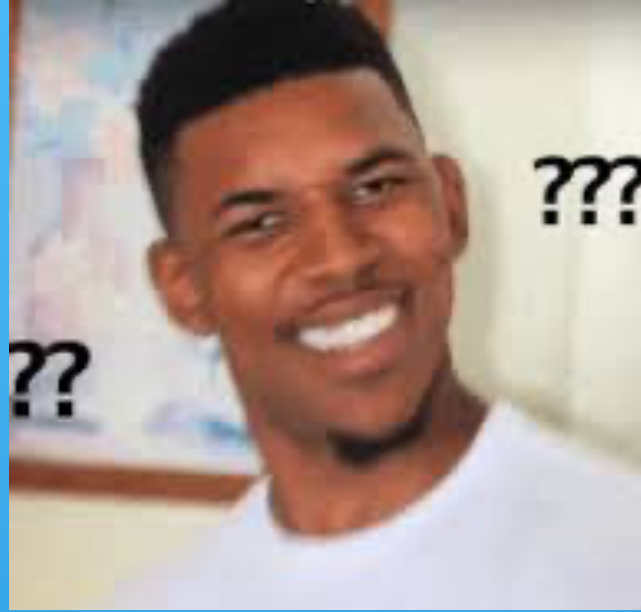
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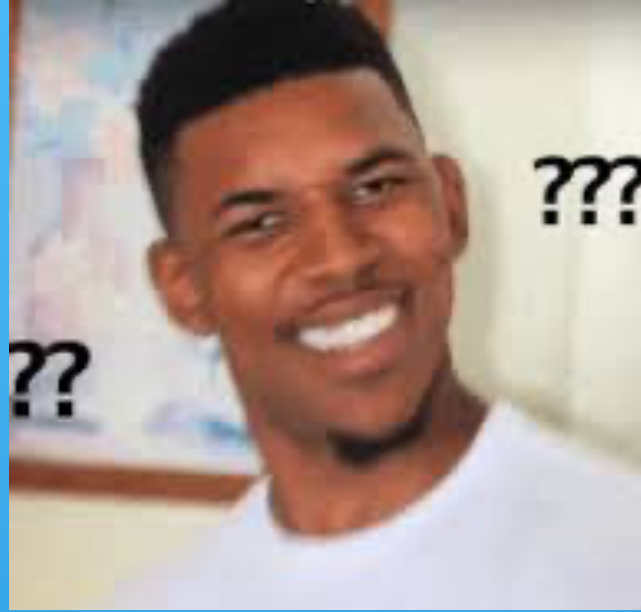
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- Introducing scalar-photon couplings may reopen the parameter space of modified gravity theories by modifying the propagation of EM waves in the same way as GWs, (Babichev et al, arXiv:2407.20339) “ with a grain of salt “

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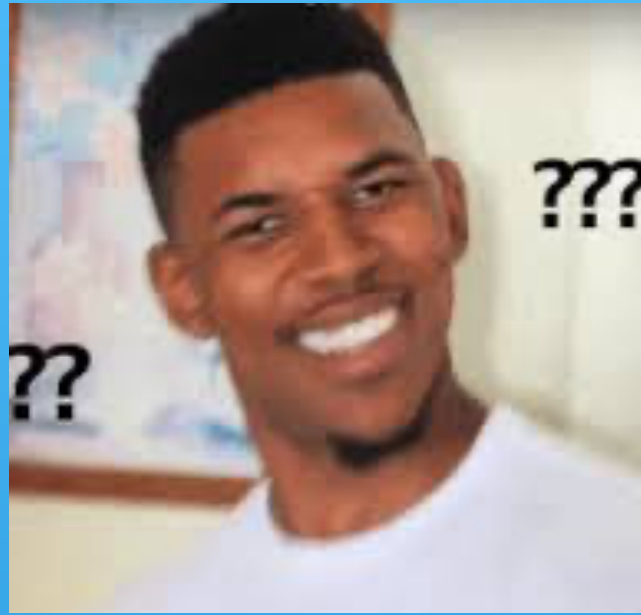


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Hold on ! **Black holes aren't neutral**
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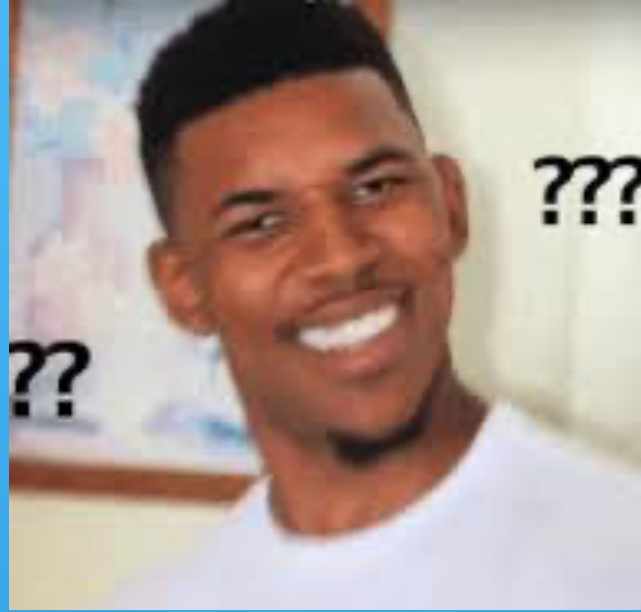
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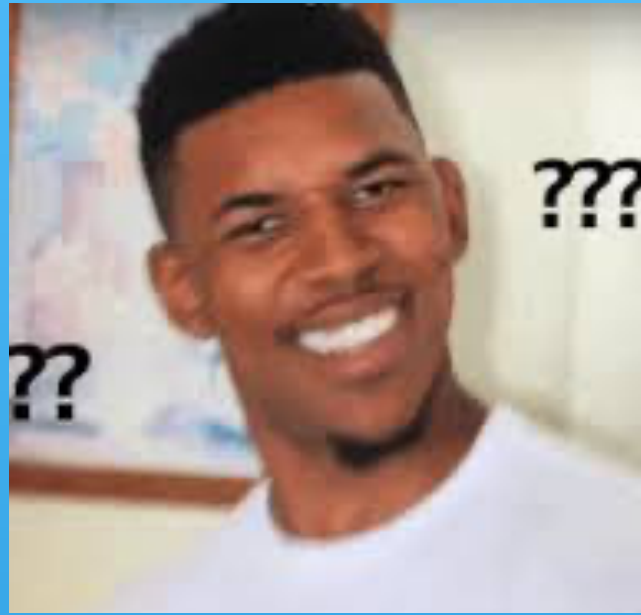
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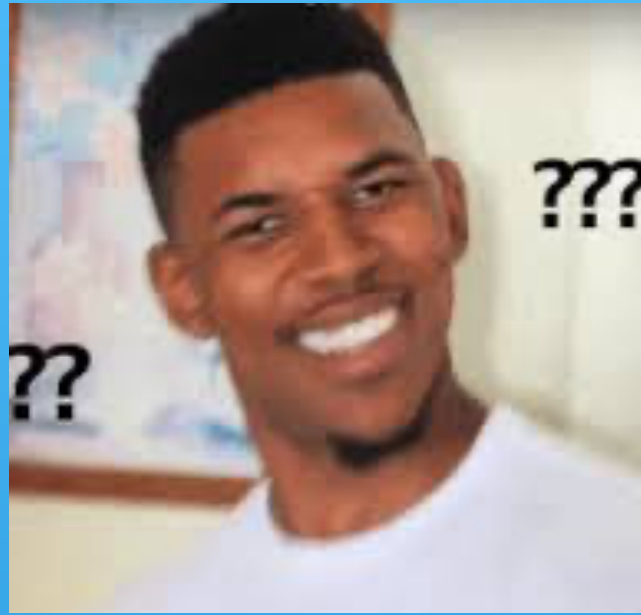
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- We can consider **Magnetically** charged black holes. motivated by hypothetical magnetic monopoles (a lot of interest since the paper by Maldacena arXiv : 2004.06084)

Charge of the Black holes



Hold on ! **Black holes aren't neutral**
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- We can consider it as a **toy** model. Also RN black holes are interesting because they admit an extremal limit, so they can also serve as for understanding rotating solutions.

The theory setup

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Regularity on the horizon



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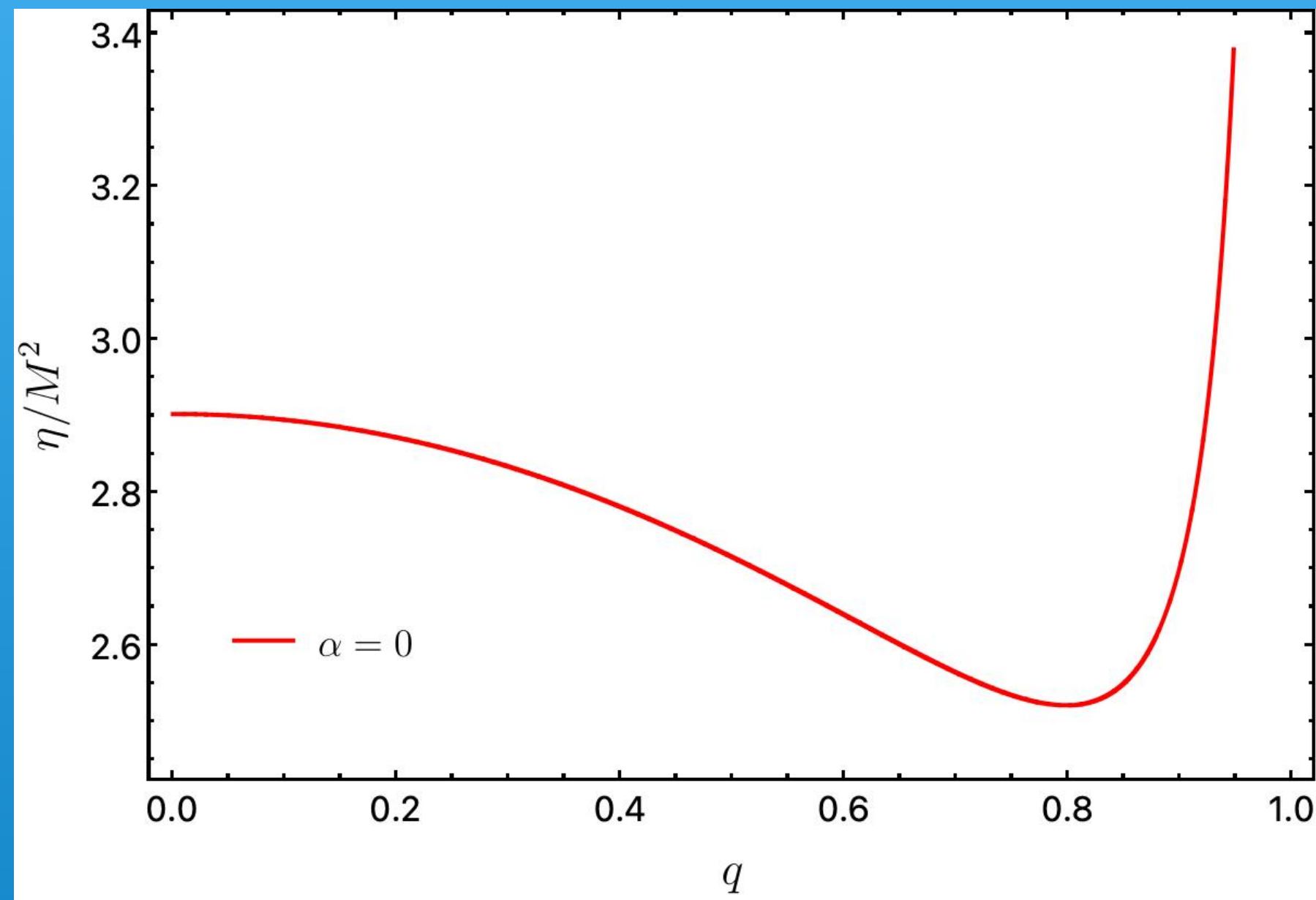
$$\rho^6((q^2 + (-2 + \rho)\rho)u''(\rho) + 2(-1 + \rho)u'(\rho)) = (q^2\rho^4\alpha - 2(5q^4 - 12q^2\rho + 6\rho^2)\gamma)u(\rho).$$

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We solve the dimensionless linearised KG equation numerically to obtain the behavior of the lower limit of the dimensionless Gauss-Bonnet coupling constant $\gamma = \eta/M^2$, which is necessary to guarantee the instability of the Reissner-Nordström black hole (wrt the reduced electric charge $q = Q/M$)

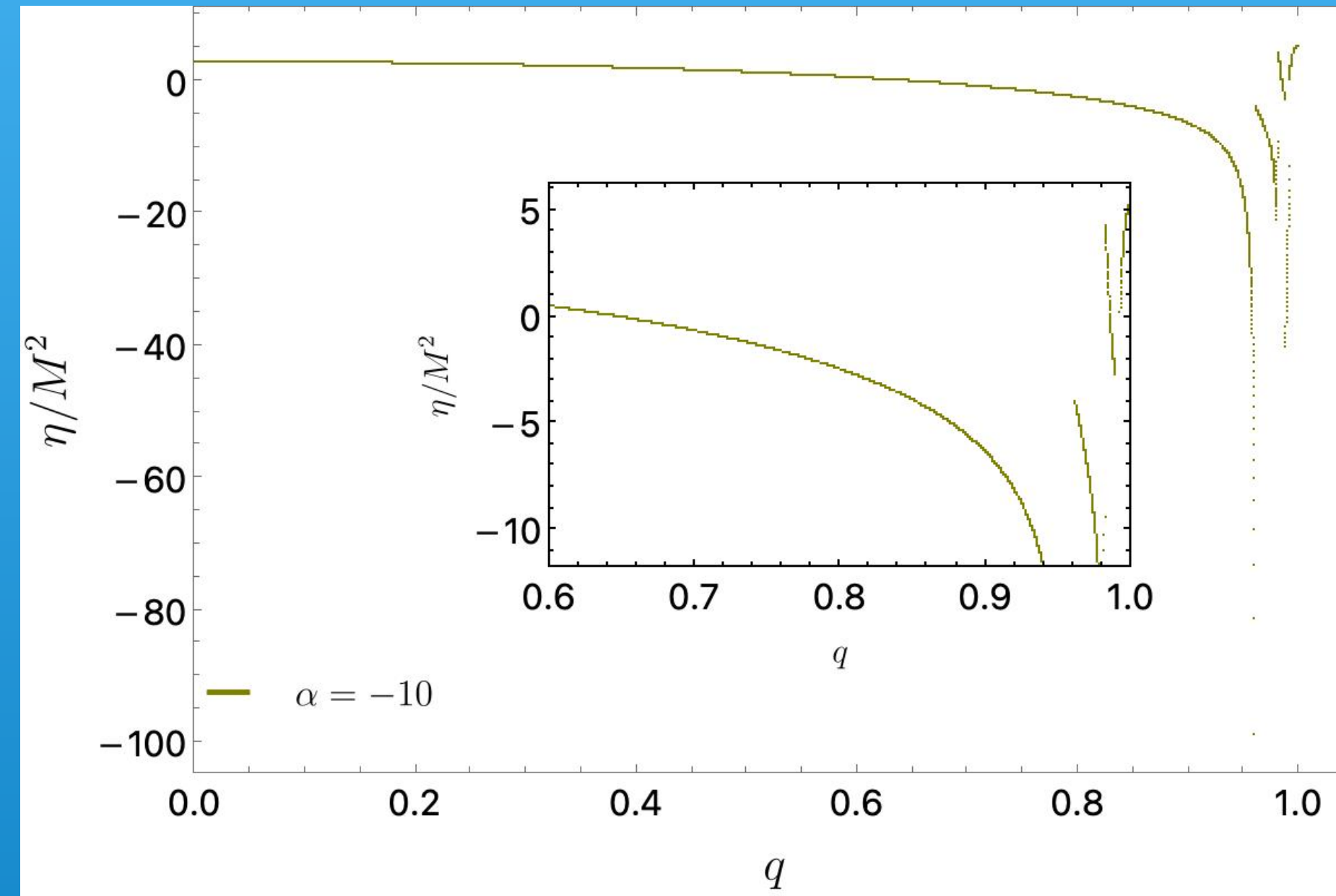
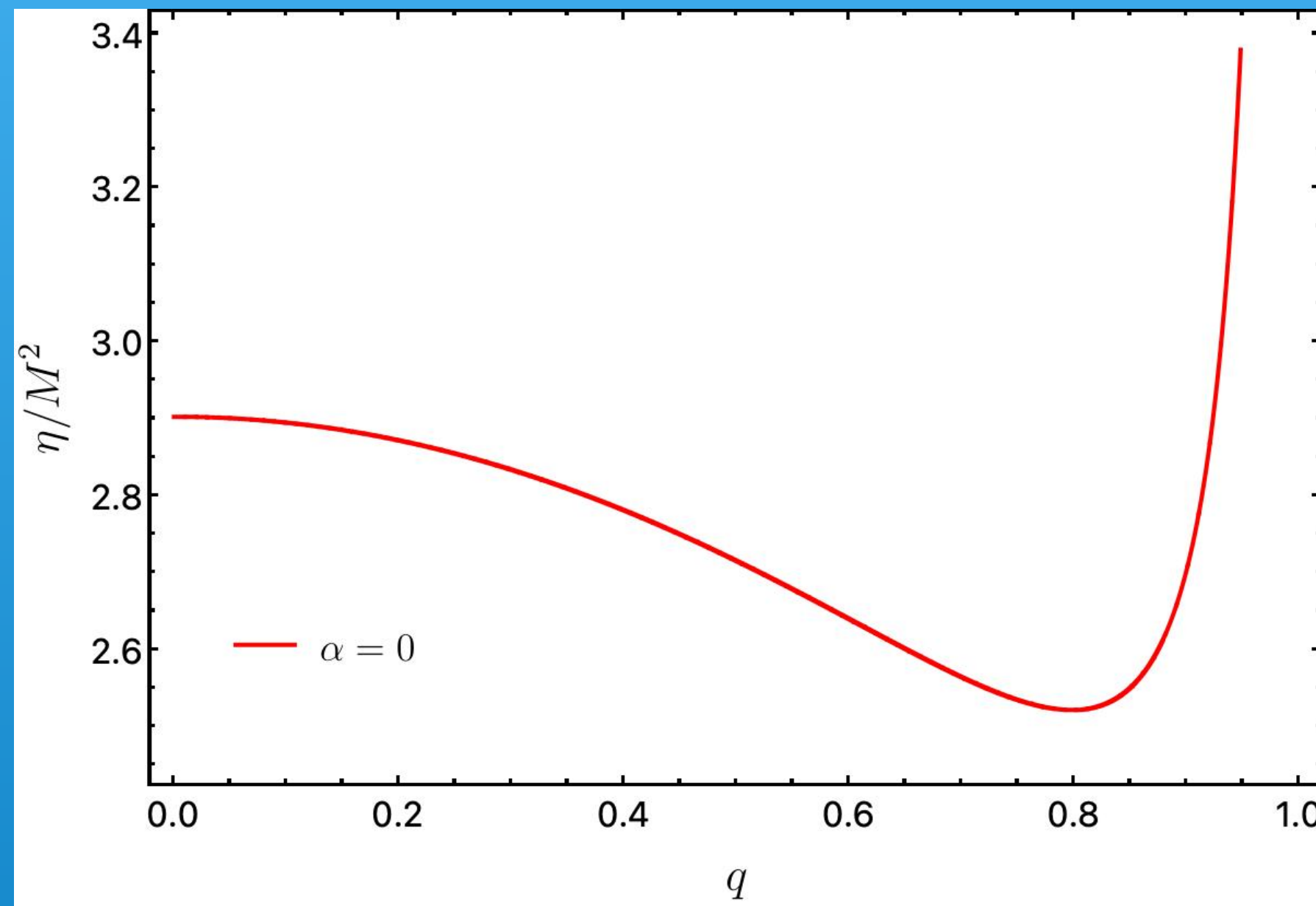
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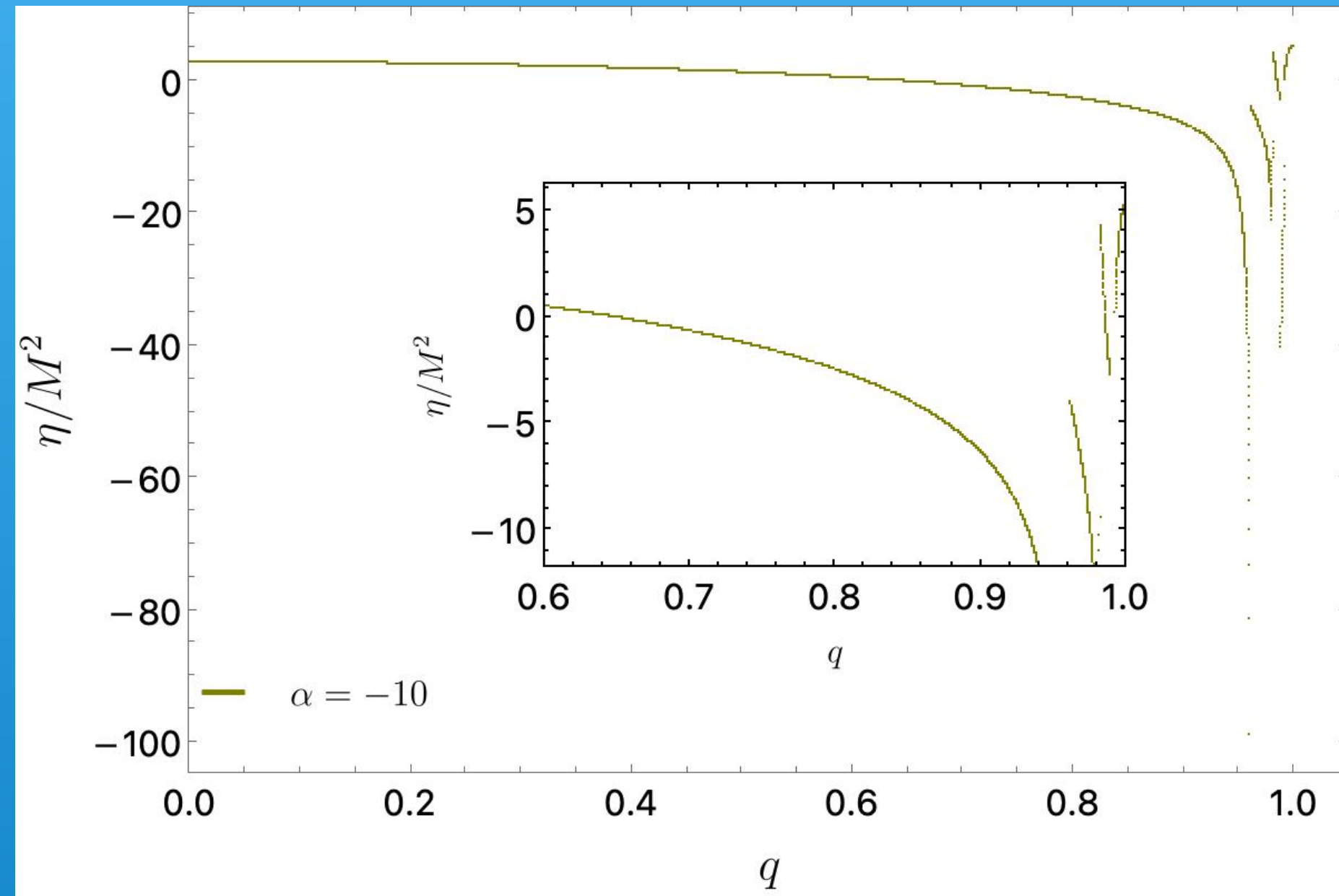
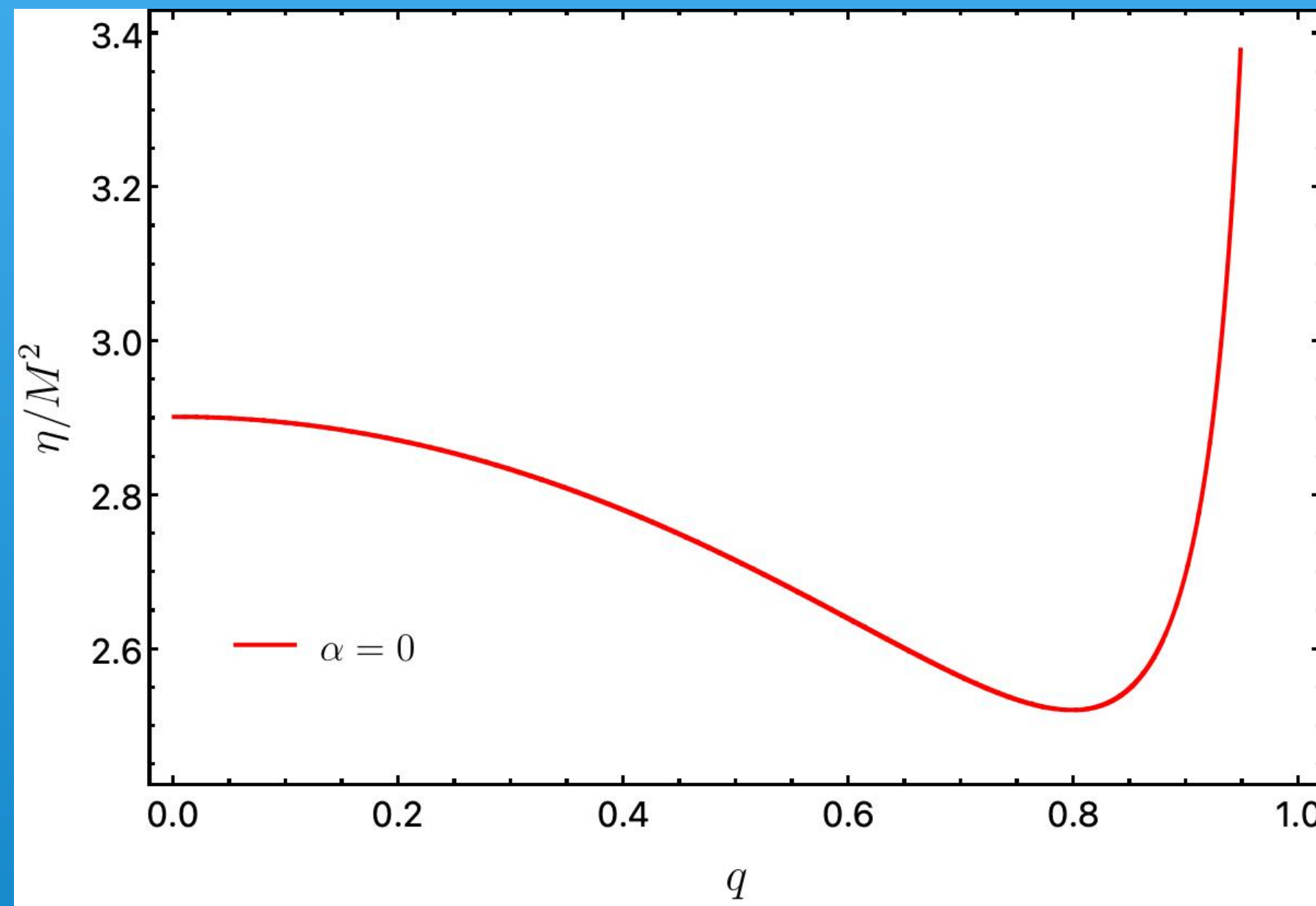
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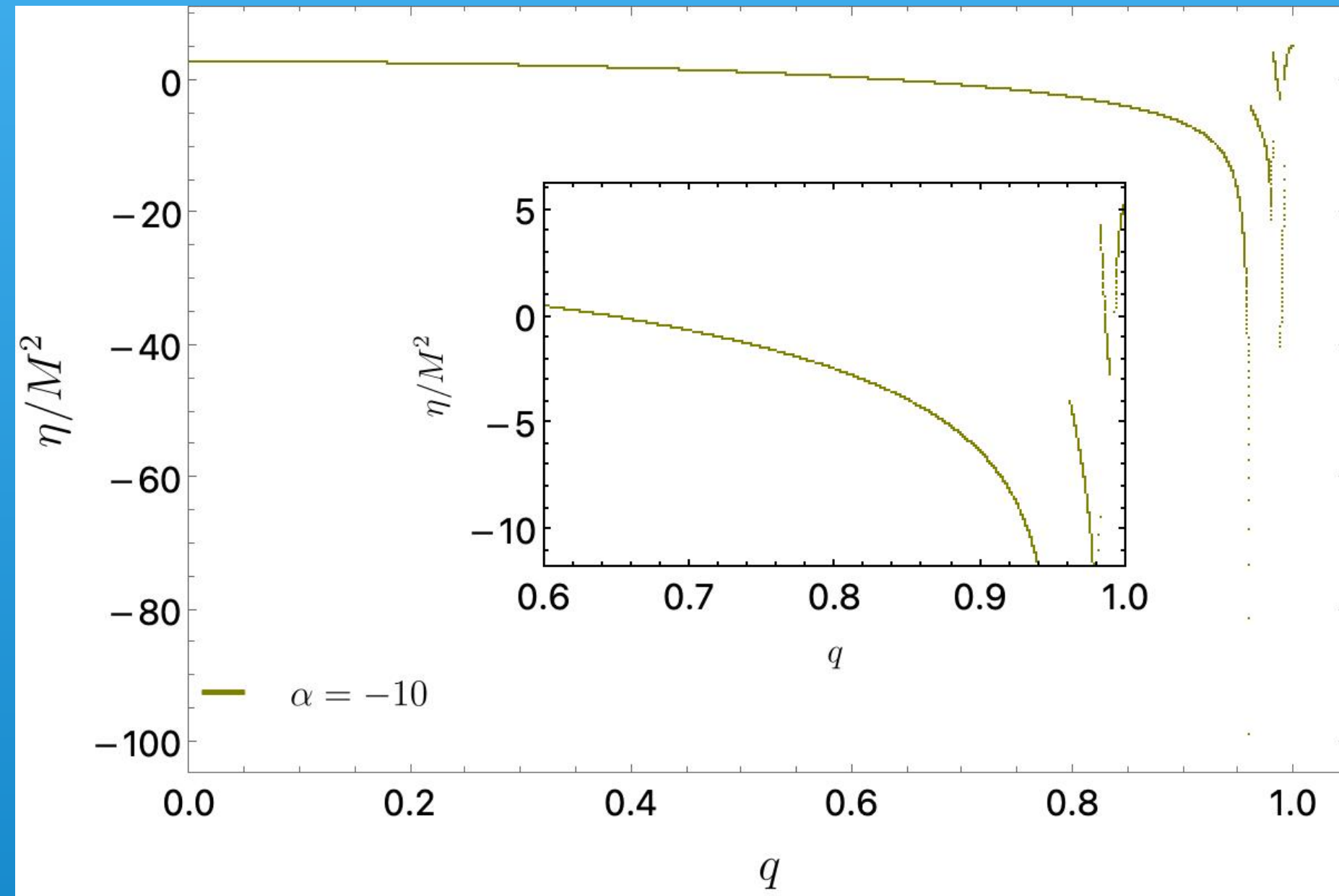
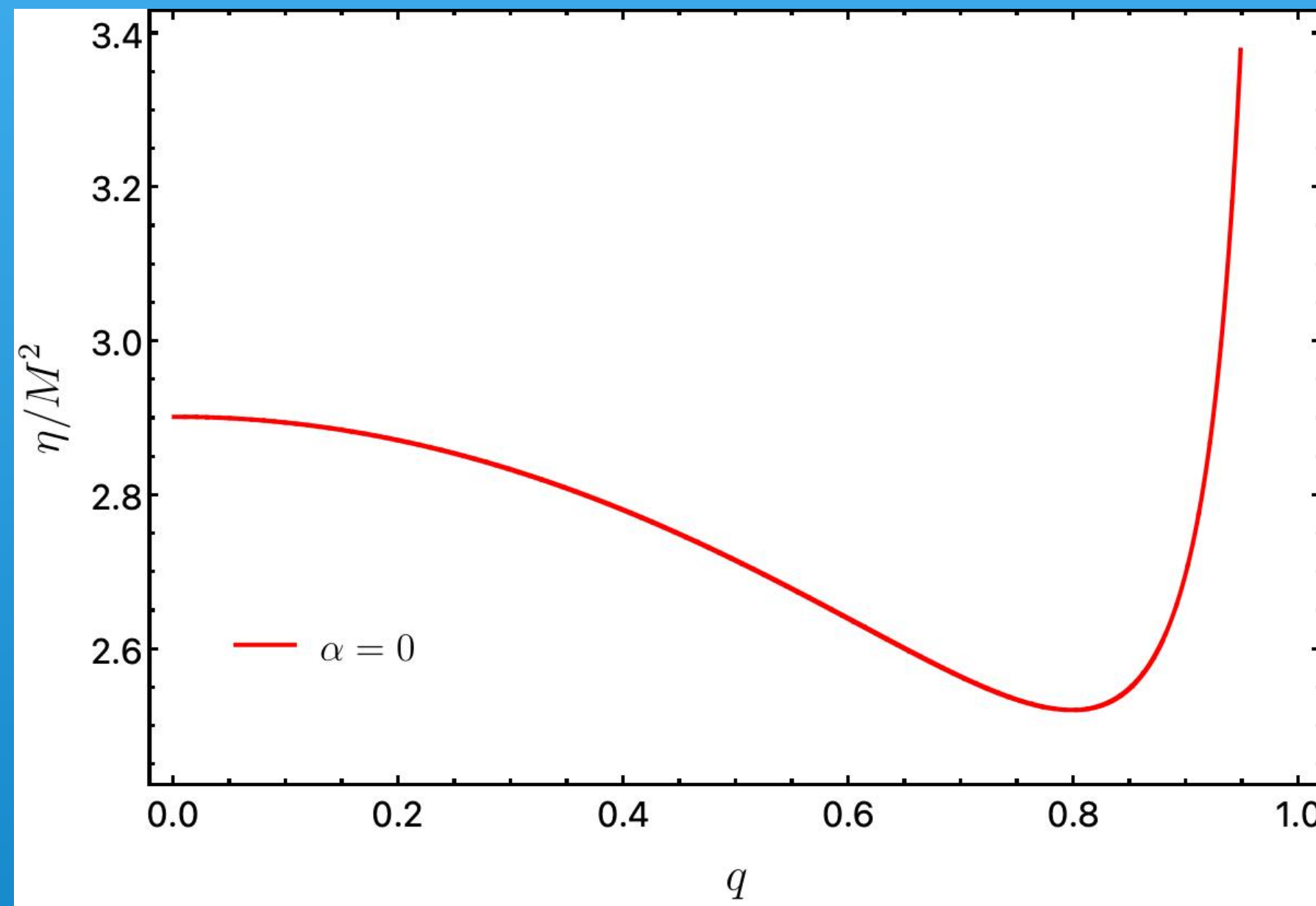
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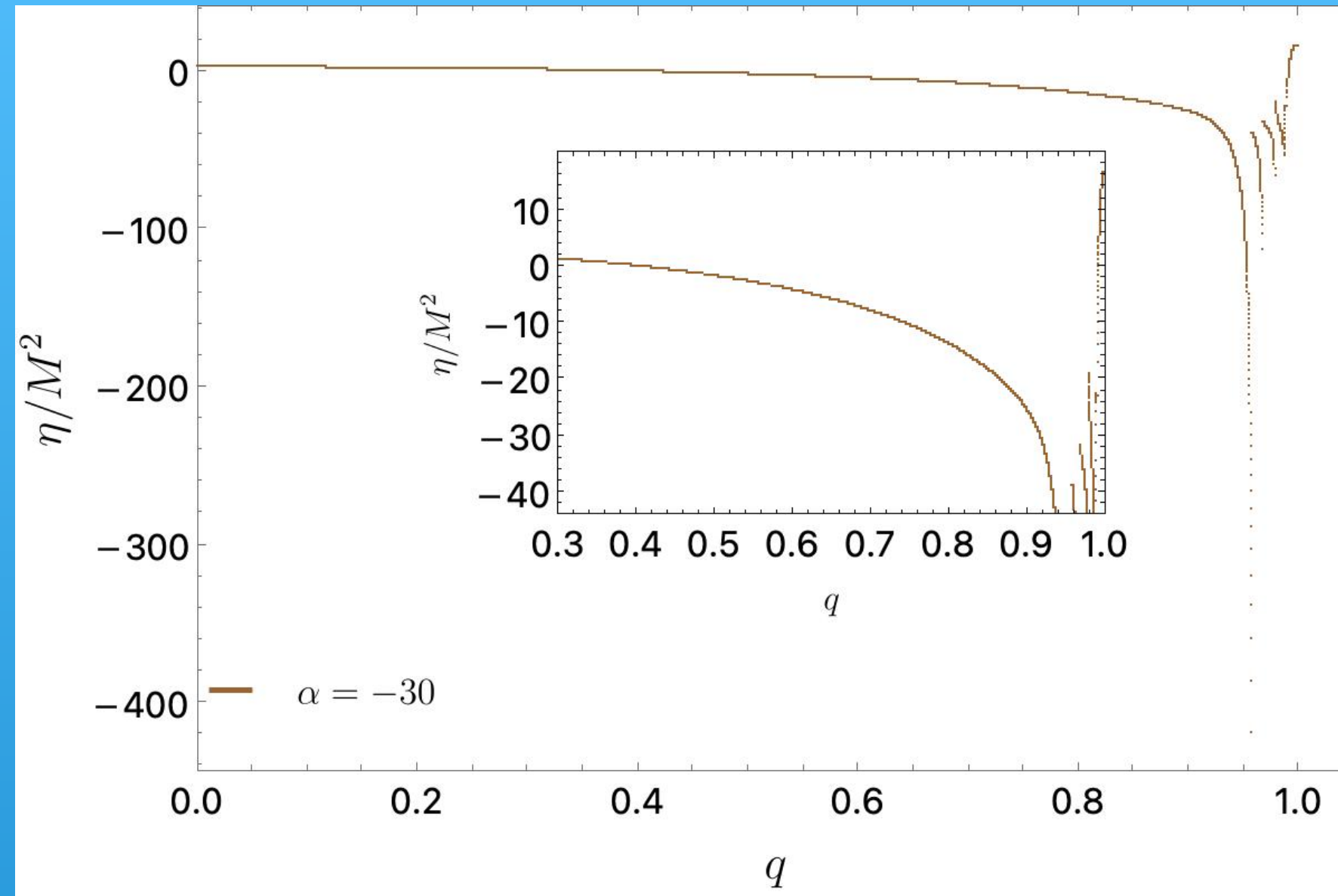
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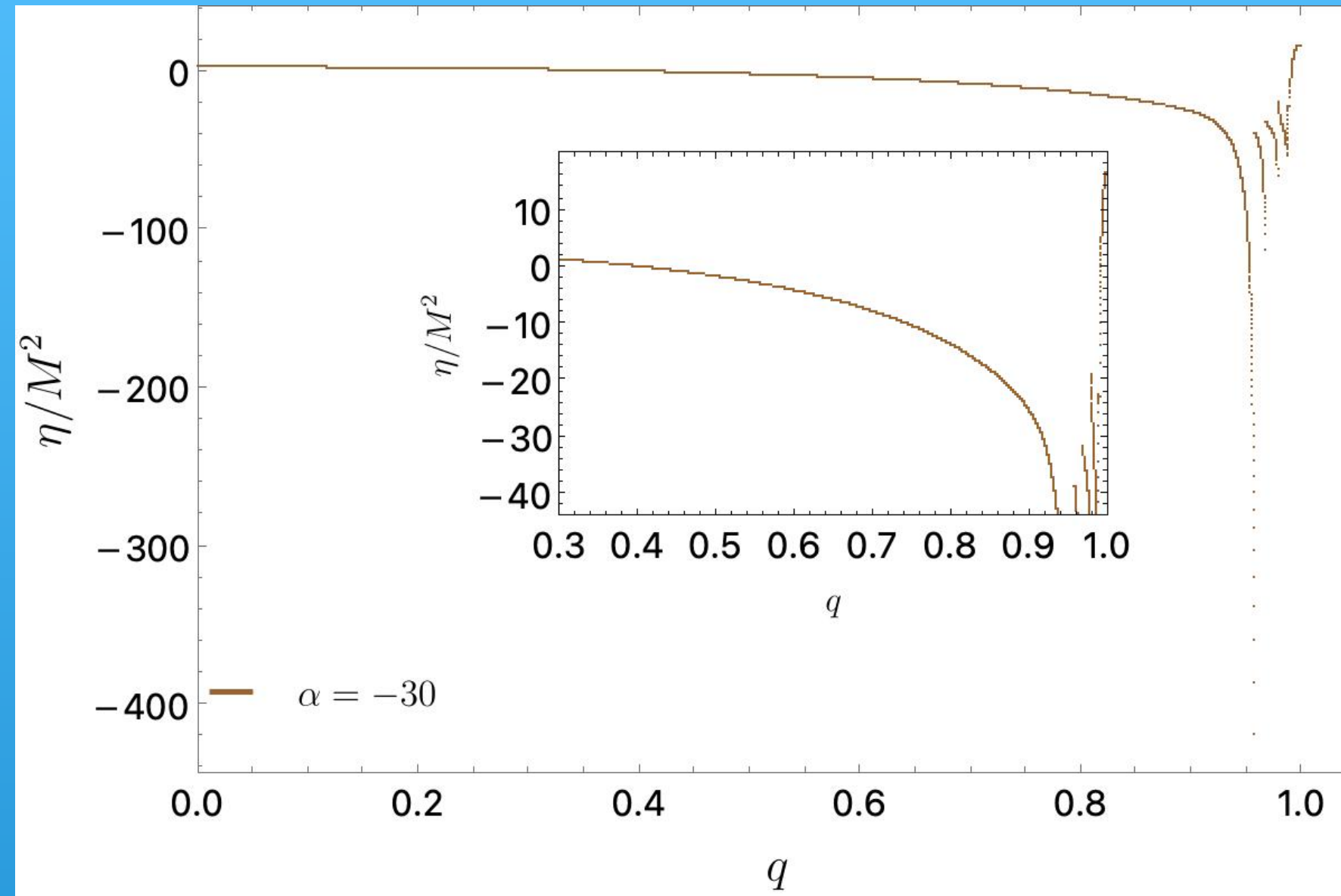


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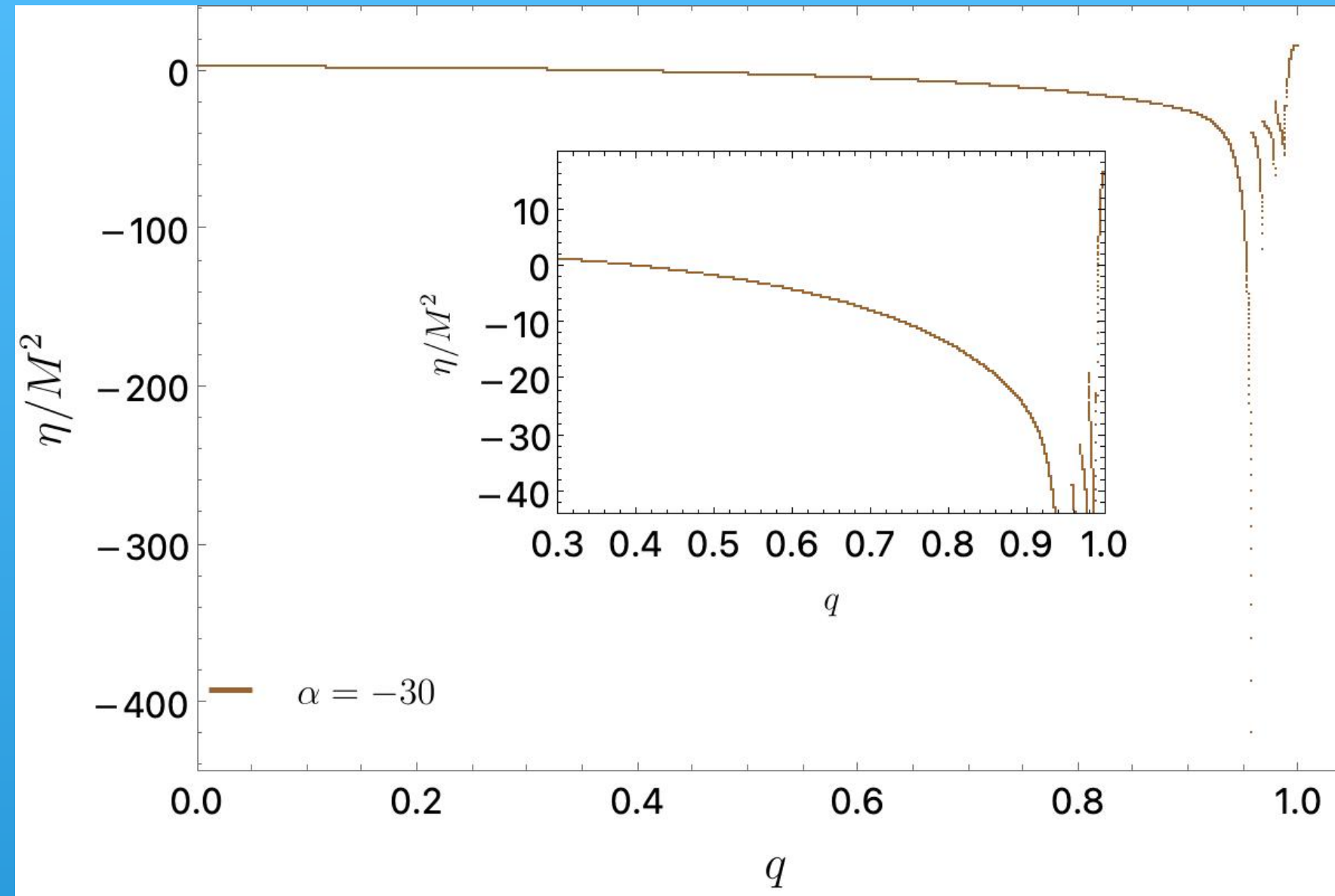


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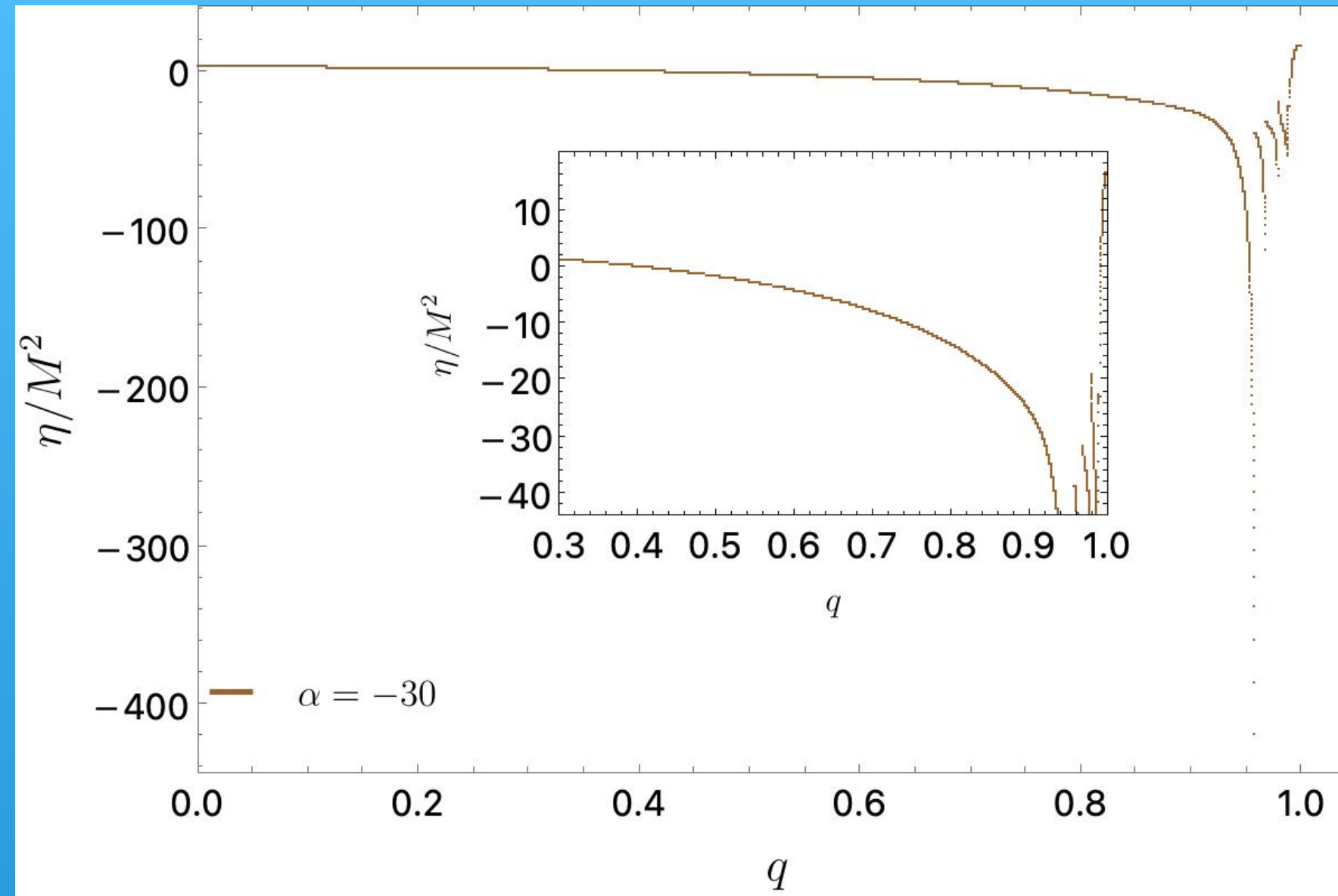
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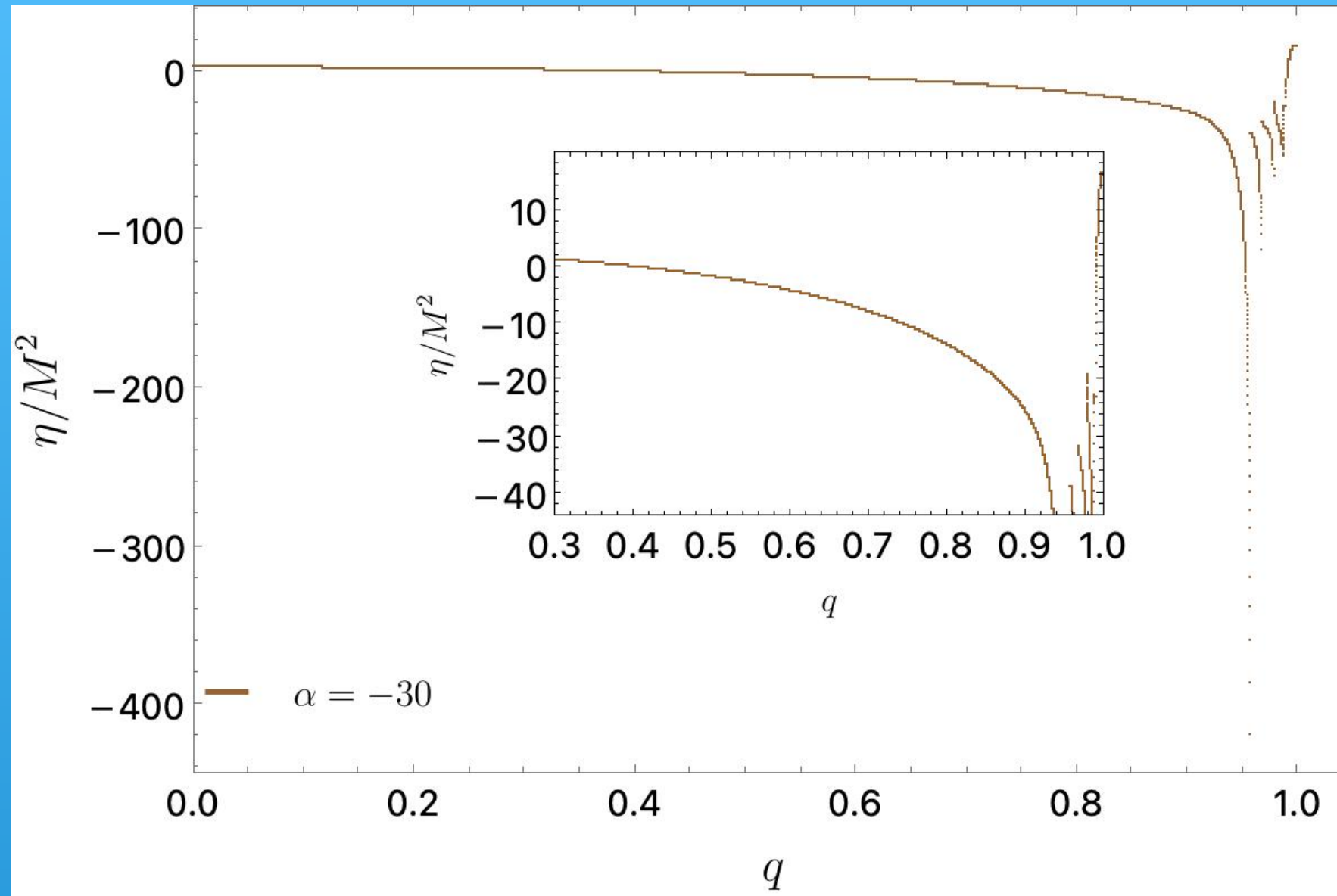
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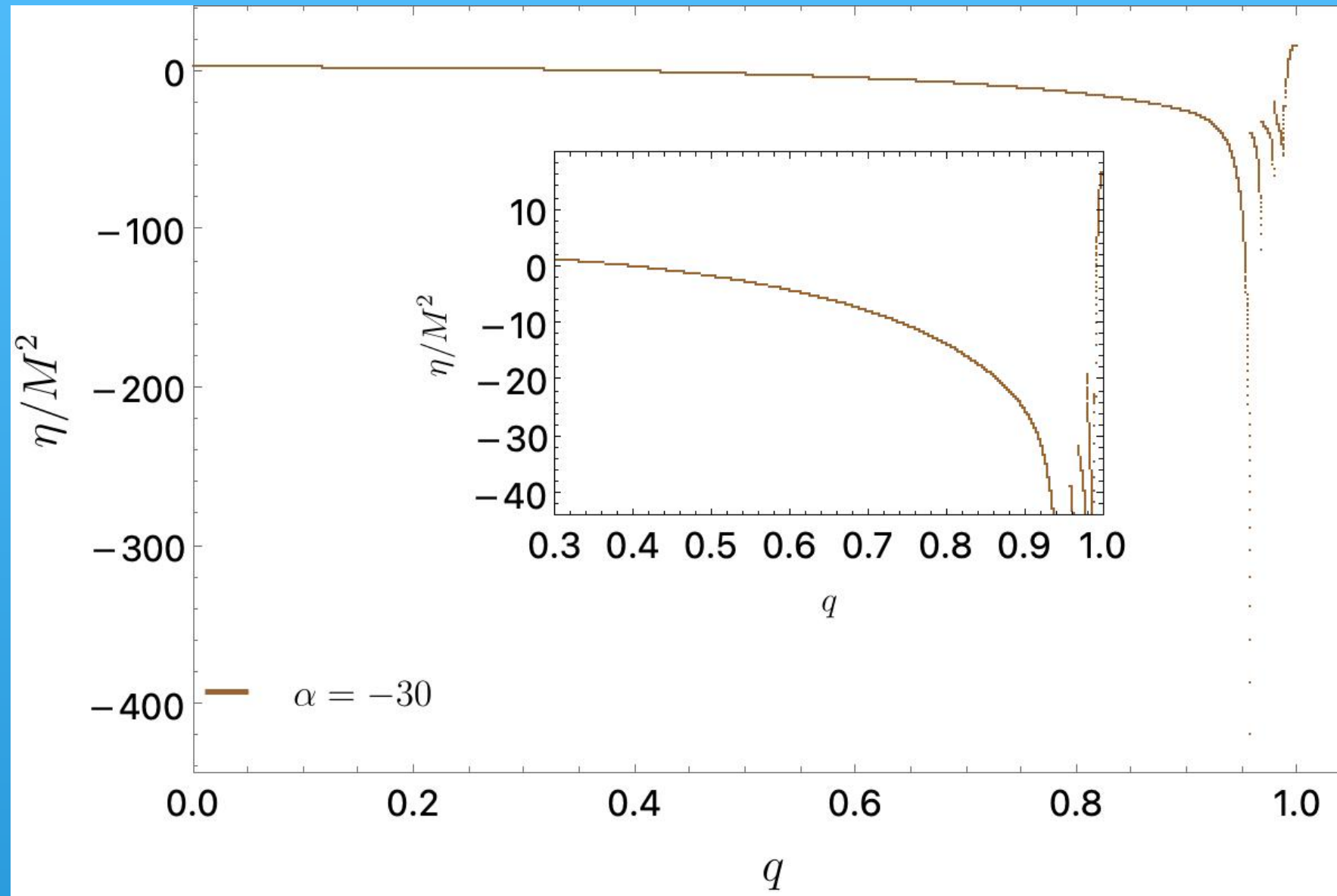
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- Additional branches : Activating the Maxwell coupling generates extra bifurcation branches once $\alpha \leq -2$.

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- If the solver fail we return to the previous solution and adjust the seize. We check also the sign of Δ and mass and charge, and local integration error are around 10^{-10}

Black hole solutions

Black hole solutions

- we focus only on the case where η is positive and α is negative

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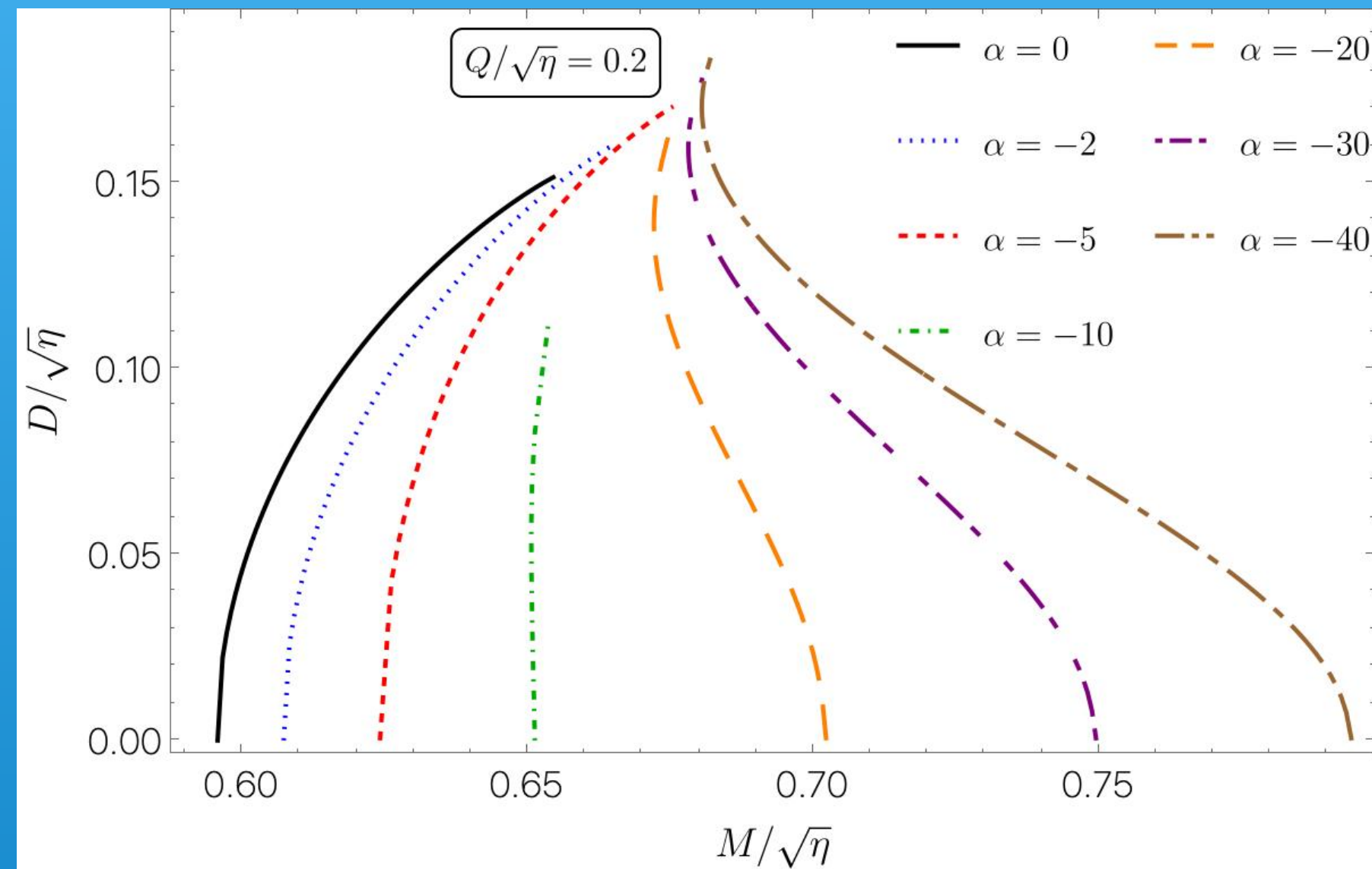
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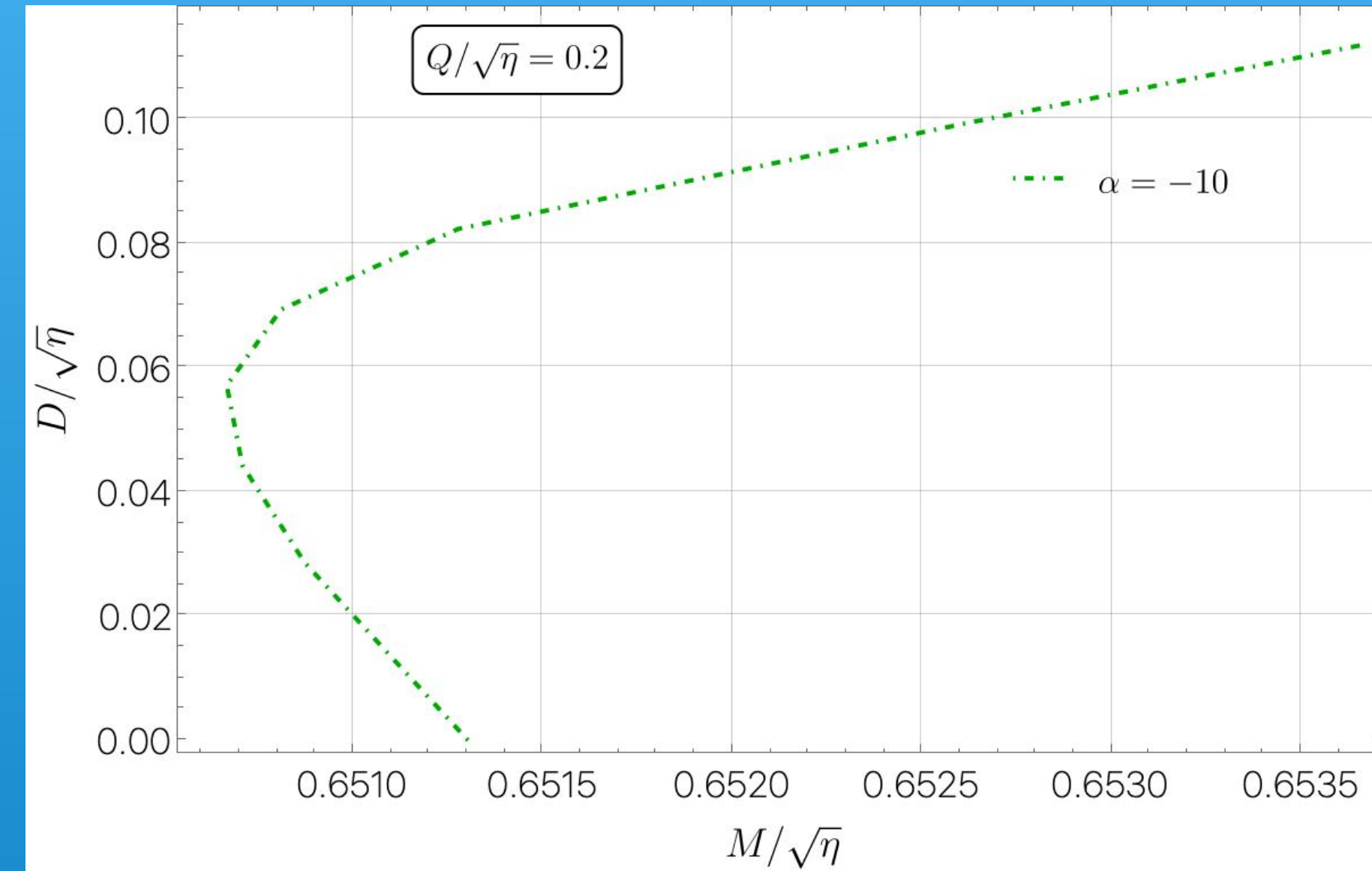
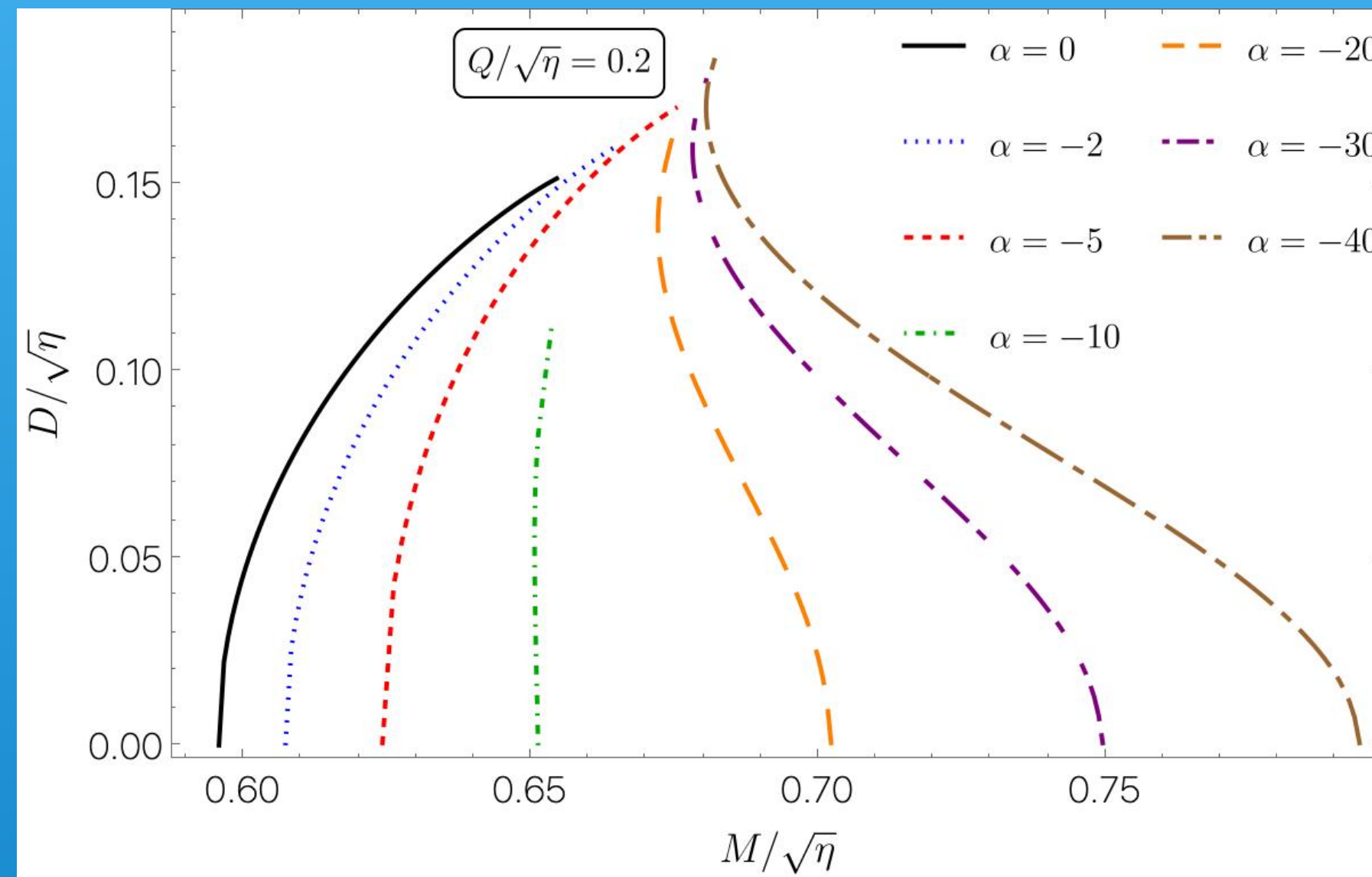
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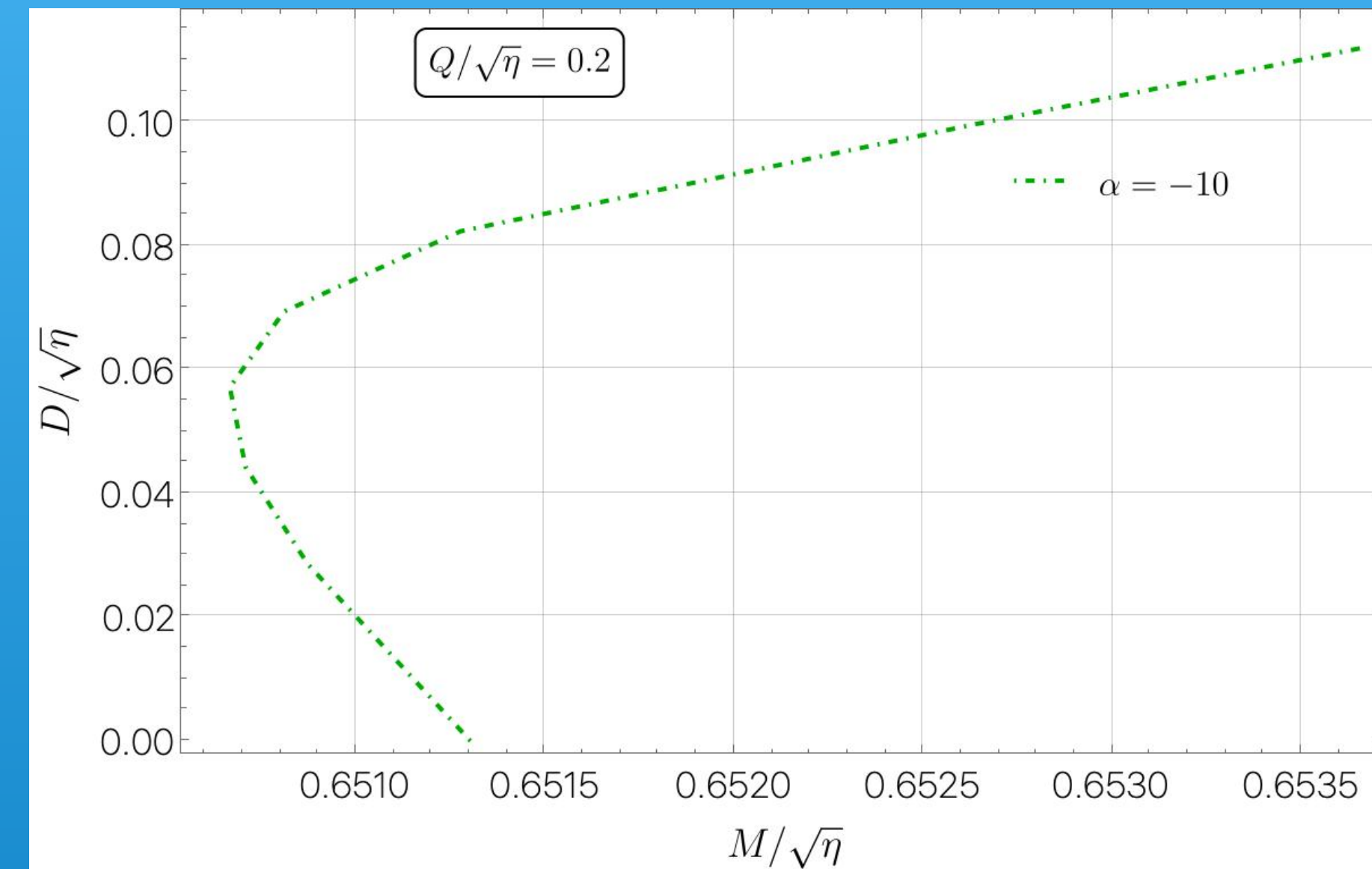
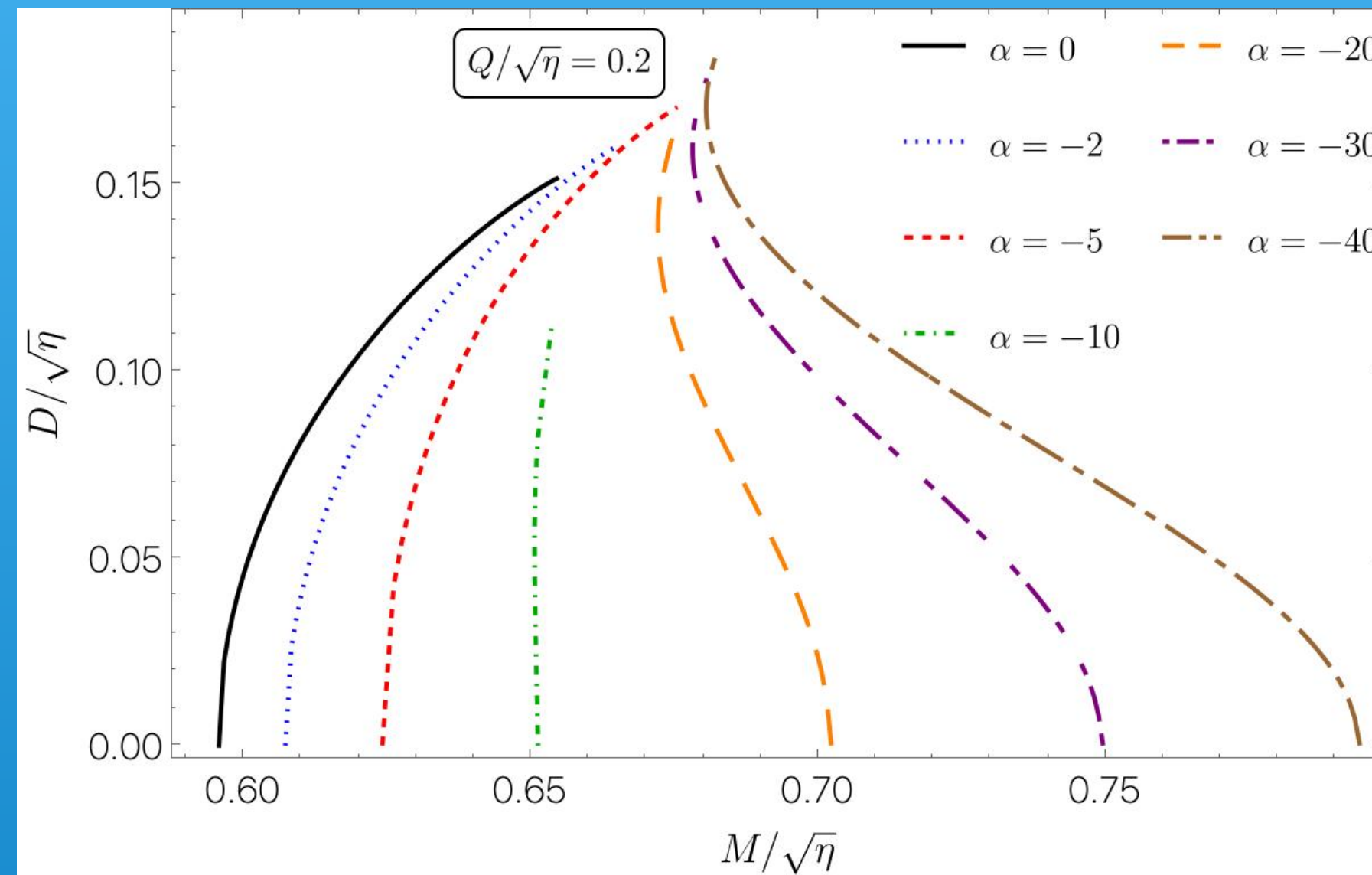
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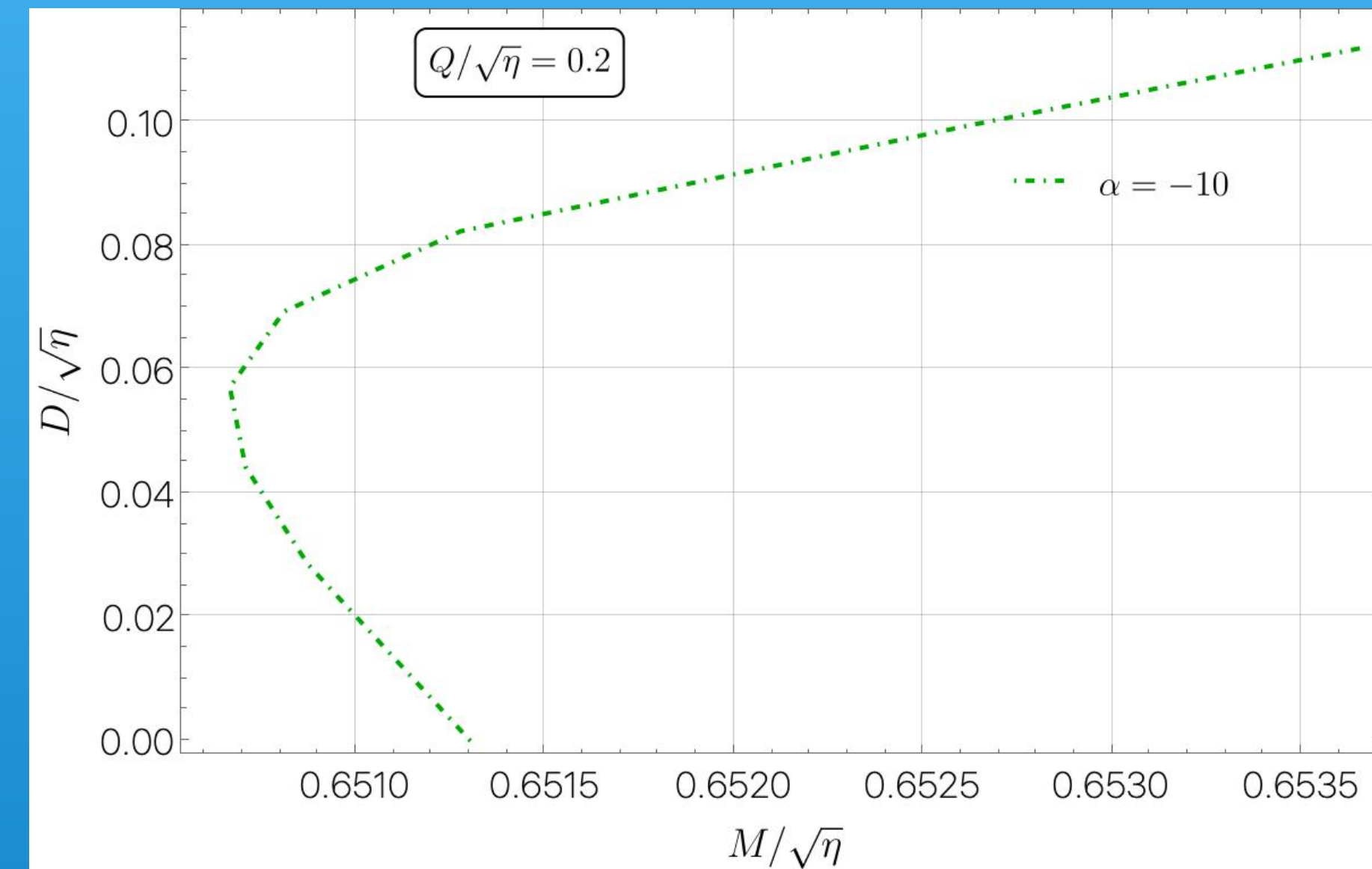
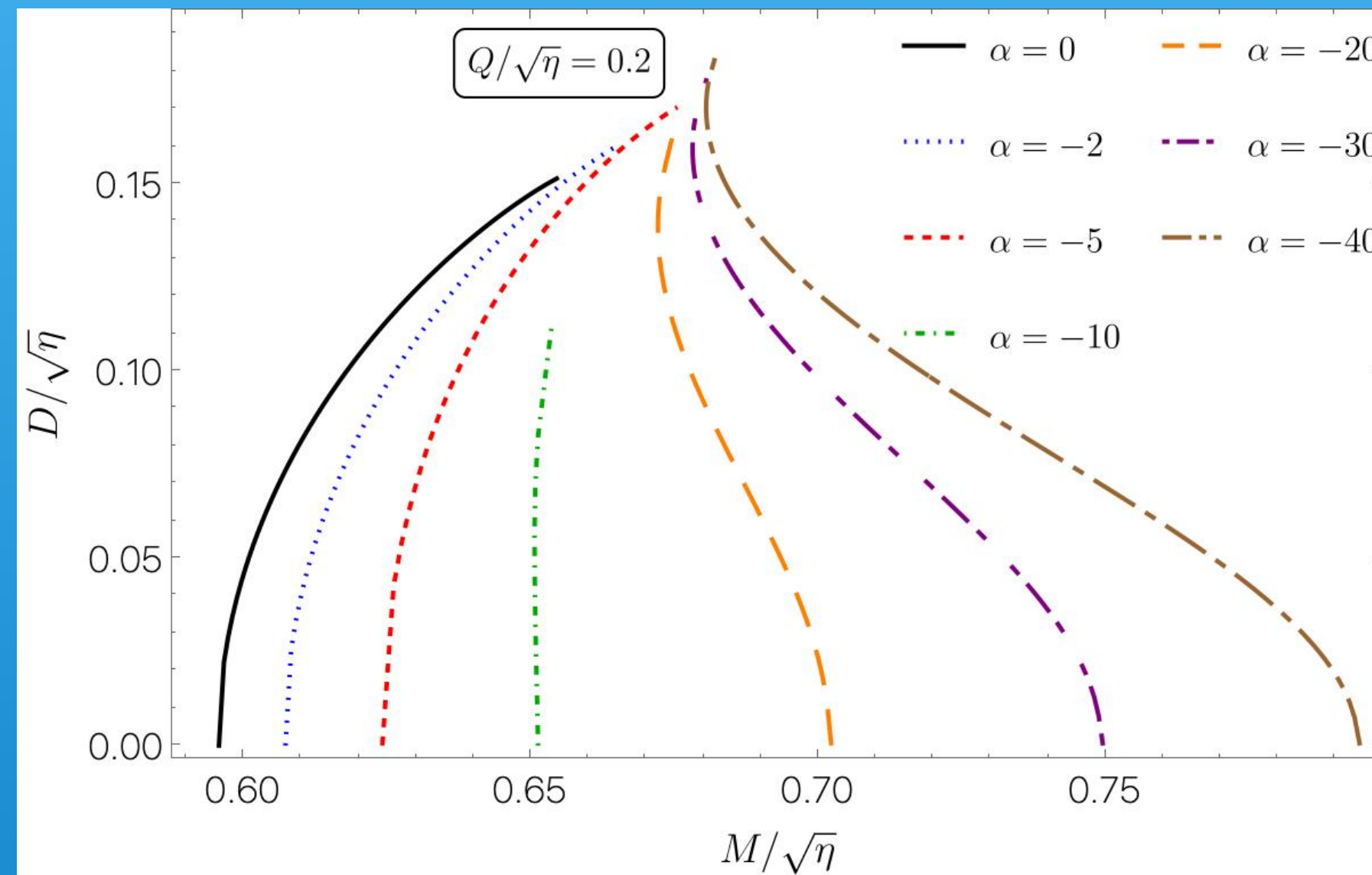


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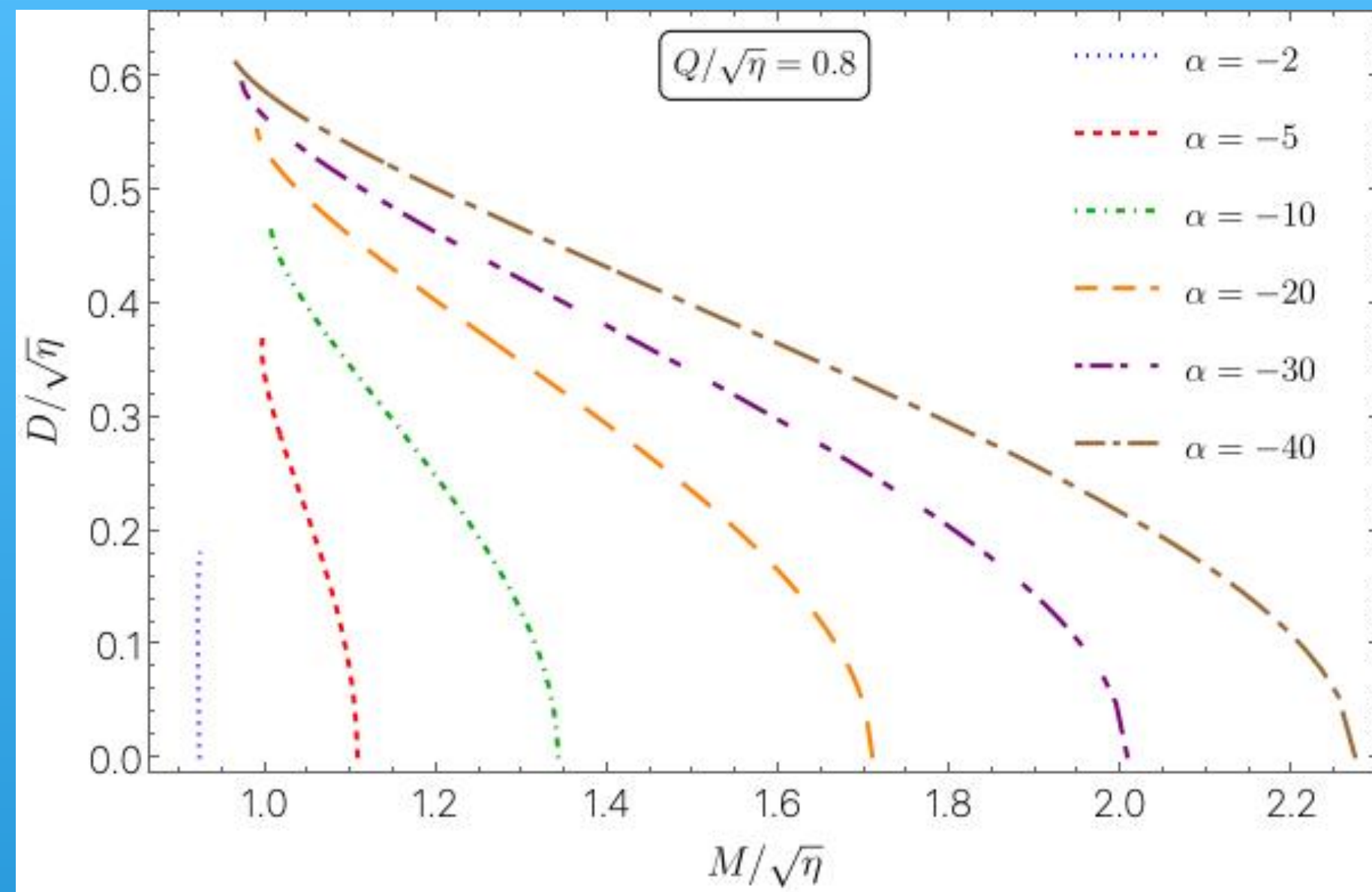
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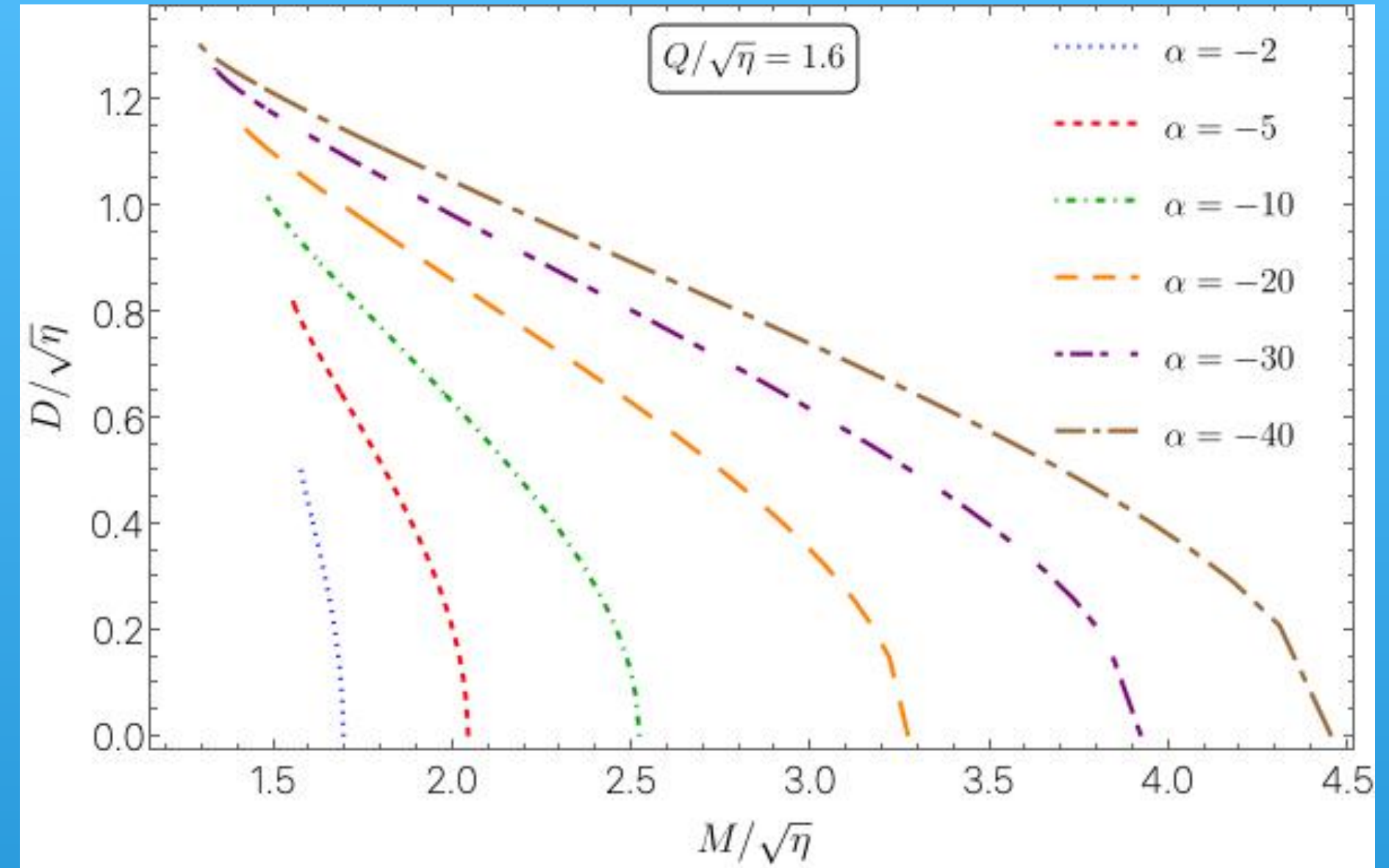
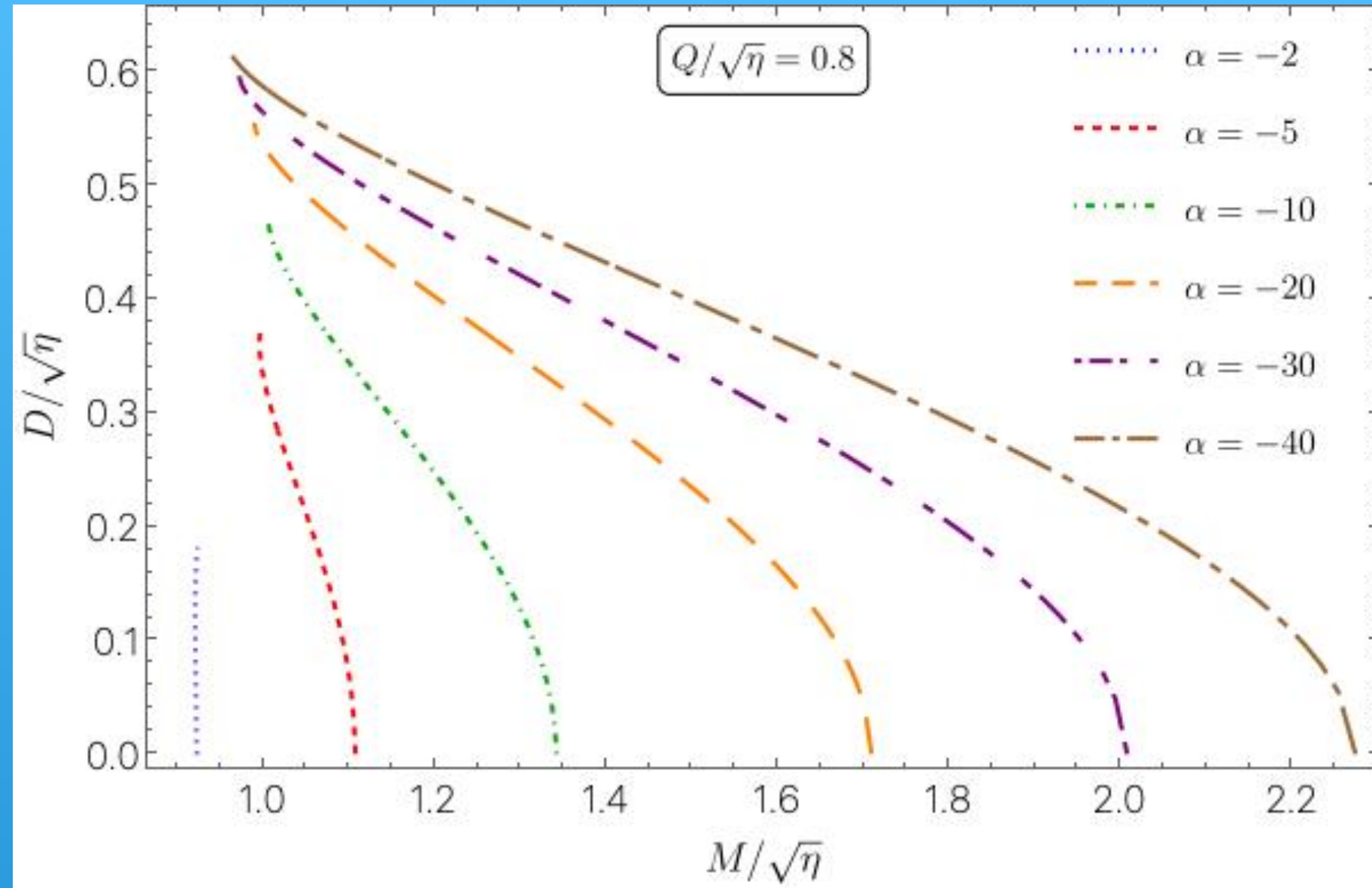
- Varying threshold masses : Each value of leads to a distinct onset mass for scalarization, unlike simpler EsGB models or EsGB with a Ricci , where all branches share one bifurcation point.
- At Low electric charge : at $Q/\sqrt{\eta} = 0.2$ solutions with $\alpha = 0, -2, -5$ move toward higher masses , while the solutions with $\alpha = -20, -30, -40$ extend to lower masses, the case of $\alpha = -10$ has a turning point

Black hole solutions

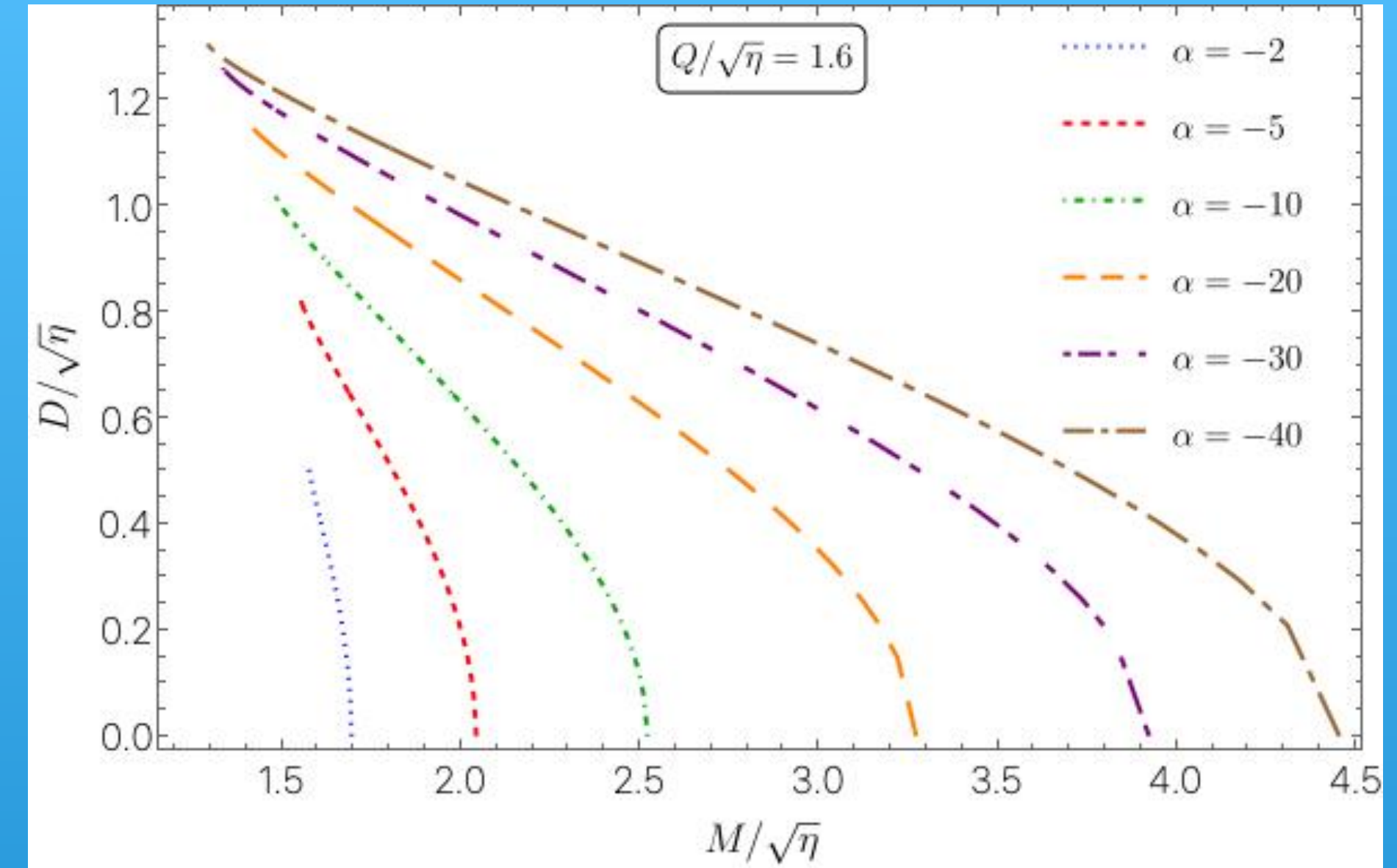
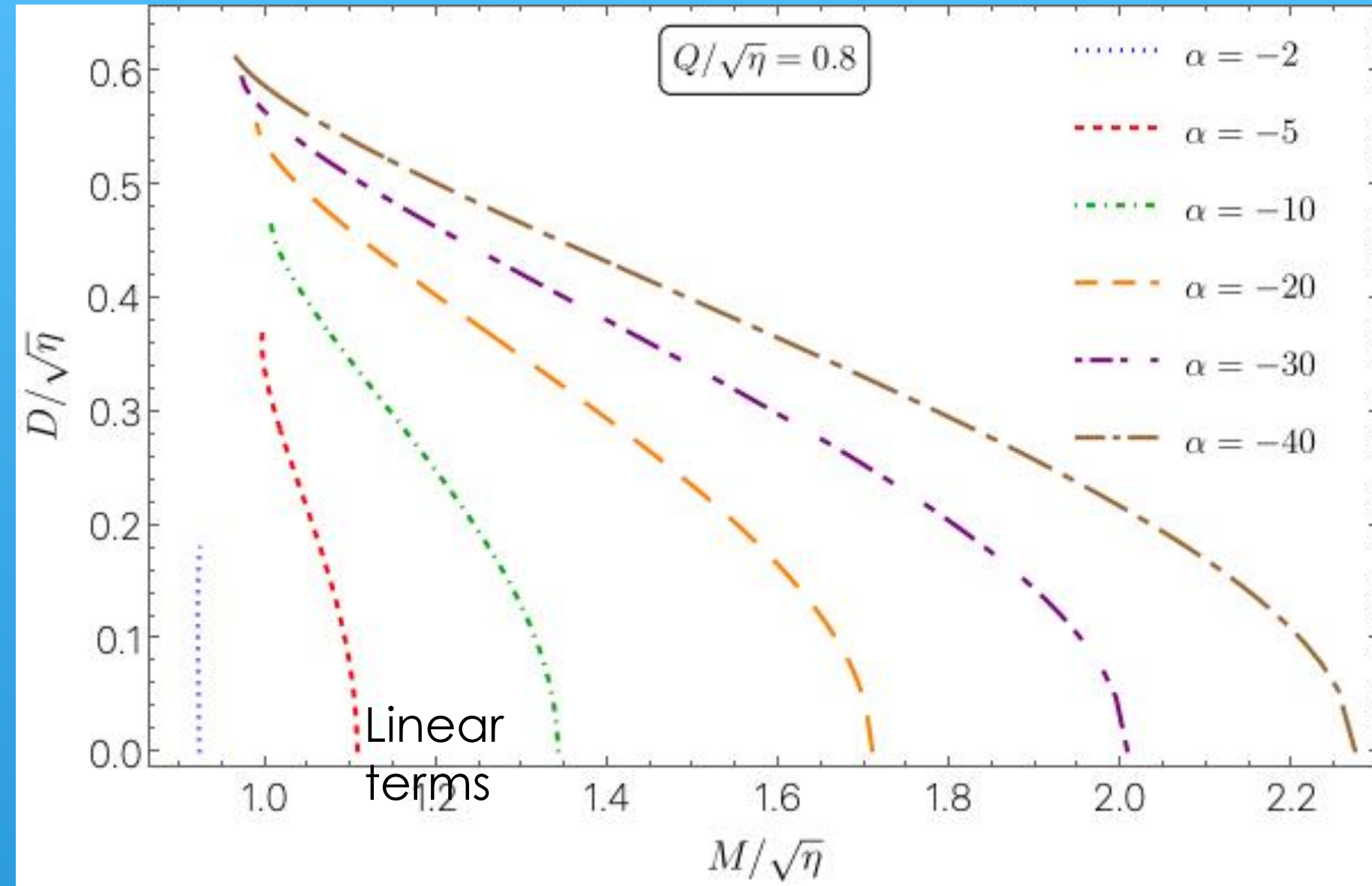
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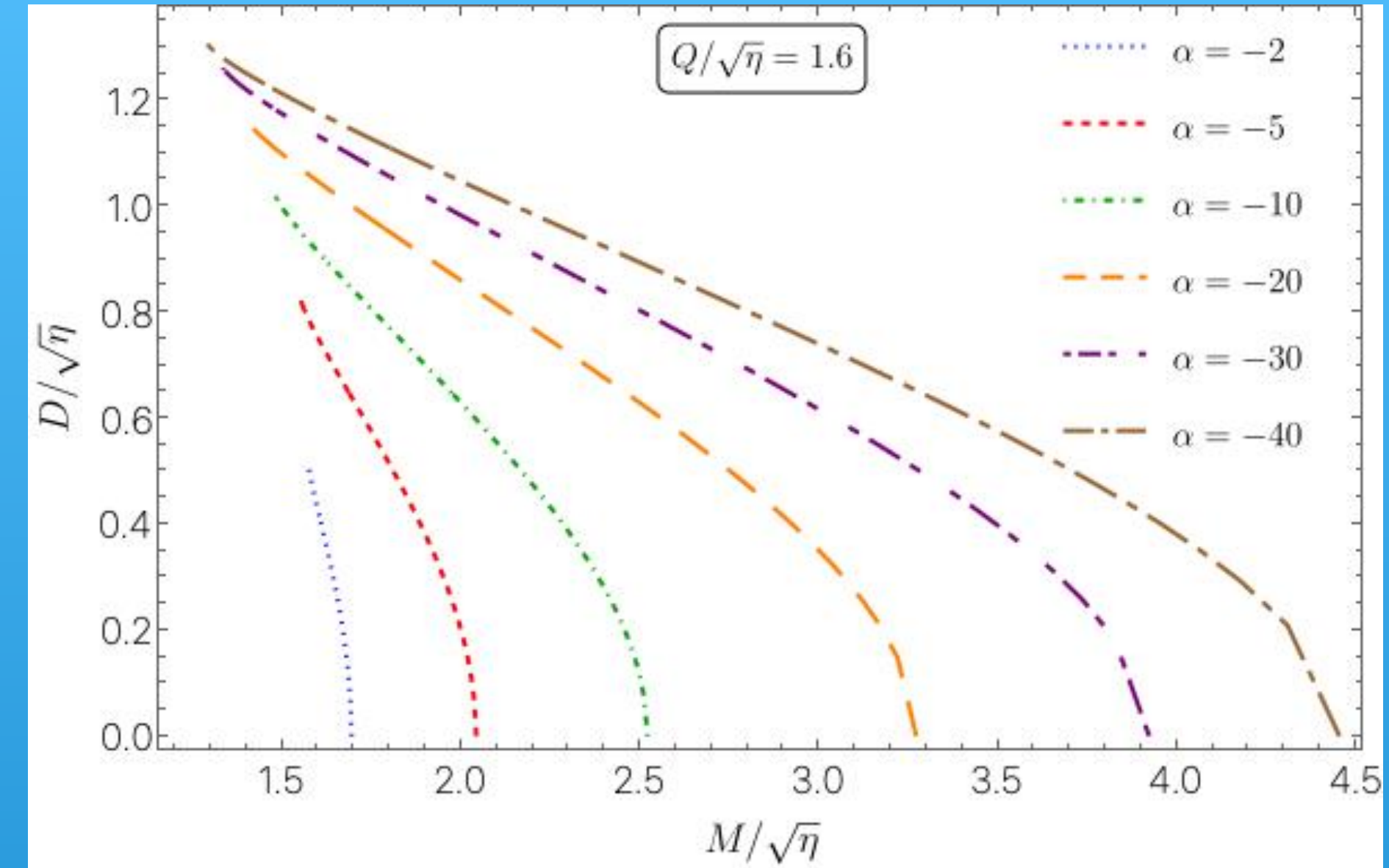
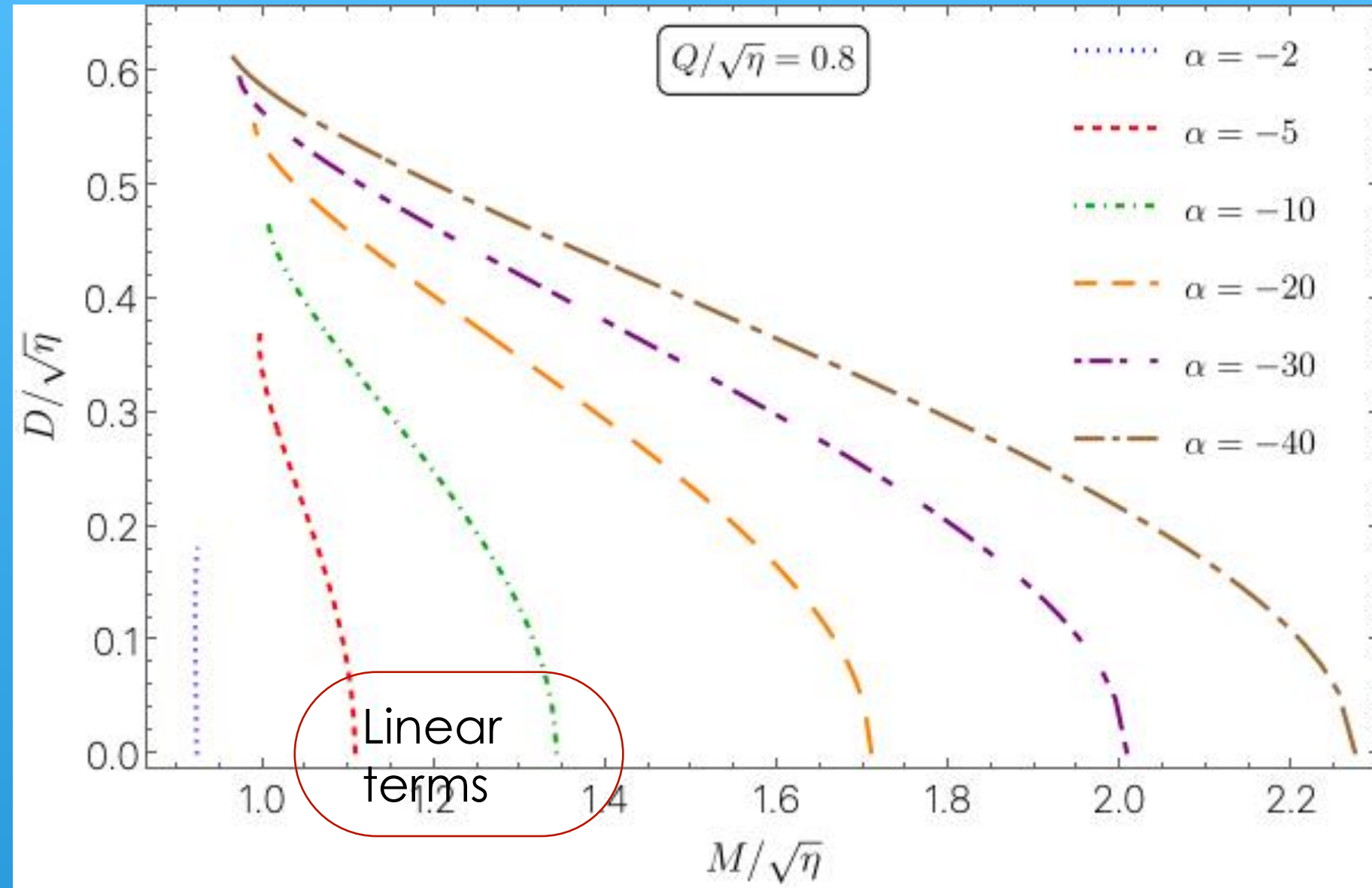
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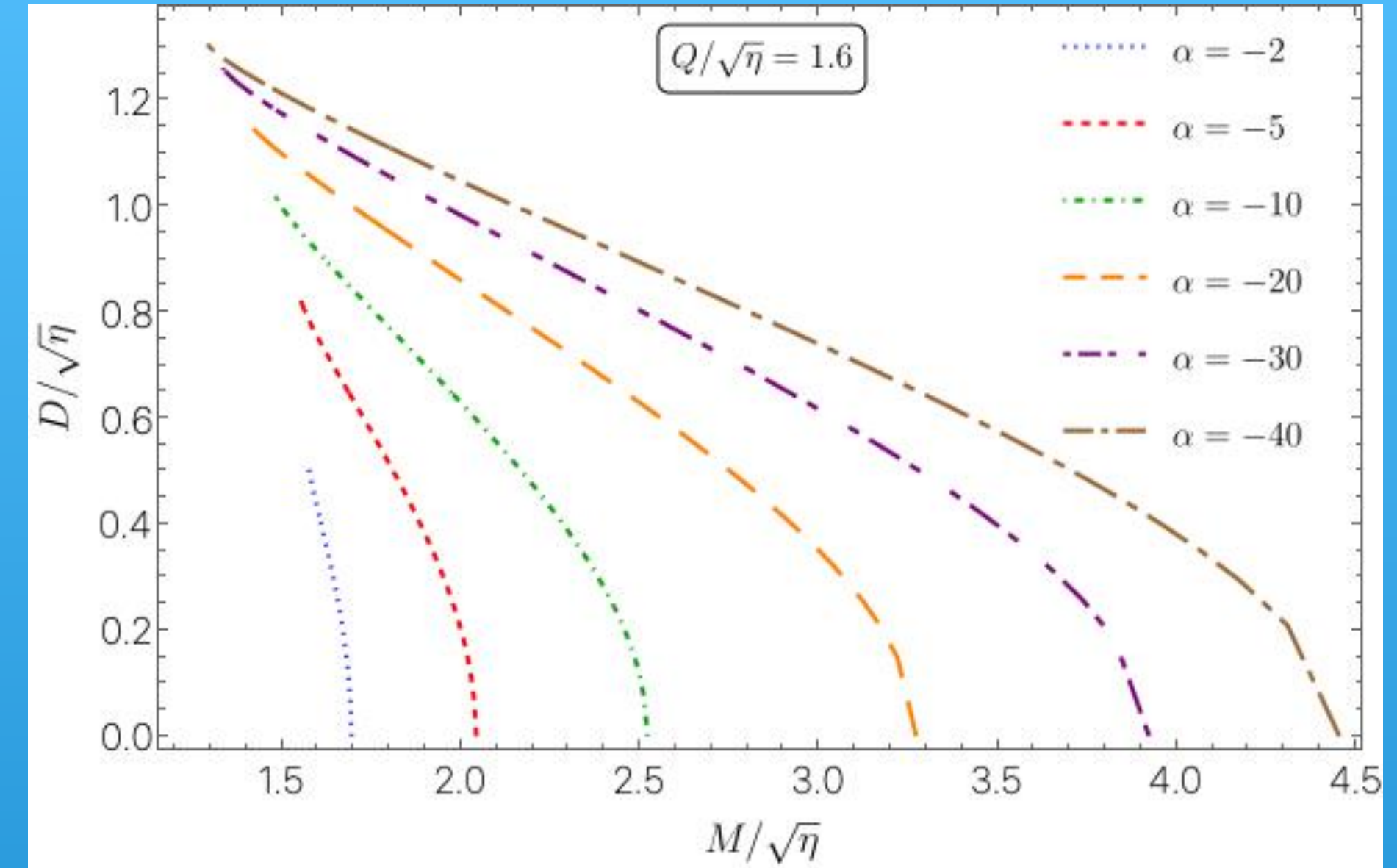
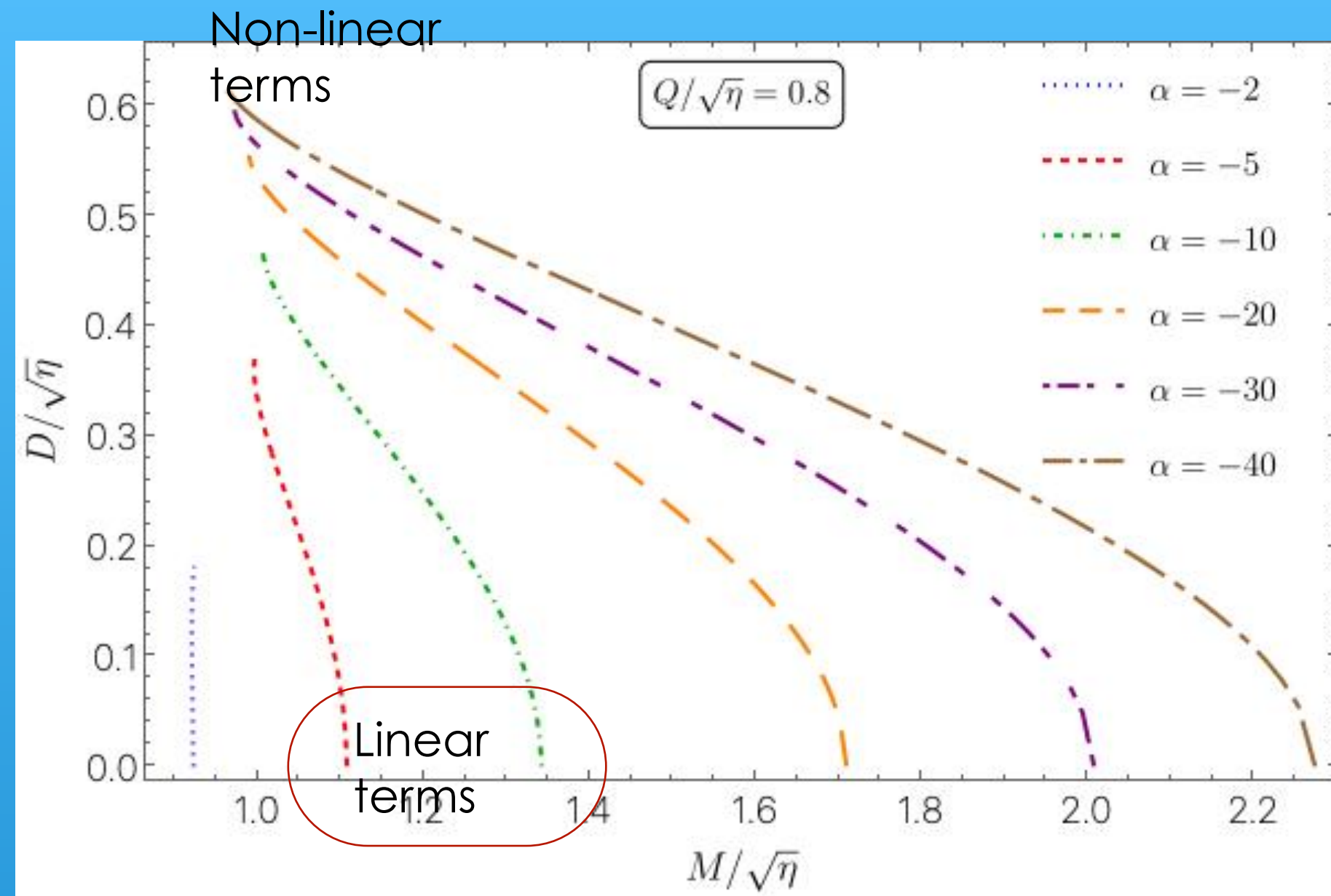
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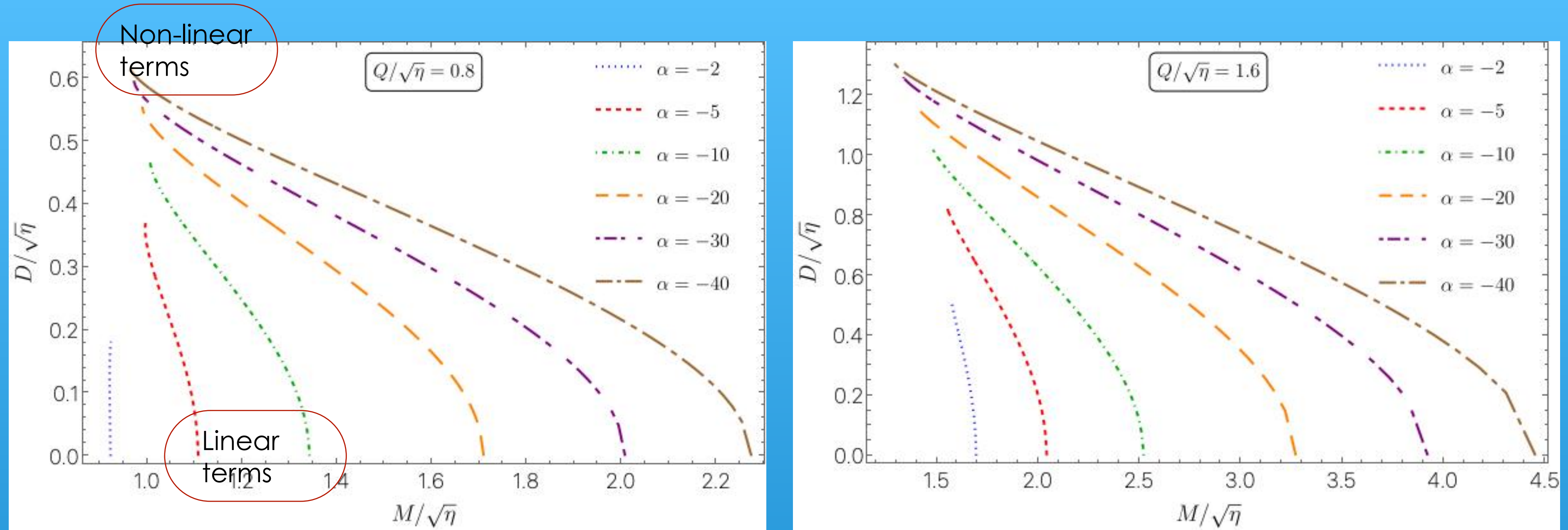
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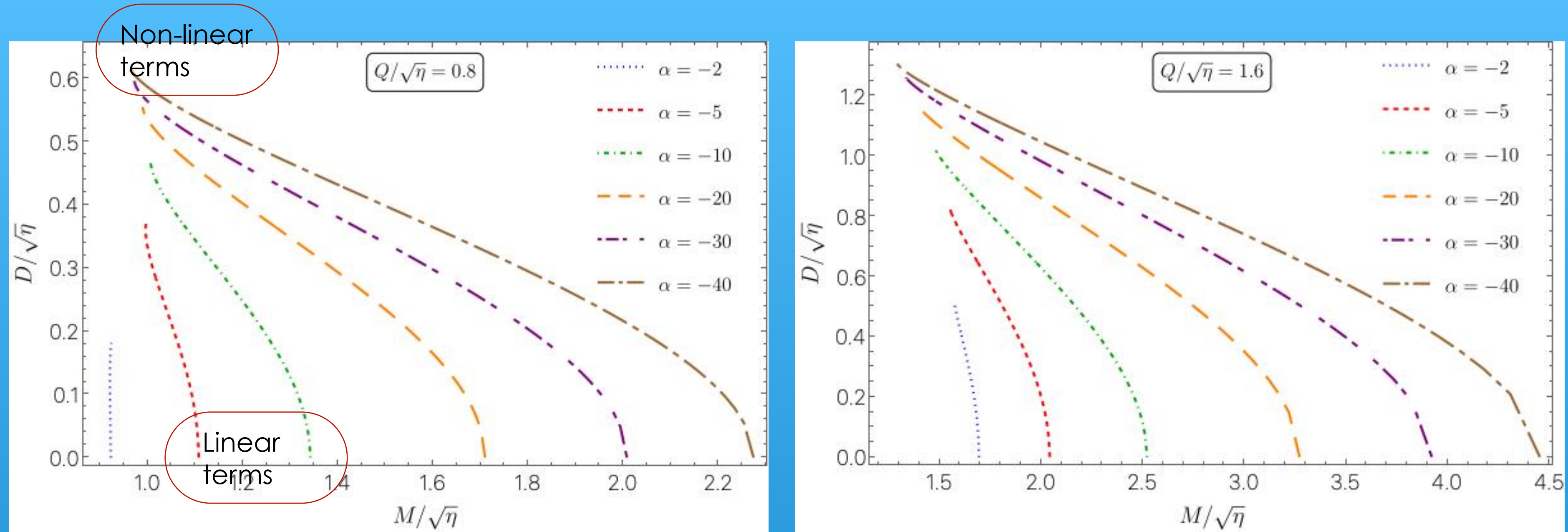
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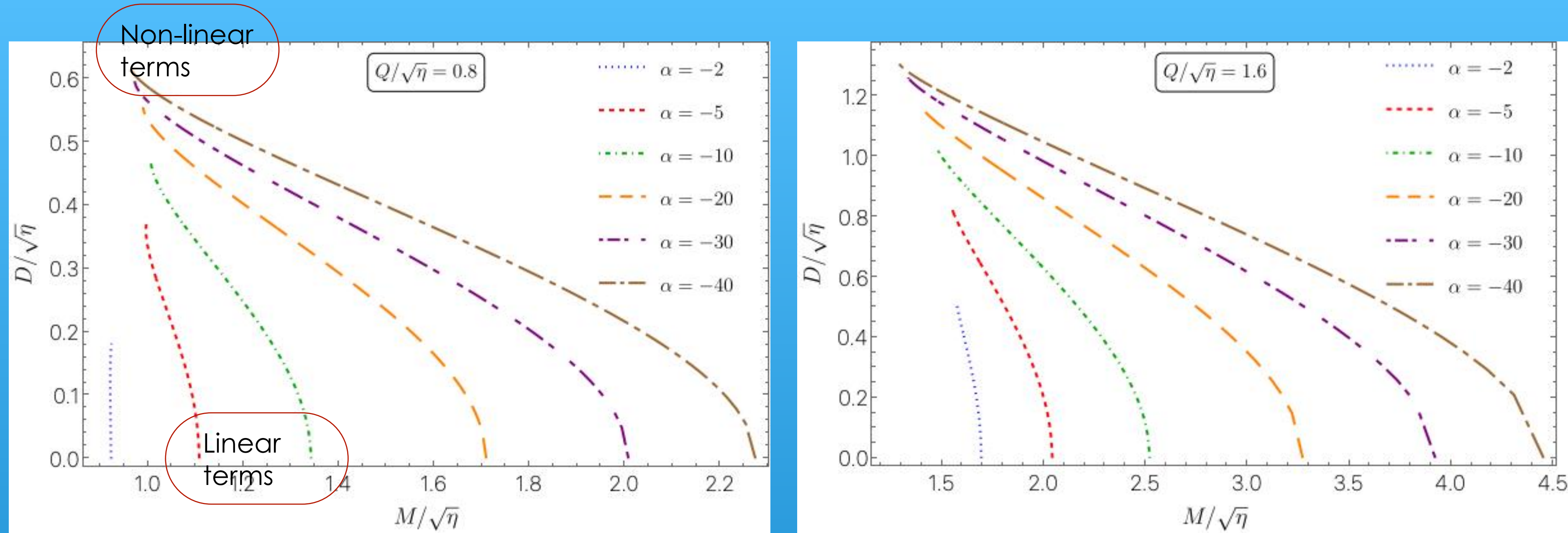


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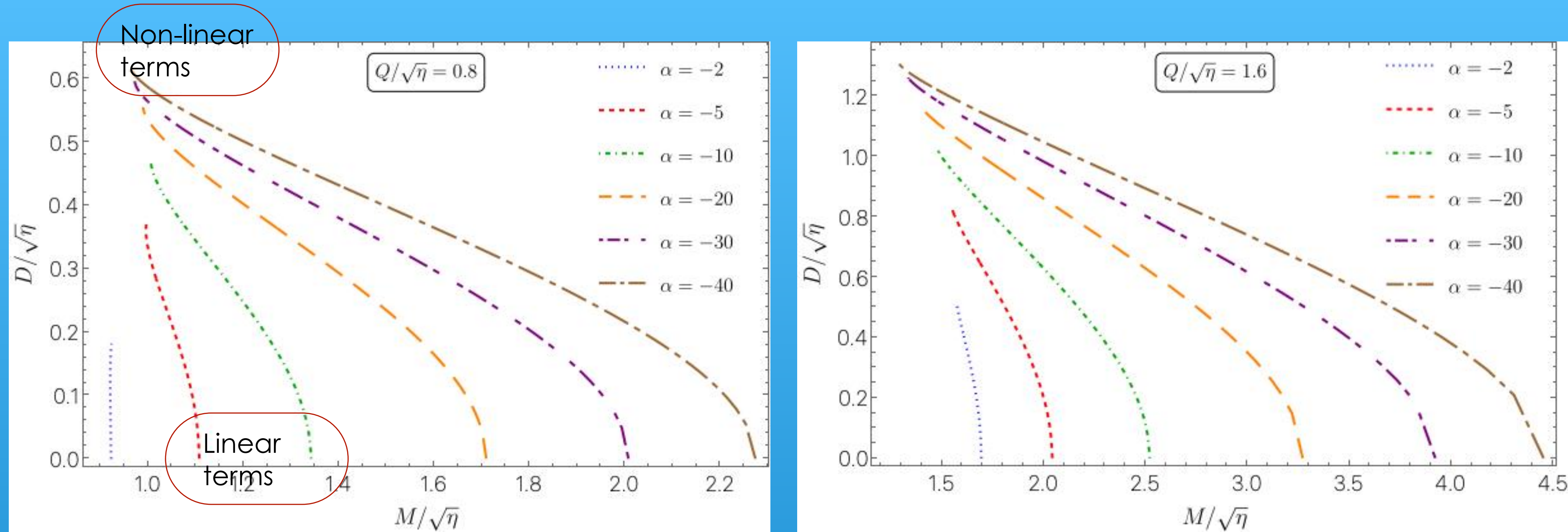
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- Convergent end-point : Despite different onset masses and trajectories, many branches end comparable final black hole states. (it may indicate a presence of an attractor).

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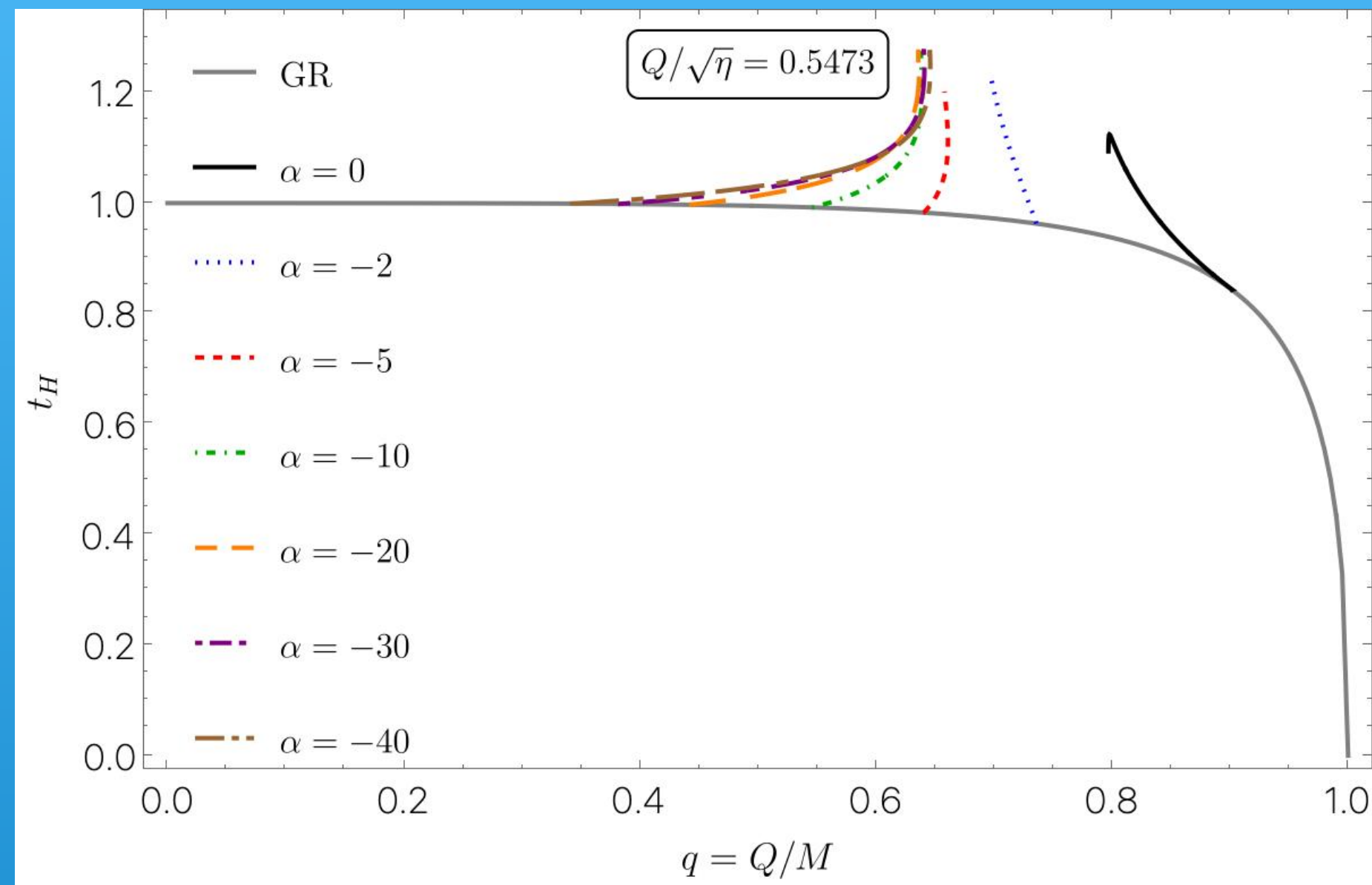
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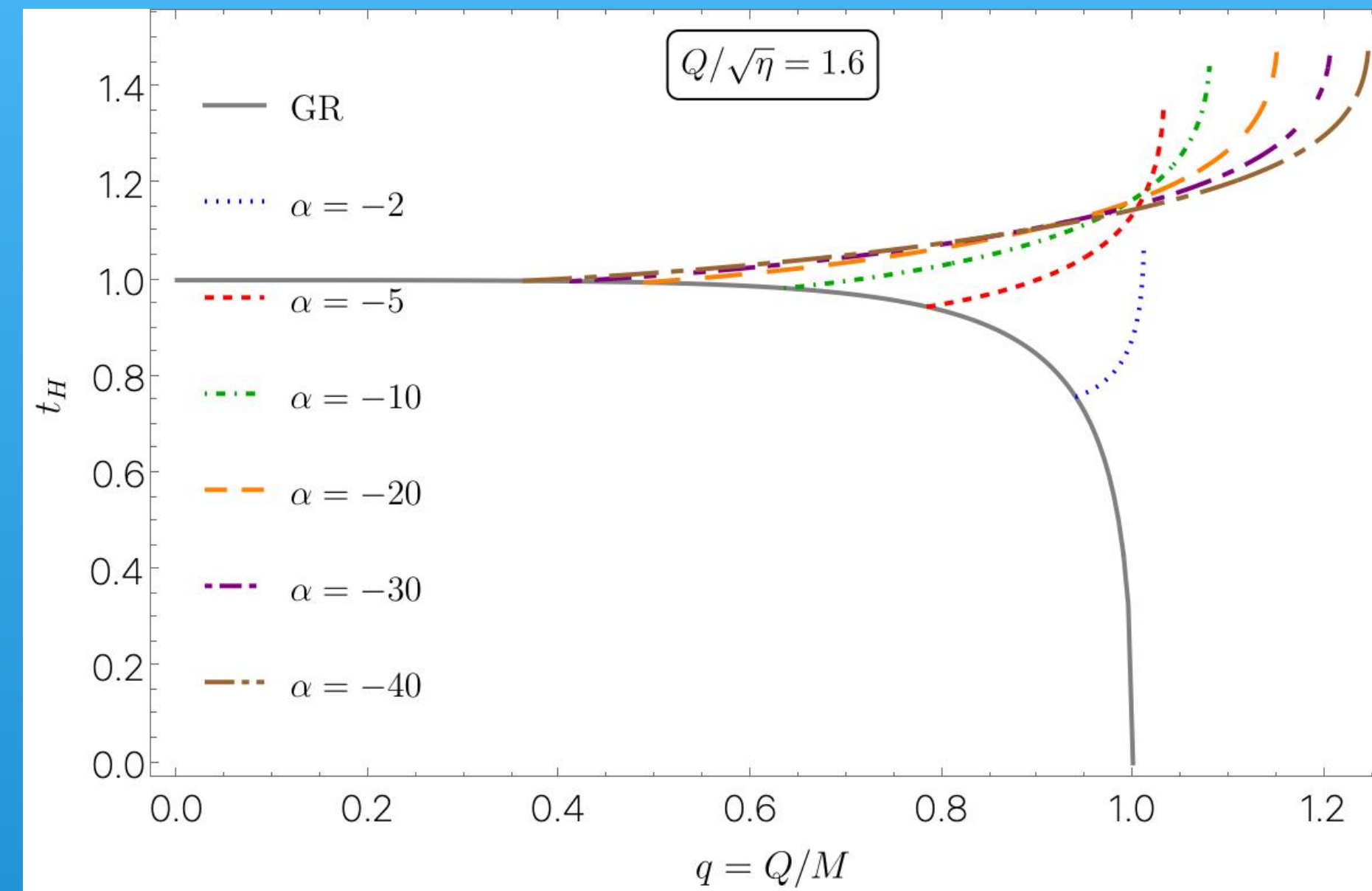
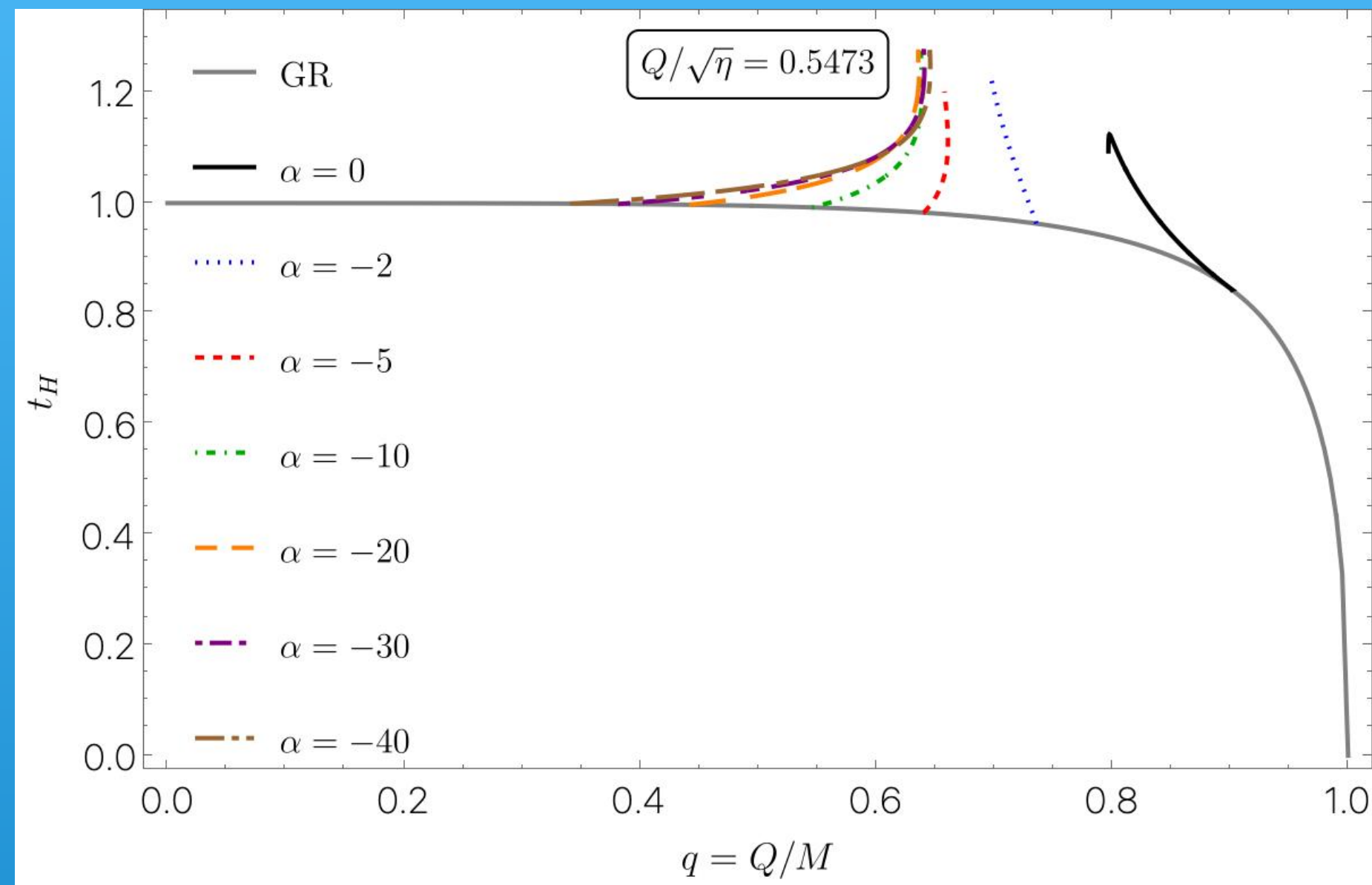
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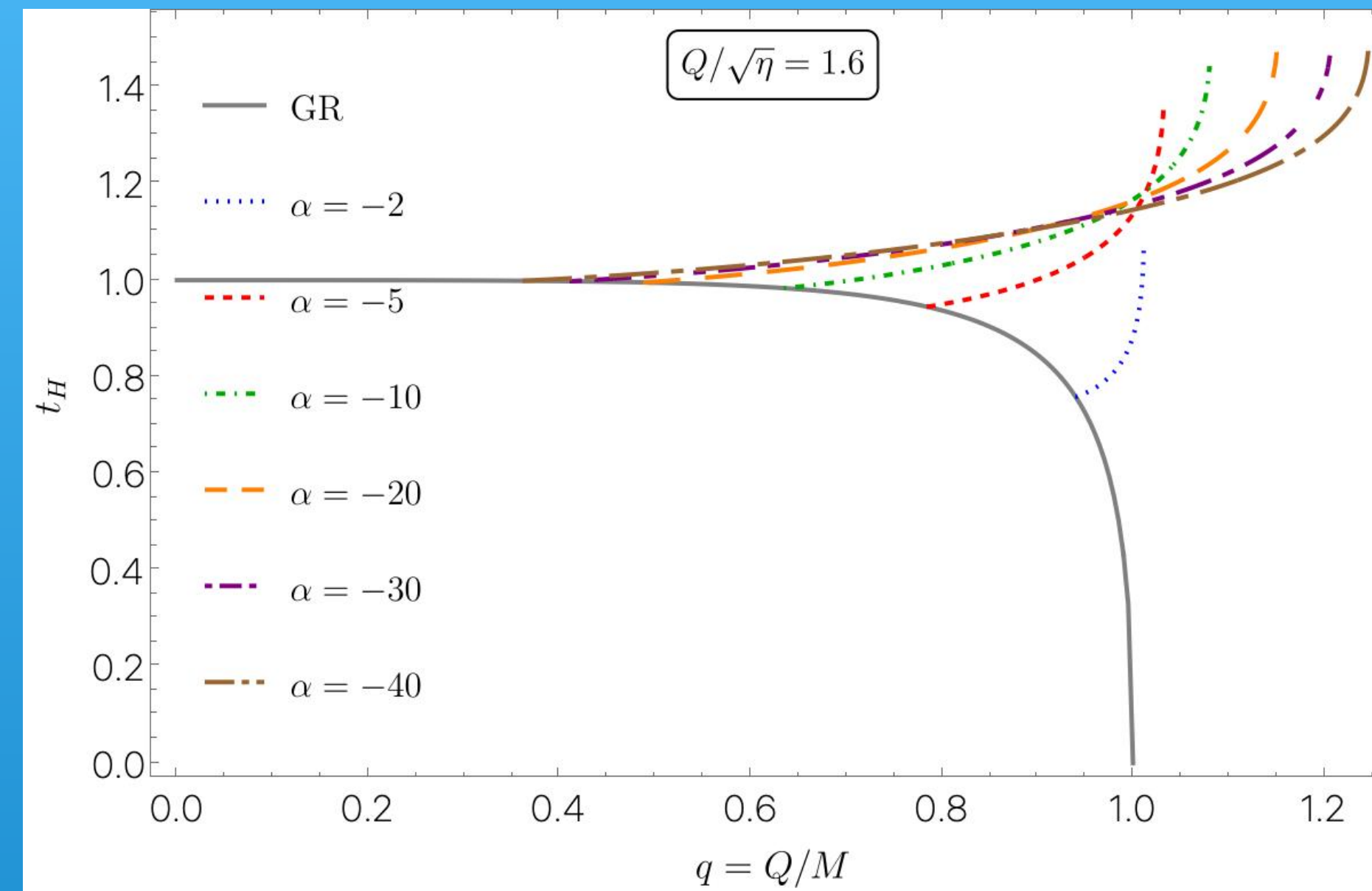
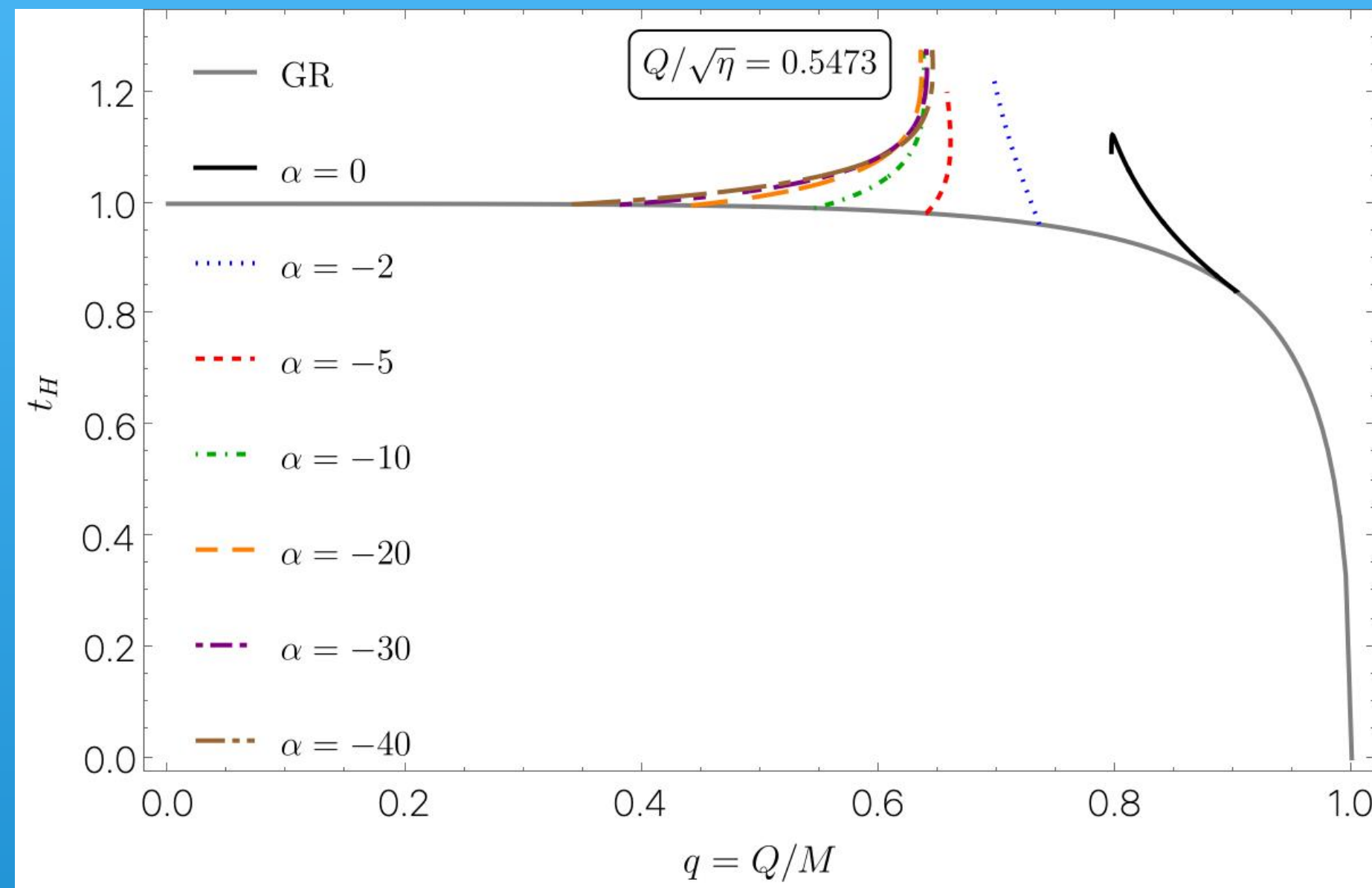
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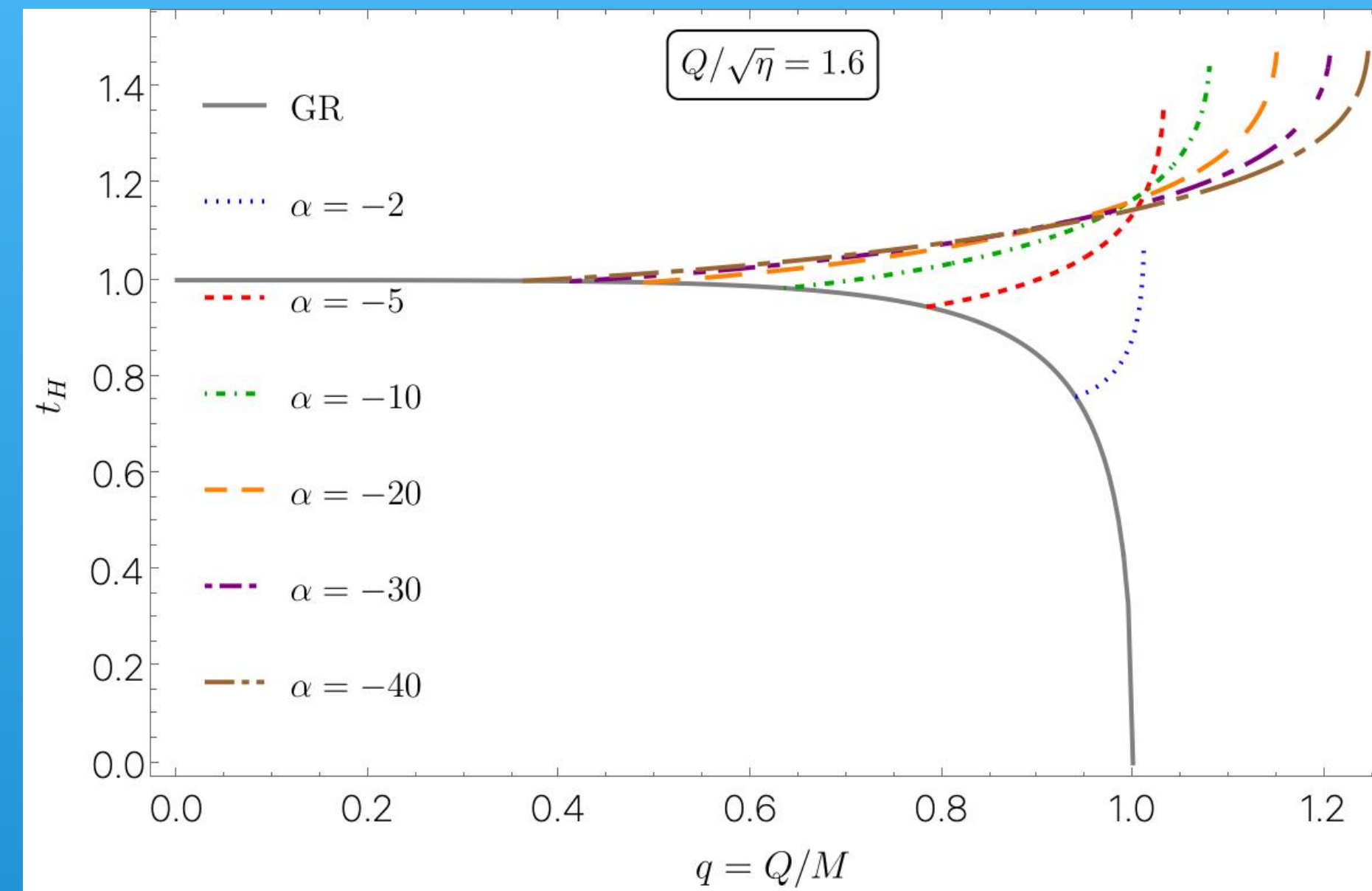
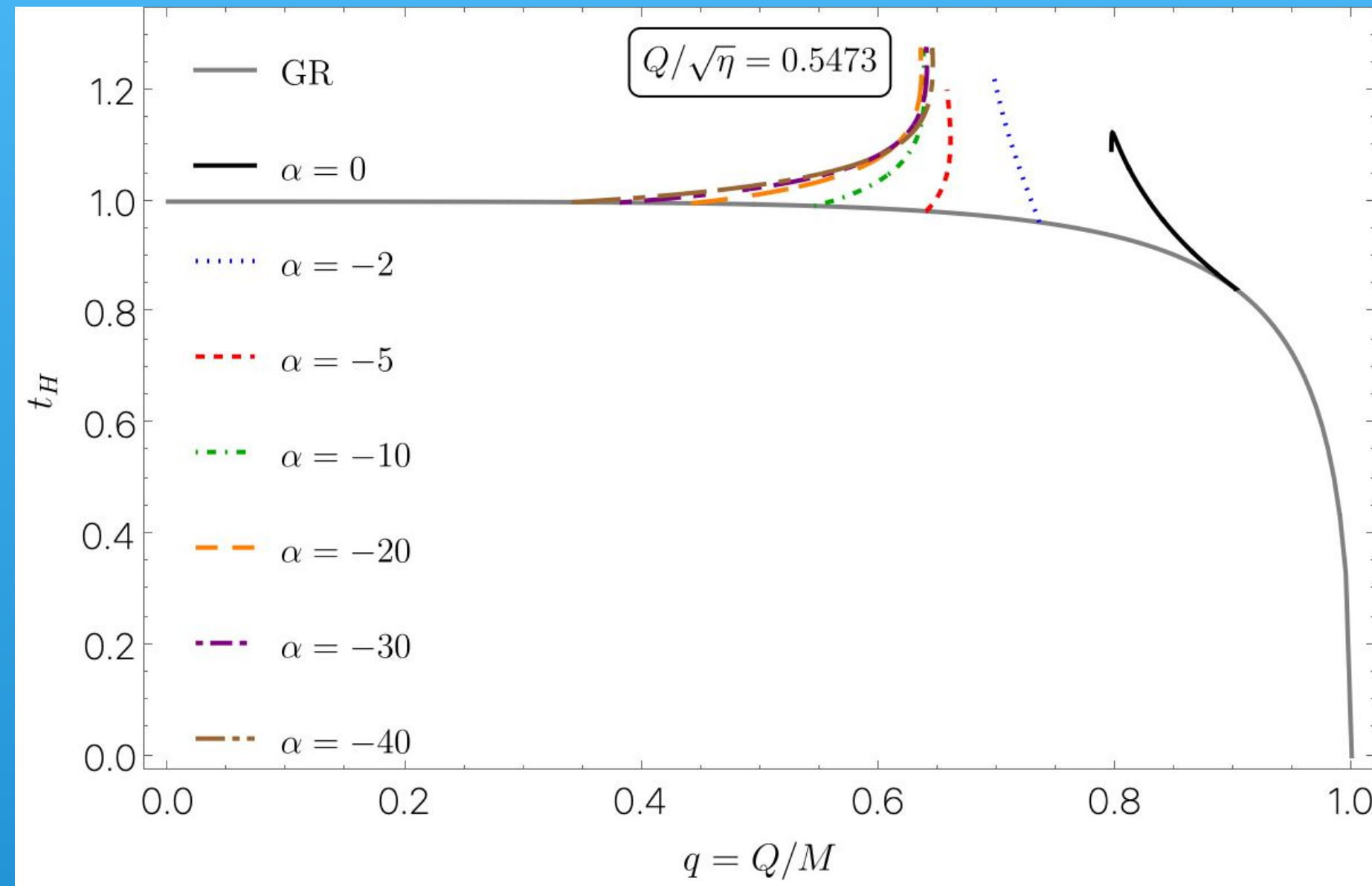


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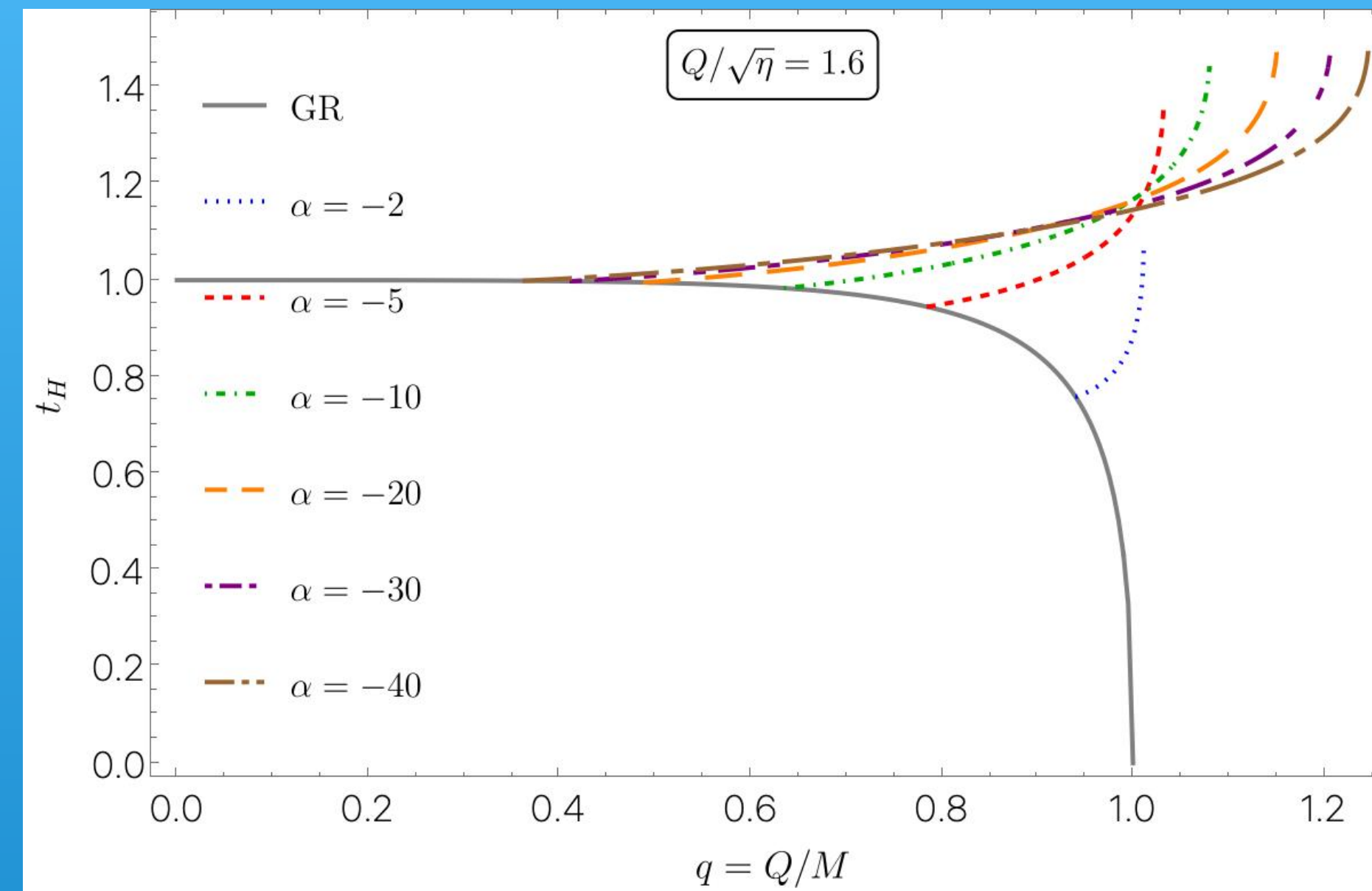
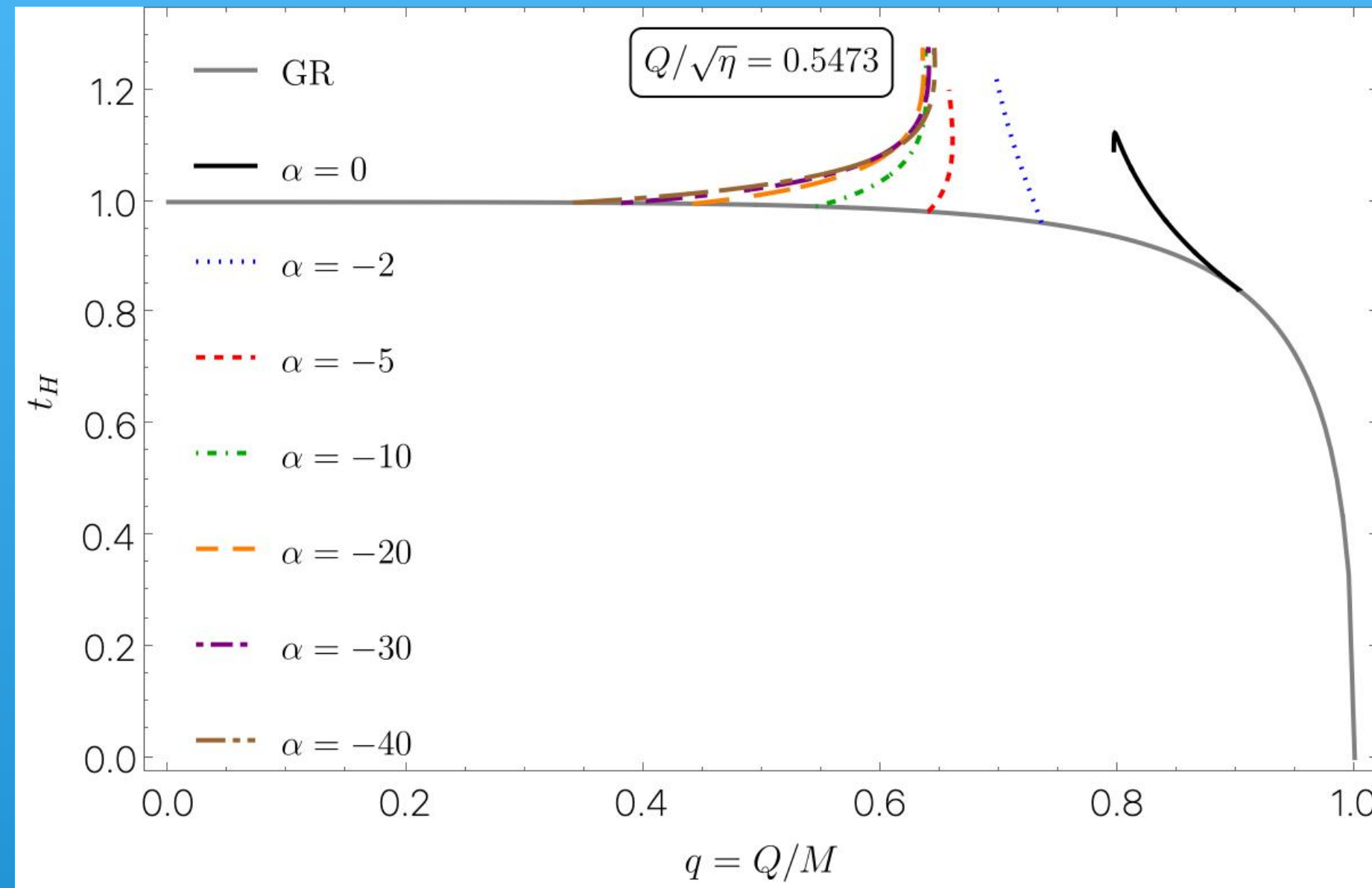


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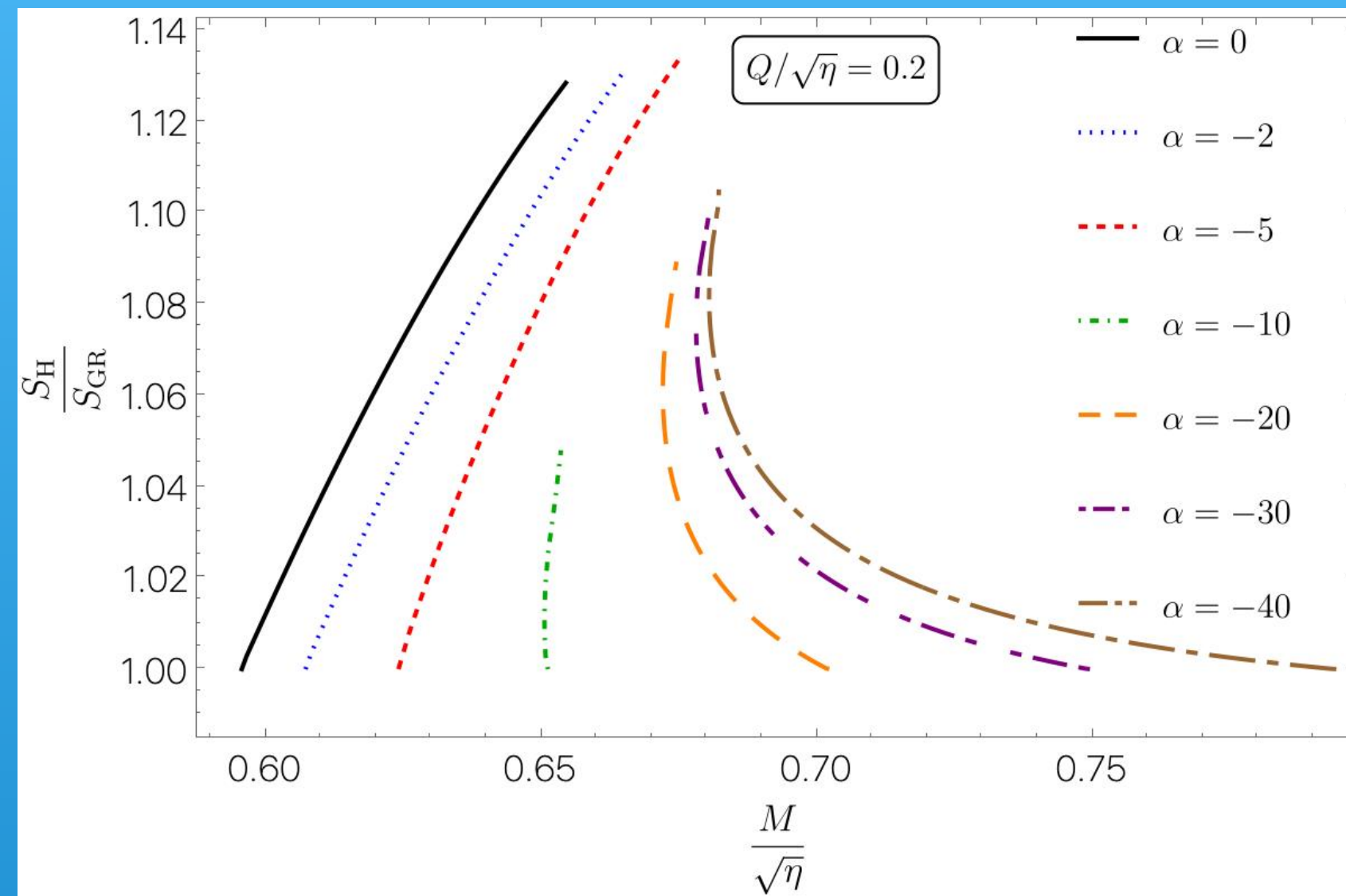
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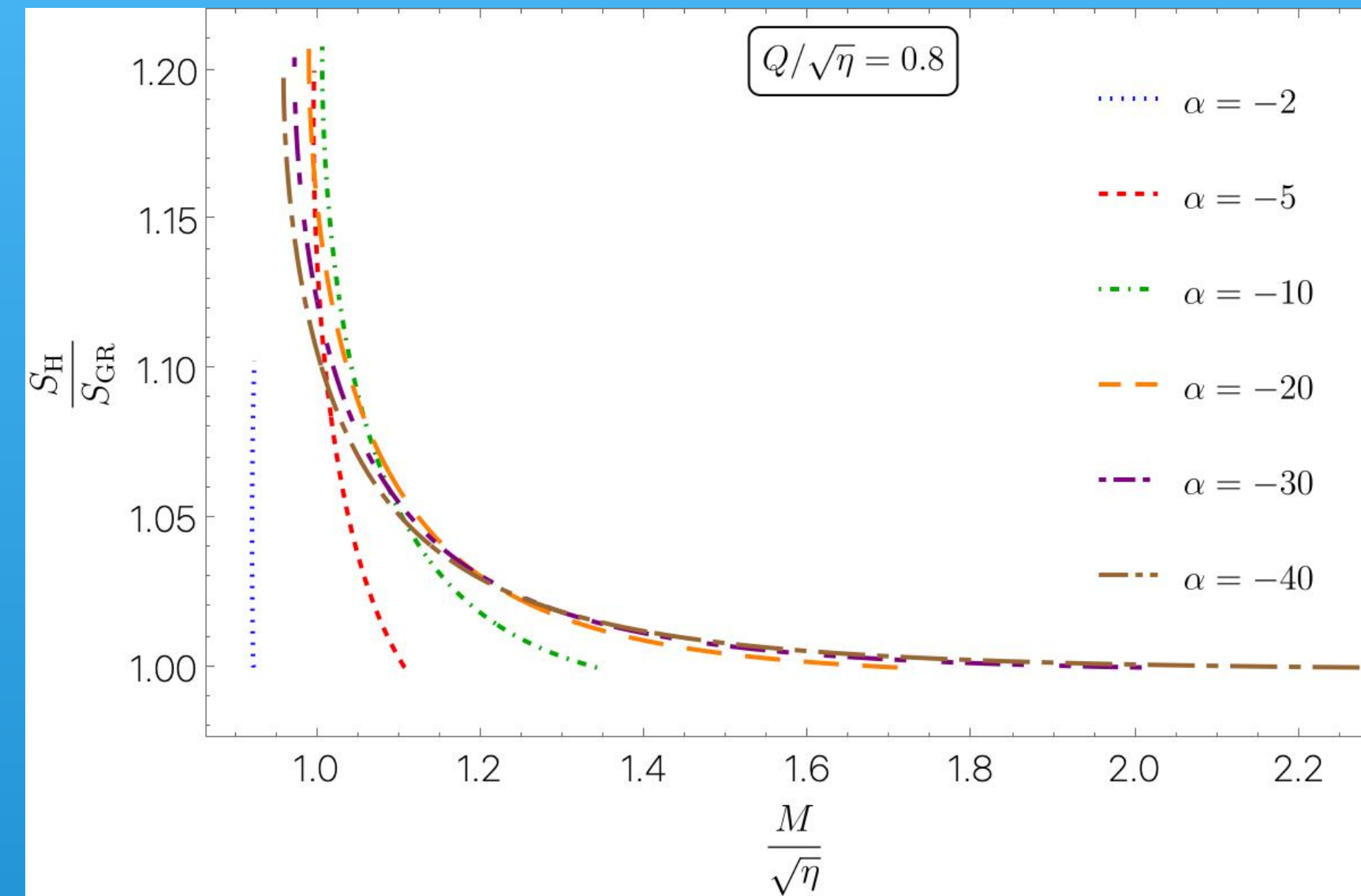
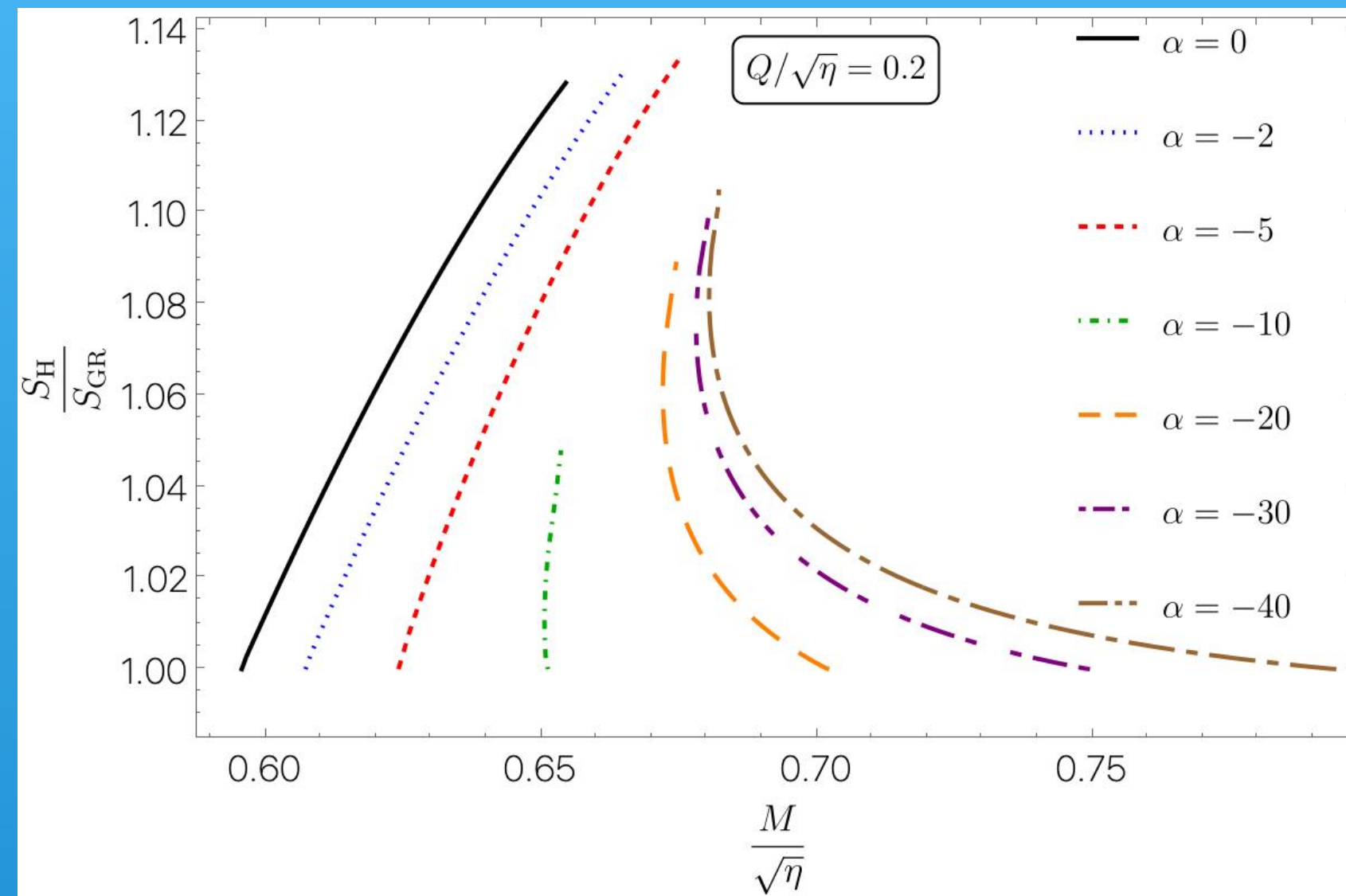
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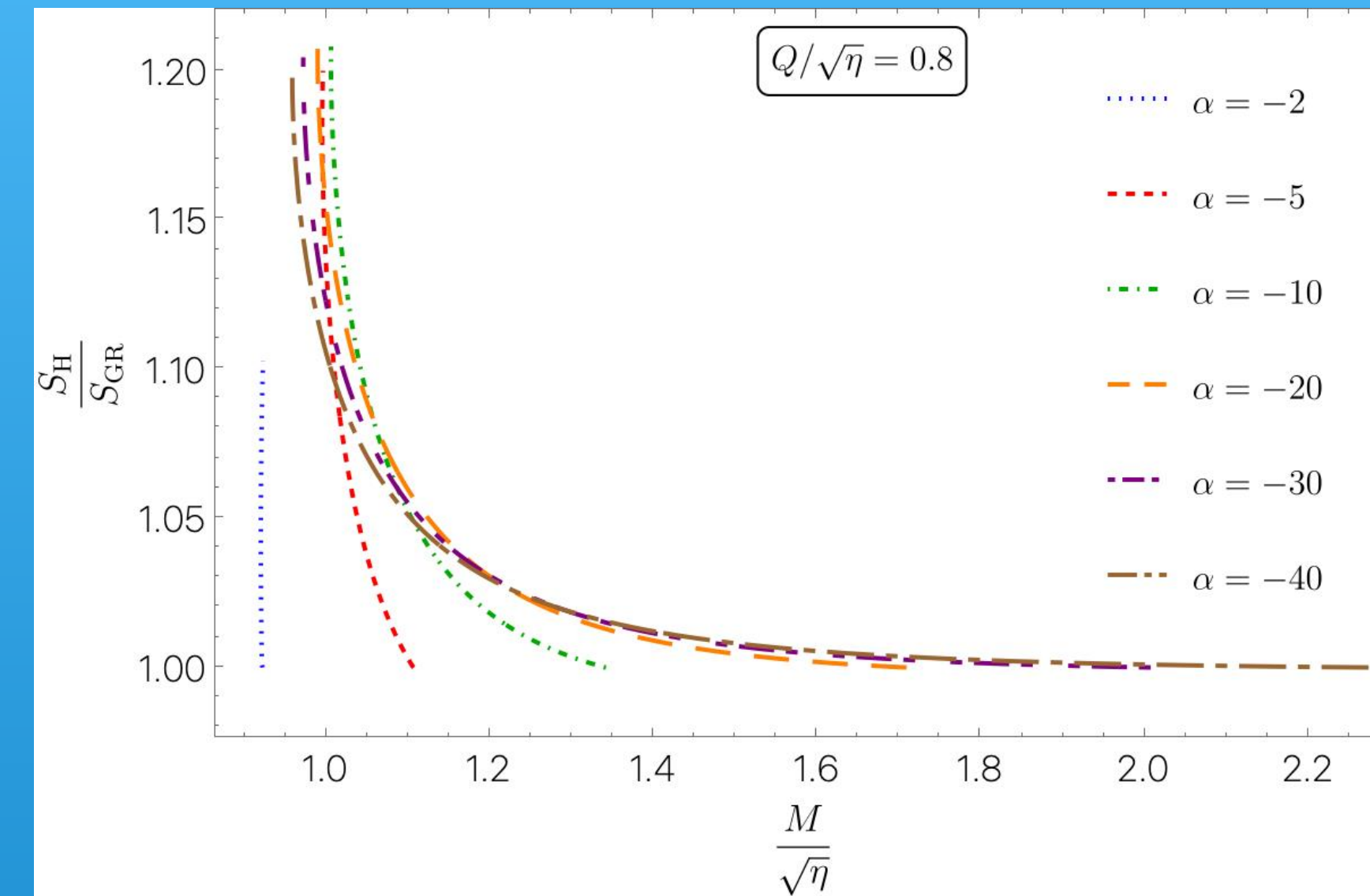
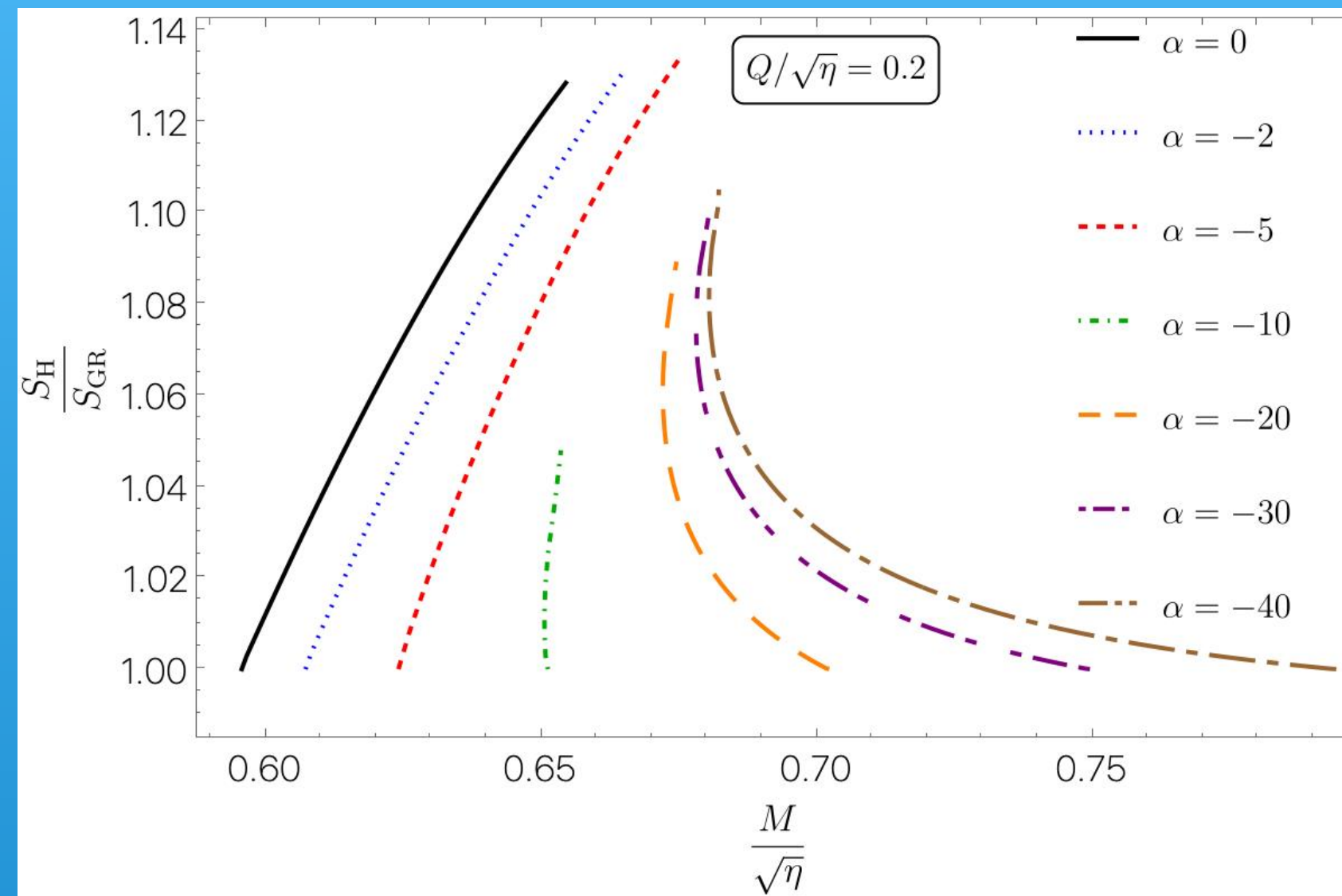
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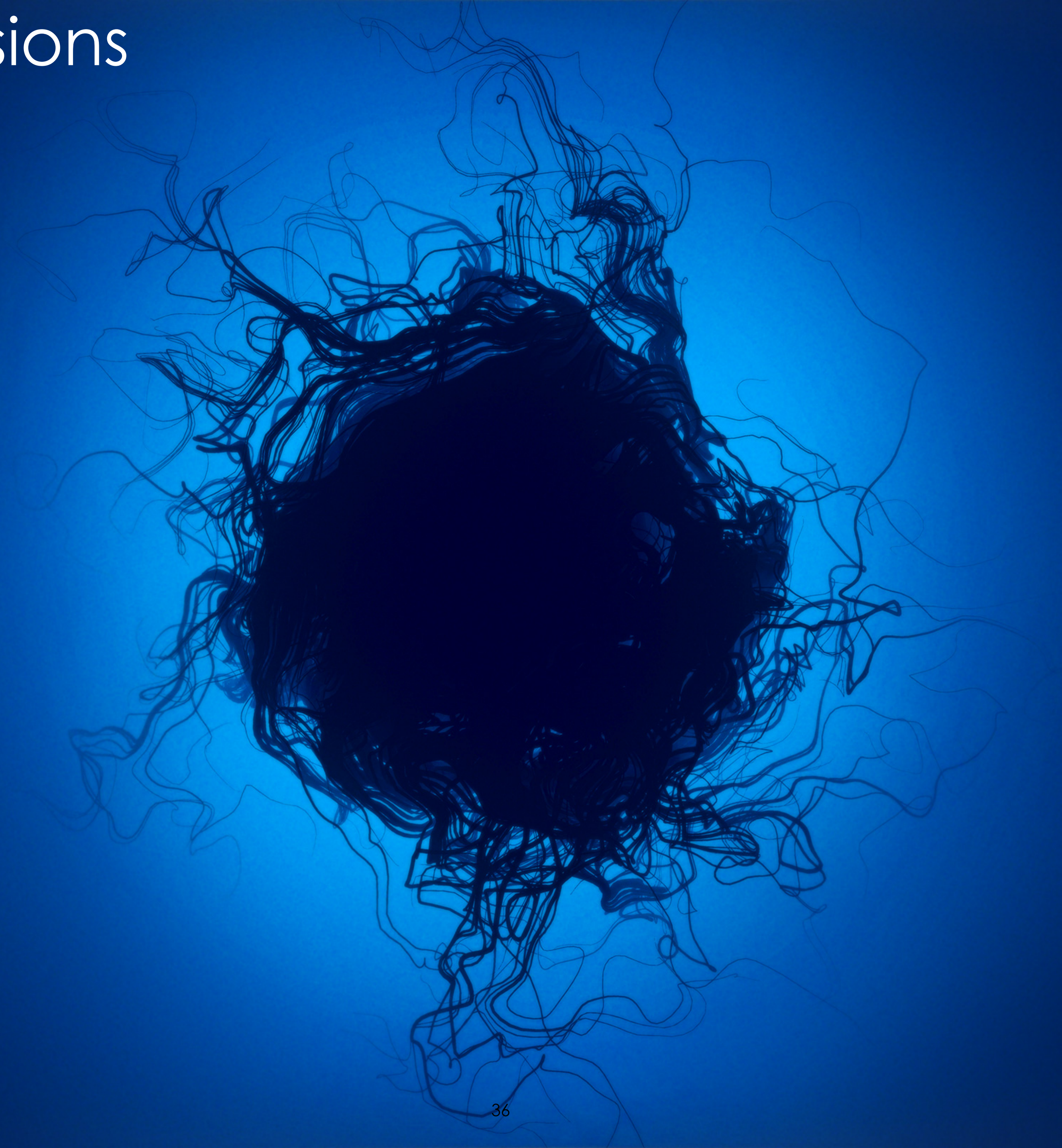
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- scalarized solutions have higher entropy than their GR counterparts, which means they might be thermodynamically preferred

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- We built a new general class of models with new black holes solutions which can be promising candidates for further studies on deviation of GR in strong gravity .

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Thank you for your
attention !

Backup slides

Summary of no-scalar-hair theorems and asymptotically flat scalar-hairy BHs

Theory Lagrangian density $\mathcal{L}$	No-hair theorem	Known scalar hairy BHs with regular geometry on and outside $\mathcal{H}$ (primary or secondary hair; regularity)
Scalar-vacuum $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi$	Chase ^[22]	
Massive-scalar-vacuum $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - \frac{1}{2}\mu^2\Phi^2$	Bekenstein ^[11]	
Massive-complex-scalar-vacuum $\frac{1}{4}R - \nabla_\mu\Phi^*\nabla^\mu\Phi - \mu^2\Phi^*\Phi$	Pena– Sudarsky ^[61]	Herdeiro–Radu ^[136,137] (primary, regular); generalizations. ^[159]
Conformal-scalar-vacuum $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - \frac{1}{12}R\Phi^2$	Xanthopoulos– Zannias ^[32] Zannias ^[33]	Bocharova–Bronnikov–Melnikov– Bekenstein (BBMB) ^[16,18] (secondary, diverges at $\mathcal{H}$); generalizations. ^[87]
V -scalar-vacuum $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - V(\Phi)$	Heusler ^[46,47,50] Bekenstein ^[26] Sudarsky ^[51]	Many, with non-positive definite potentials. ^[71,75,78,80] (typically secondary, regular)
P -scalar-vacuum $\frac{1}{4}R + P(\Phi, X)$	Graham– Jha ^[62]	
Einstein-Skyrme $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi^a\nabla^\mu\Phi^a$ $-\kappa \nabla_{[\mu}\Phi^a\nabla_{\nu]}\Phi^b ^2$		Droz–Heusler–Straumann ^[126] (primary but topological; regular); generalizations. ^[129,131]
Scalar-tensor theories $\varphi\hat{R} - \frac{\omega(\varphi)}{\varphi}\hat{\nabla}_\mu\varphi\hat{\nabla}^\mu\varphi - U(\varphi)$	Hawking ^[27] Saa ^[34,35] Sotiriou– Faraoni ^[31]	
Horndeski/Galileon theories Full $\mathcal{L}$ in eq. (41)	Hui– Nicolis ^[45]	Sotiriou-Zhou ^[43] (secondary; regular) Babichev–Charmousis ^[88,90] (secondary ^[88] or primary, ^[90] diverges at $\mathcal{H}^+$ or $\mathcal{H}^-$); generalizations. ^[91,93]

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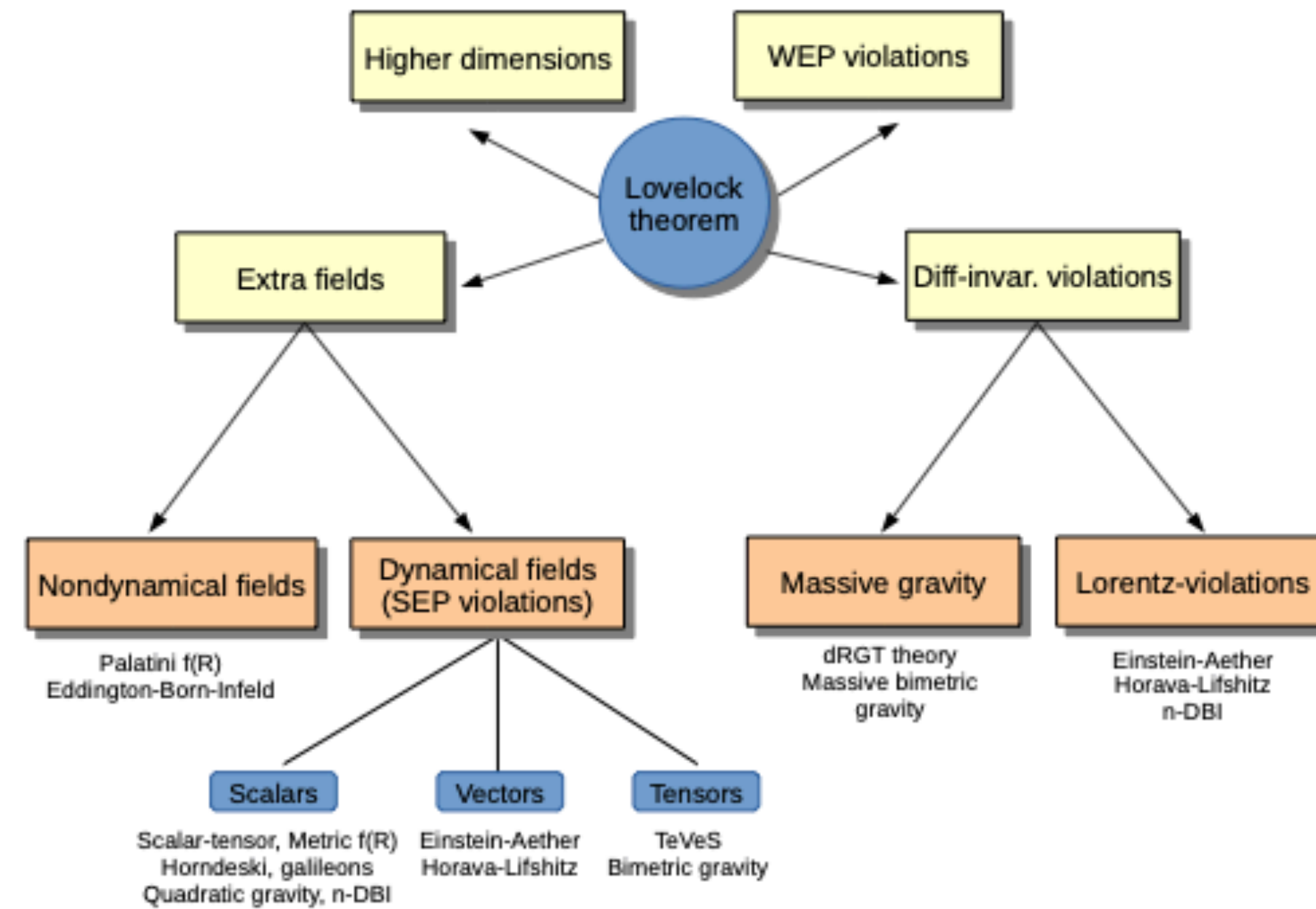
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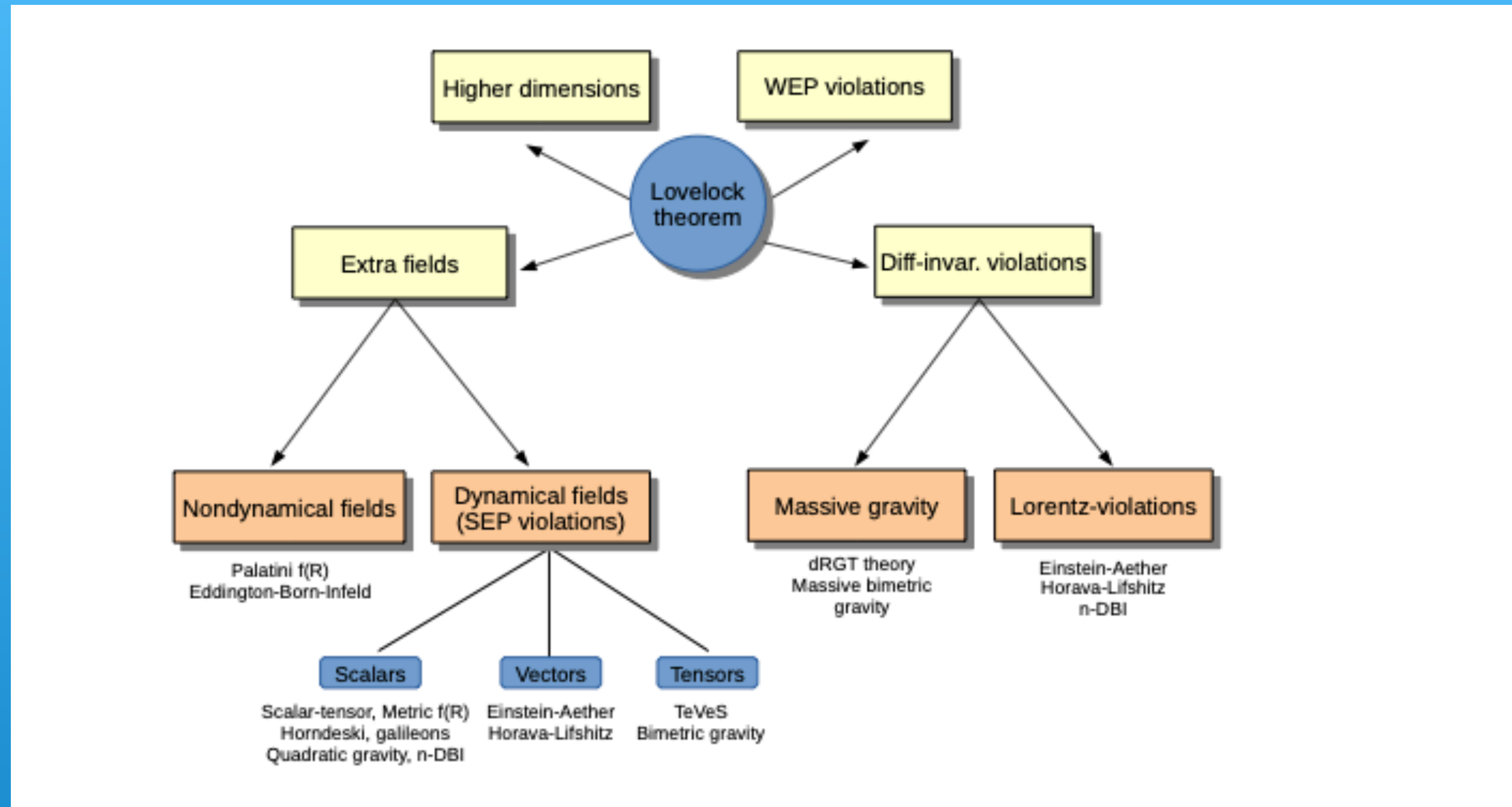
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Modified gravity from Lovelock theorem



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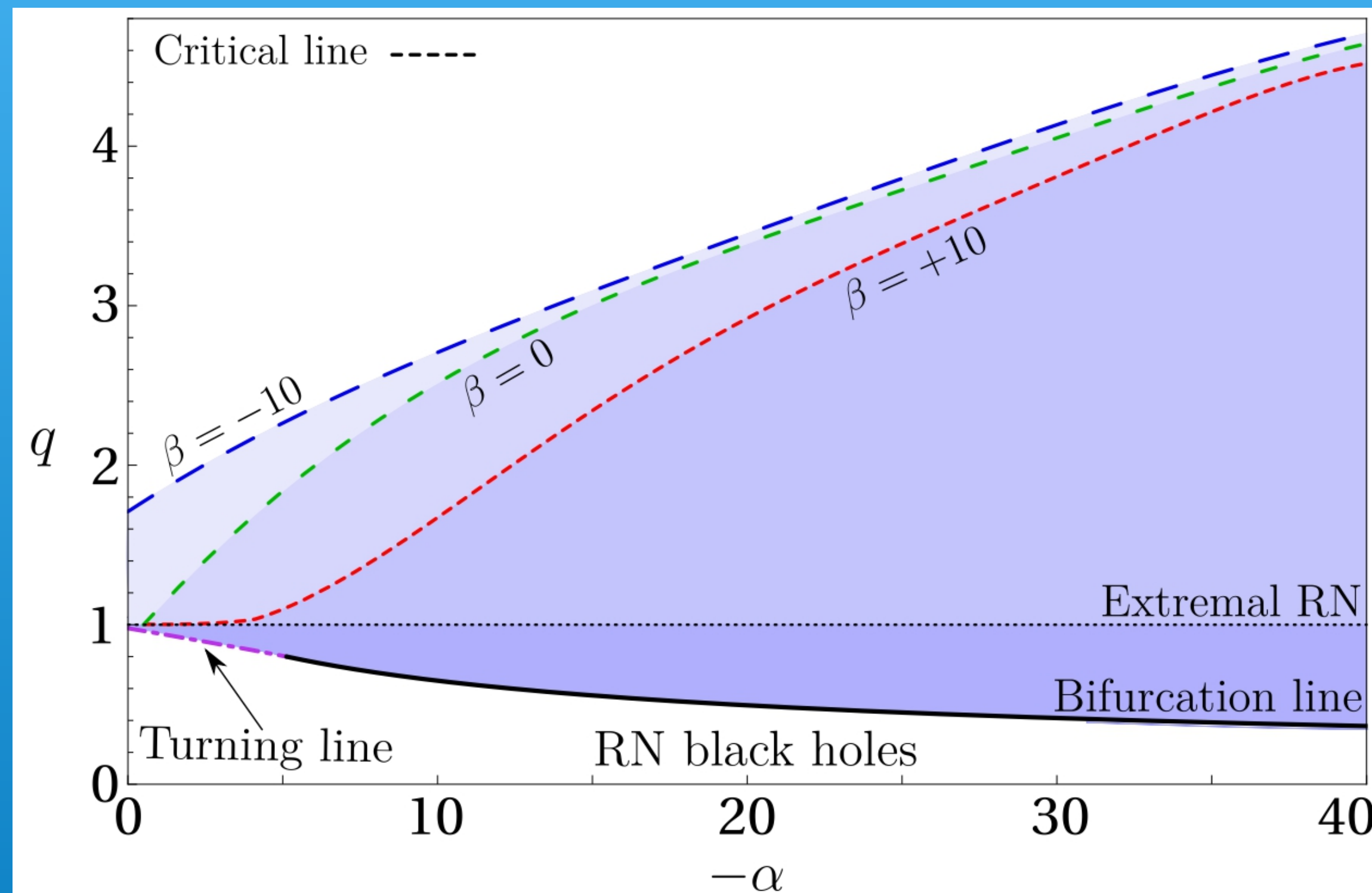


Berti et al 2015

Phase space / domaine of existence of solutions

$$f(\phi) = e^{-\alpha\phi^2 - \beta\phi^4}$$

Fixed α solutions

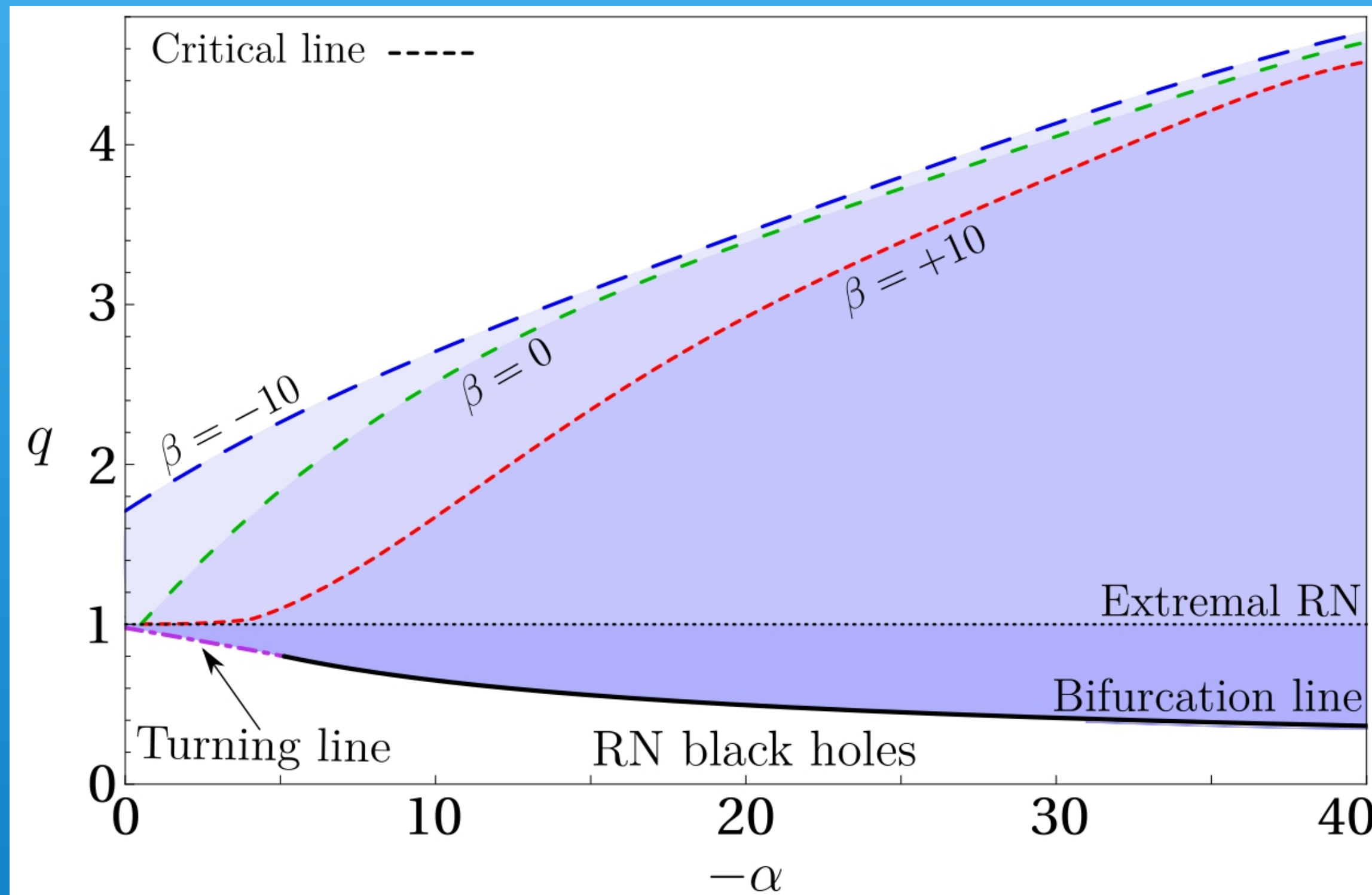


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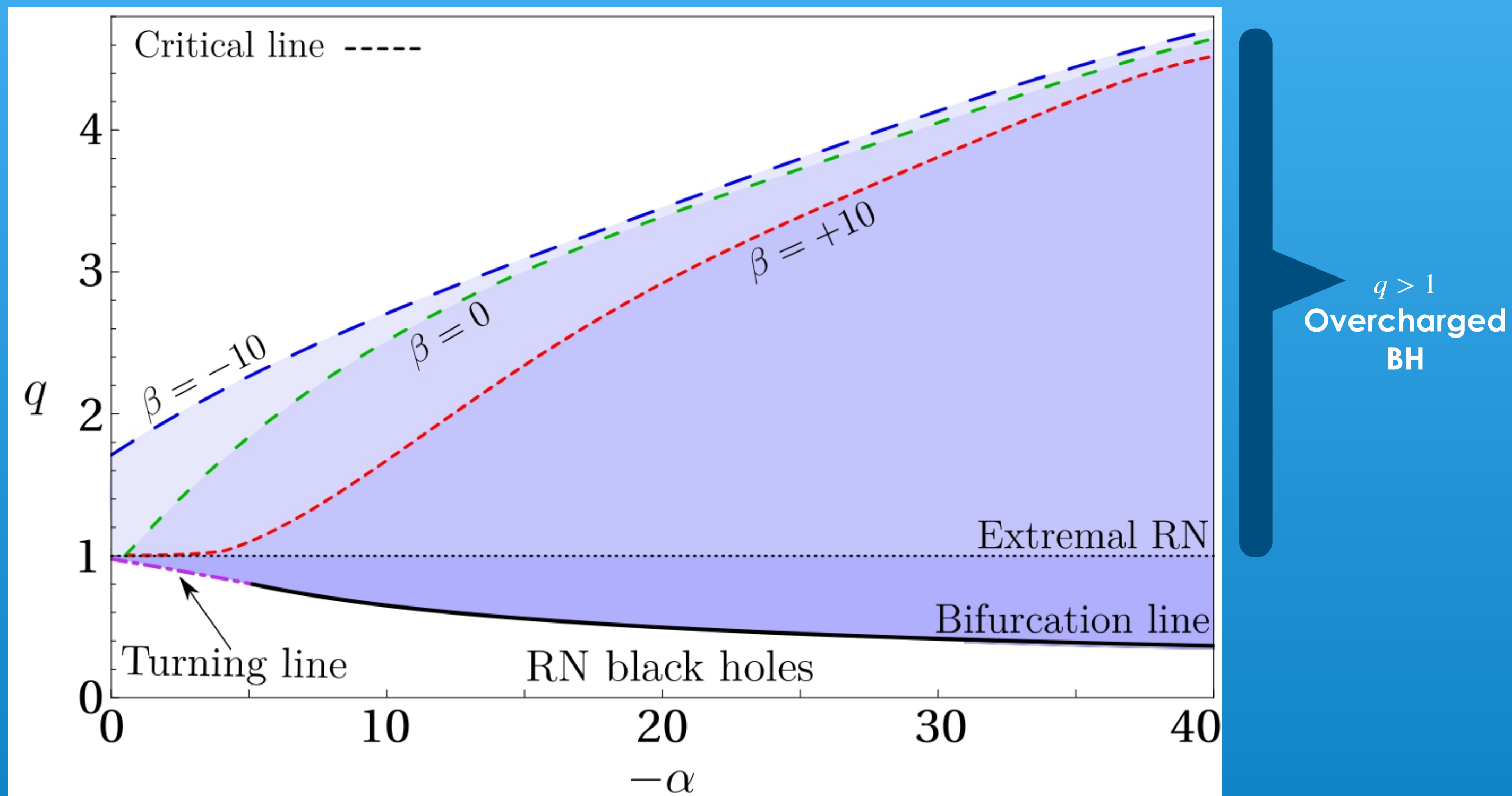


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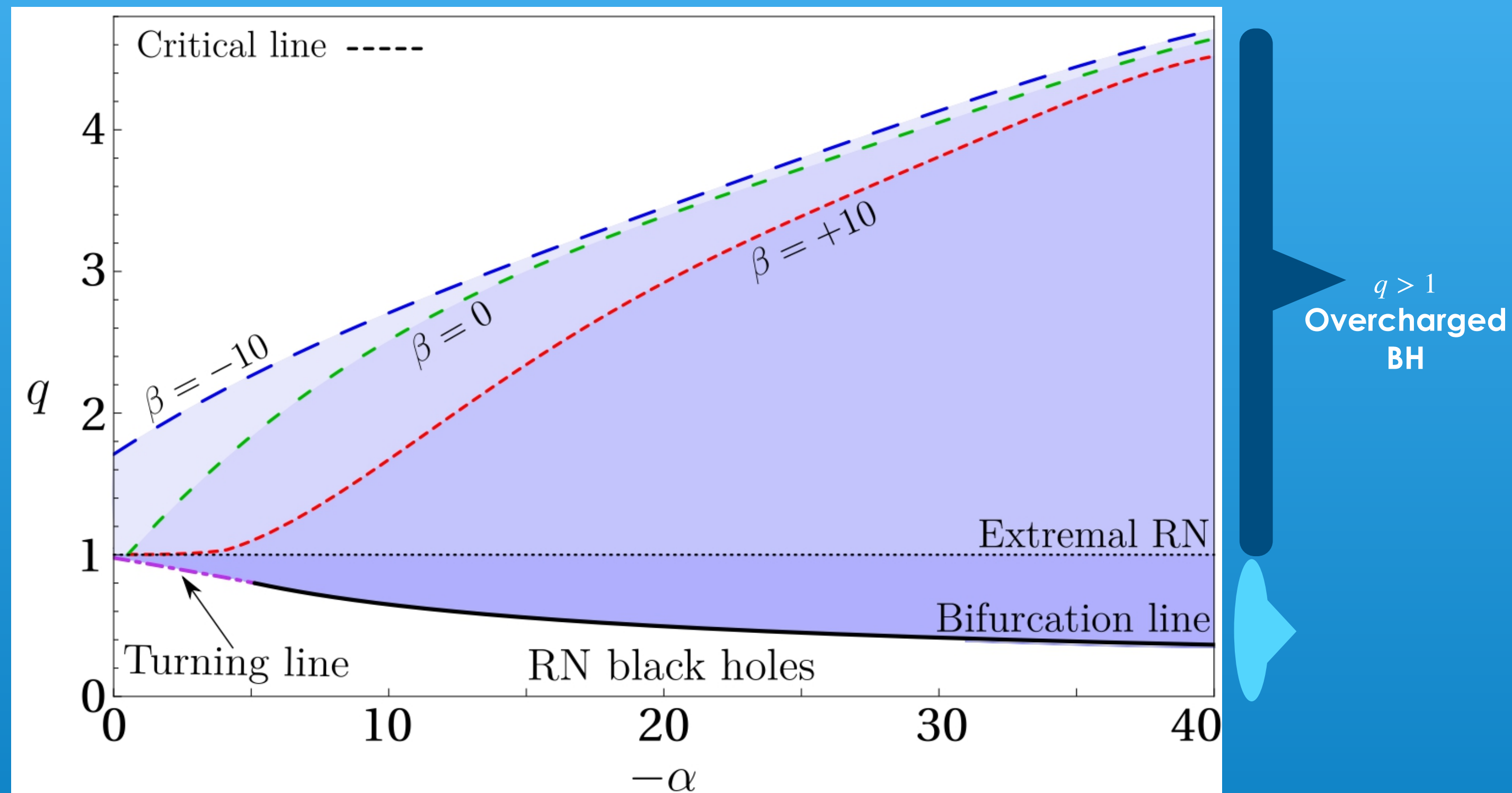


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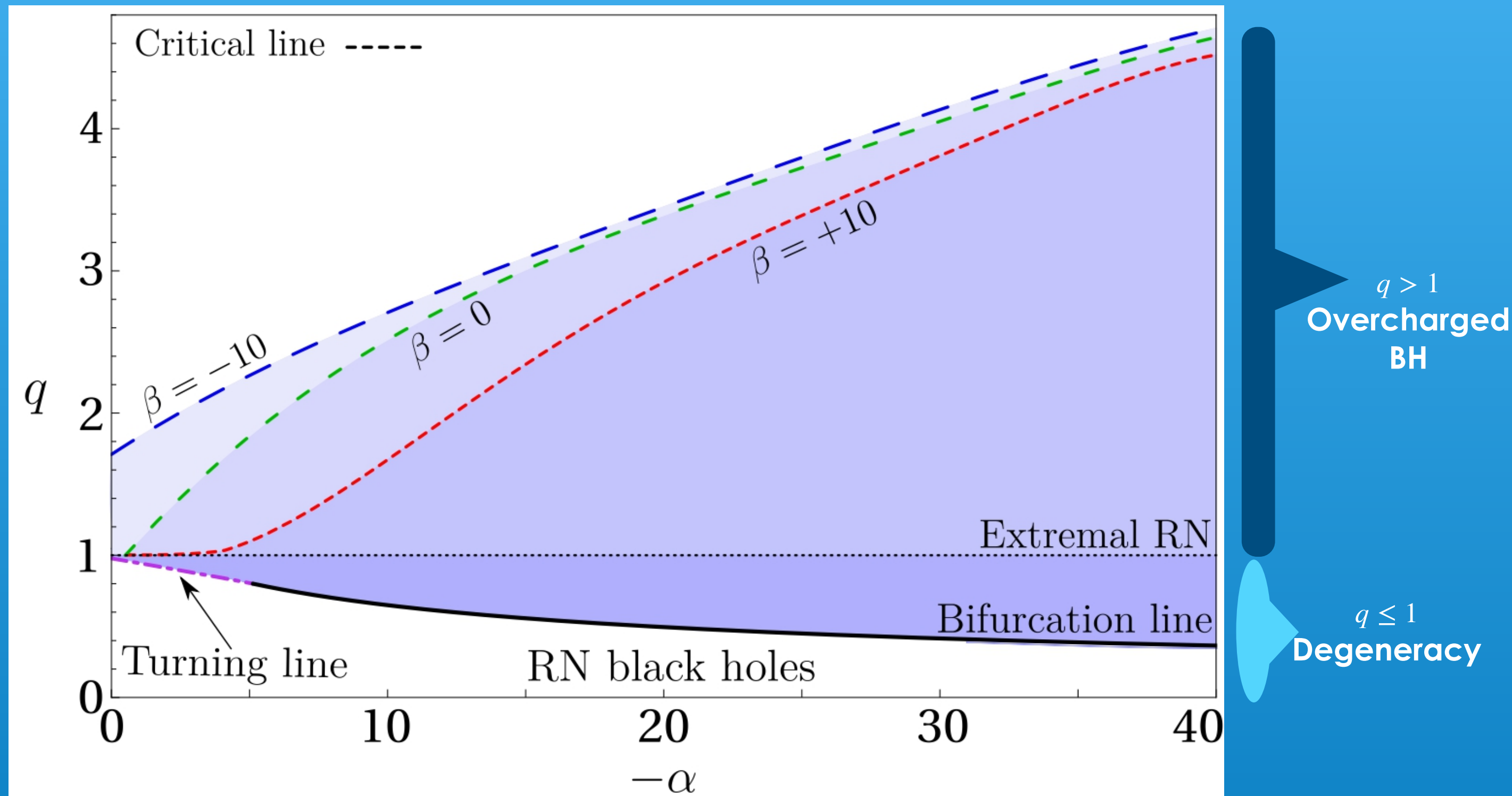


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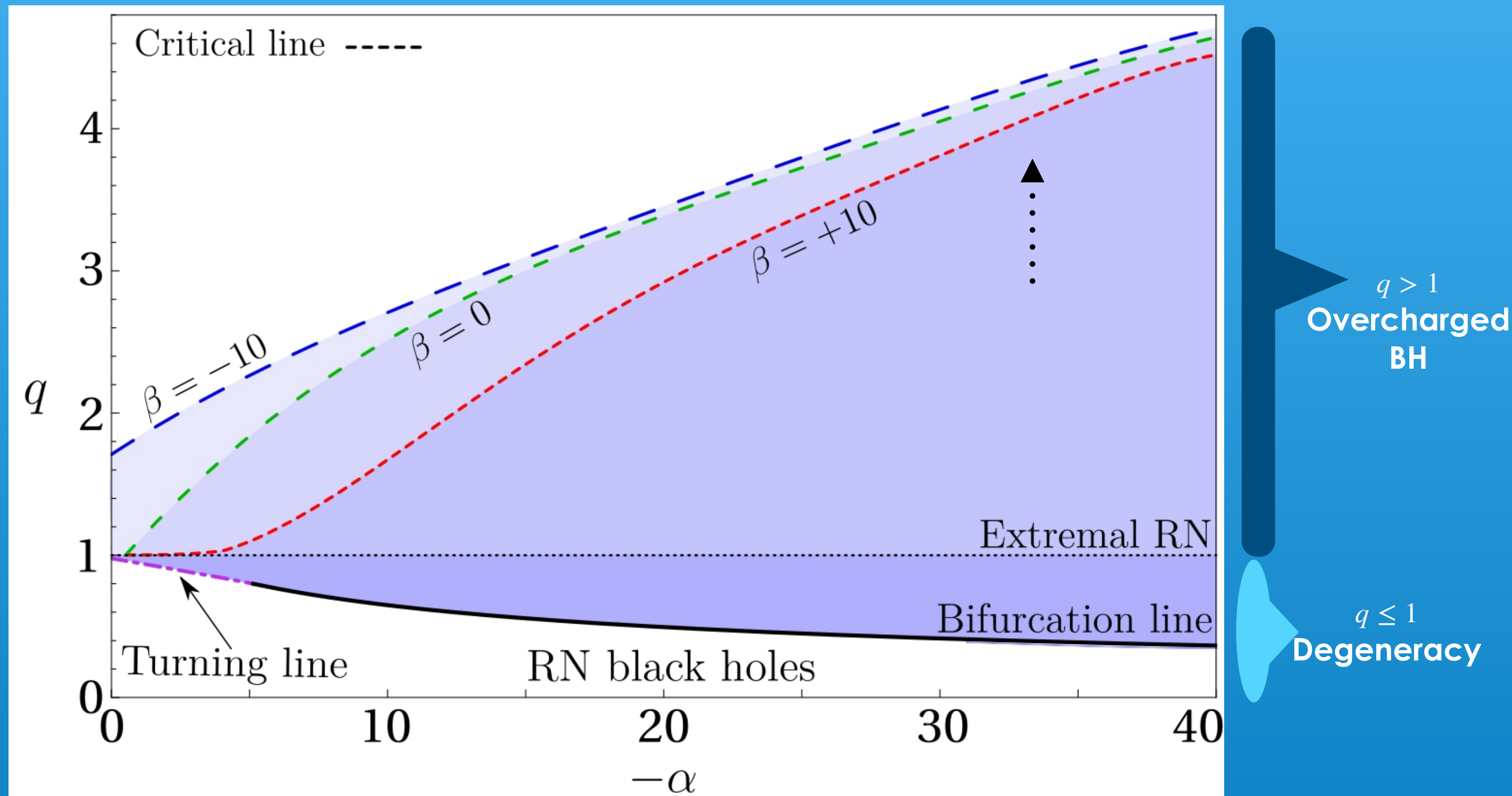


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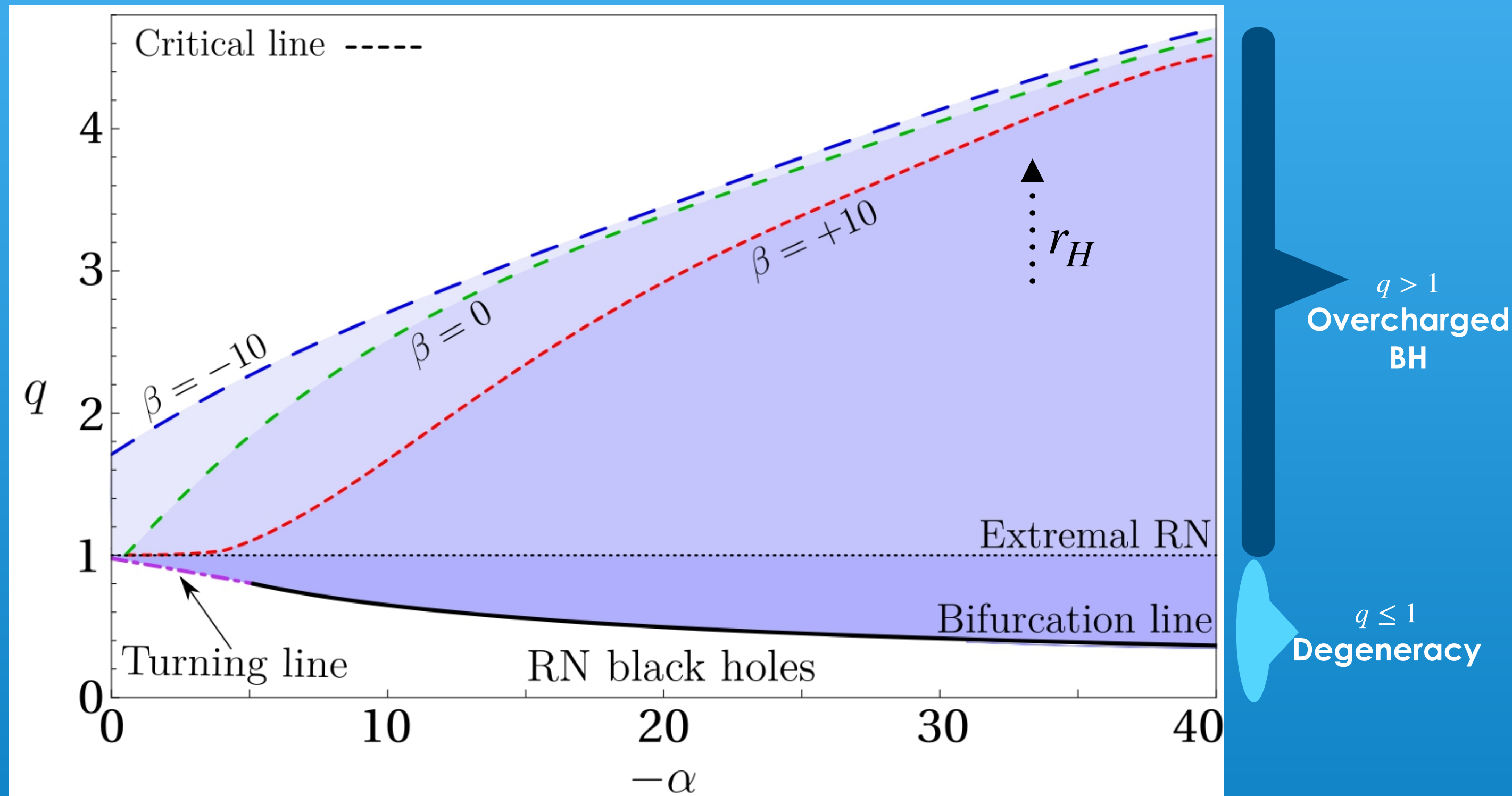


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$$f(\phi) = e^{-\alpha\phi^2 - \beta\phi^4}$$

Fixed α solutions

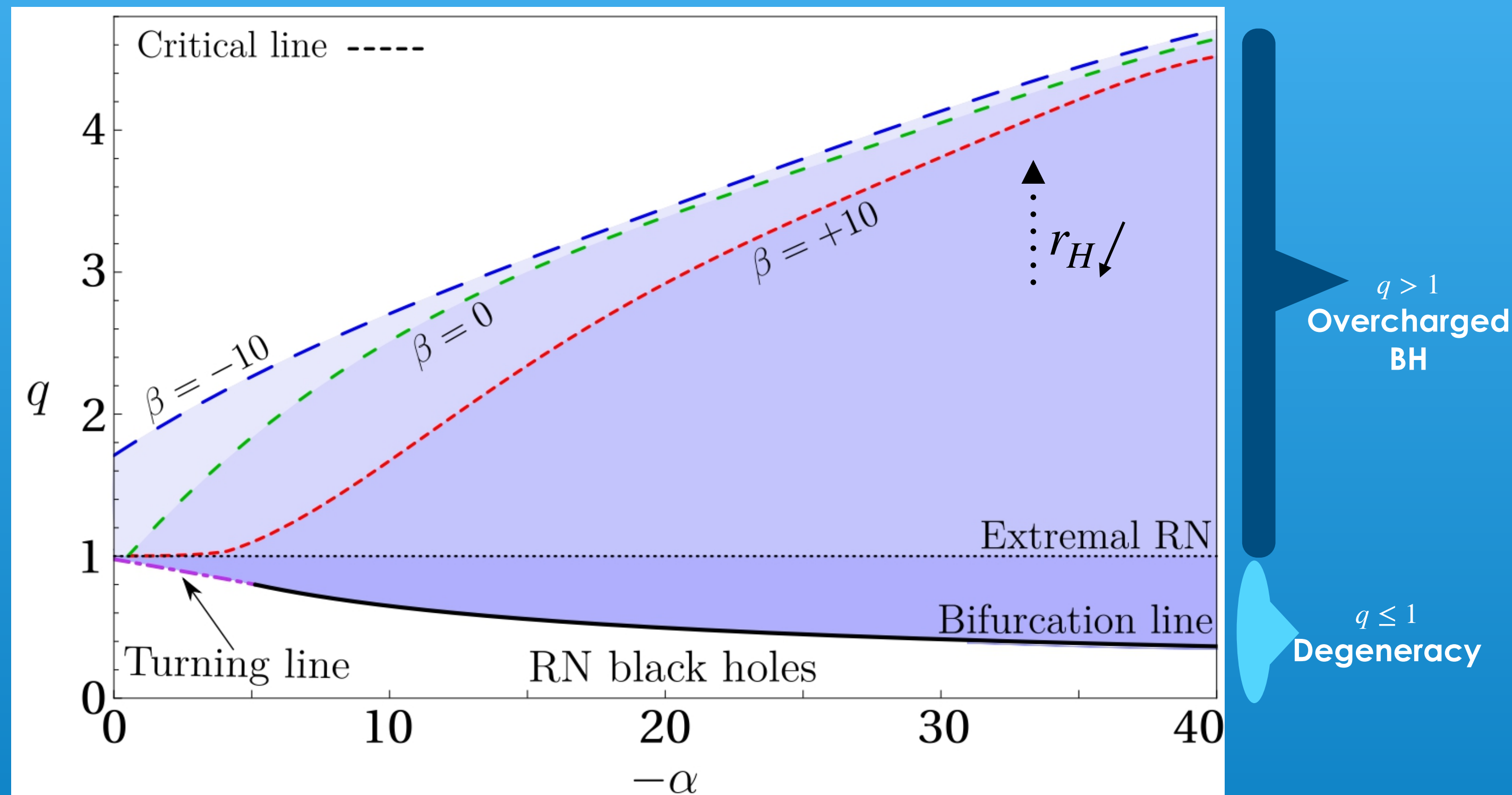


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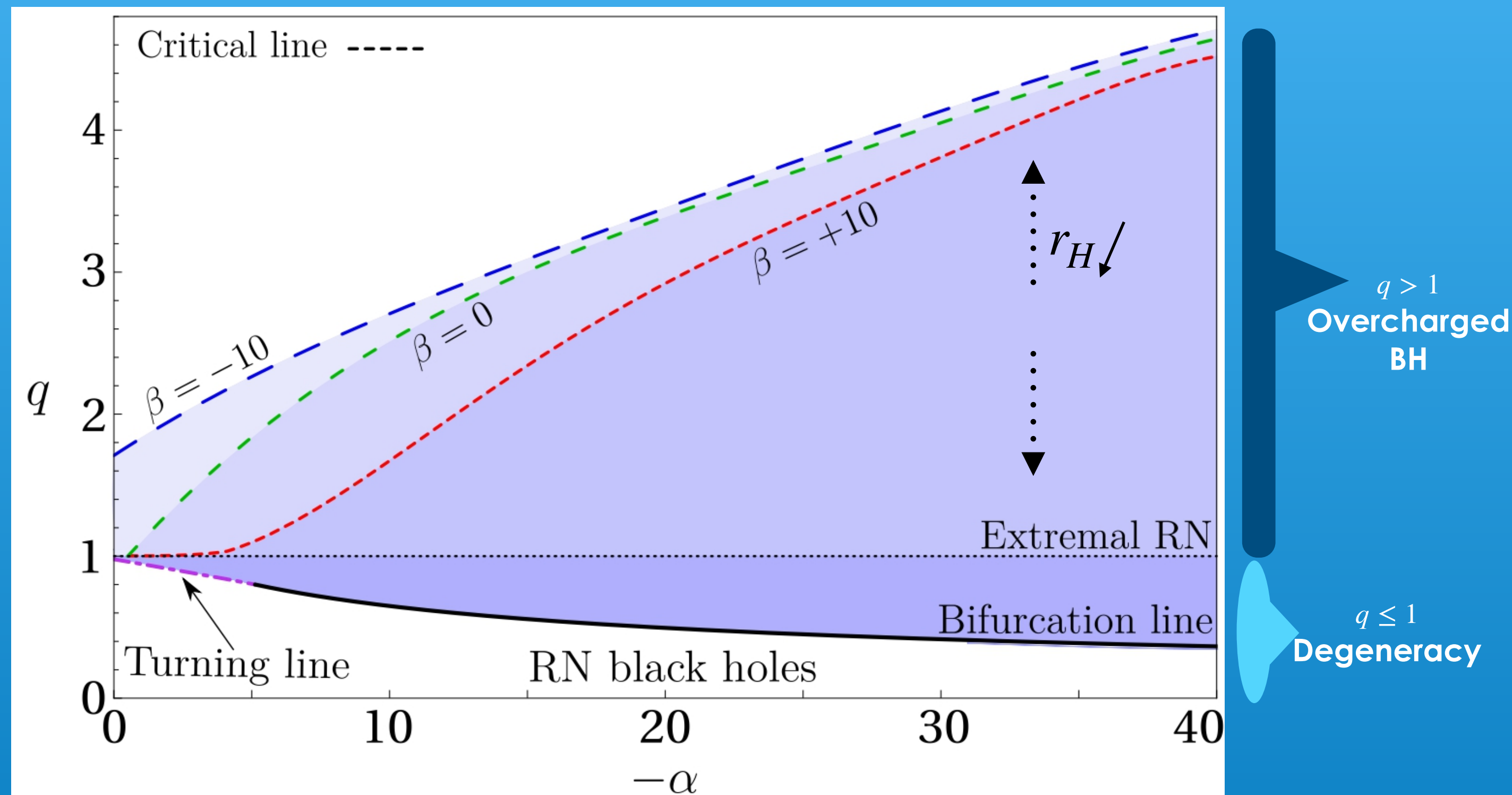


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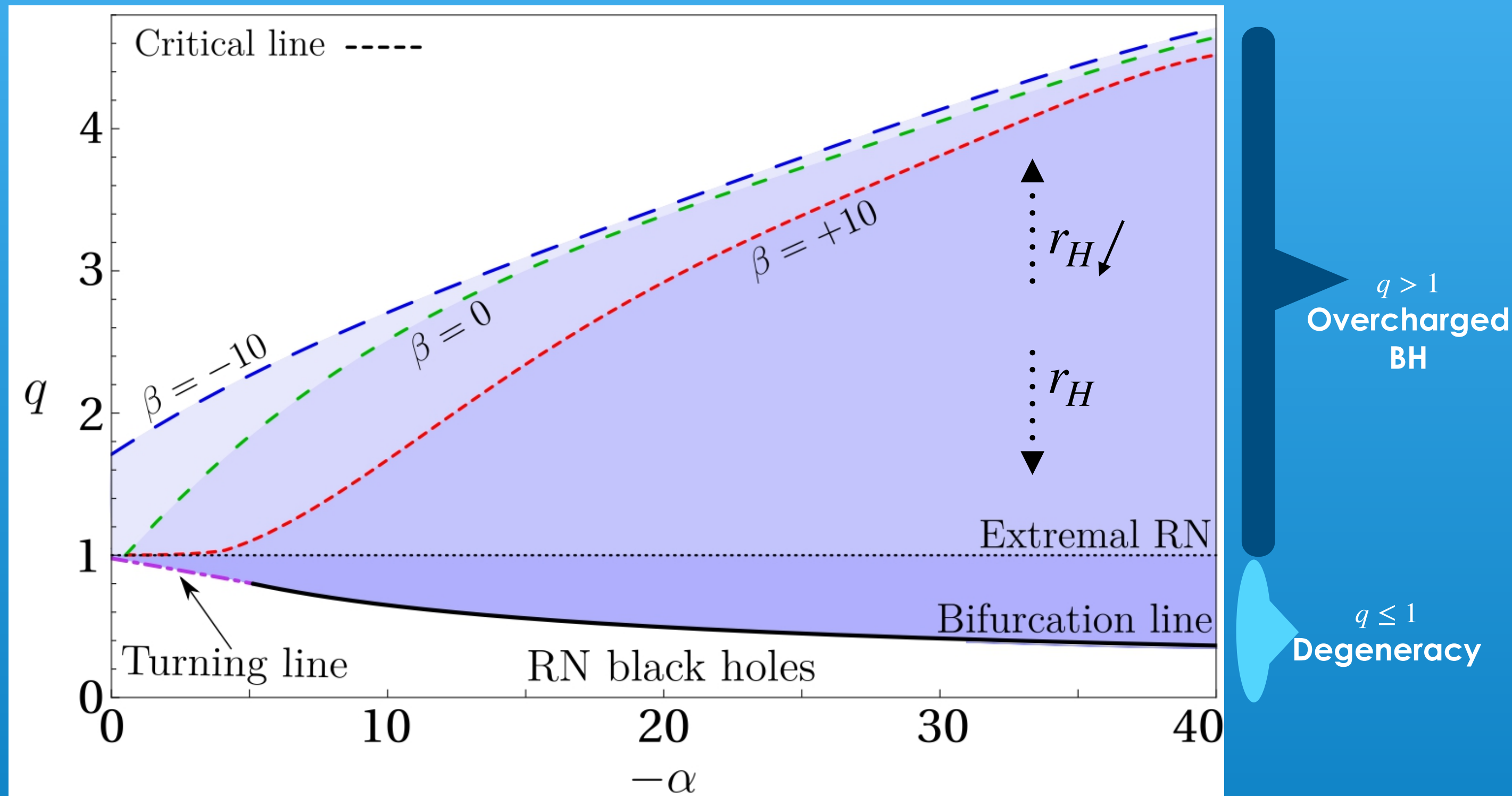


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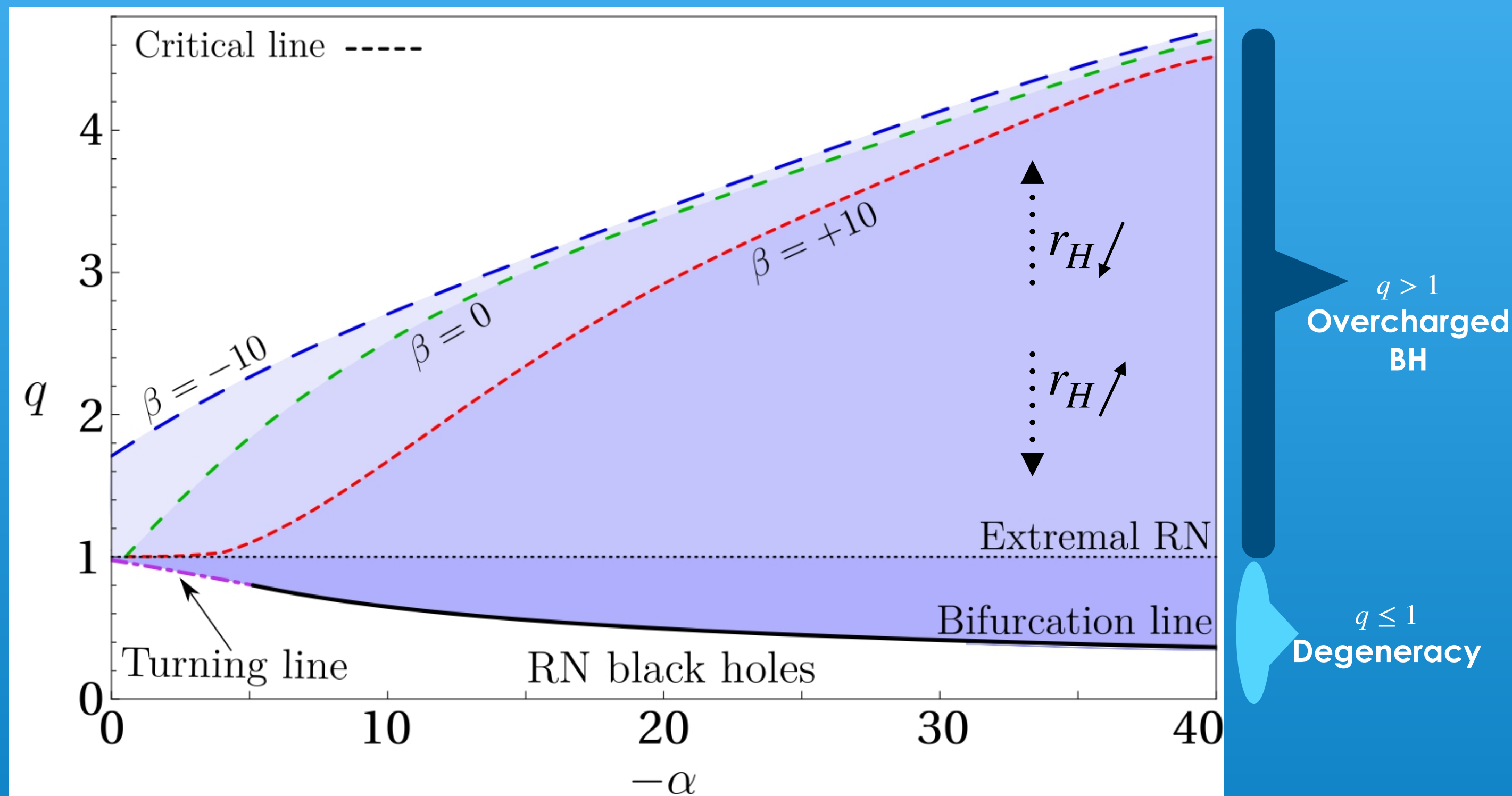


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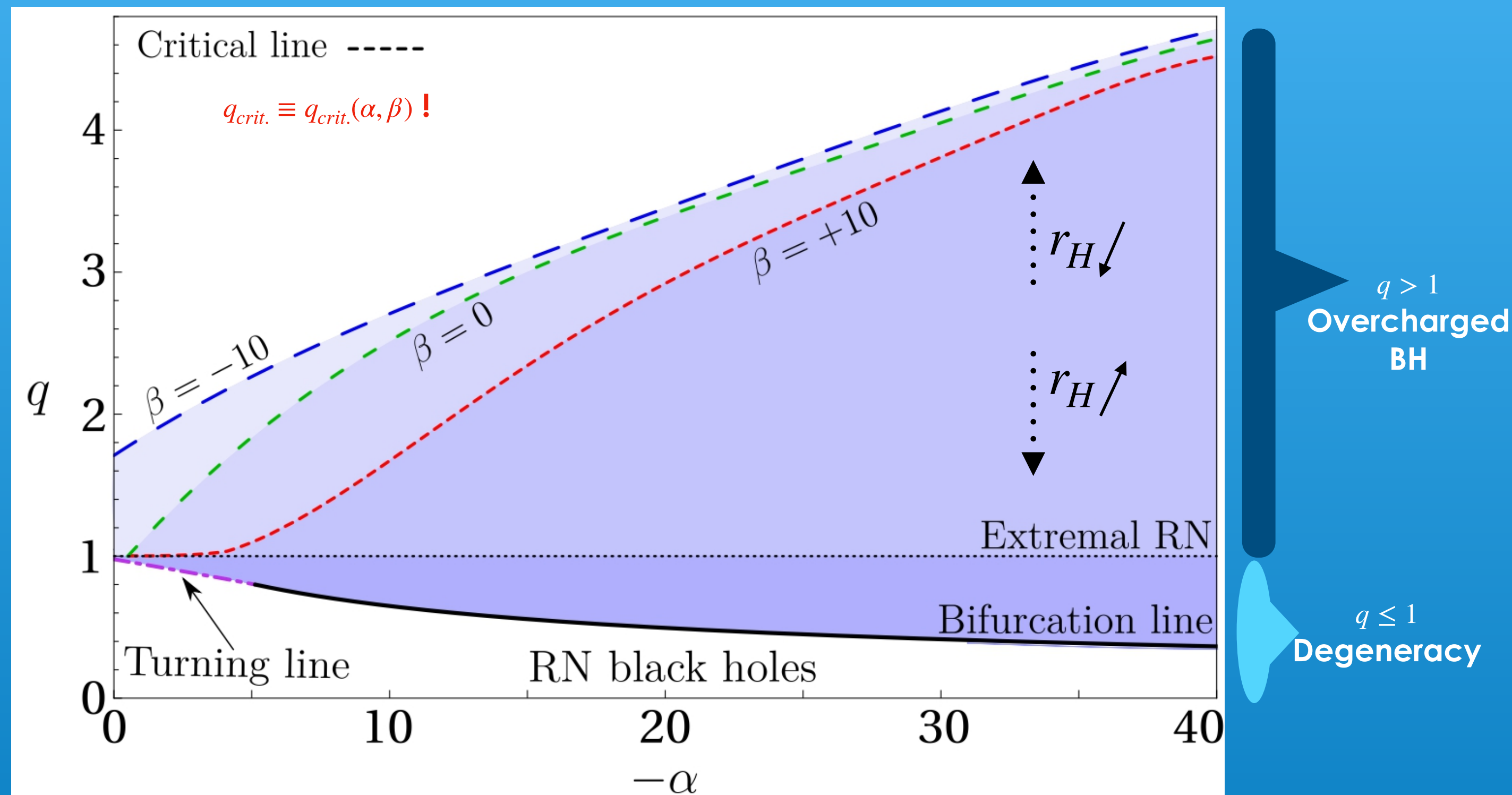


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Spontaneous scalarization in EsGB

$$f(\phi) = \frac{1}{12}(1 - e^{-6\phi^2})$$

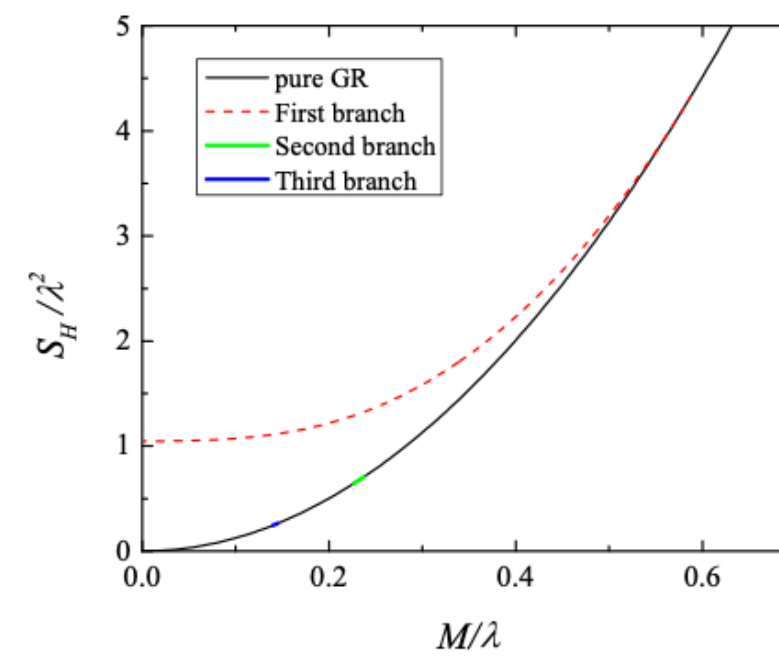
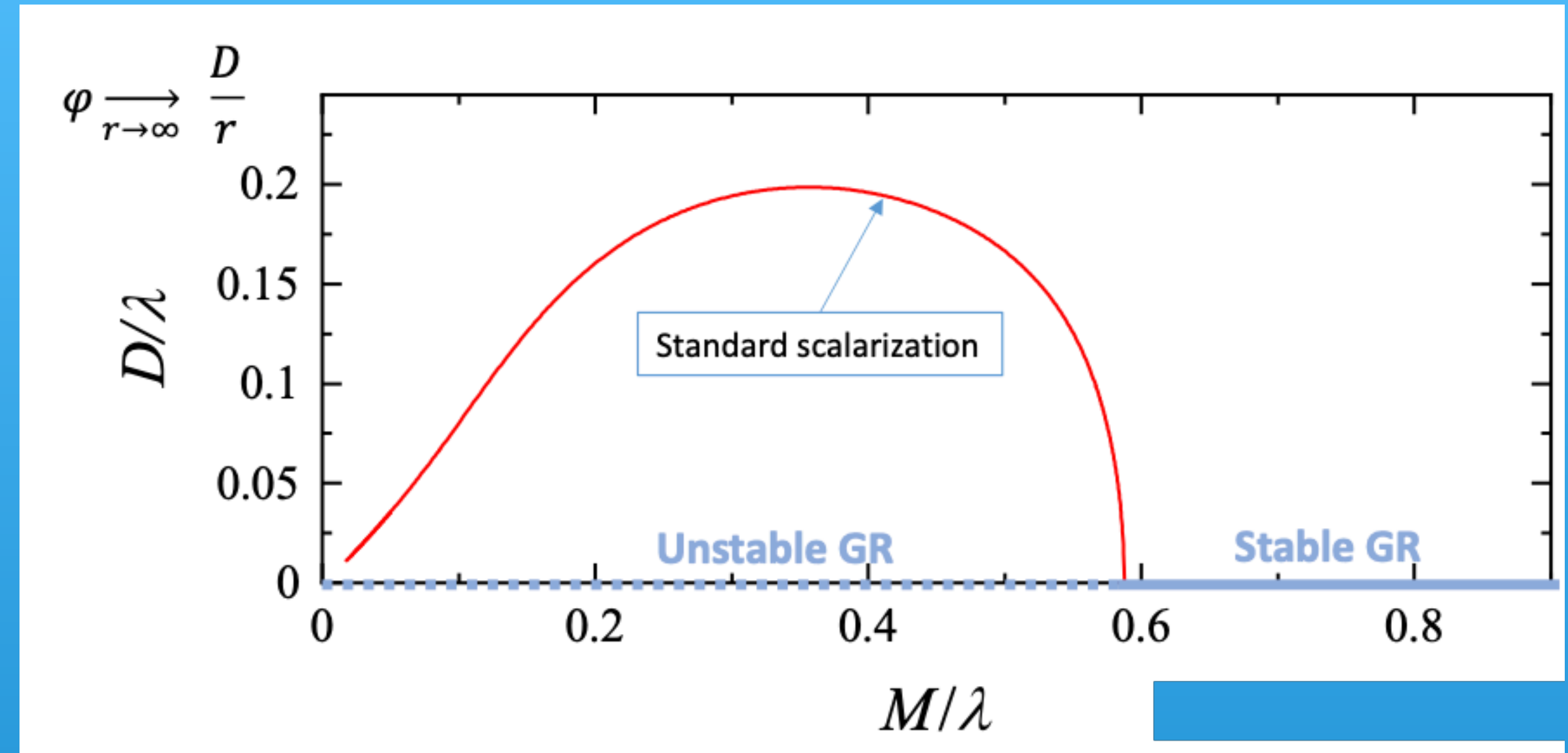
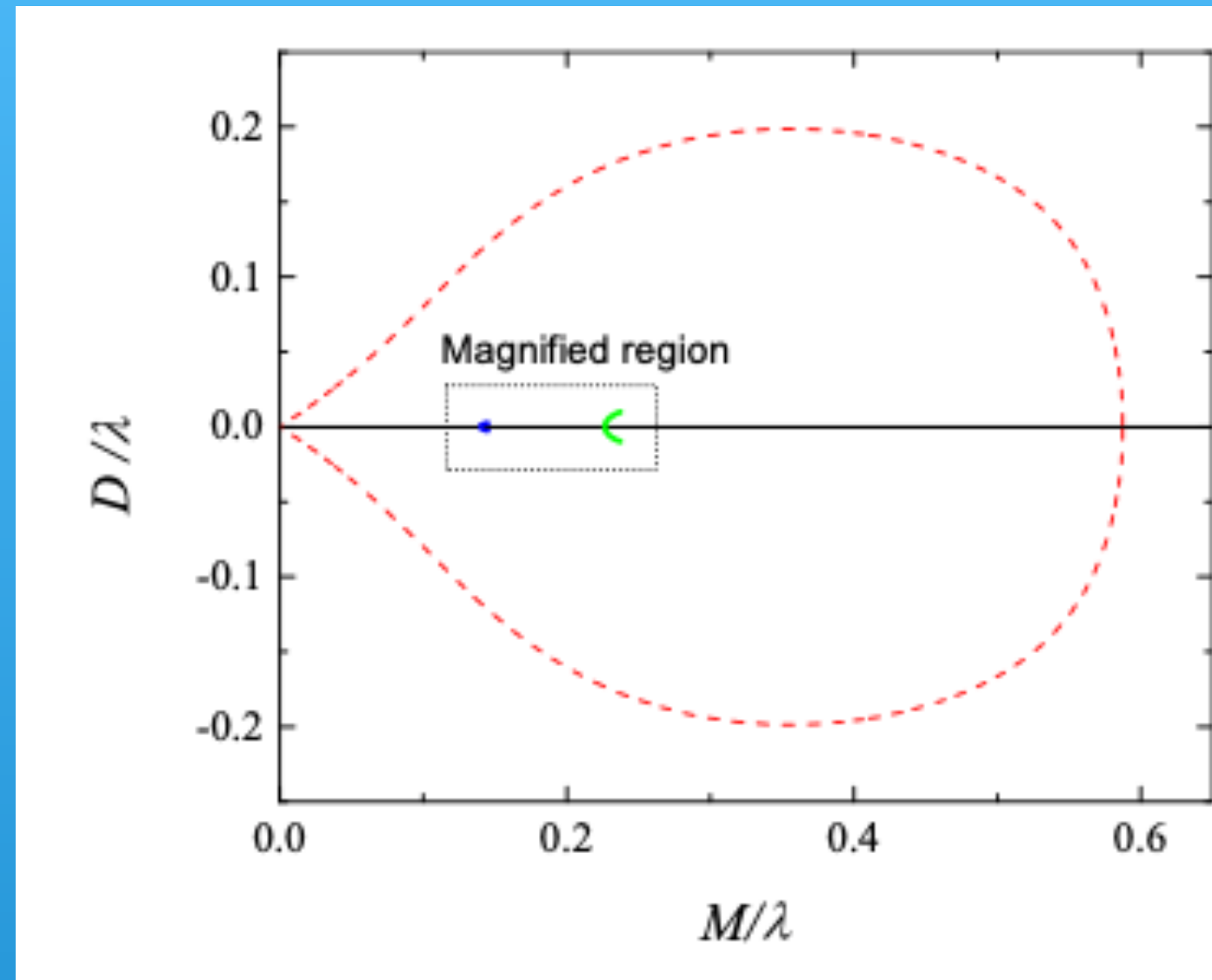
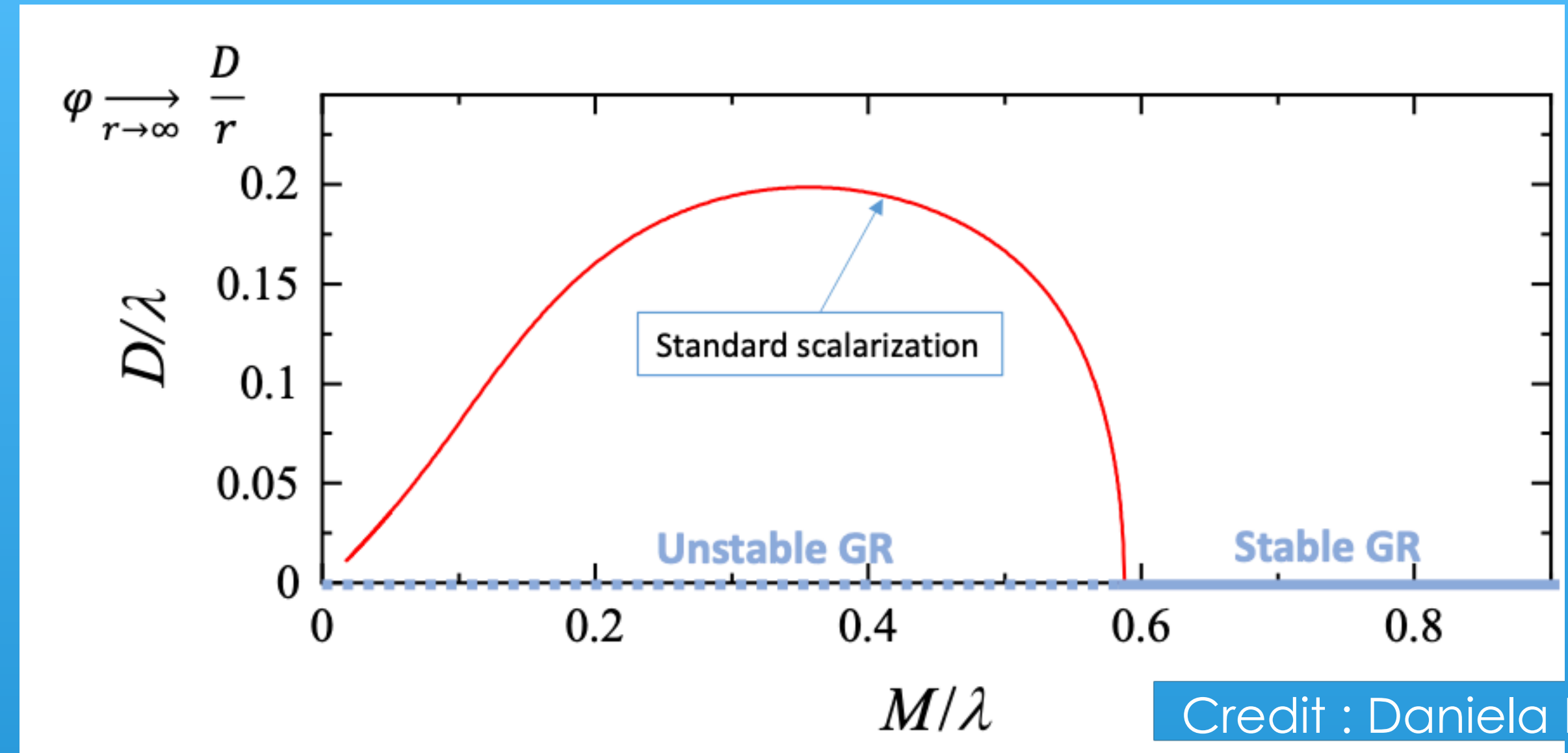
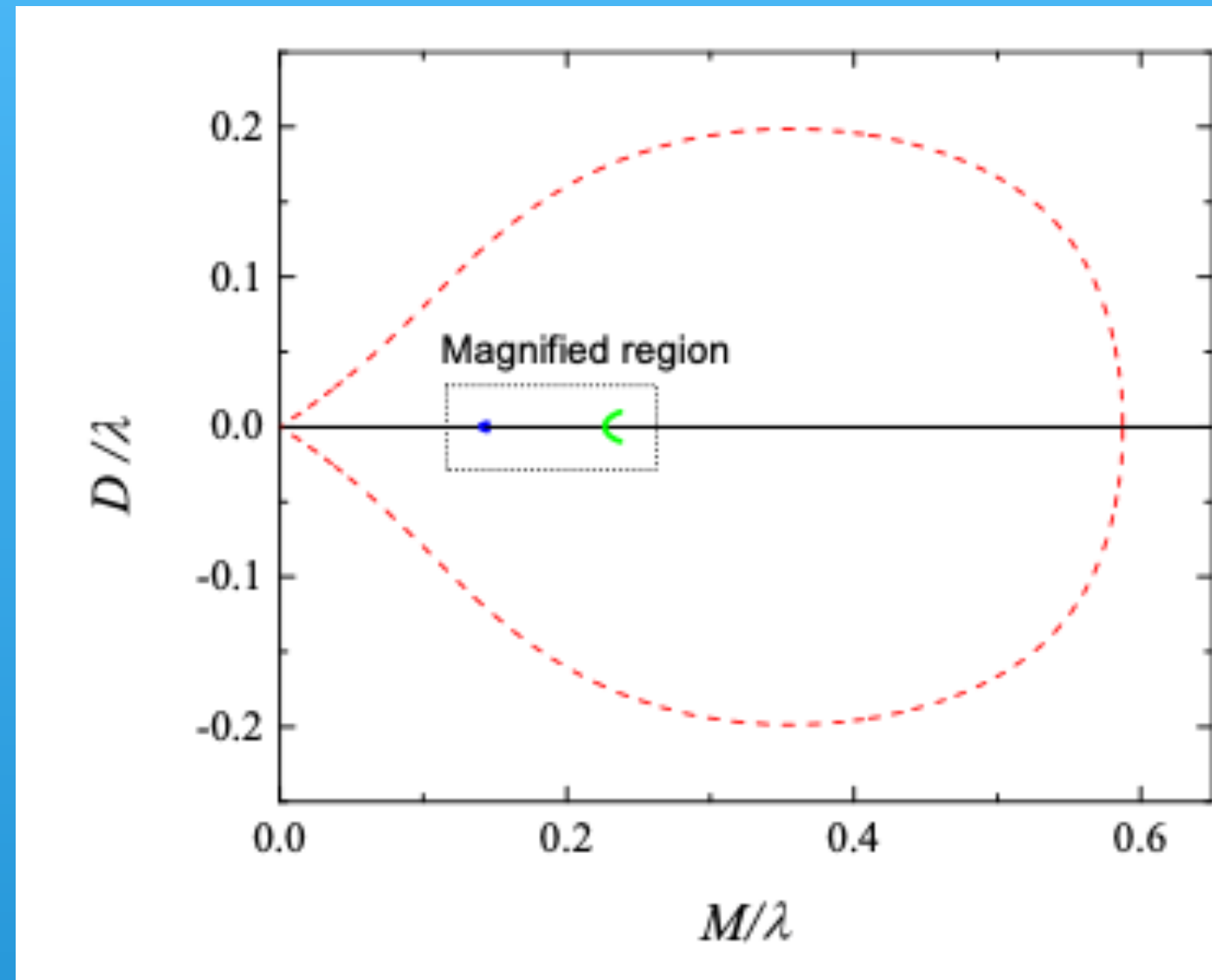


FIG. 7: The entropy of the black hole as a function of its mass. .

Spontaneous scalarization in EsGB

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Credit : Daniela Doneva

Doneva & Yazadjiev 2017
arXiv:1711.01187

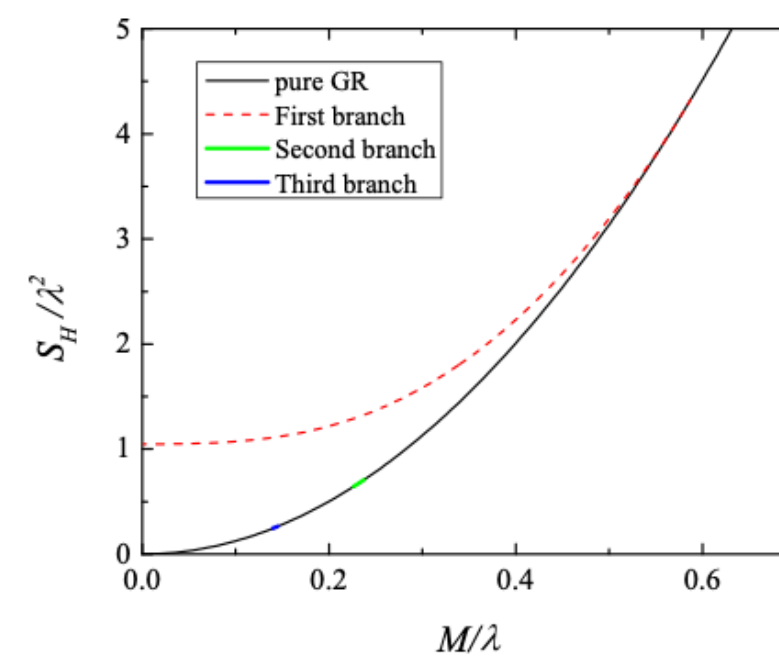


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Spontaneous scalarization in EsGB with self-interaction

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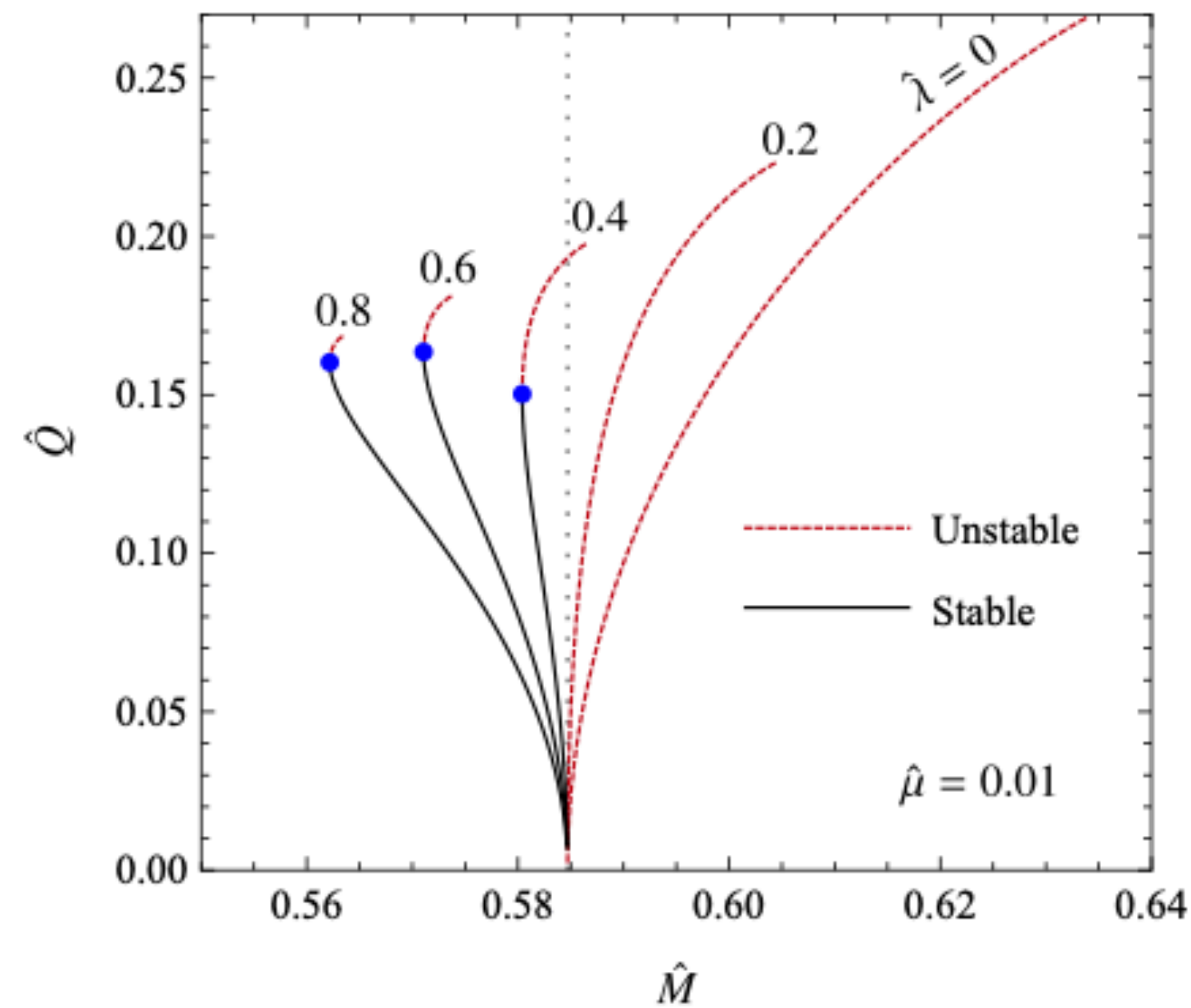


Figure 19 Scalar-charge-mass diagram for scalarized BHs with a quadratic scalar-Gauss-Bonnet model with a scalar-field potential $V(\varphi) = (\mu^2/2)\varphi^2 + (\lambda/2)\varphi^4$. Dimensionless

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Macedo et al 2019
[arXiv:1903.06784](https://arxiv.org/abs/1903.06784)

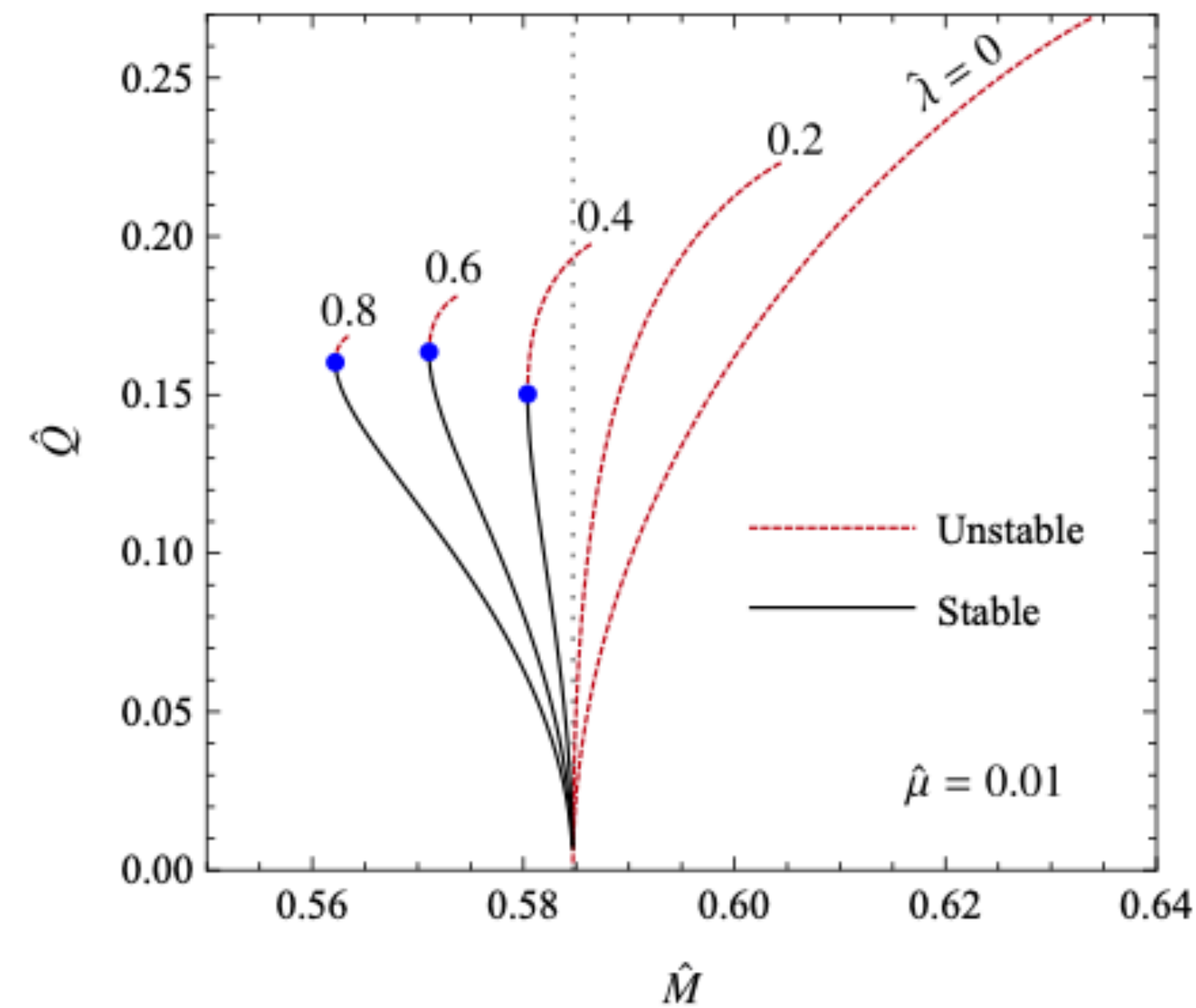
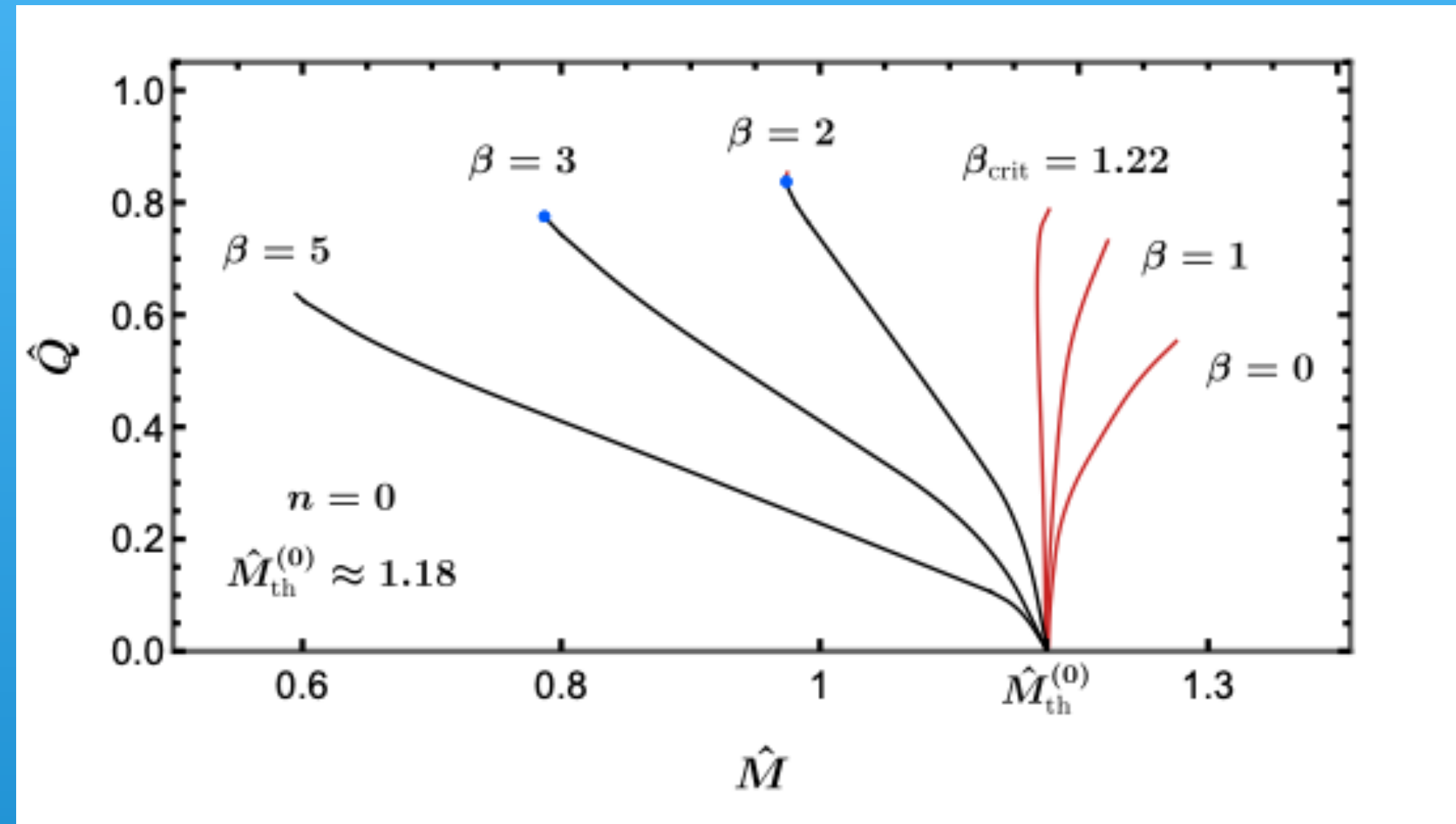


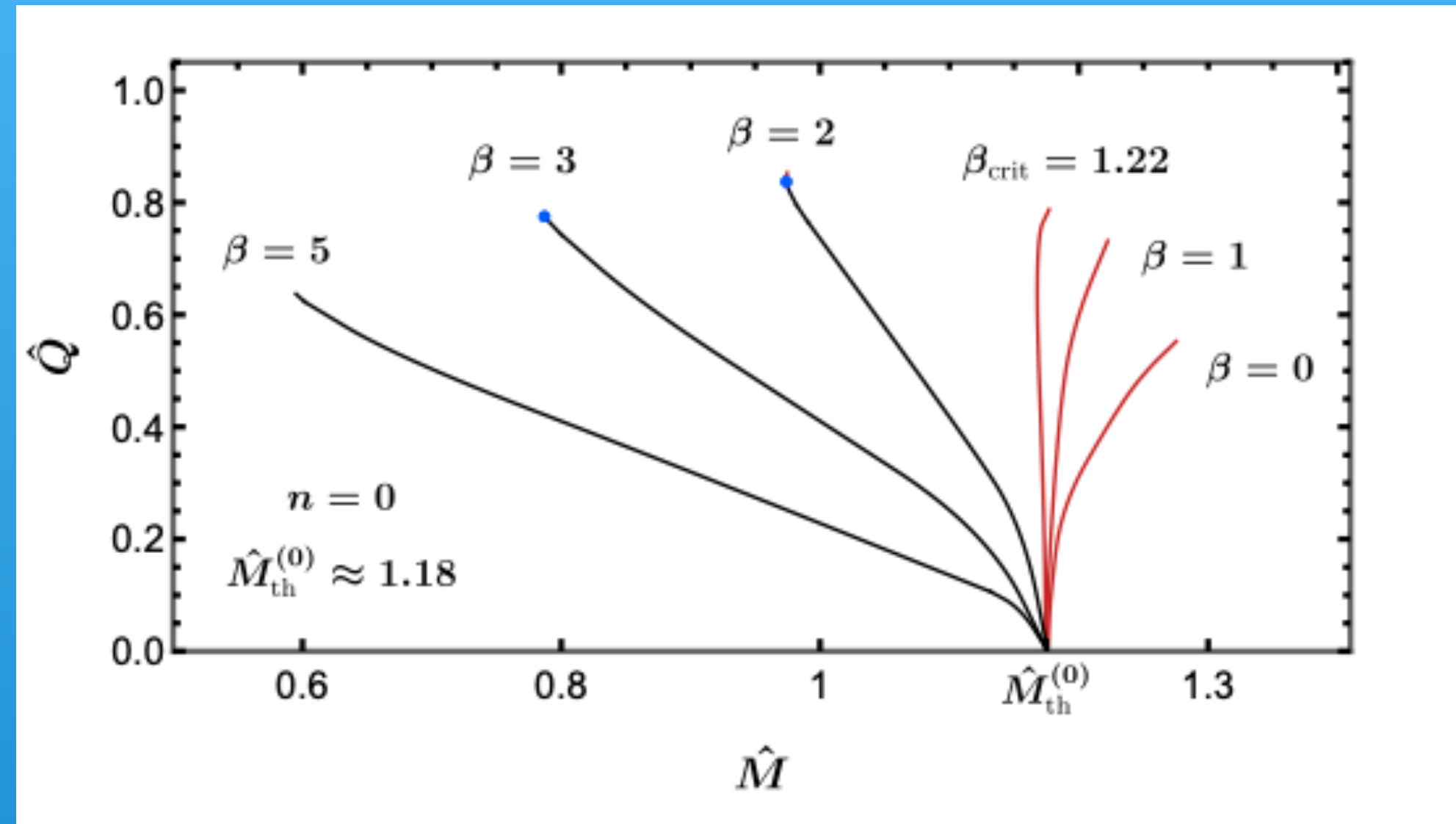
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Spontaneous scalarization in EsGBR



Spontaneous scalarization in EsGBR

Antoniou et al 2021
arXiv:2105.04479



Spontaneous scalarization in EMsGB

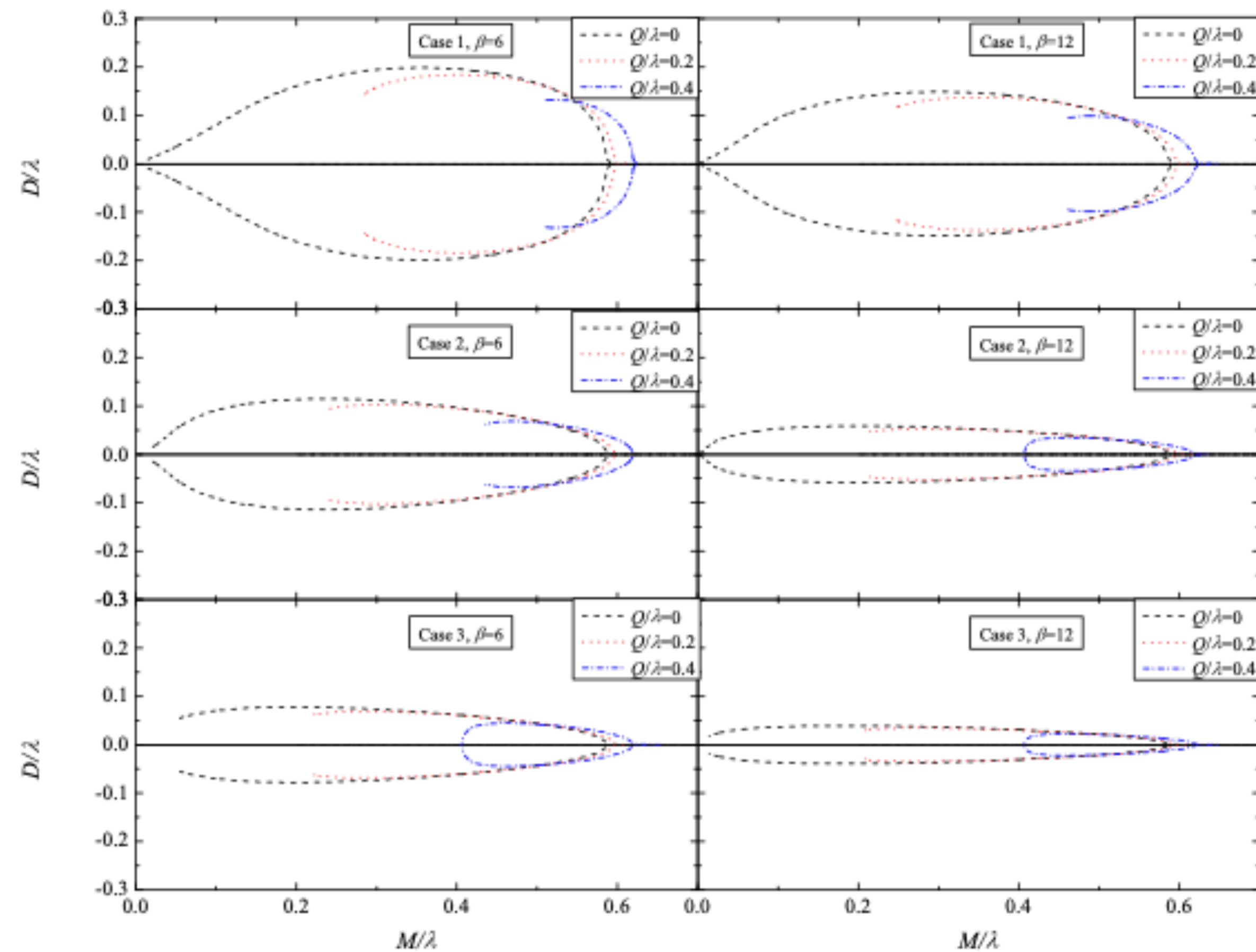


FIG. 4: The scalar charge of the black hole as a function of its mass. Figures for all three of the coupling functions are shown (Case I, Case II and Case III) for $\beta = 6$ and $\beta = 12$. In each figure the sequences with black hole charge $Q/\lambda = 0$, $Q/\lambda = 0.2$ and $Q/\lambda = 0.4$ are shown. All the quantities are normalized to the parameter λ .

Spontaneous scalarization in EMsGB

Doneva et al 2021
arXiv:1809.00844

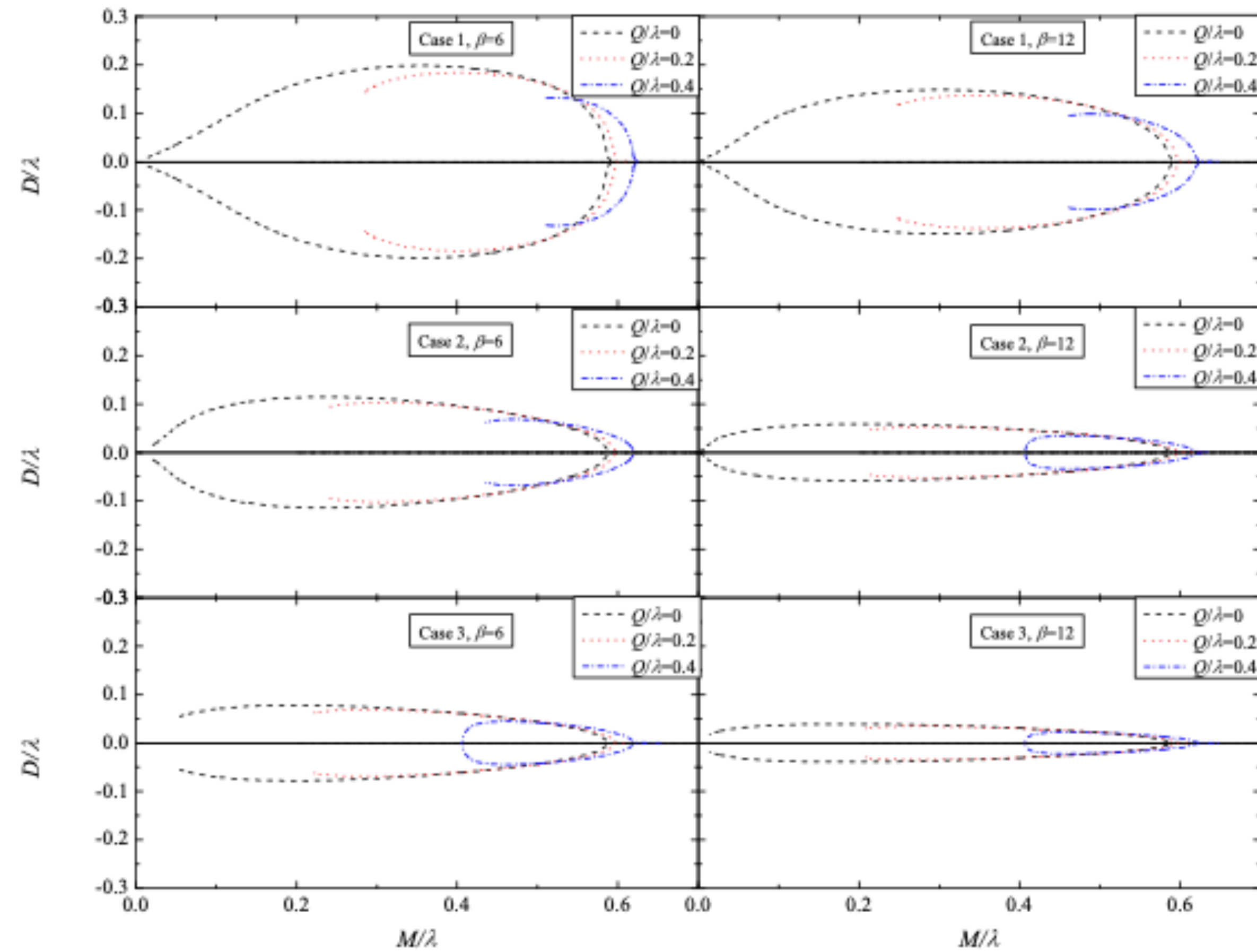


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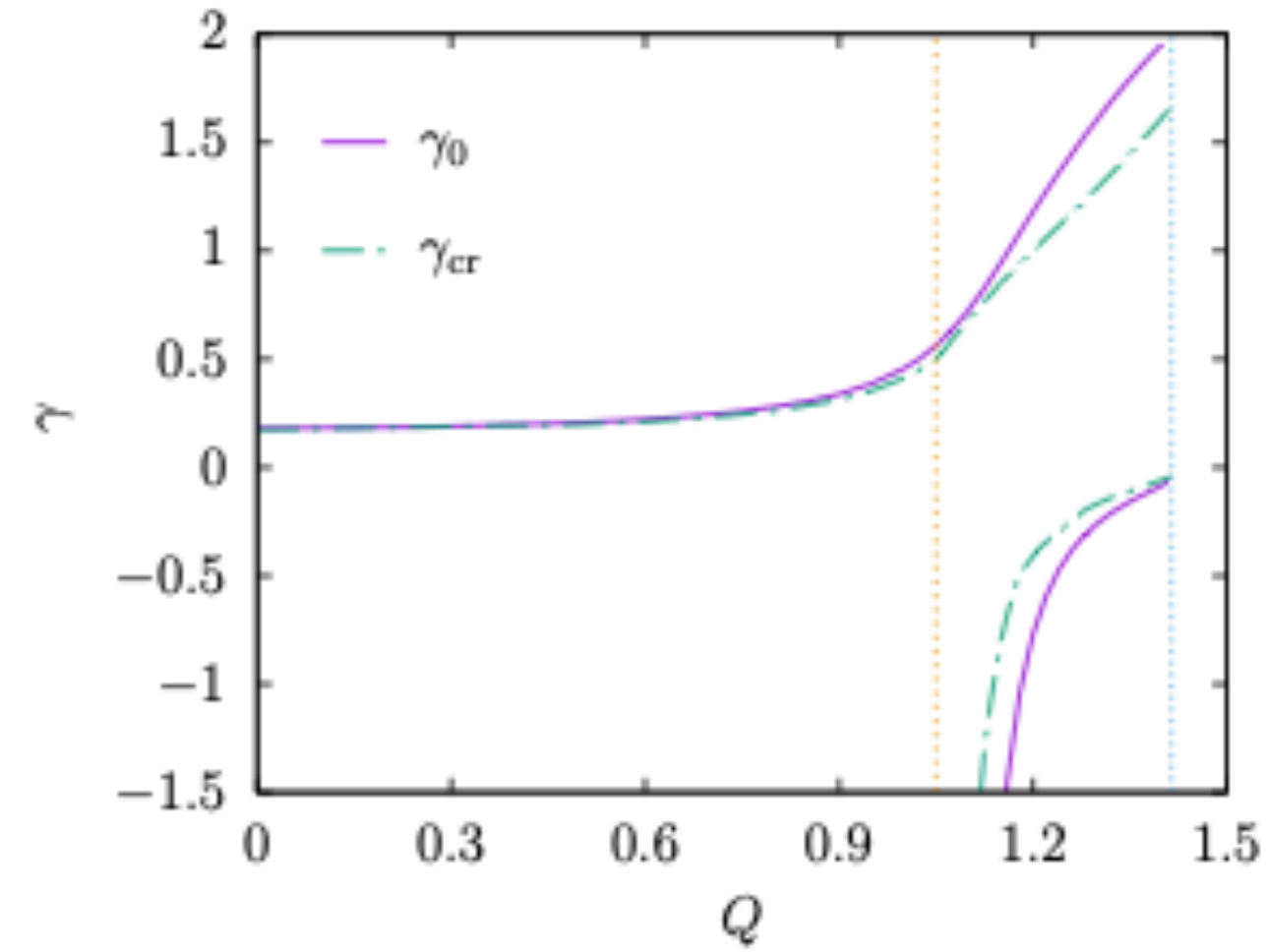


Figure 4: We show the domain of existence of scalarized, charged black holes in the γ - Q -plane. Solutions exist for values $\gamma \in [\gamma_0 : \gamma_{cr}]$, where γ_0 (solid purple) corresponds to the value of γ for which $\phi(r) \ll 1$ or – equivalently – to the value of γ for which solutions of the scalar field equation in the background of the RN solution exist (see Section 3). γ_{cr} (dotted-dashed green) corresponds to the critical value of γ to where scalarized black holes exist with the maximal possible value of $\phi(r_h)$. The division of the domain into the two different approaches to criticality is indicated by a vertical line (dashed orange) at $Q = 1.05$, while the extremal value of the charge is indicated by a vertical line (dashed blue) at $Q = \sqrt{2} \approx 1.414$.

Spontaneous scalarization in EMsGB

Brihaye et Hartmann 2019
arXiv:1902.05760

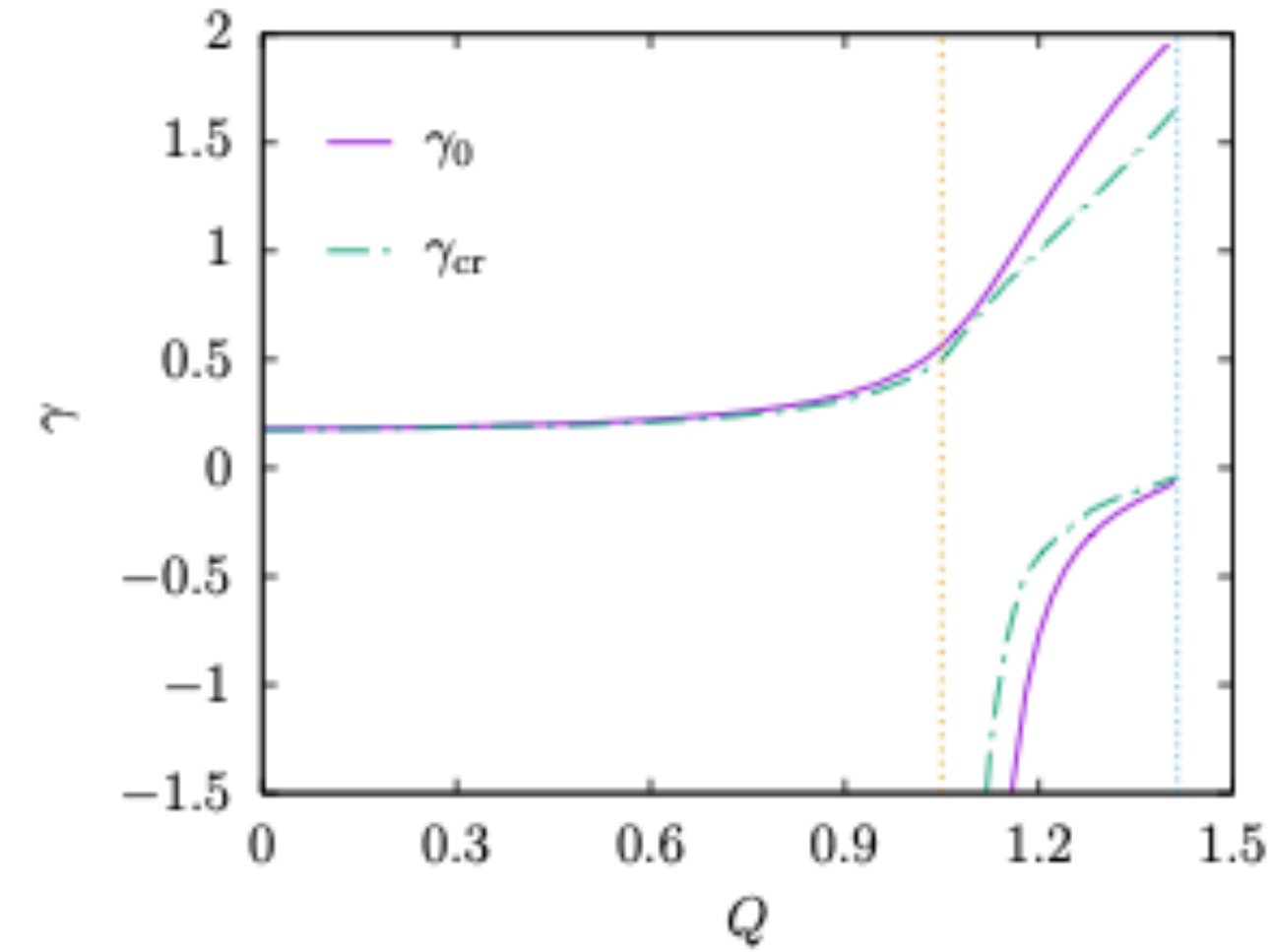


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Spontaneous scalarization in EMsGB

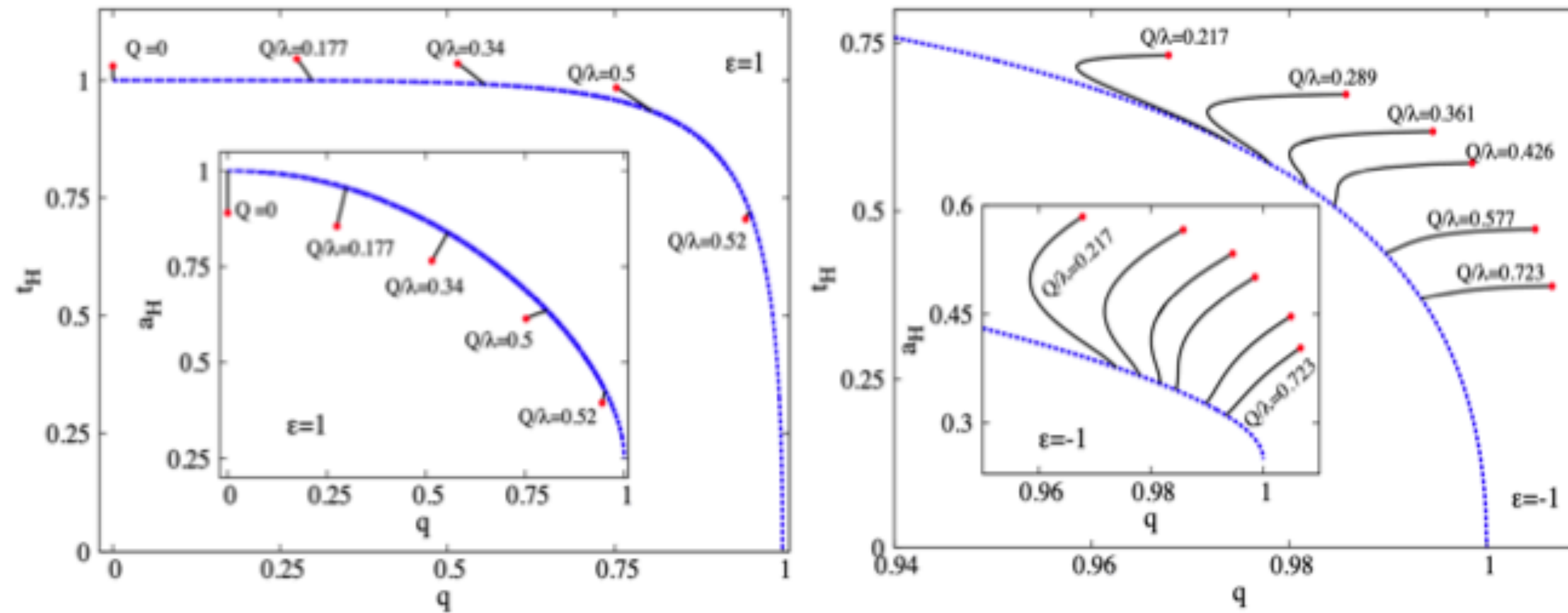


Figure 3: Charge-temperature (main panels panel) and charge-horizon area (insets) diagrams, in units set by mass, for several illustrative families of GB^ϵ scalarised RN solutions, for both $\epsilon = +1$ (left panel) and $\epsilon = -1$ (right panel). The branches of scalarised solutions bifurcate from the electrovacuum RN BHs (blue line) and terminate in critical configurations (red circles).

Spontaneous scalarization in EMsGB

Herdeiro et al 2021
[arXiv:2111.06442](https://arxiv.org/abs/2111.06442)

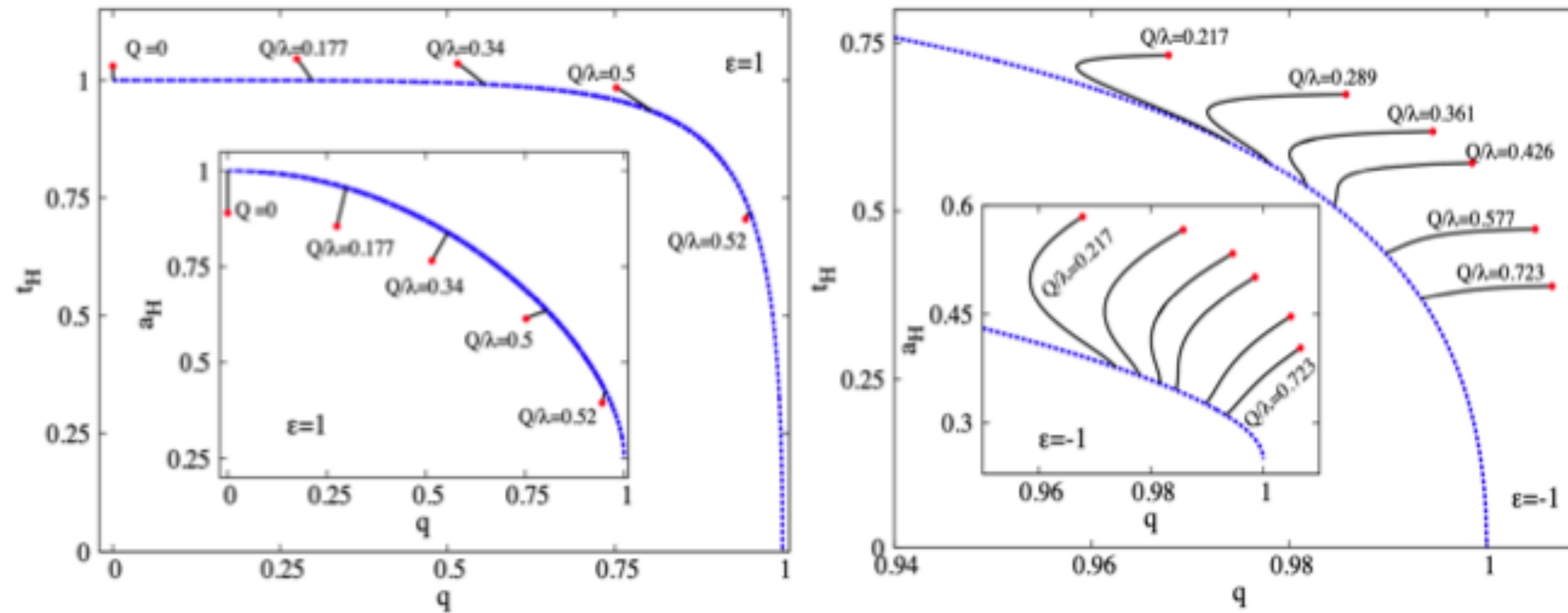


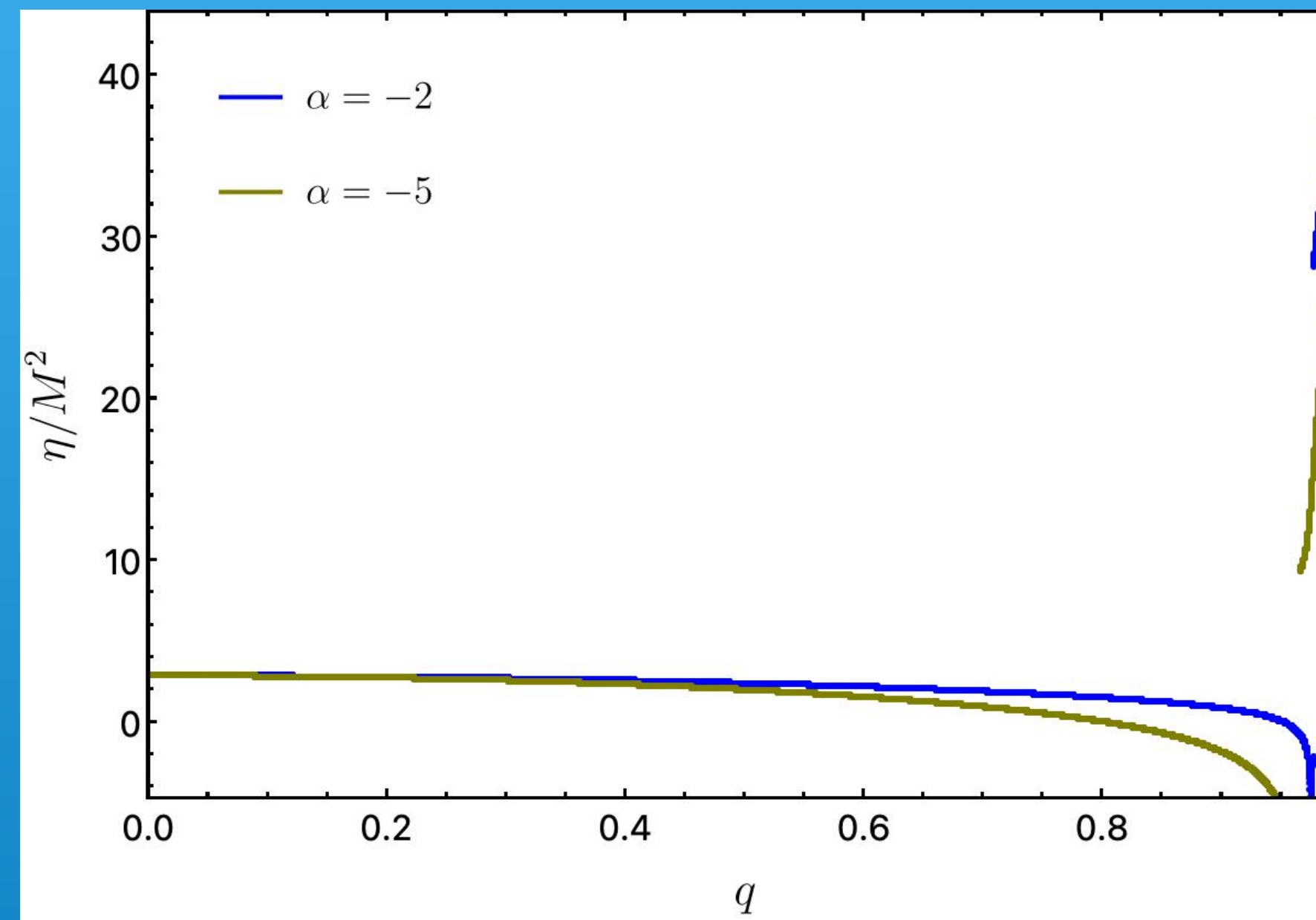
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Threshold of Scalarization for Different Values ($\alpha = 0, -2, -5$)

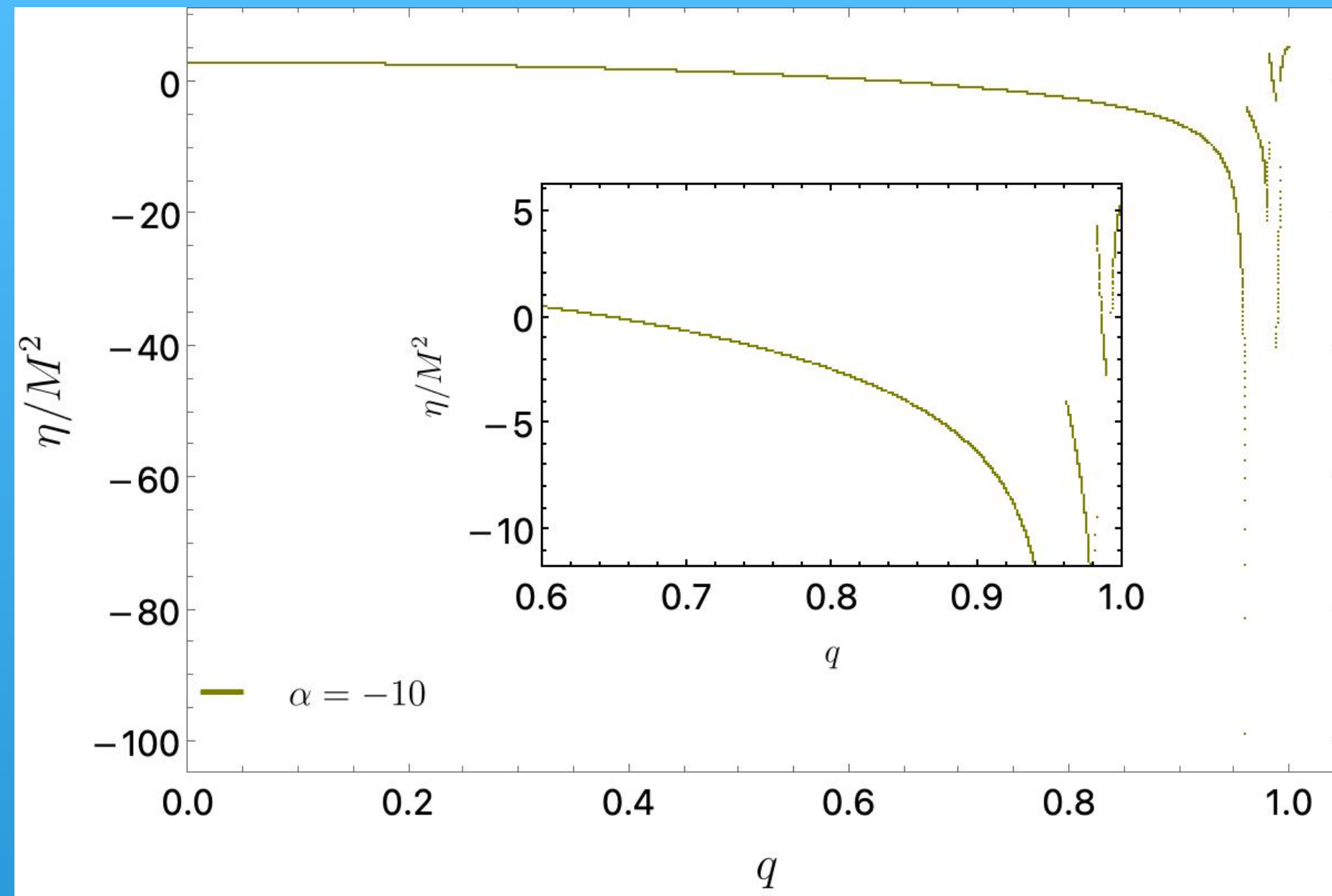
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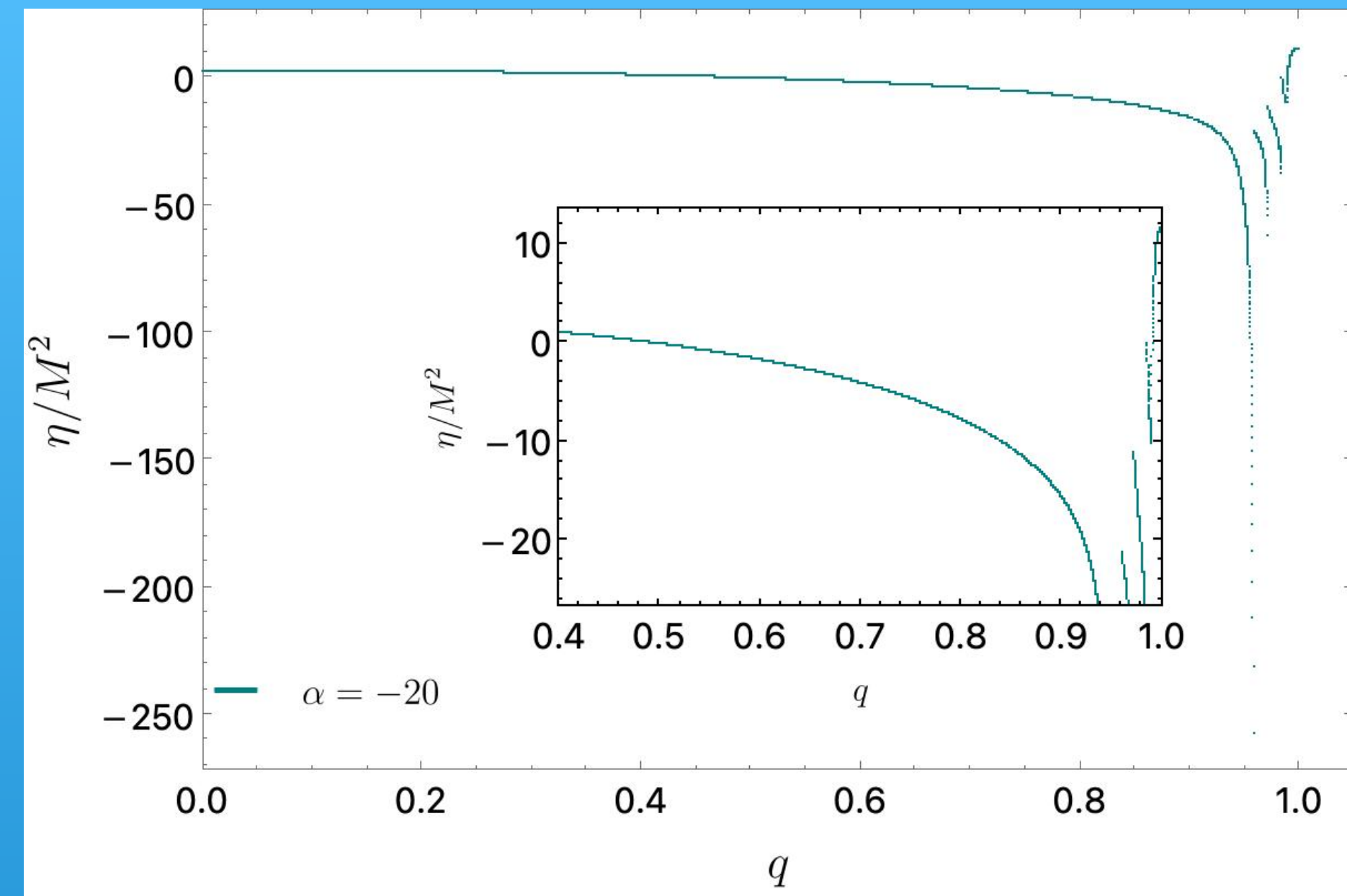
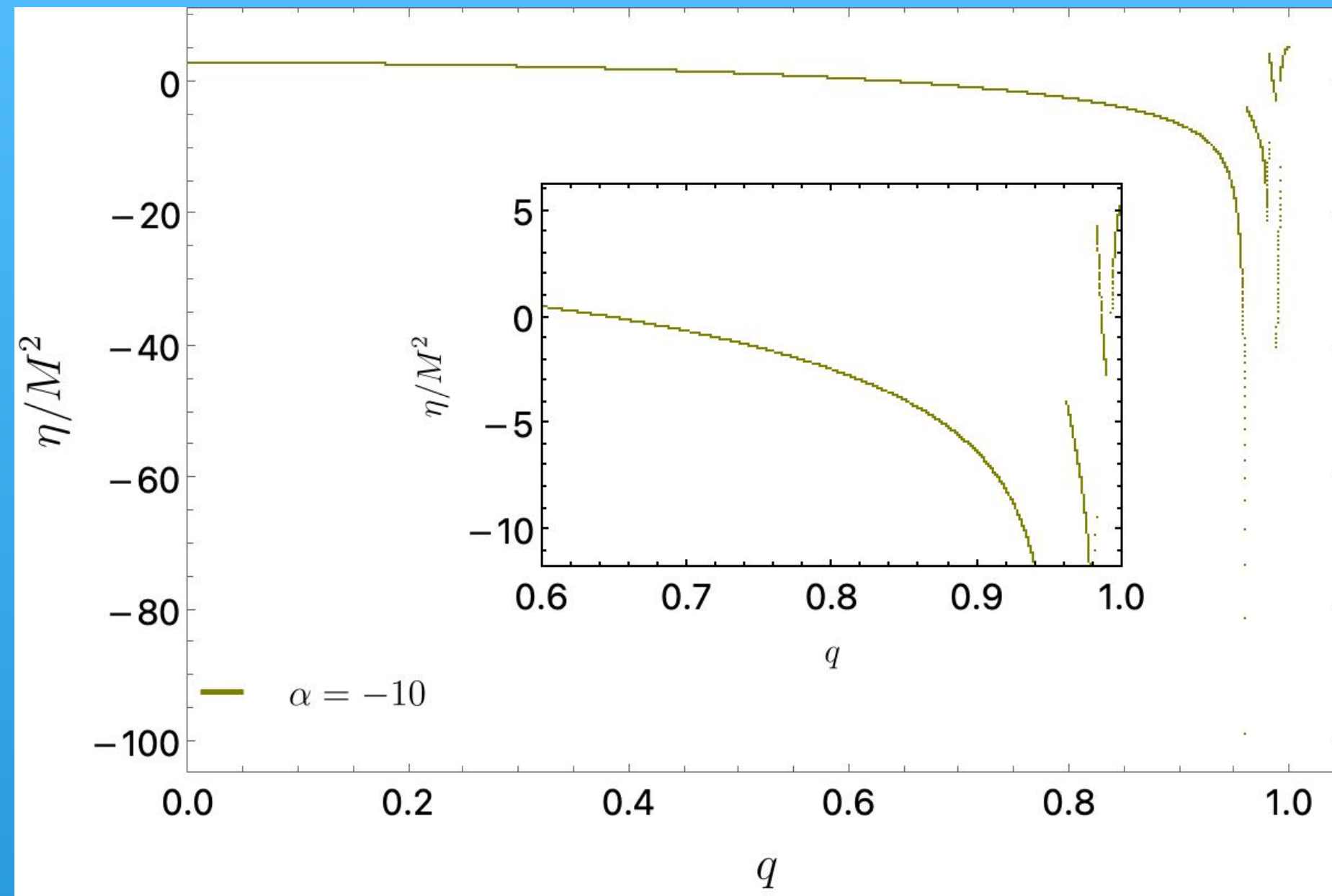
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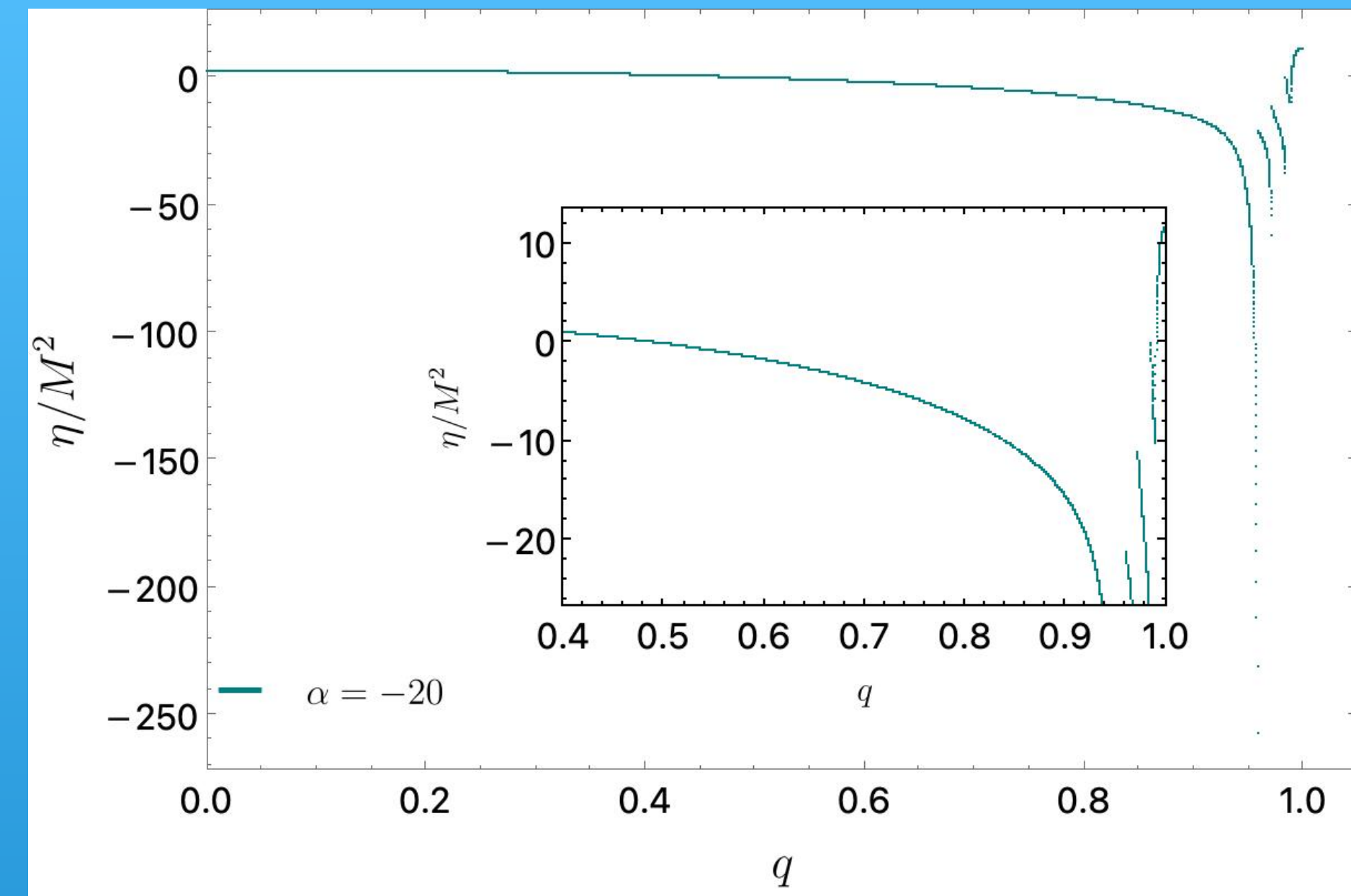
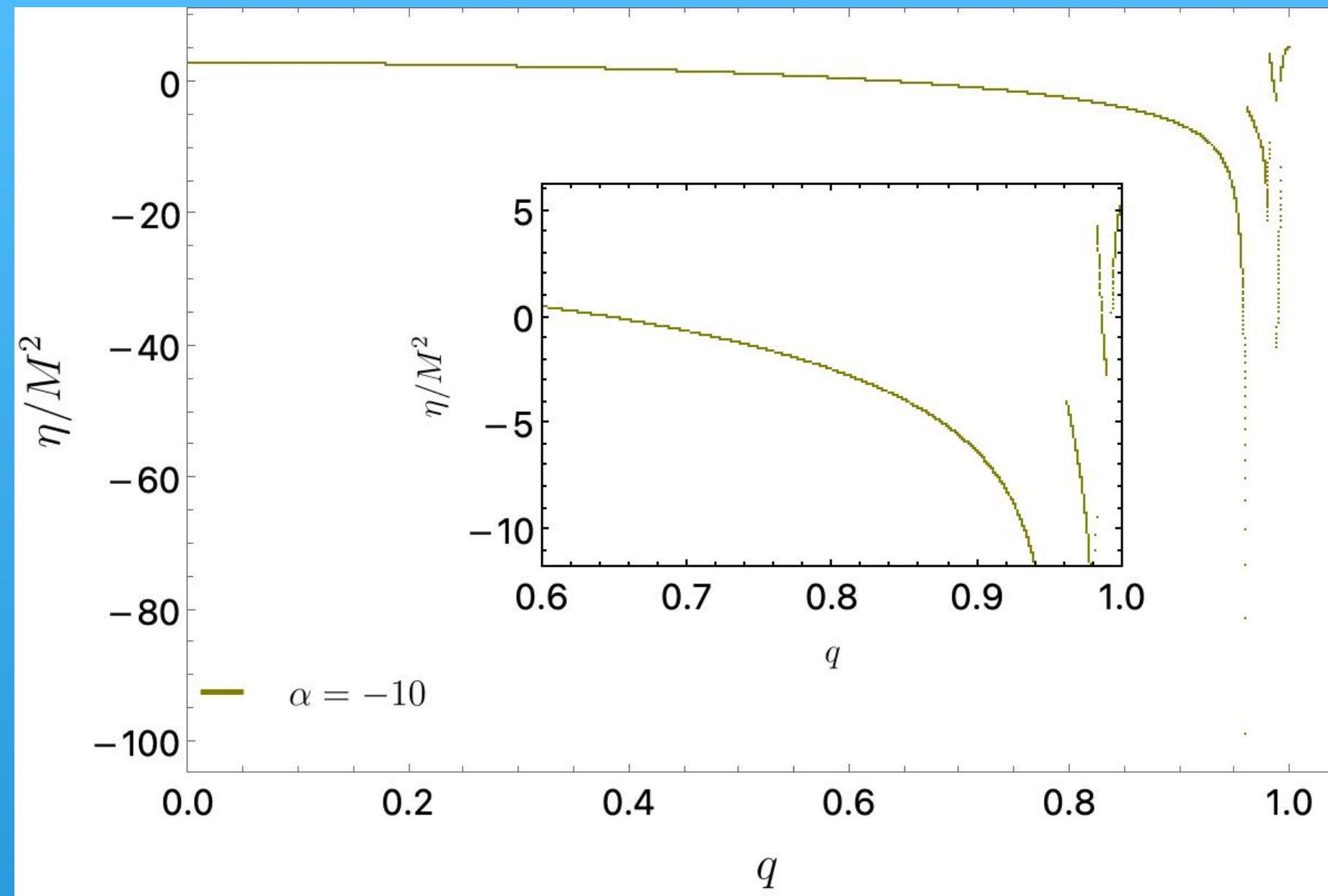
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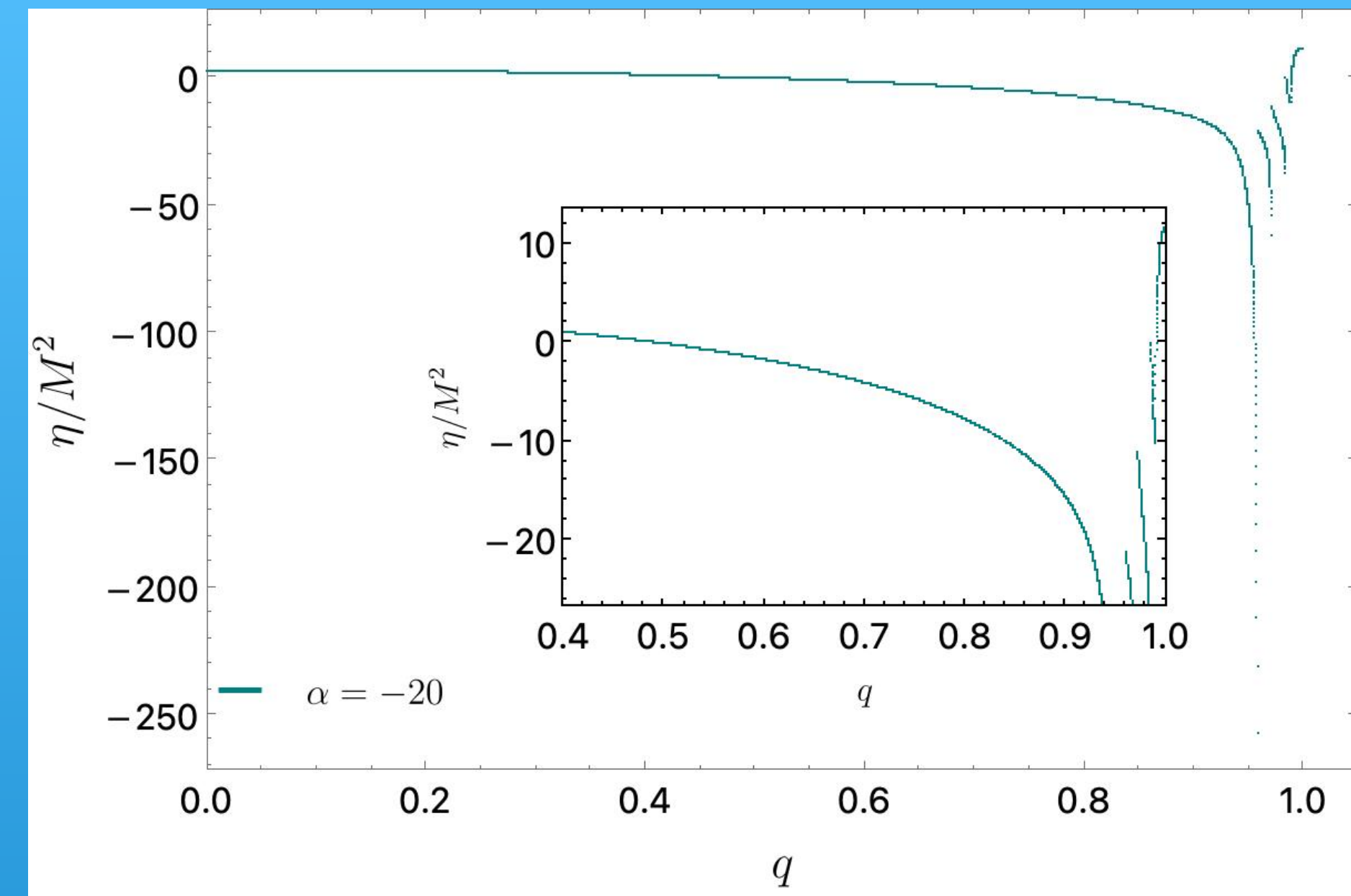
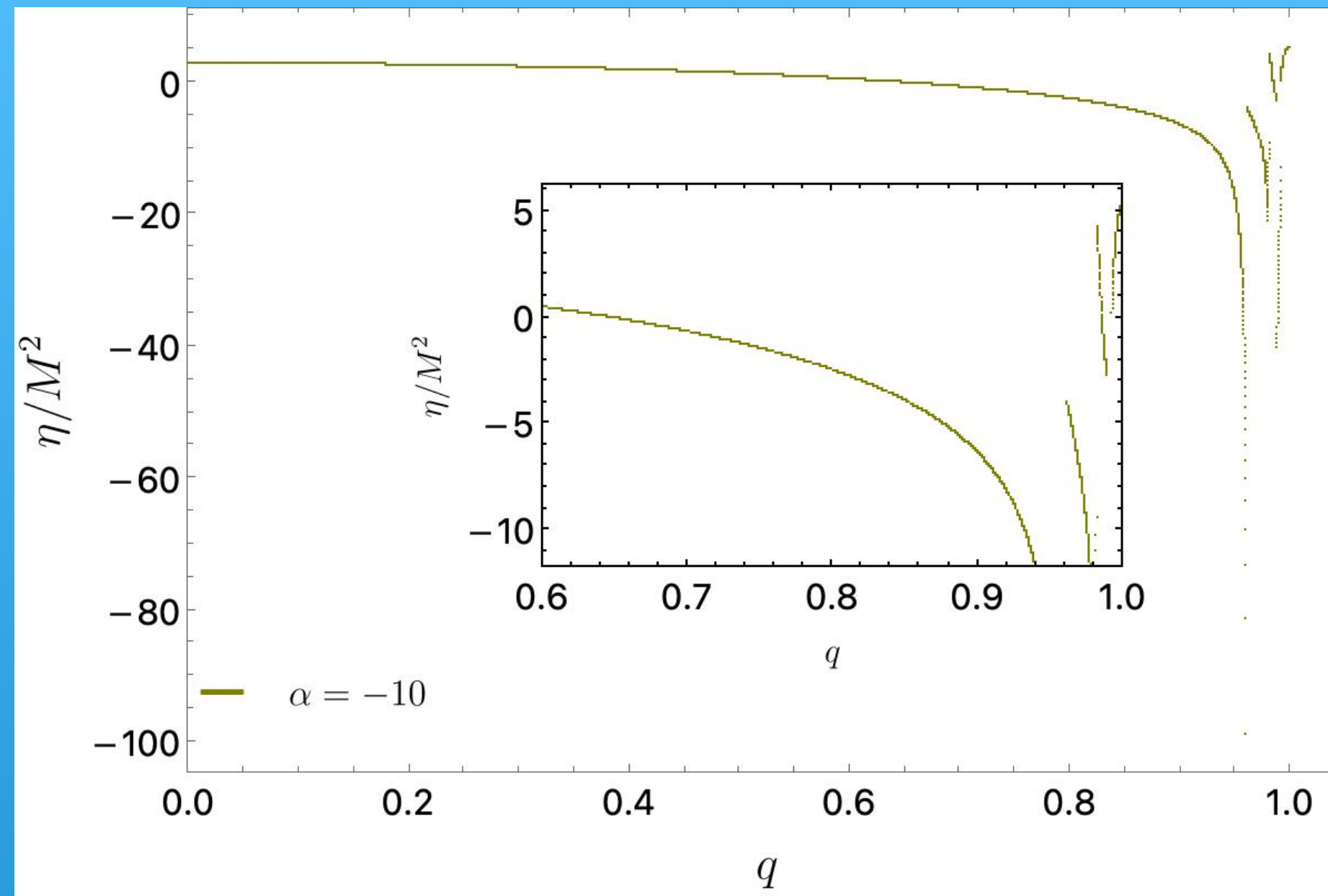


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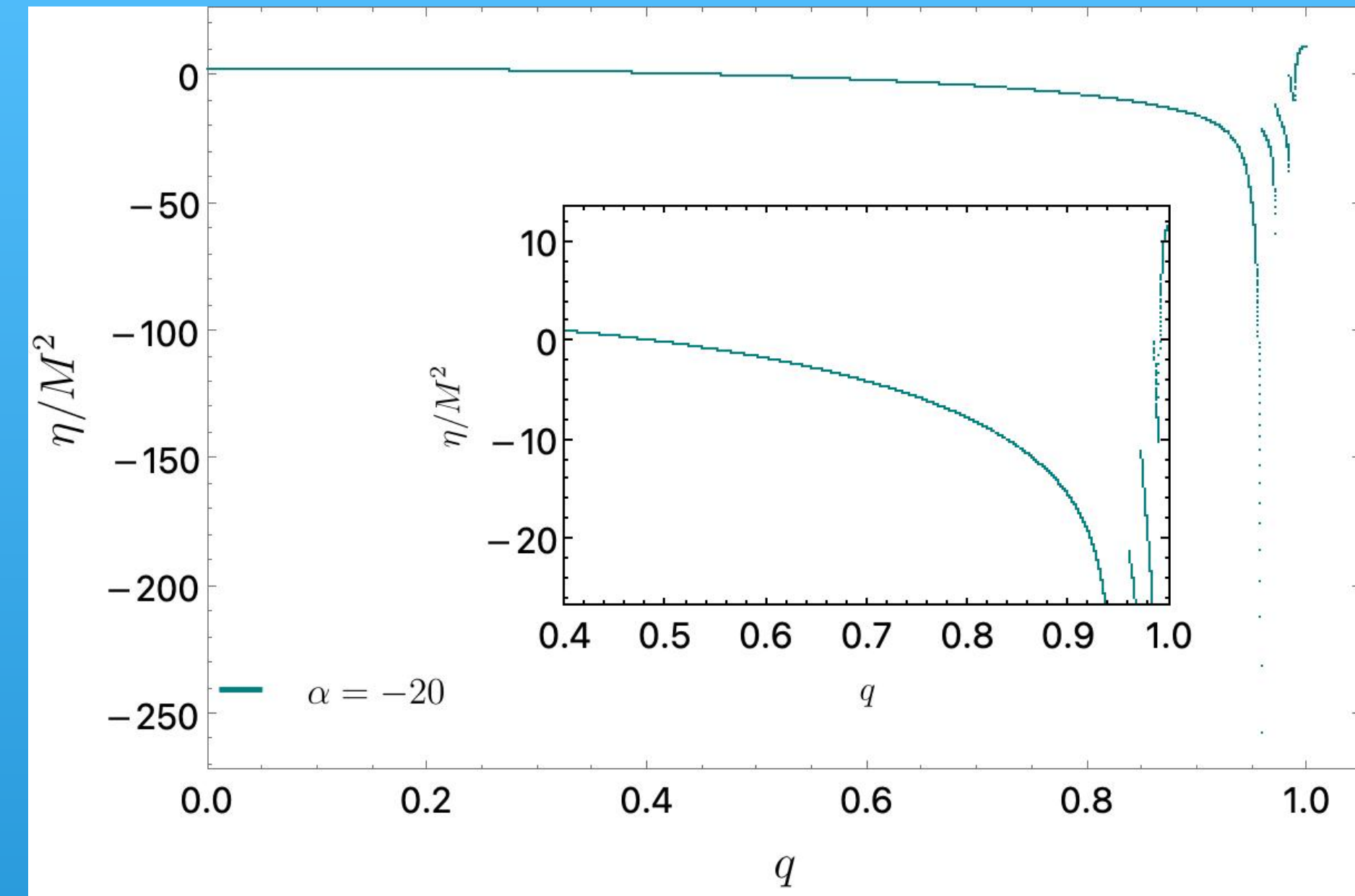
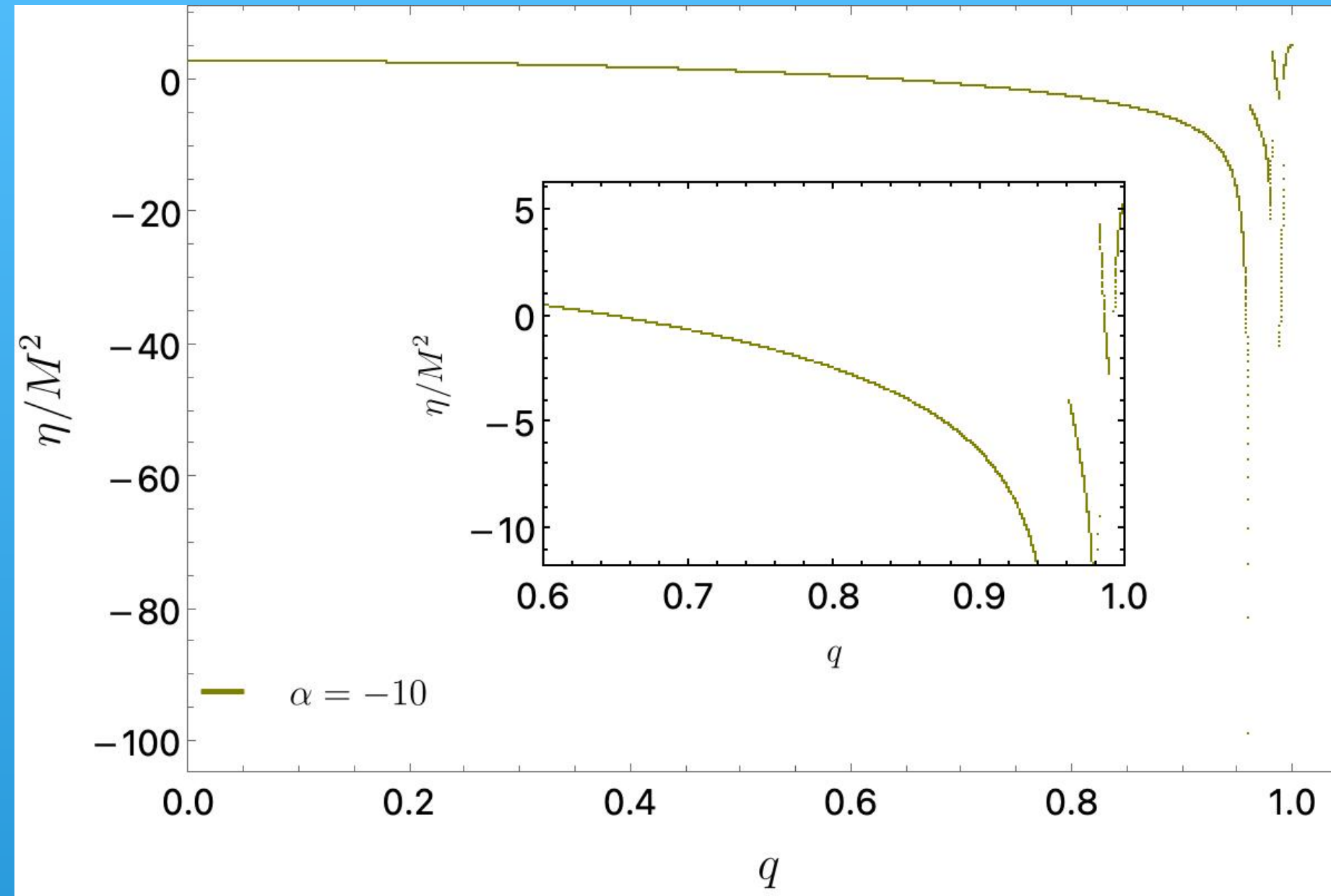
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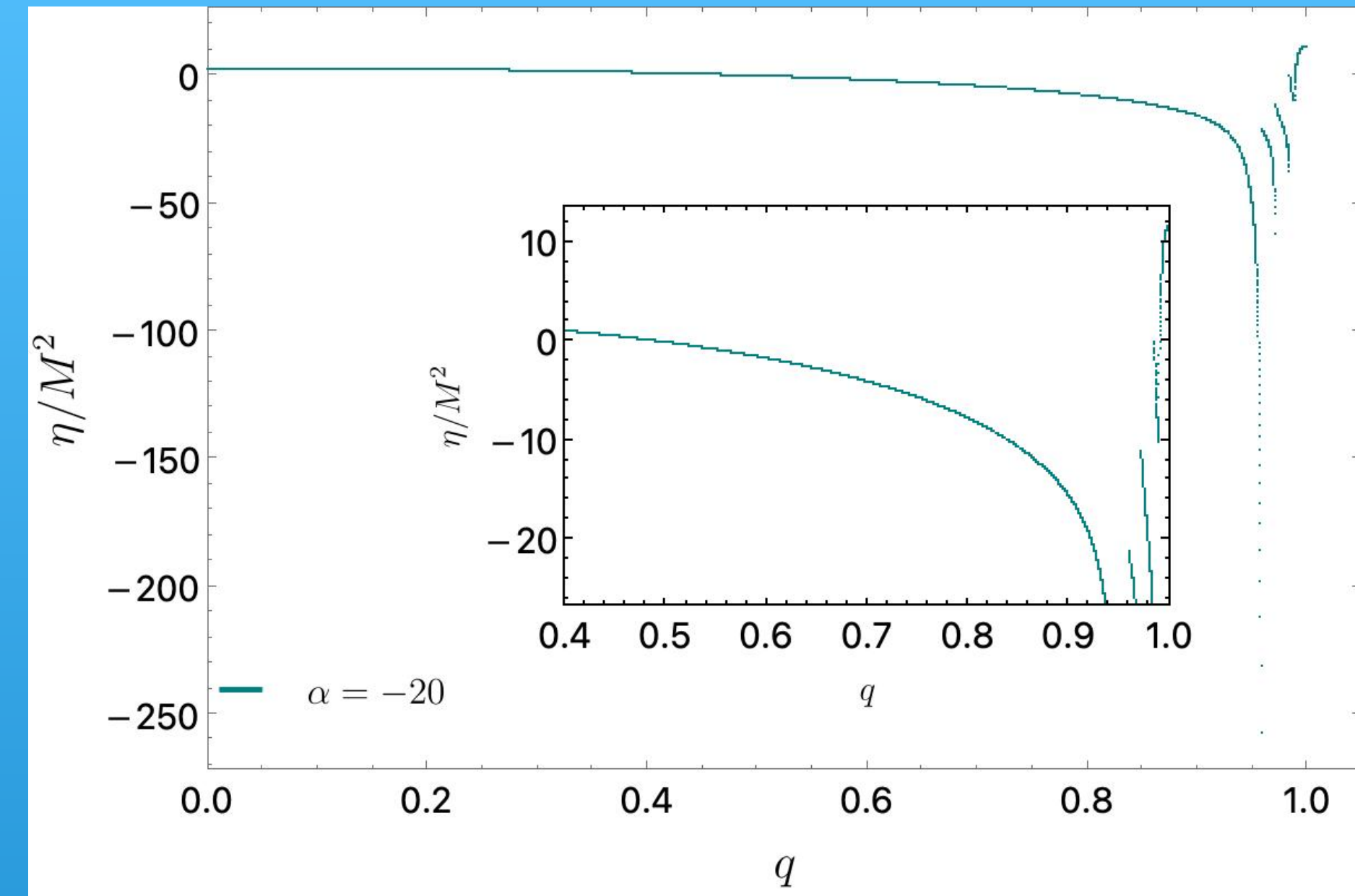
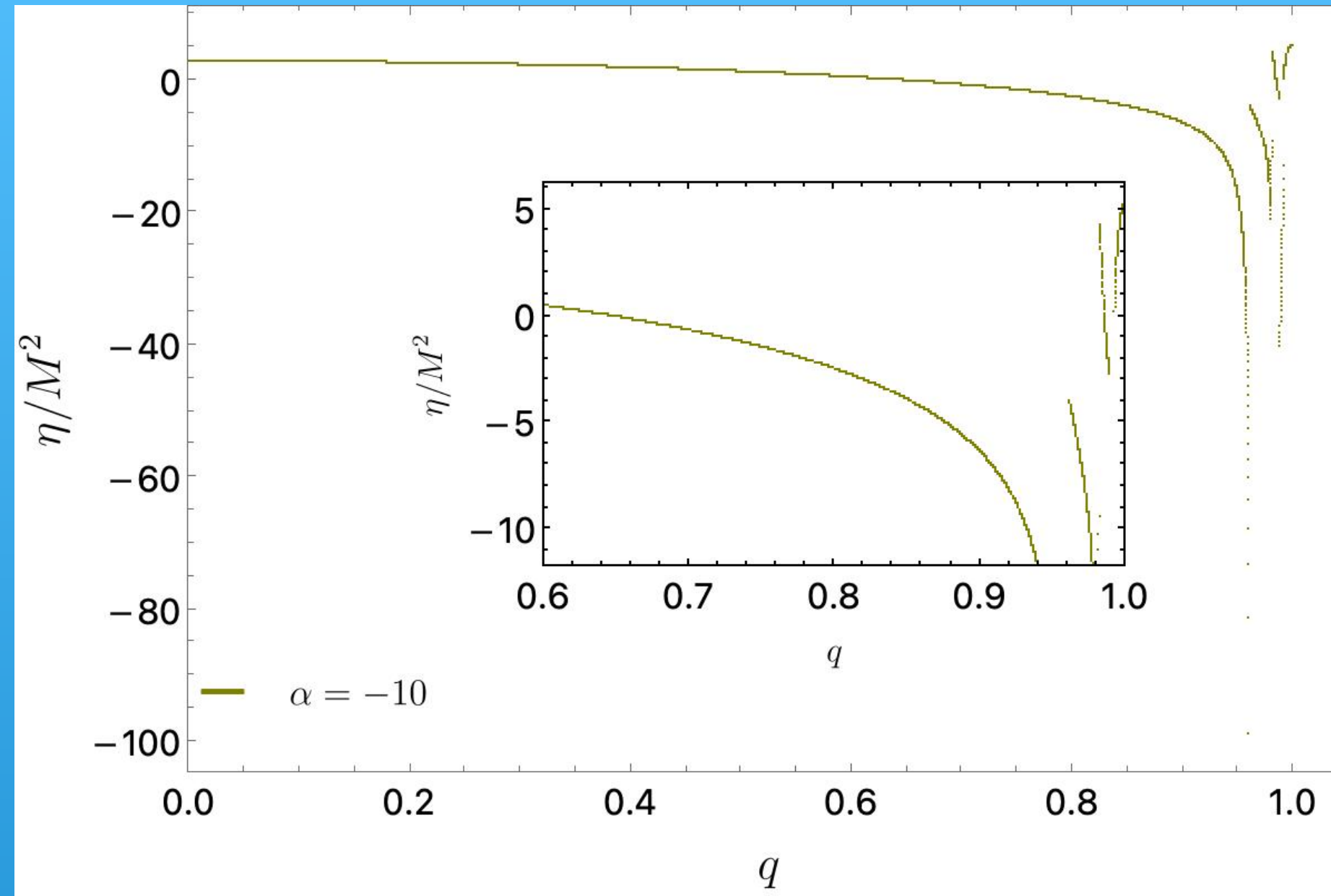
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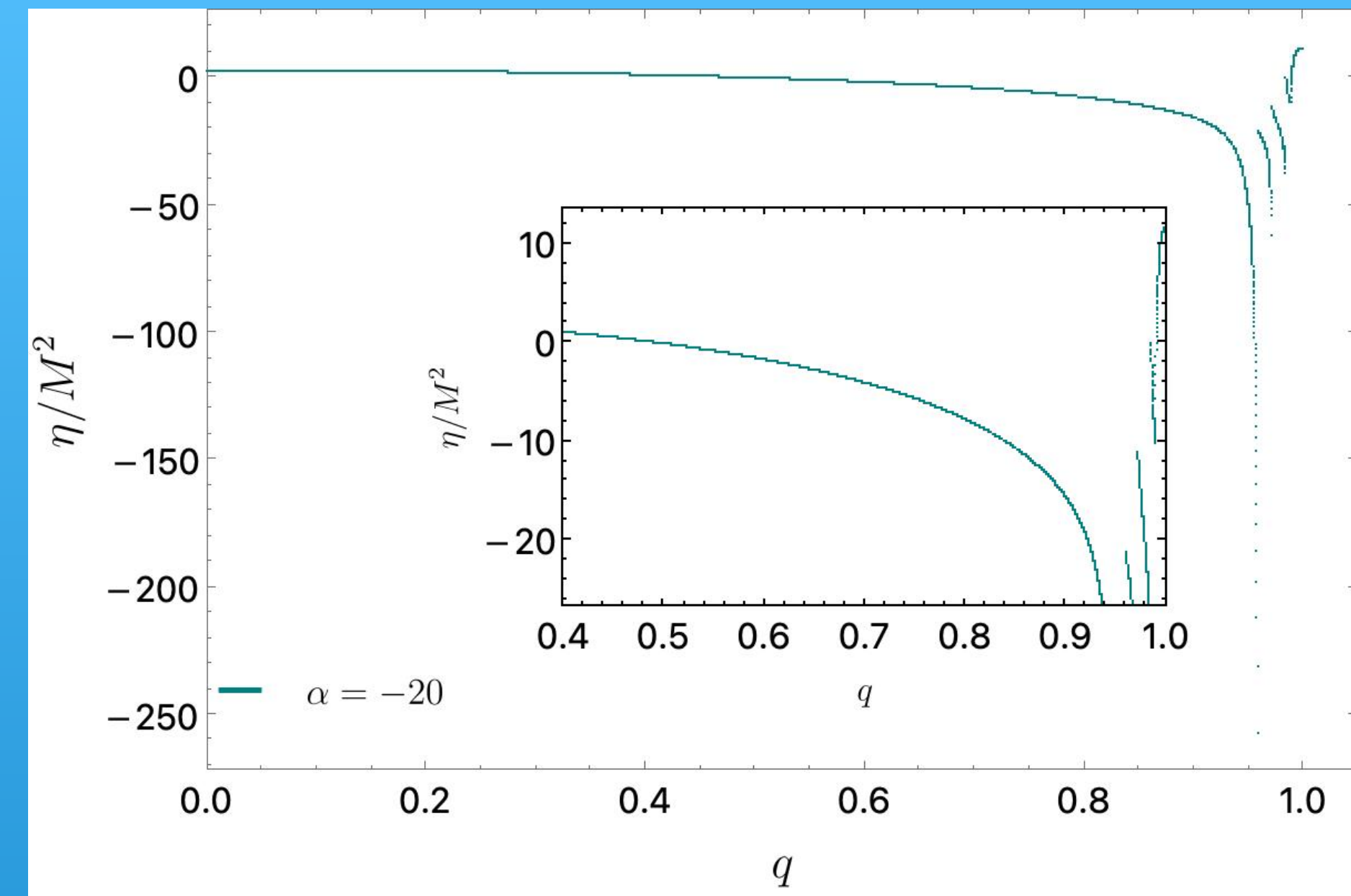
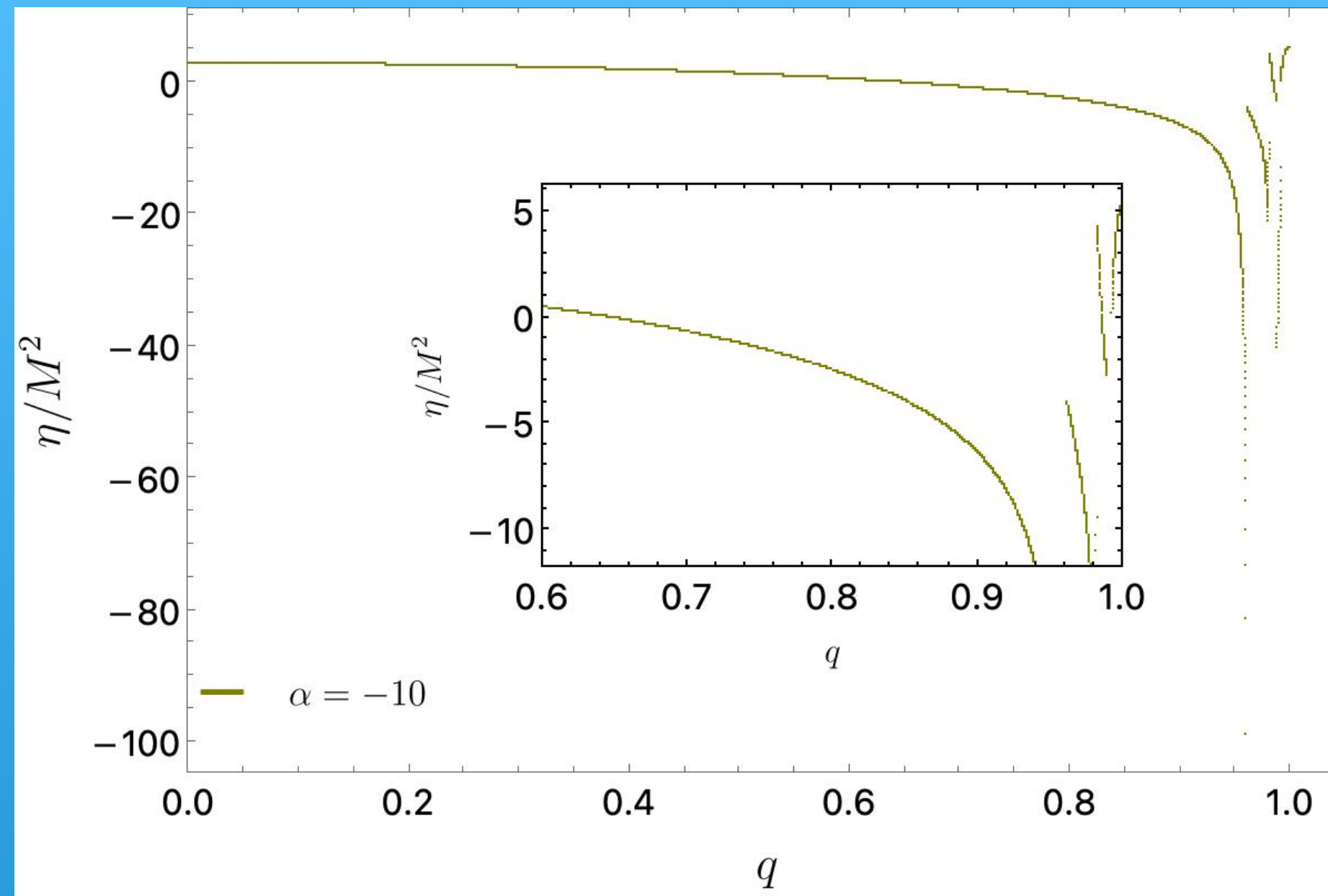
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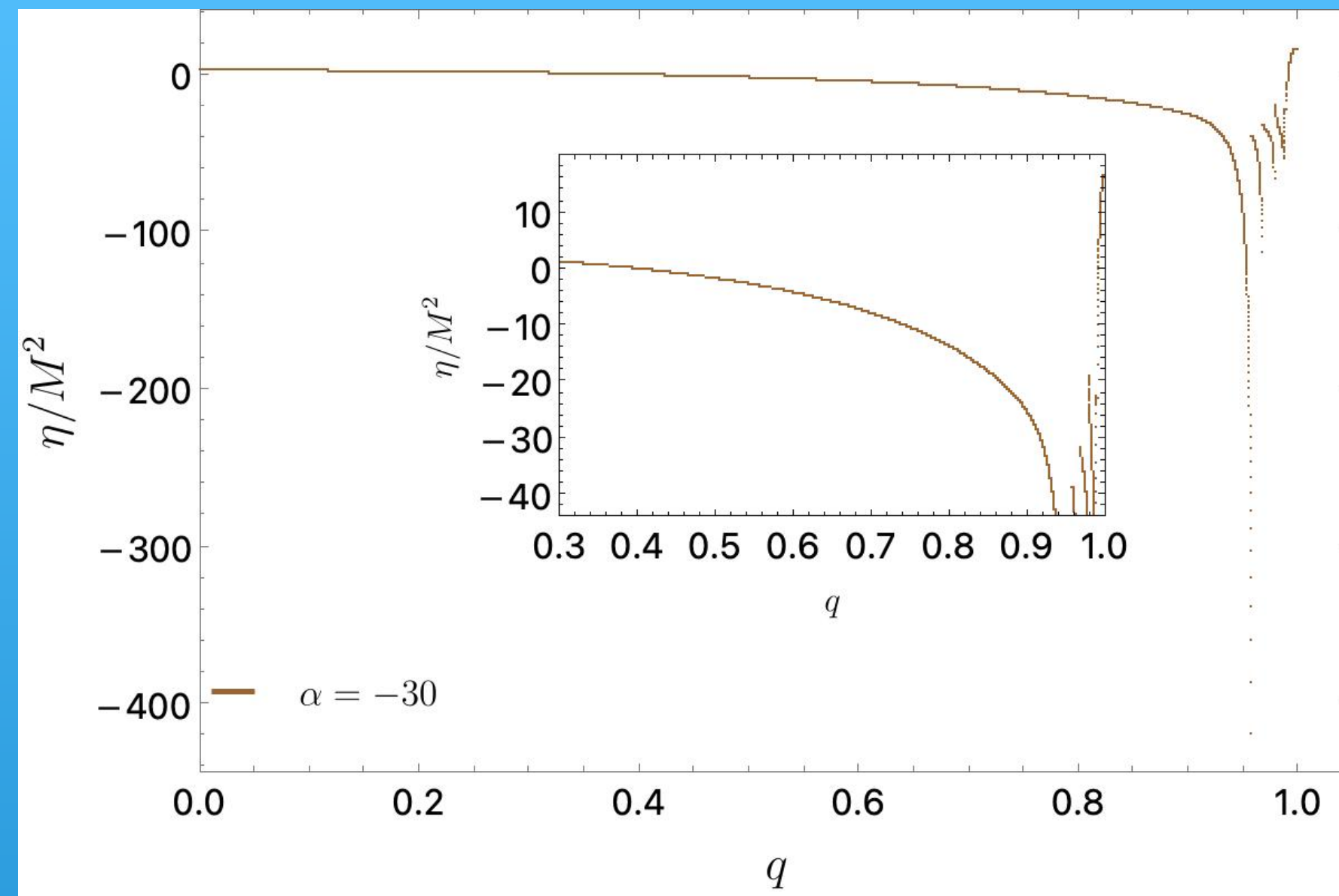
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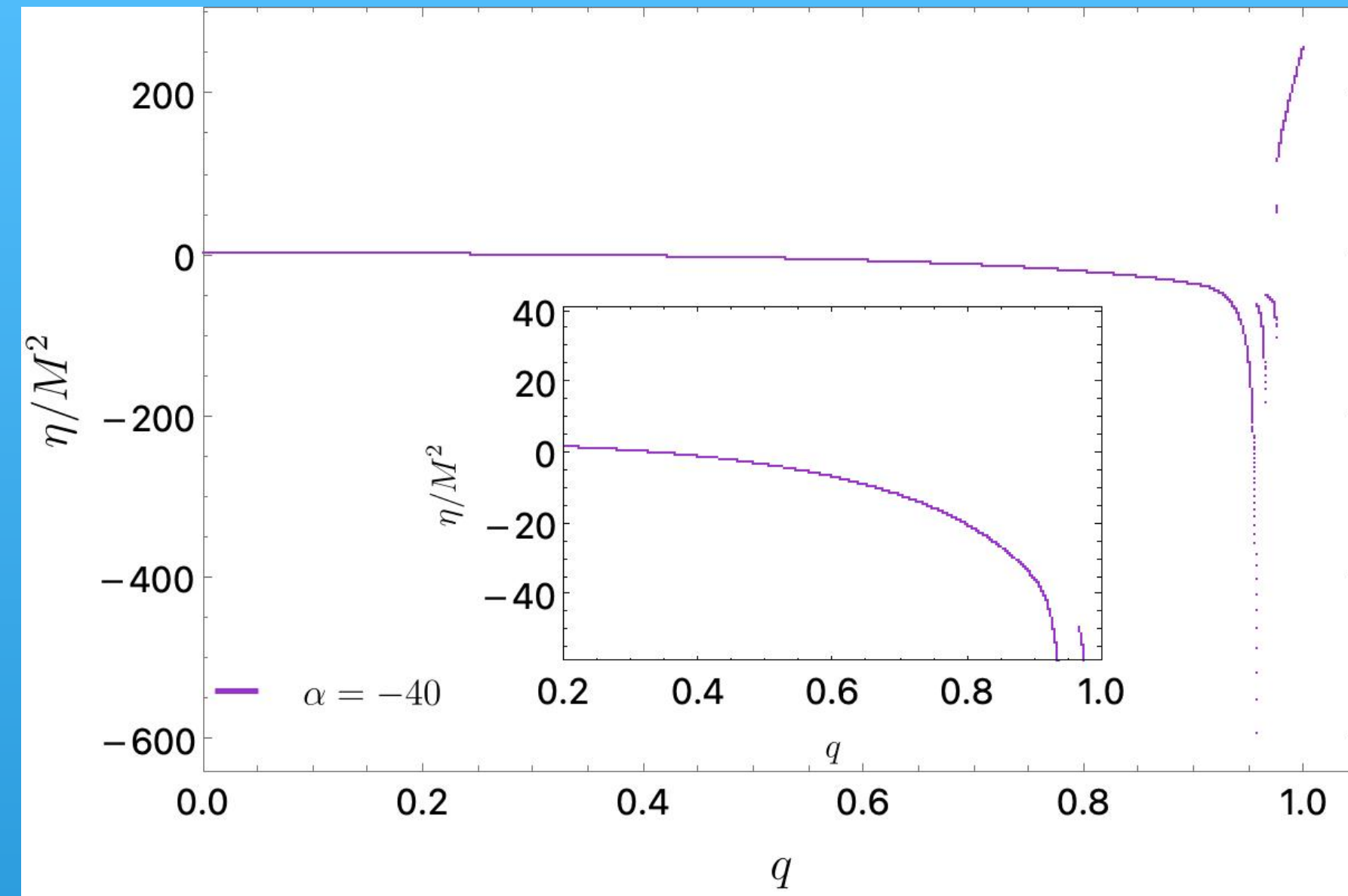
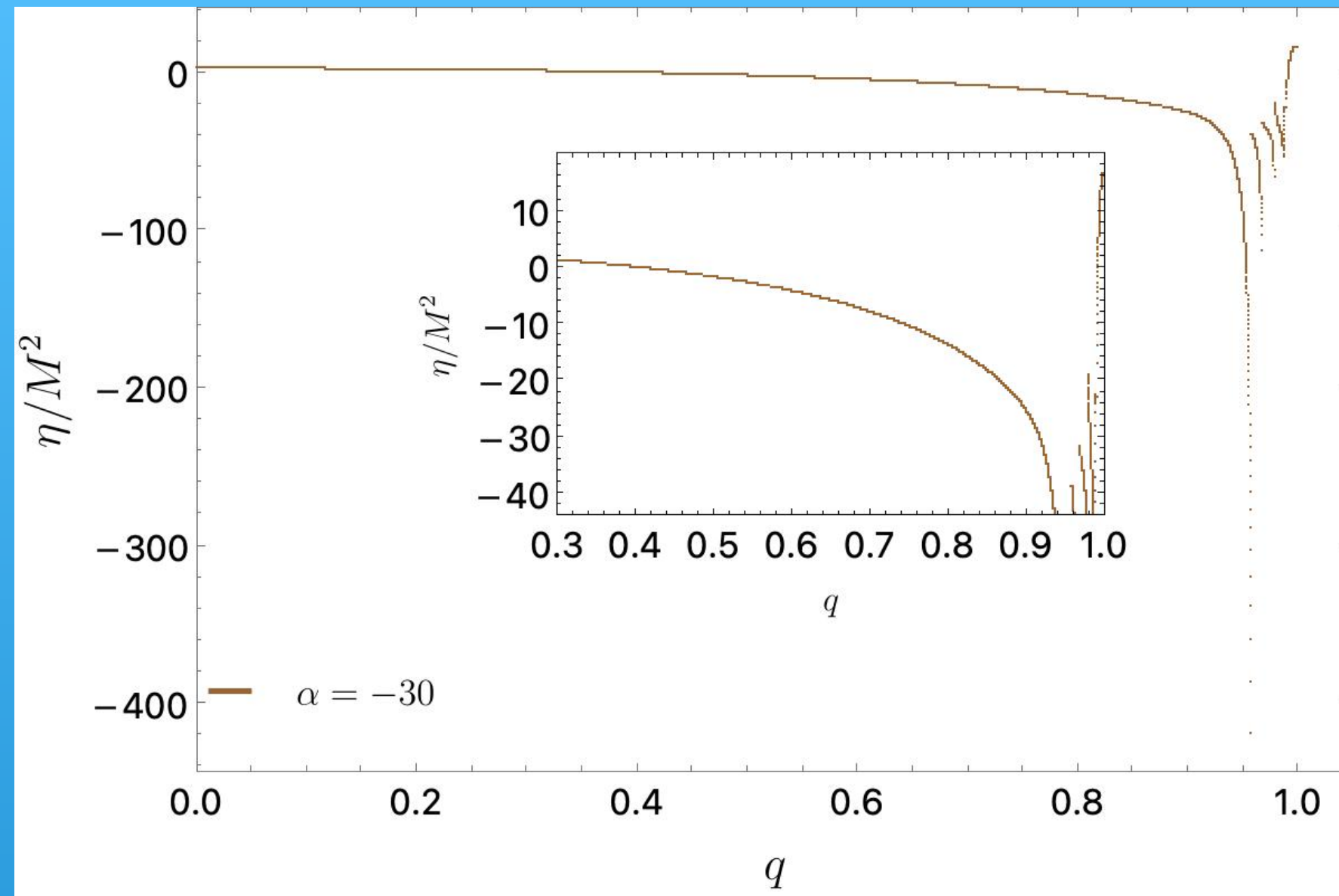
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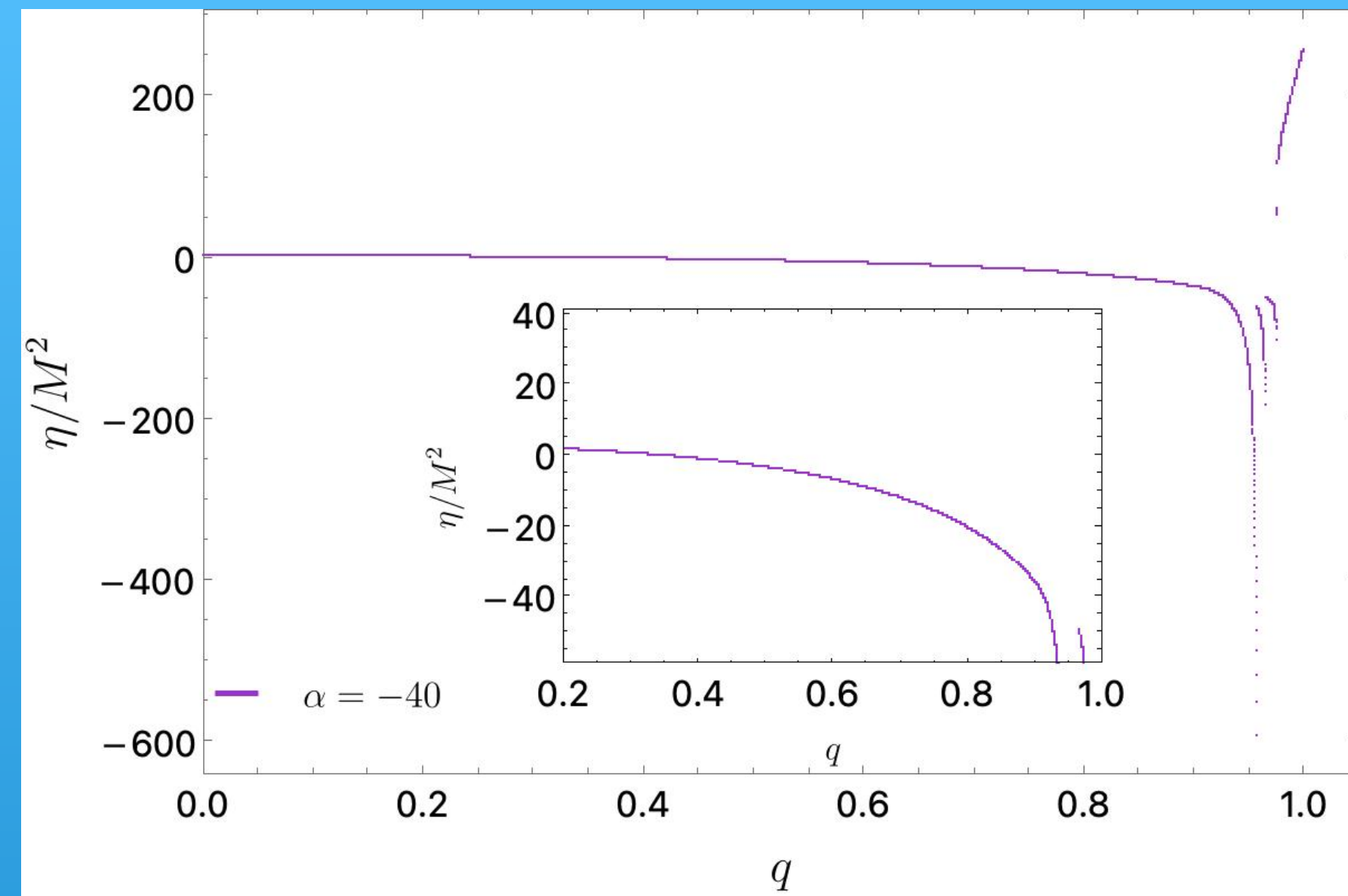
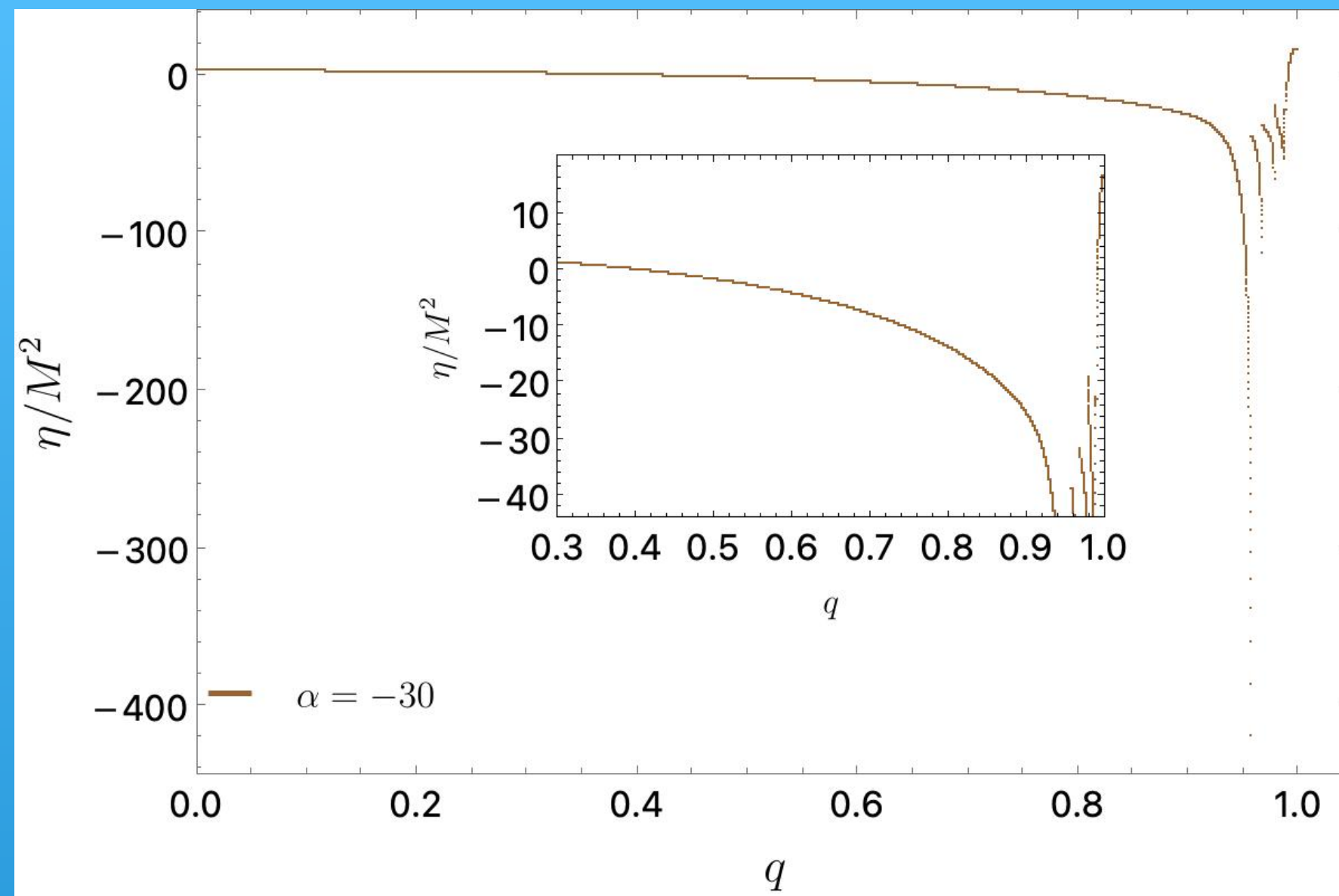
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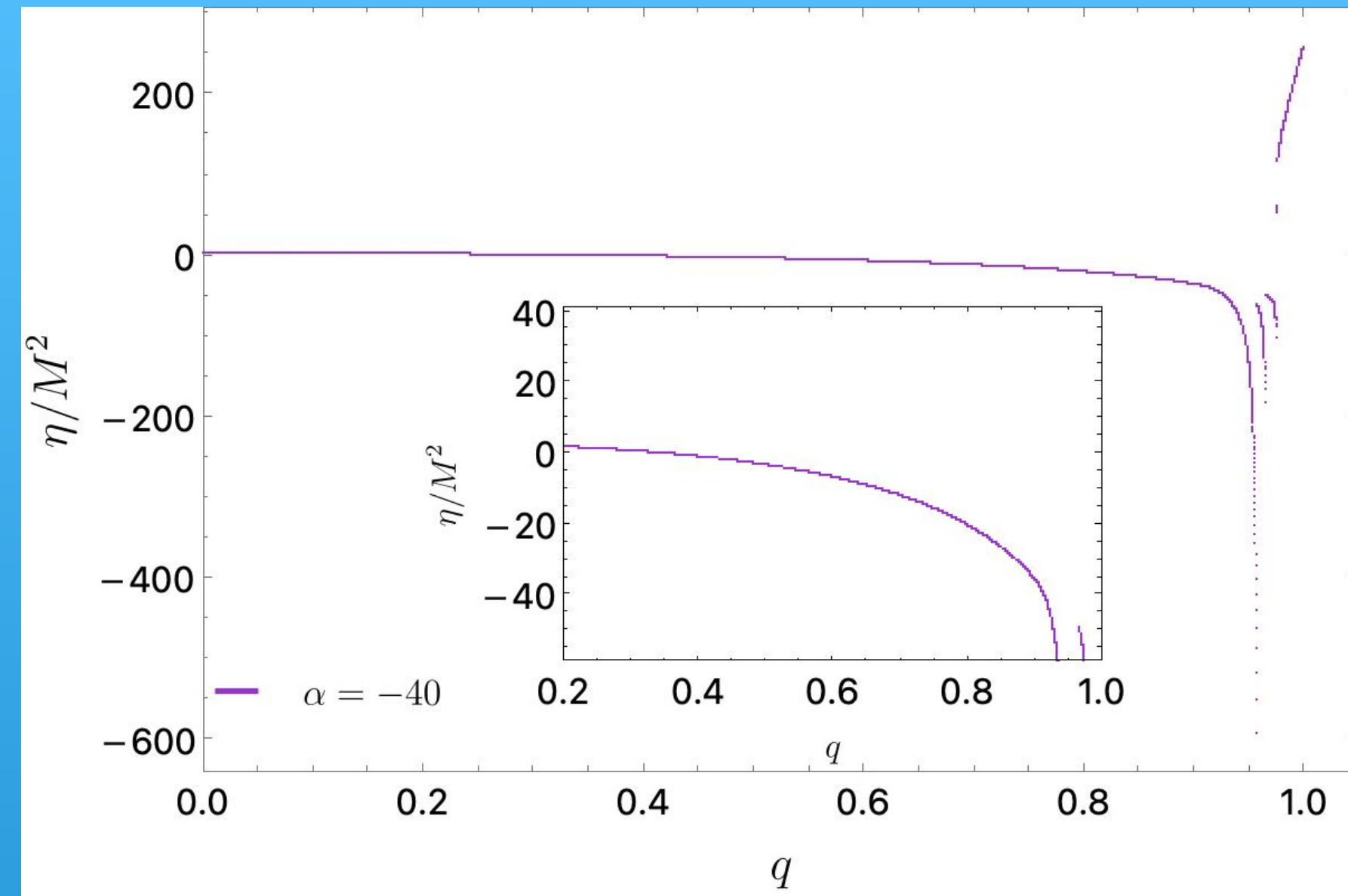
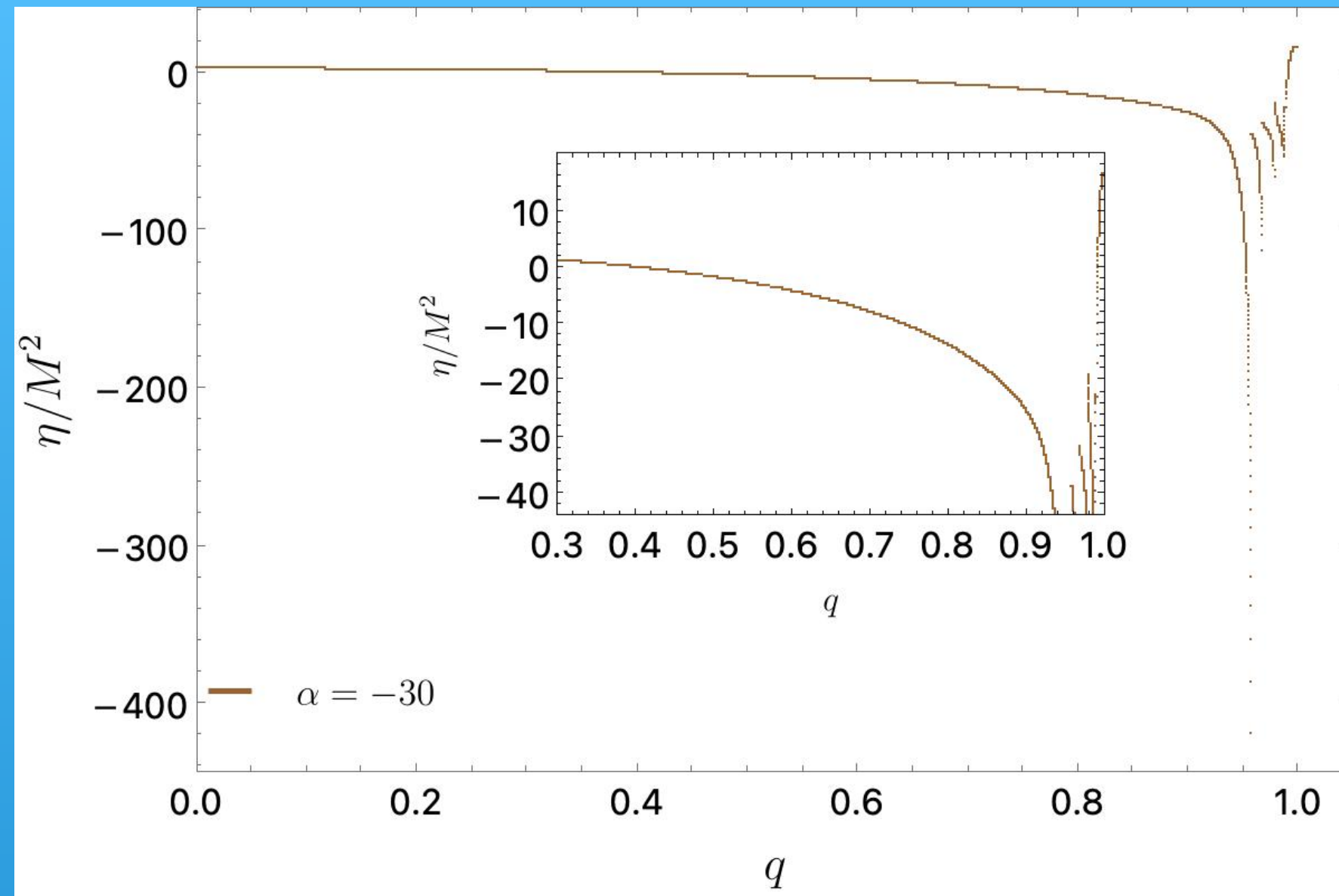


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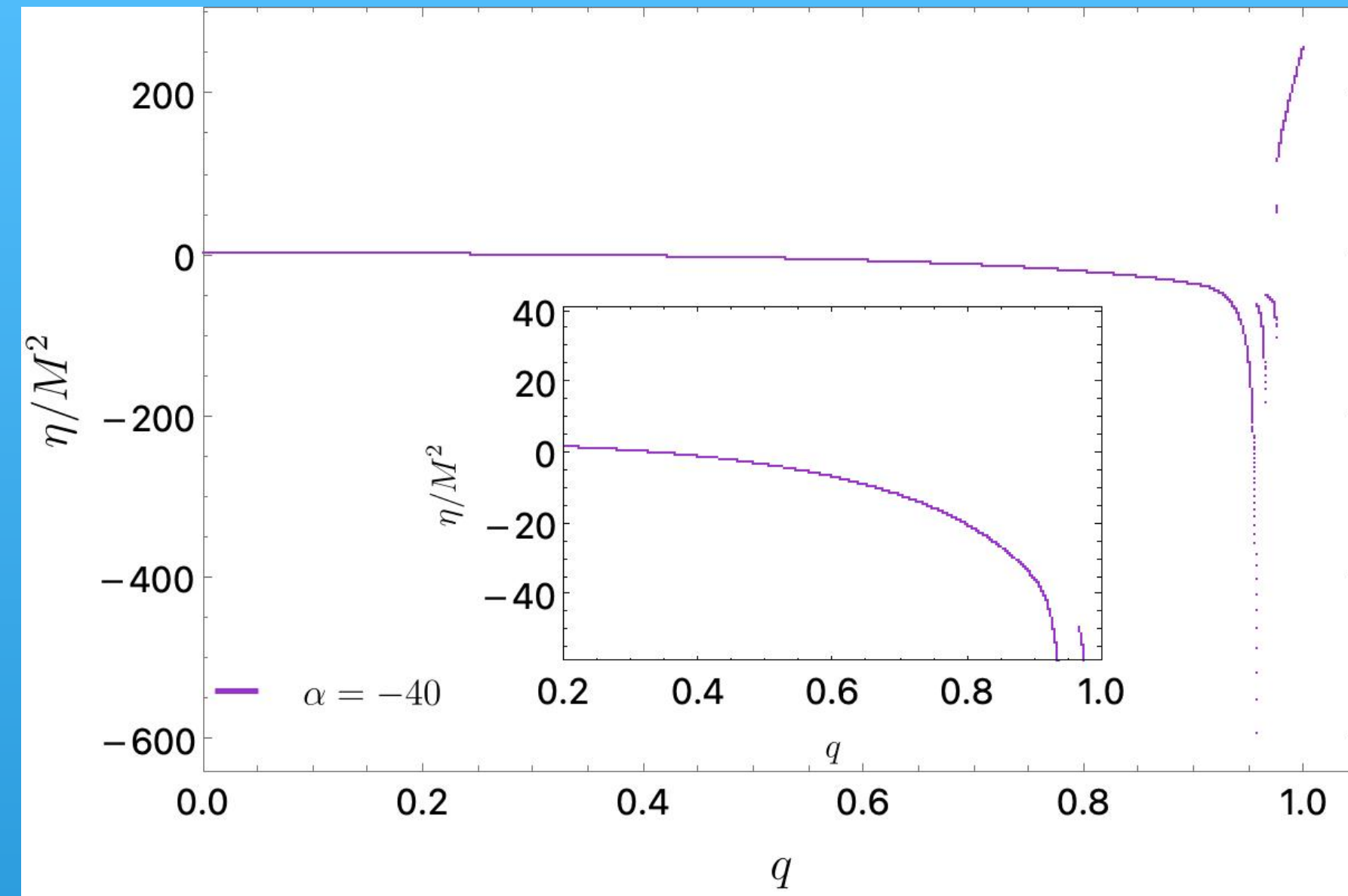
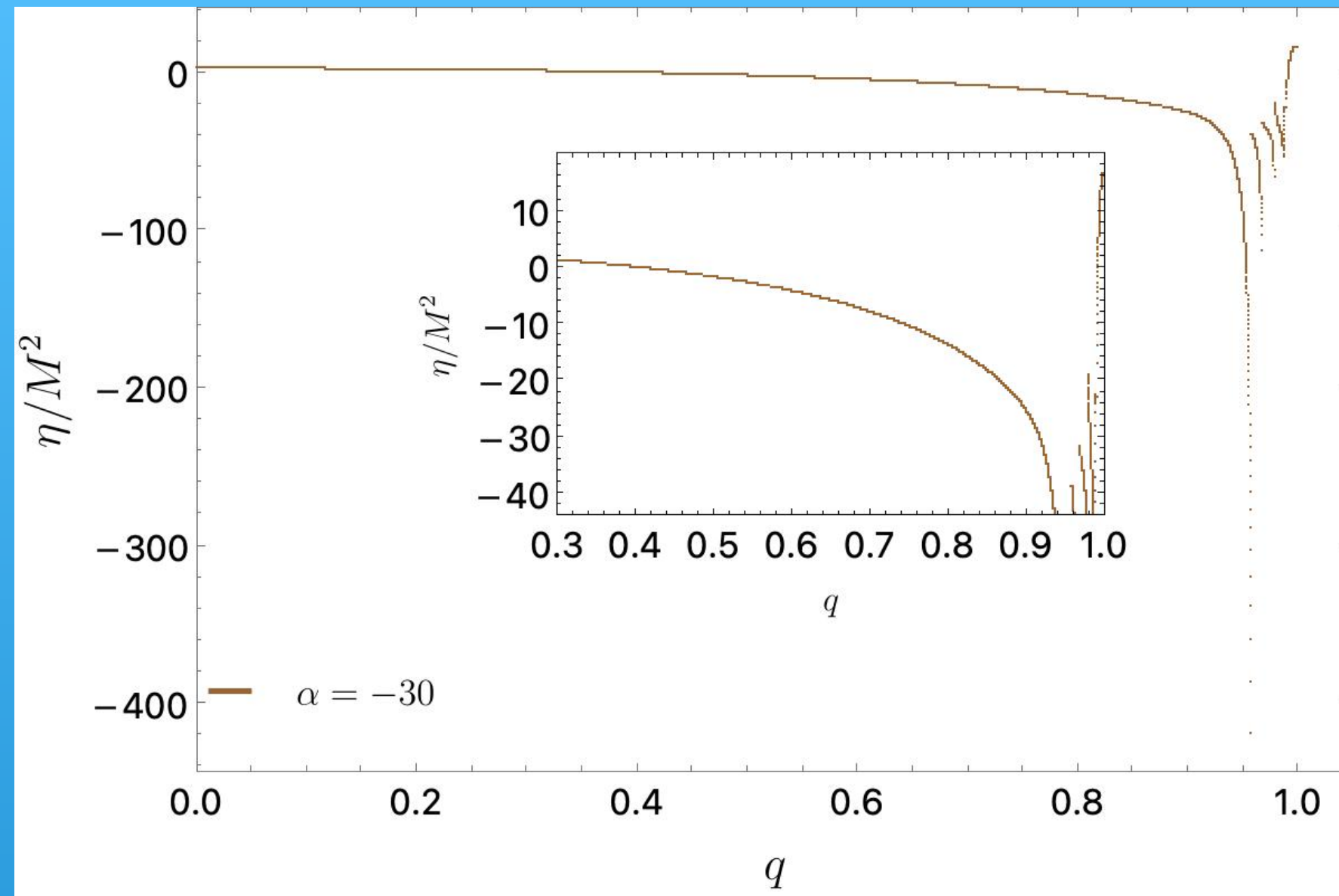
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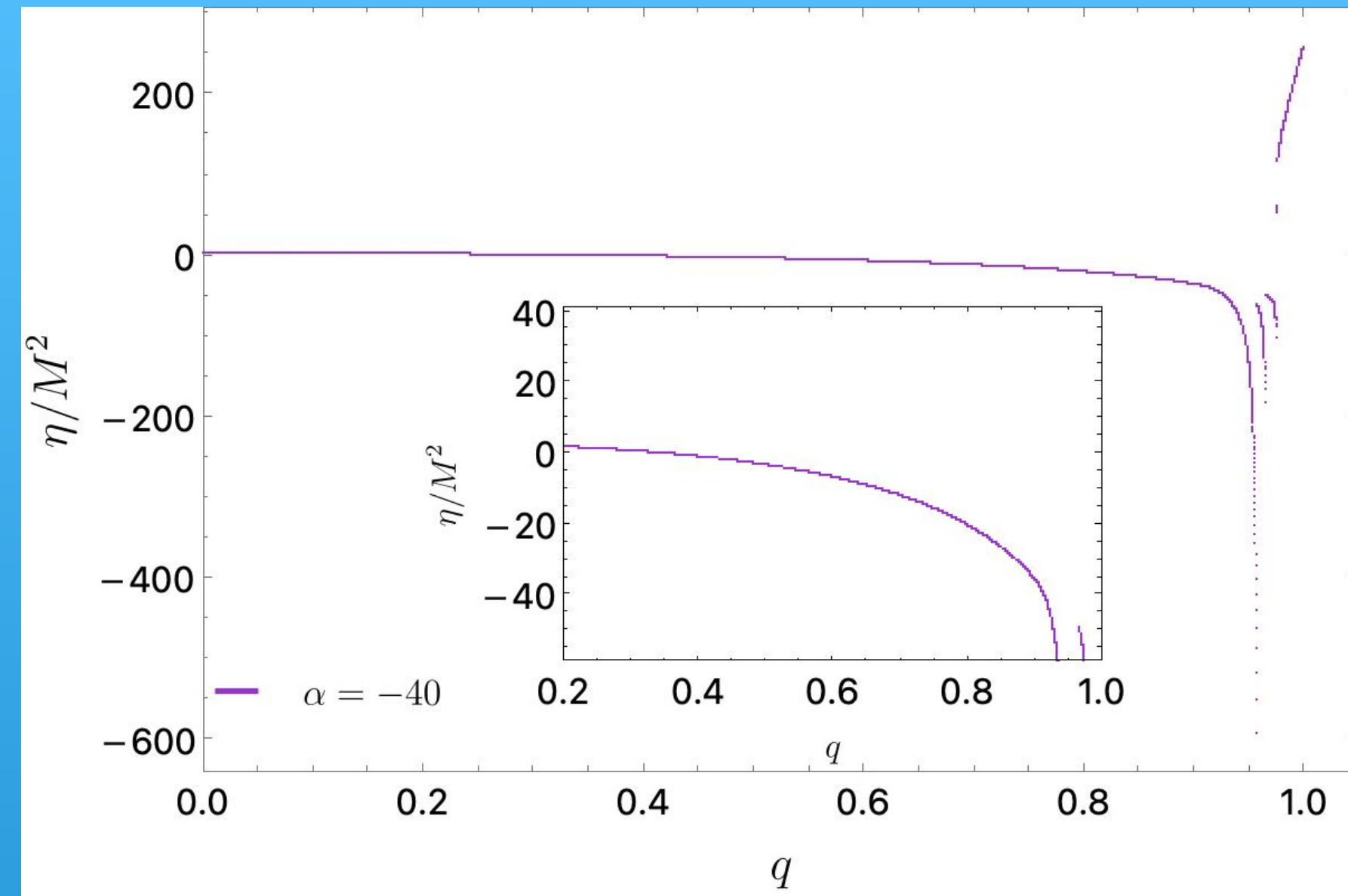
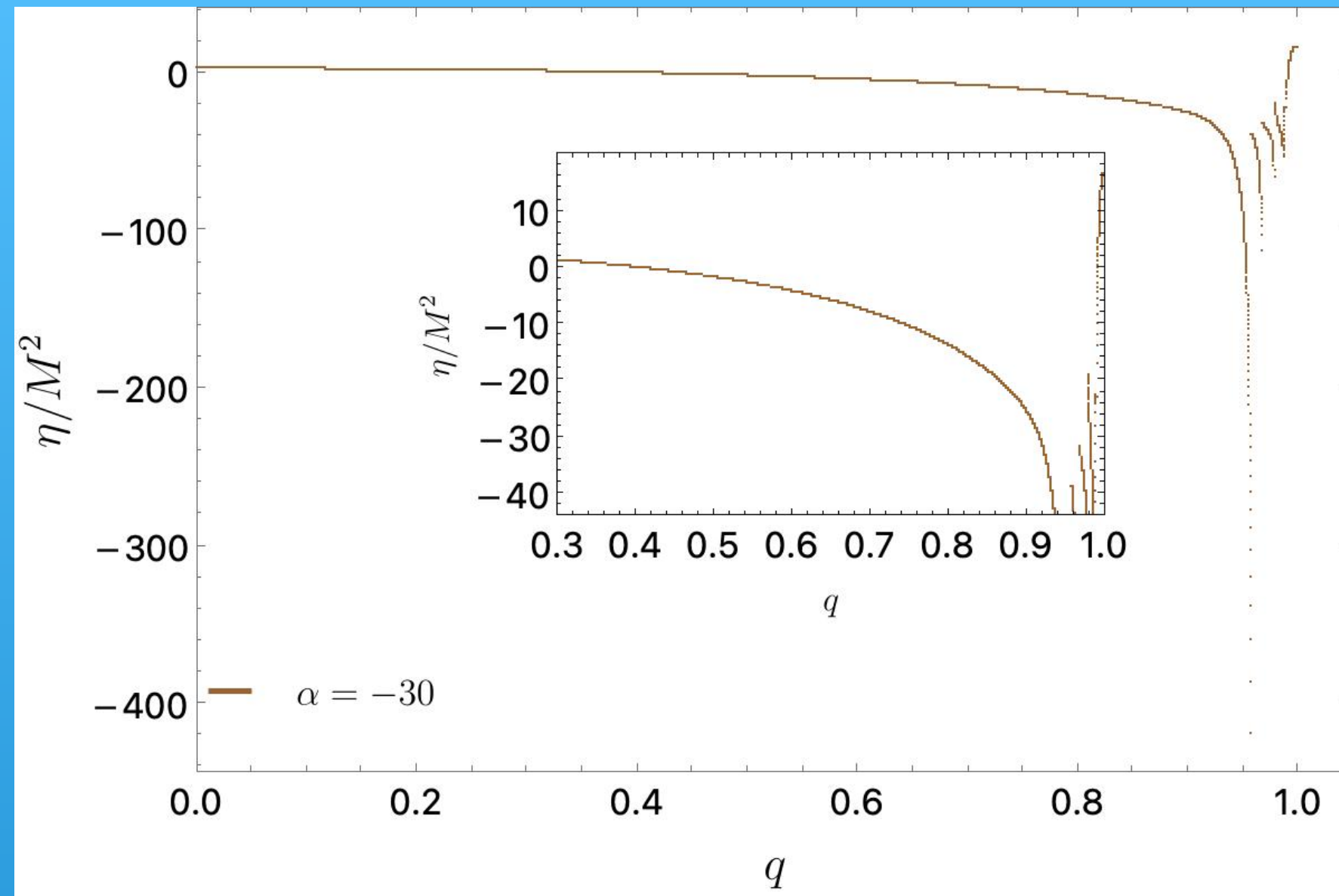
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- These results highlight that both the strength of the Maxwell coupling and the black hole charge play critical roles in determining whether and how scalarization occurs.

Thermodynamic behaviour : area

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$$A_H = 4\pi r_H^2$$

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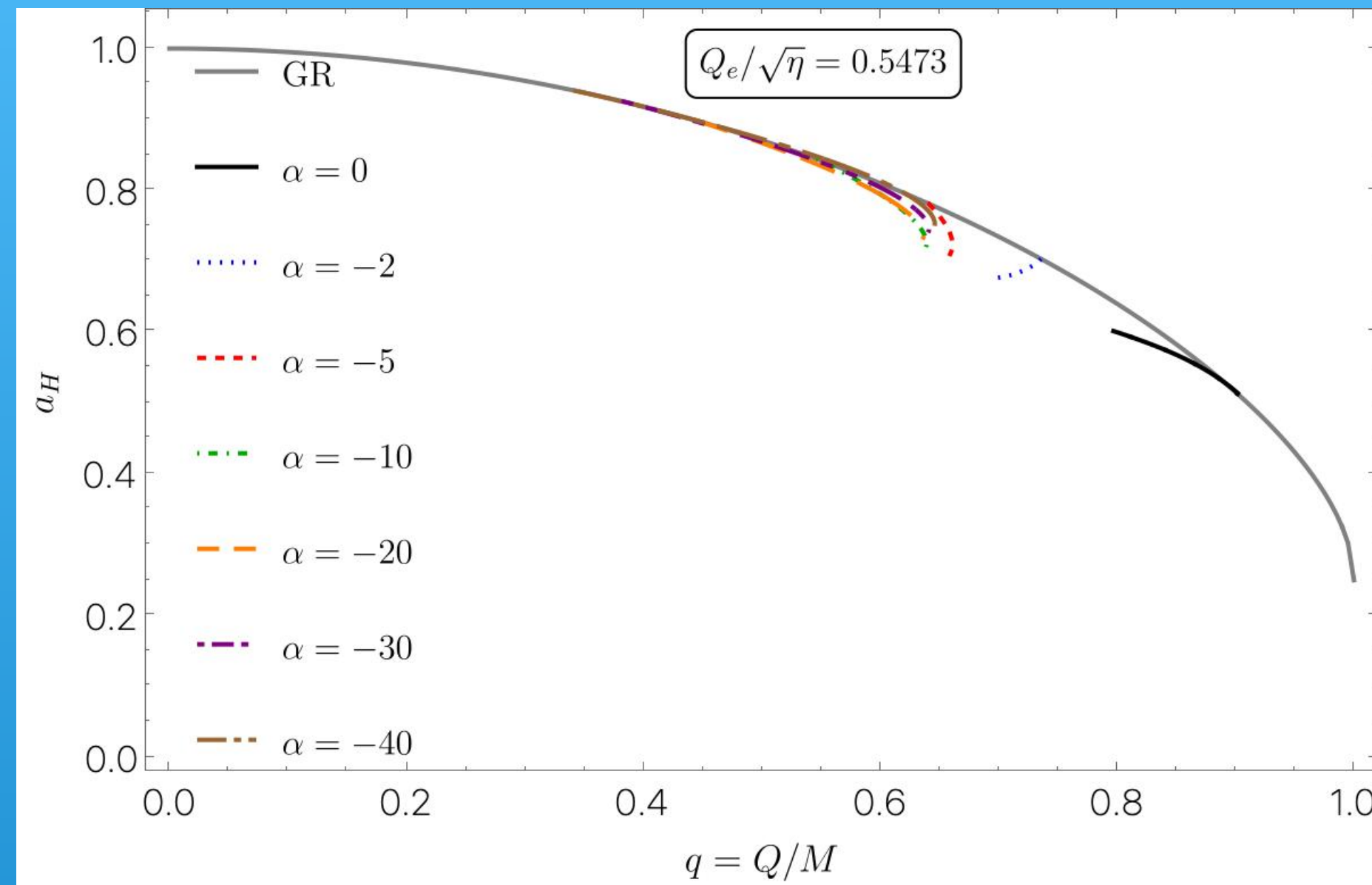
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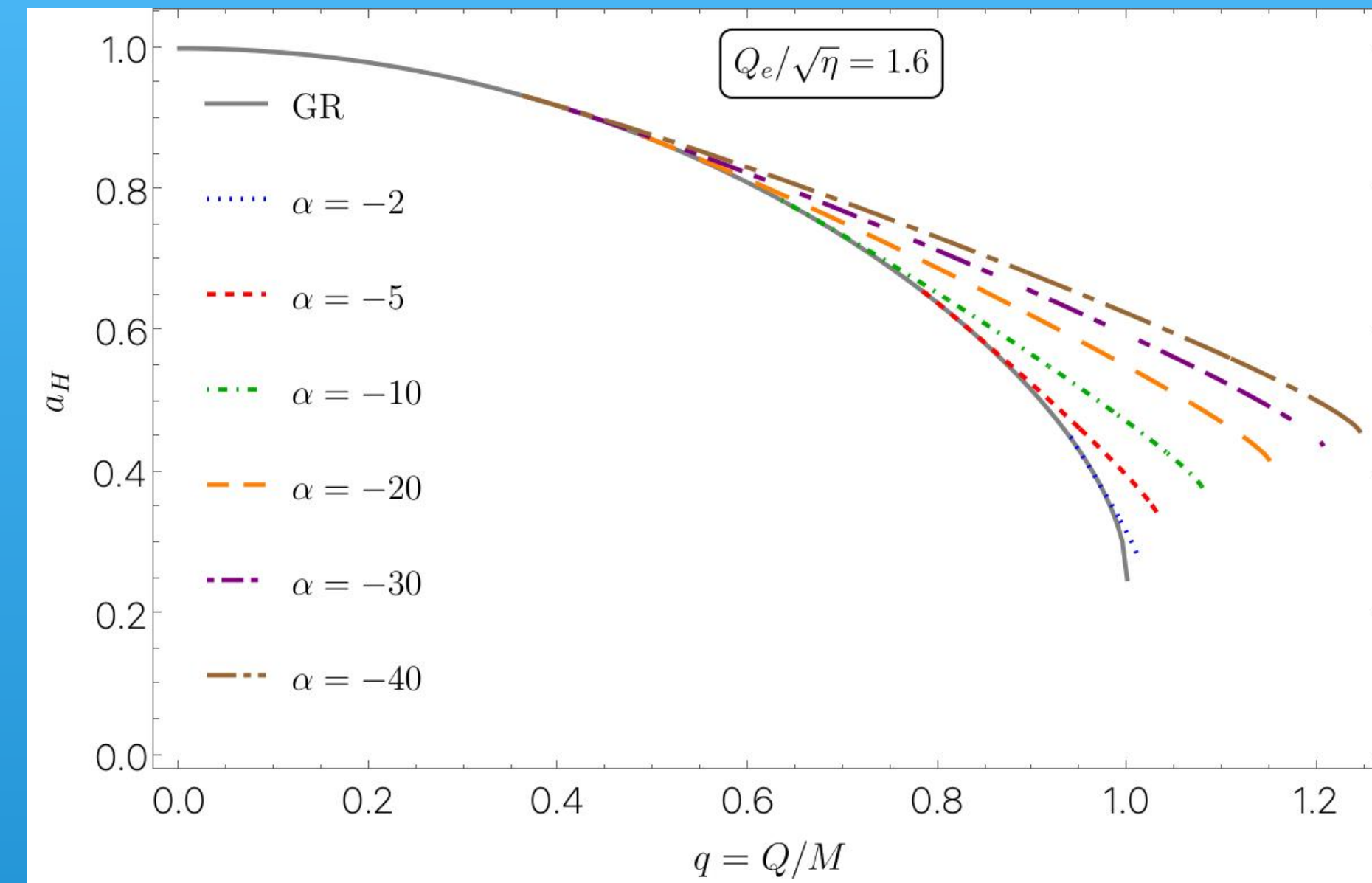
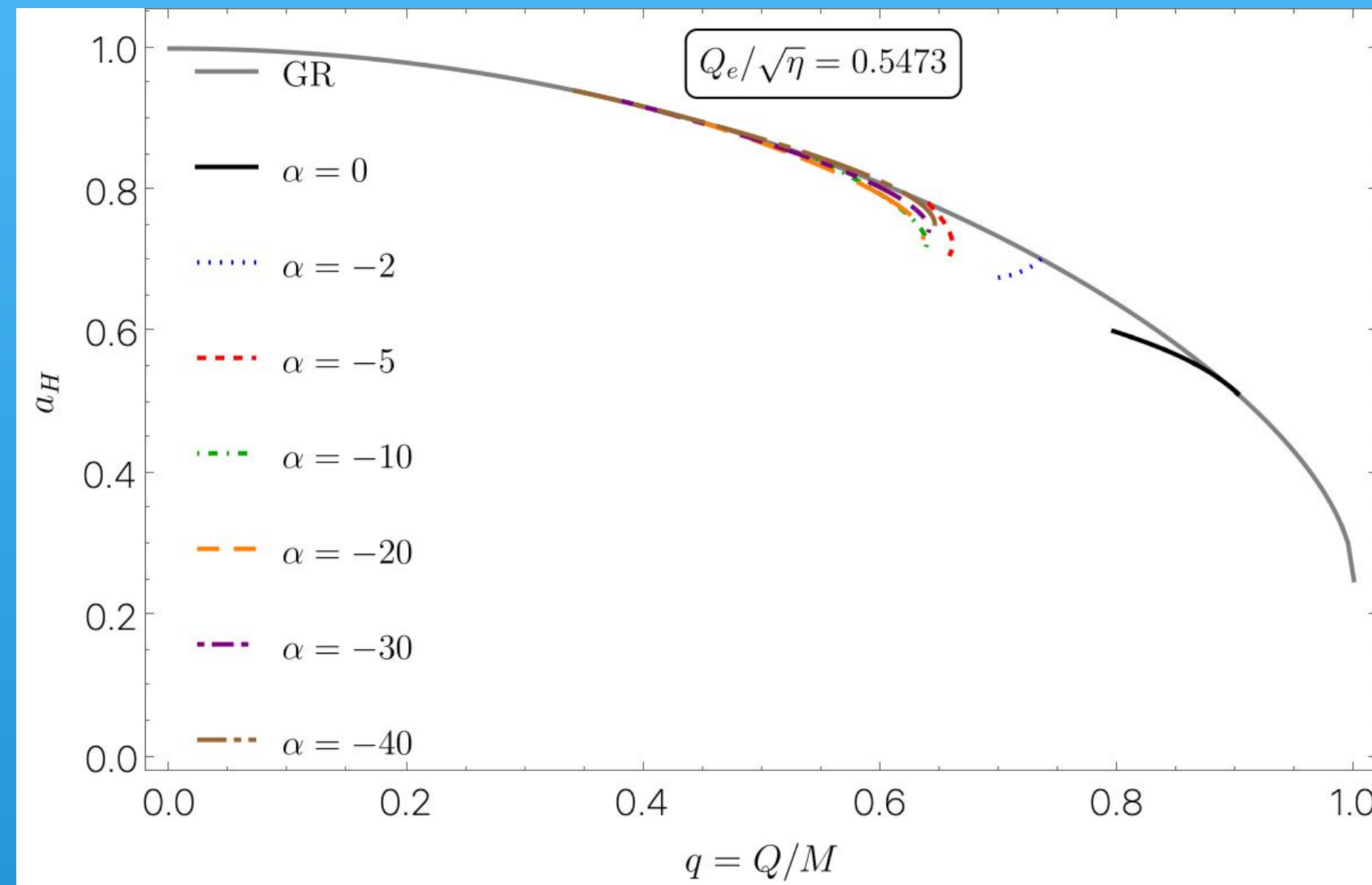
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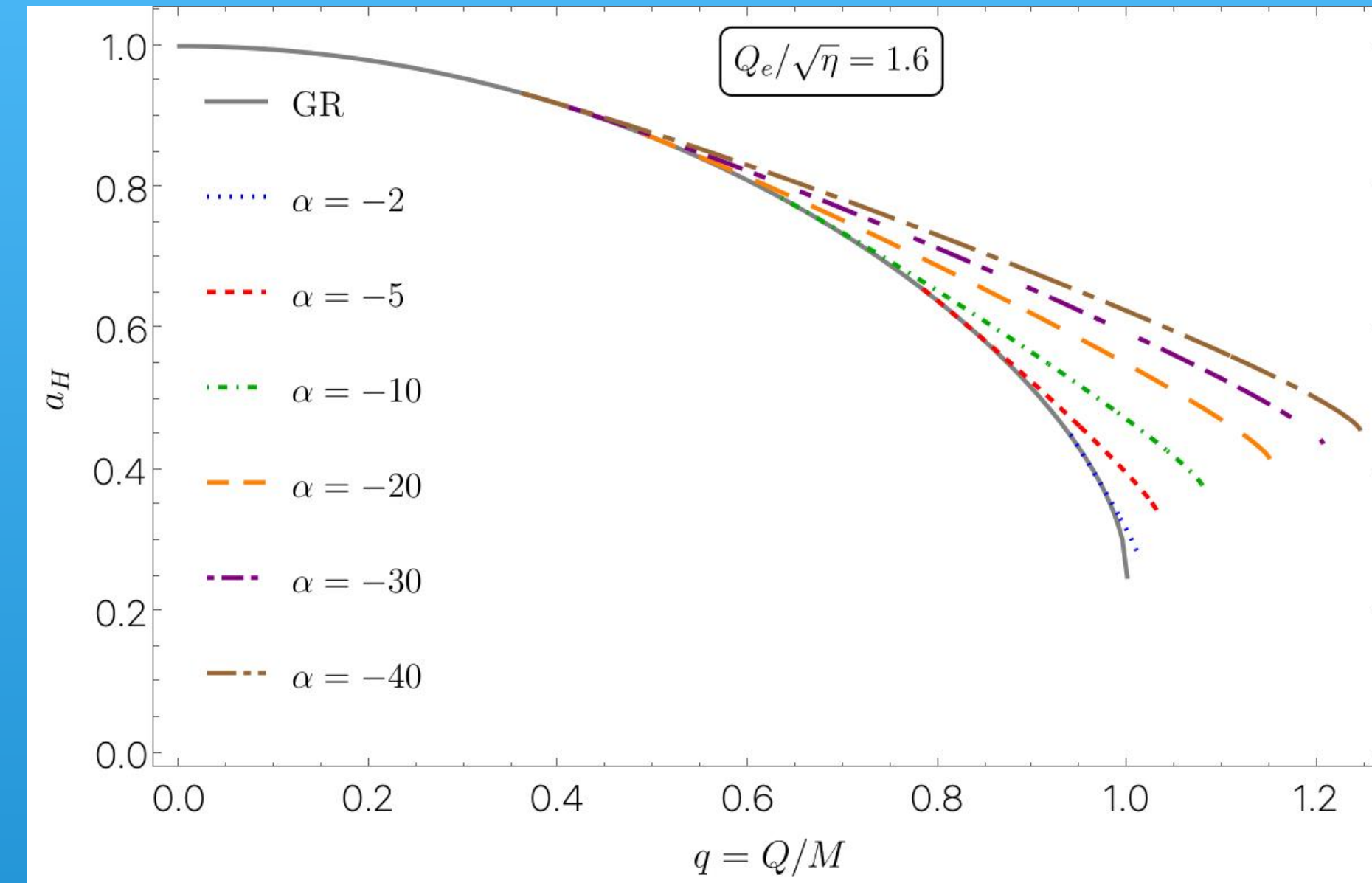
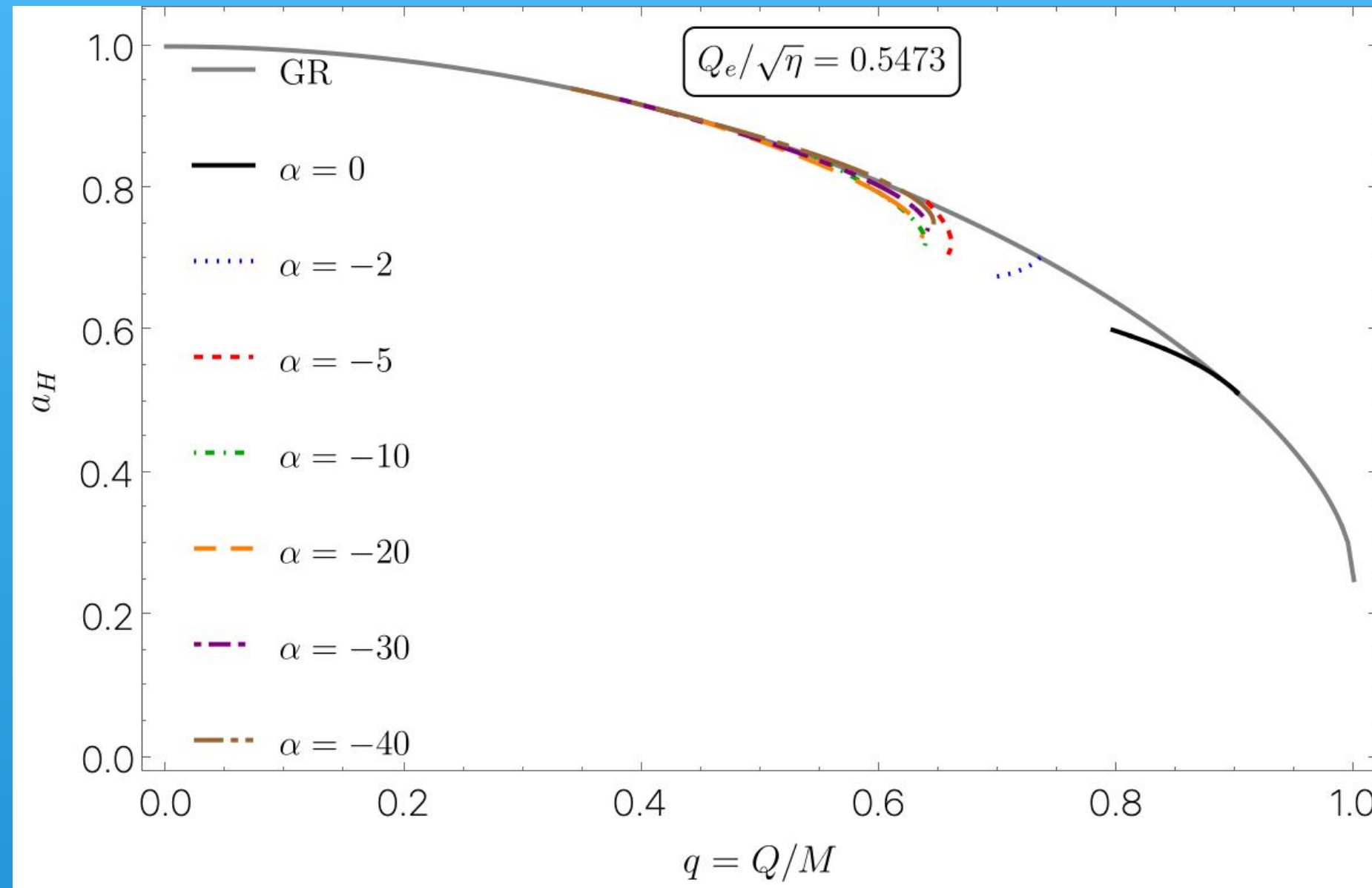
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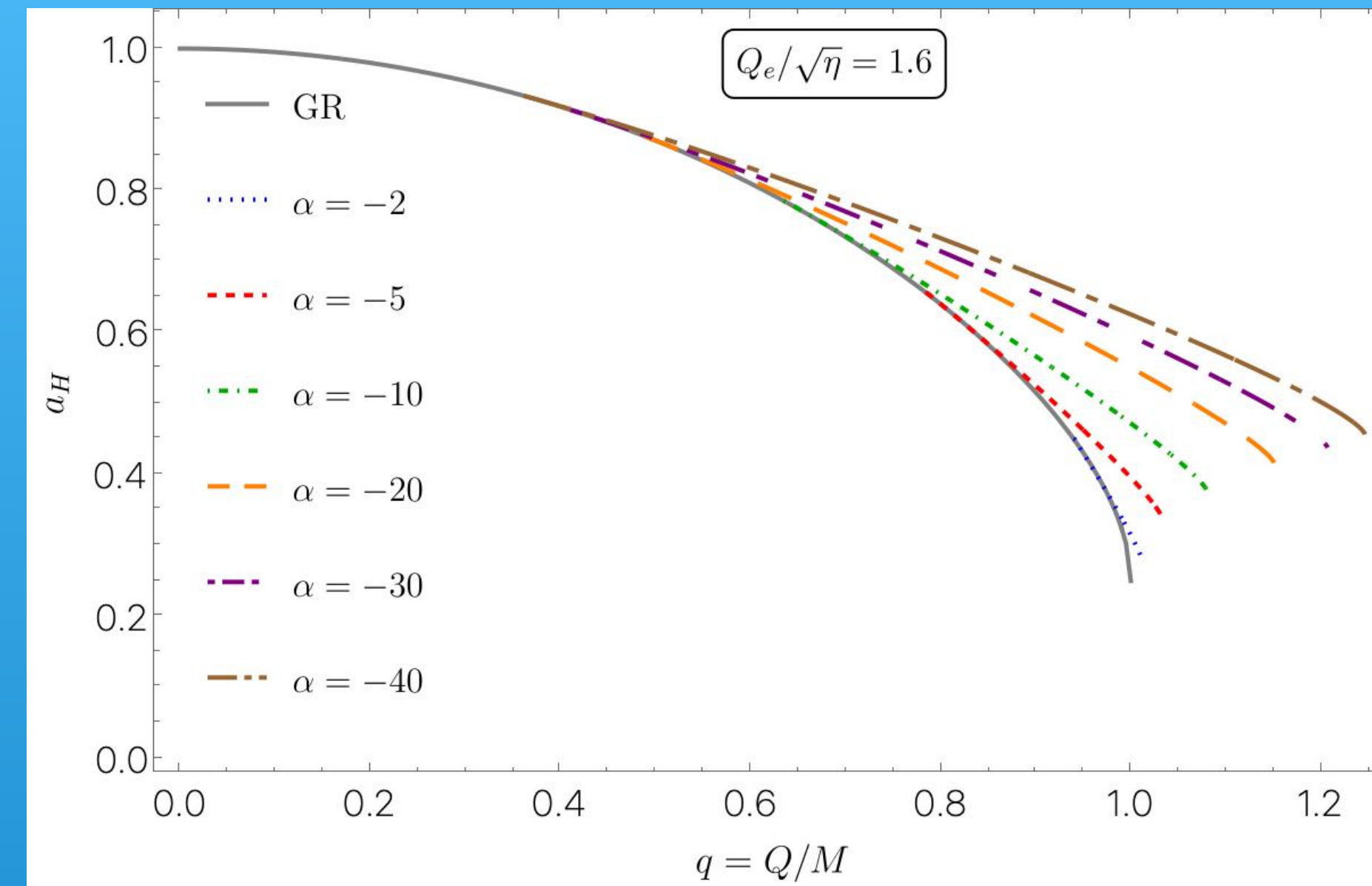
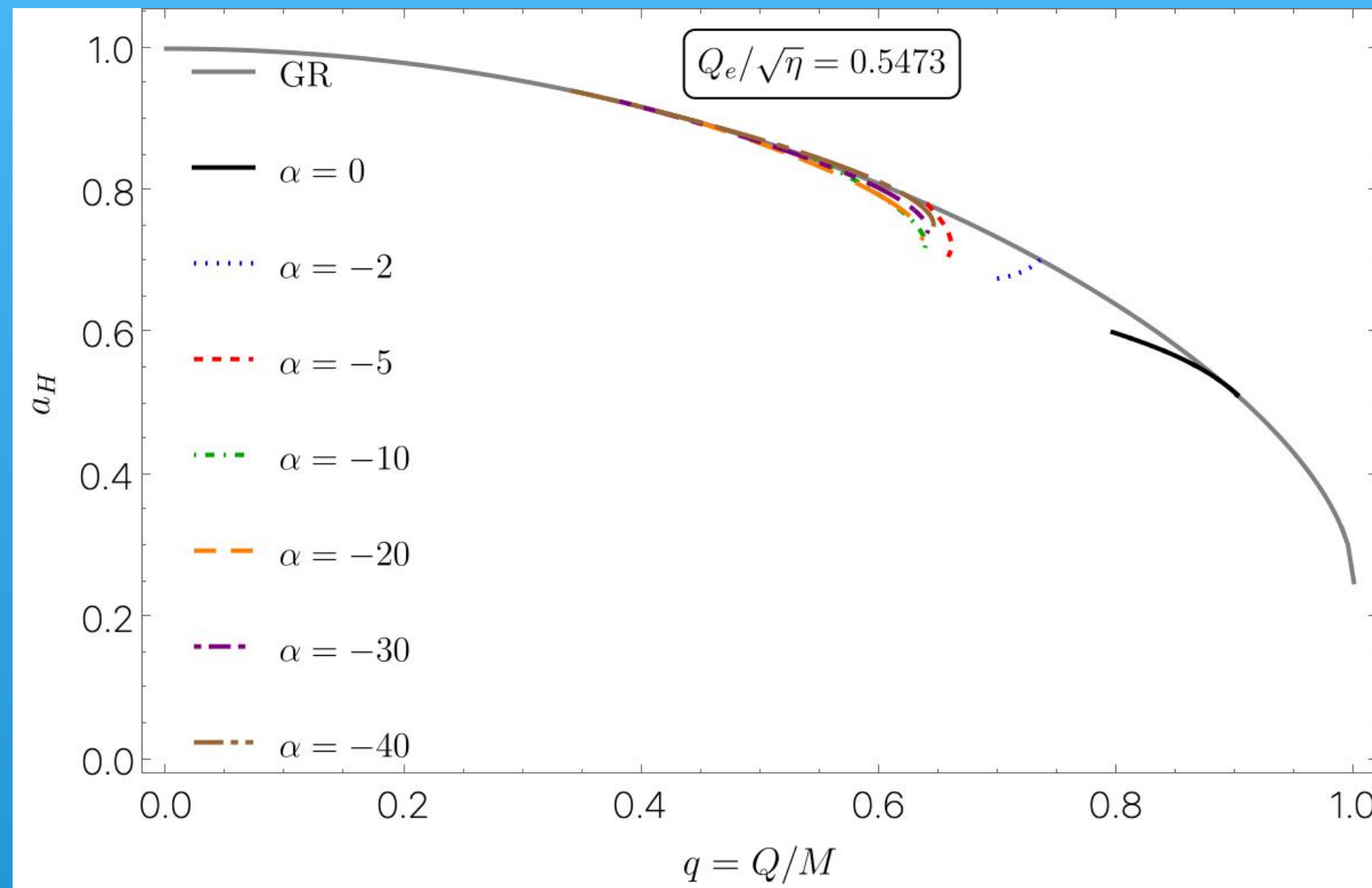


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- which is in agreement with the fact that scalarization induced by matter (Maxwell) is dominant due to the coupling to Maxwell (α).

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Thermodynamic behaviour : temperature

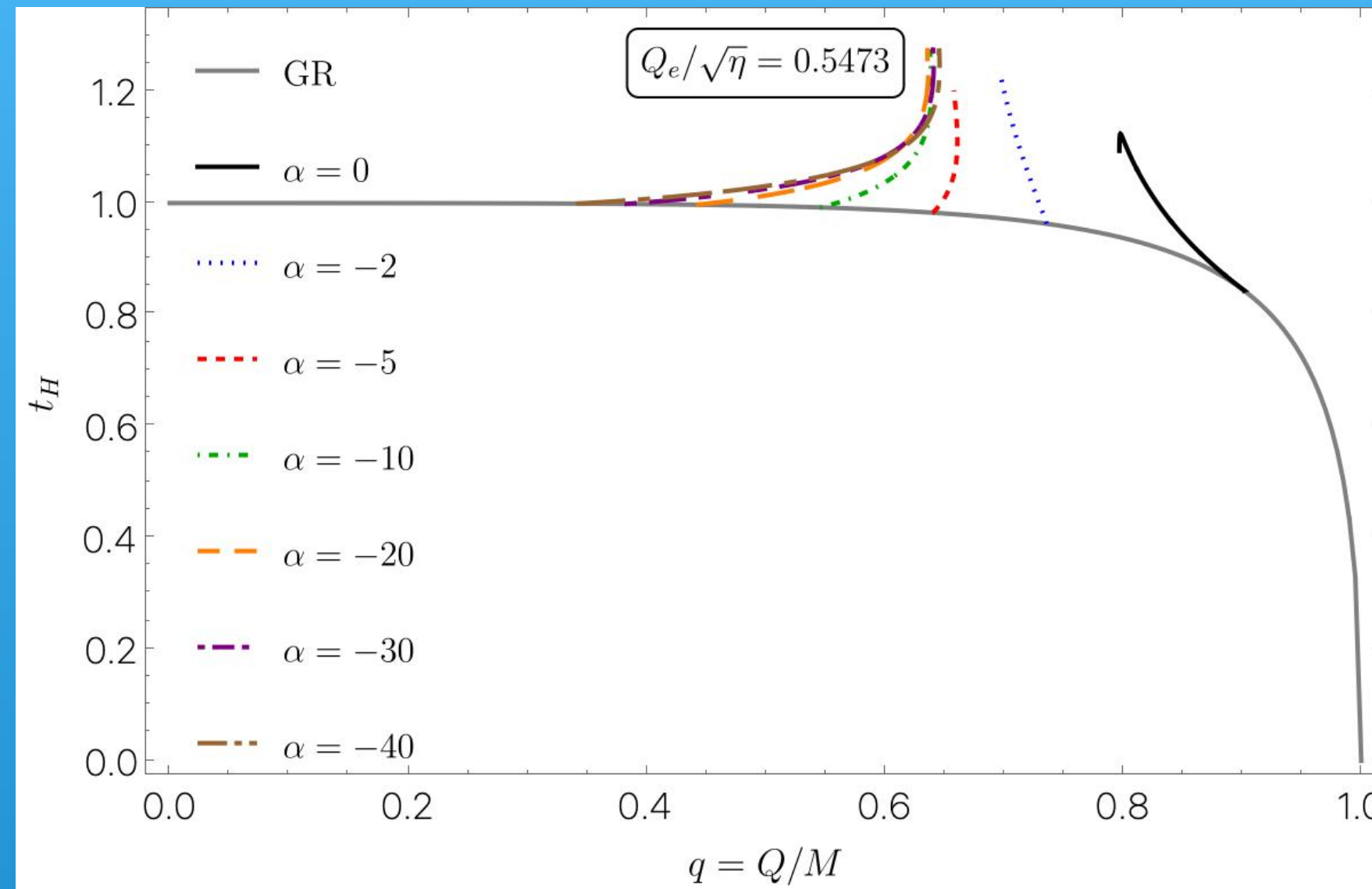
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Thermodynamic behaviour : temperature

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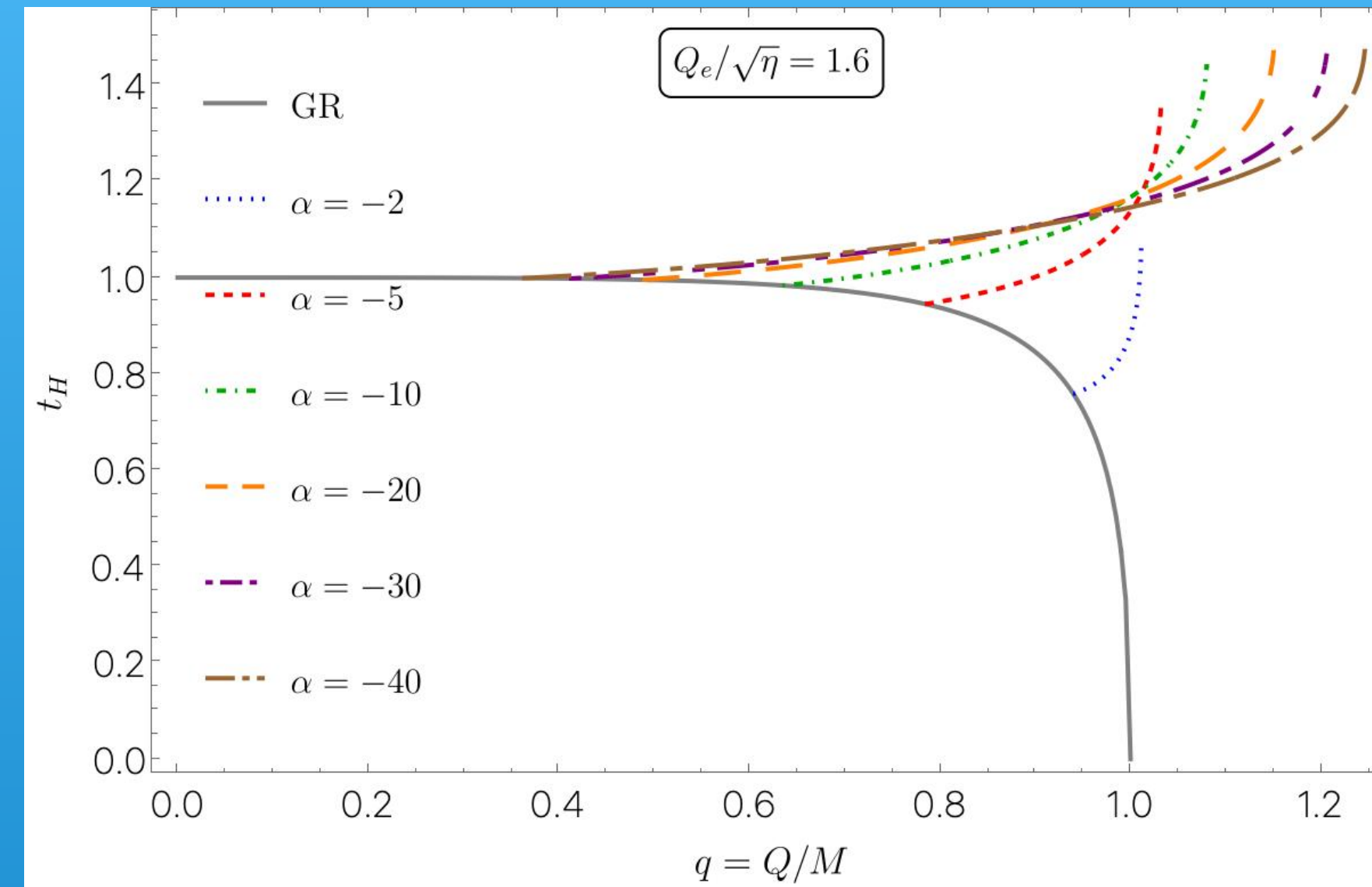
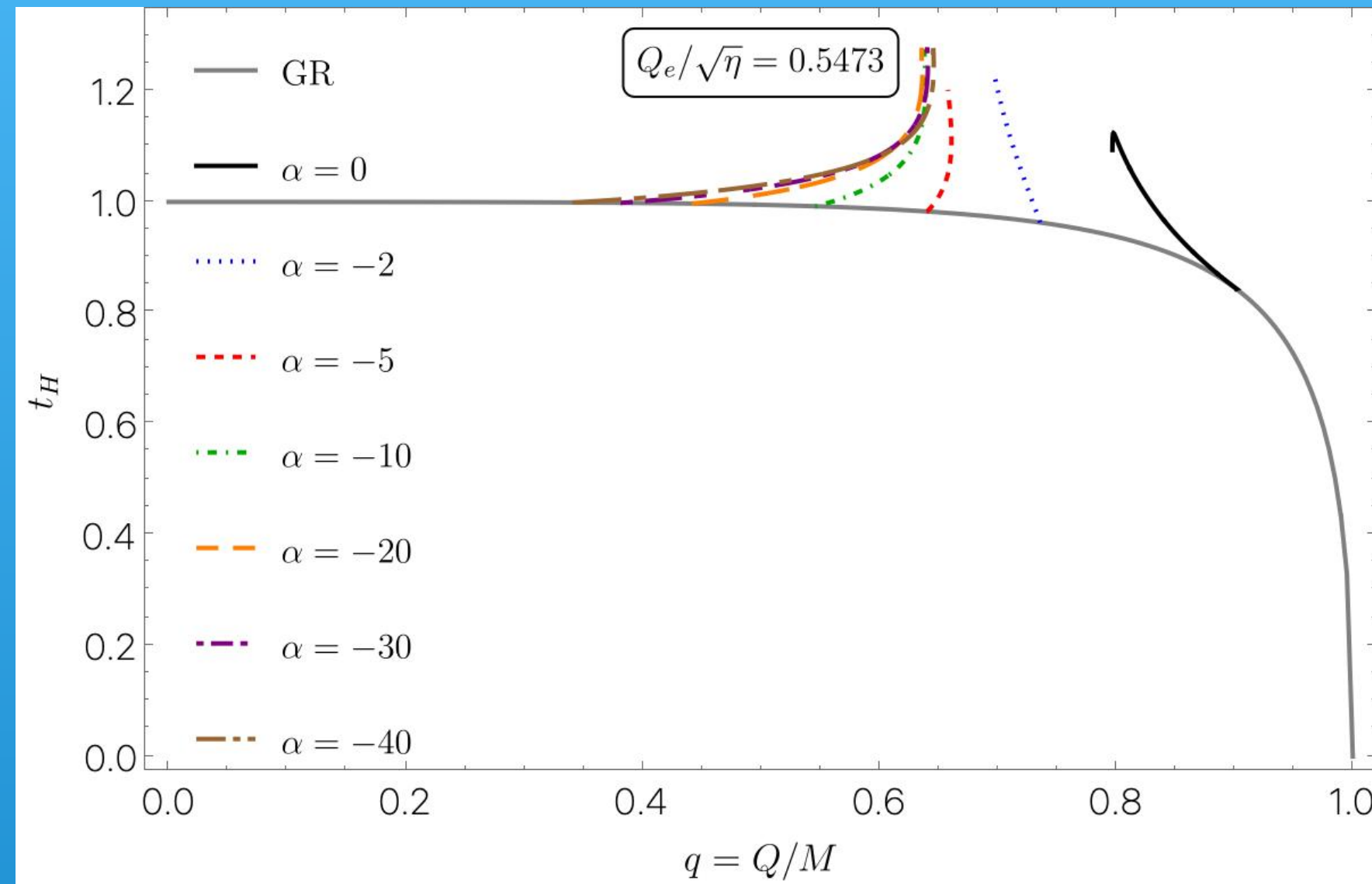
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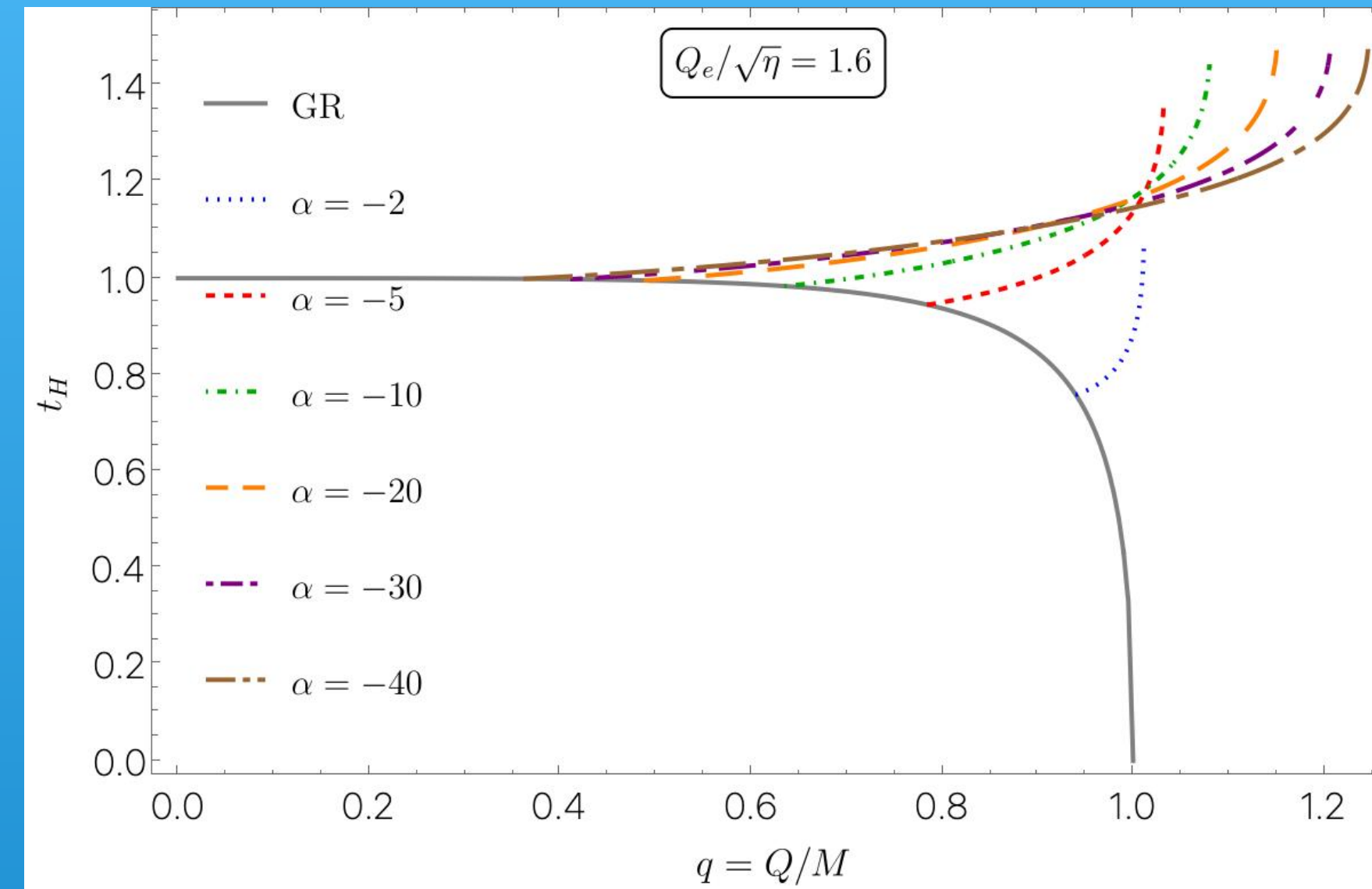
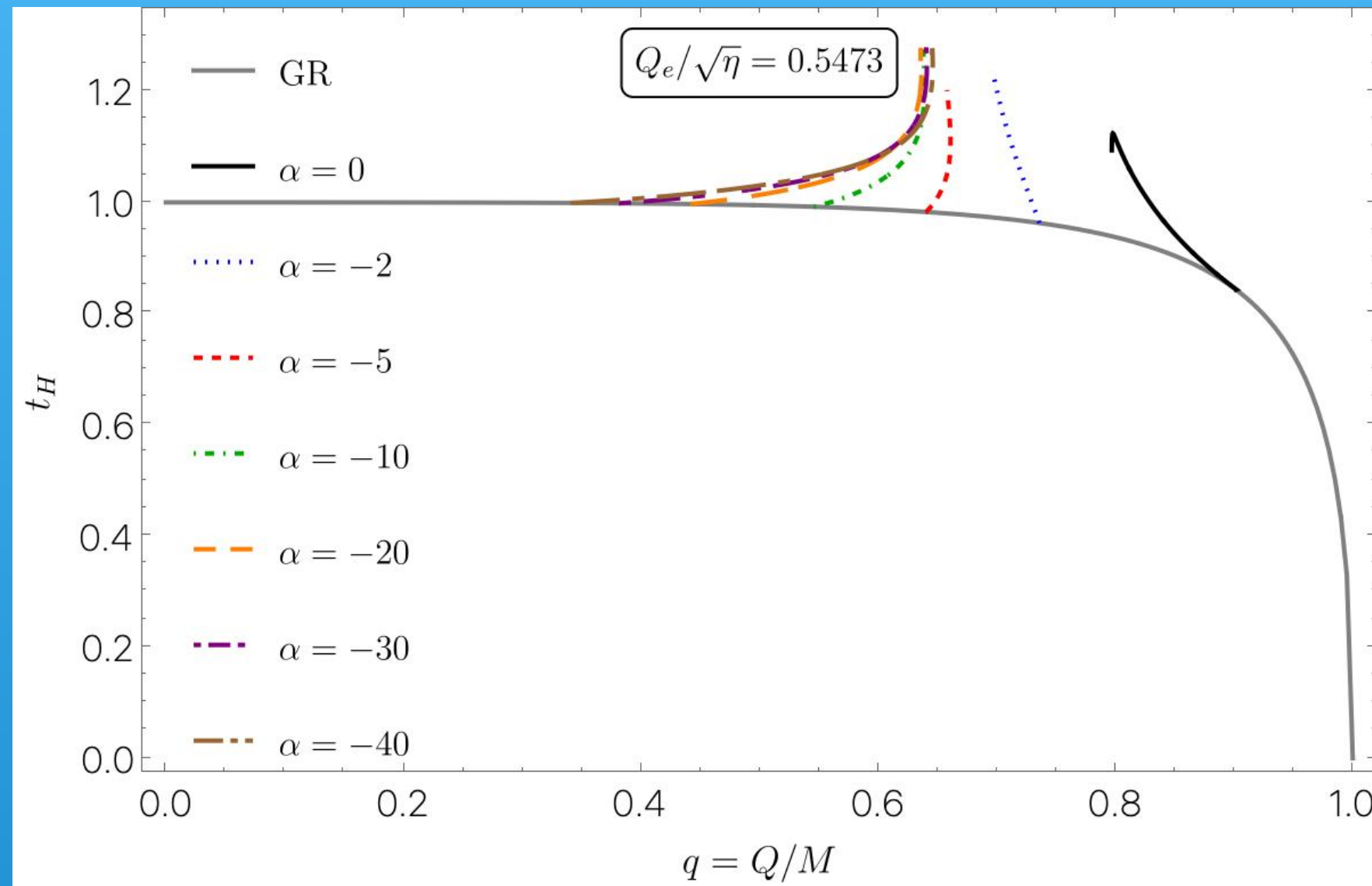
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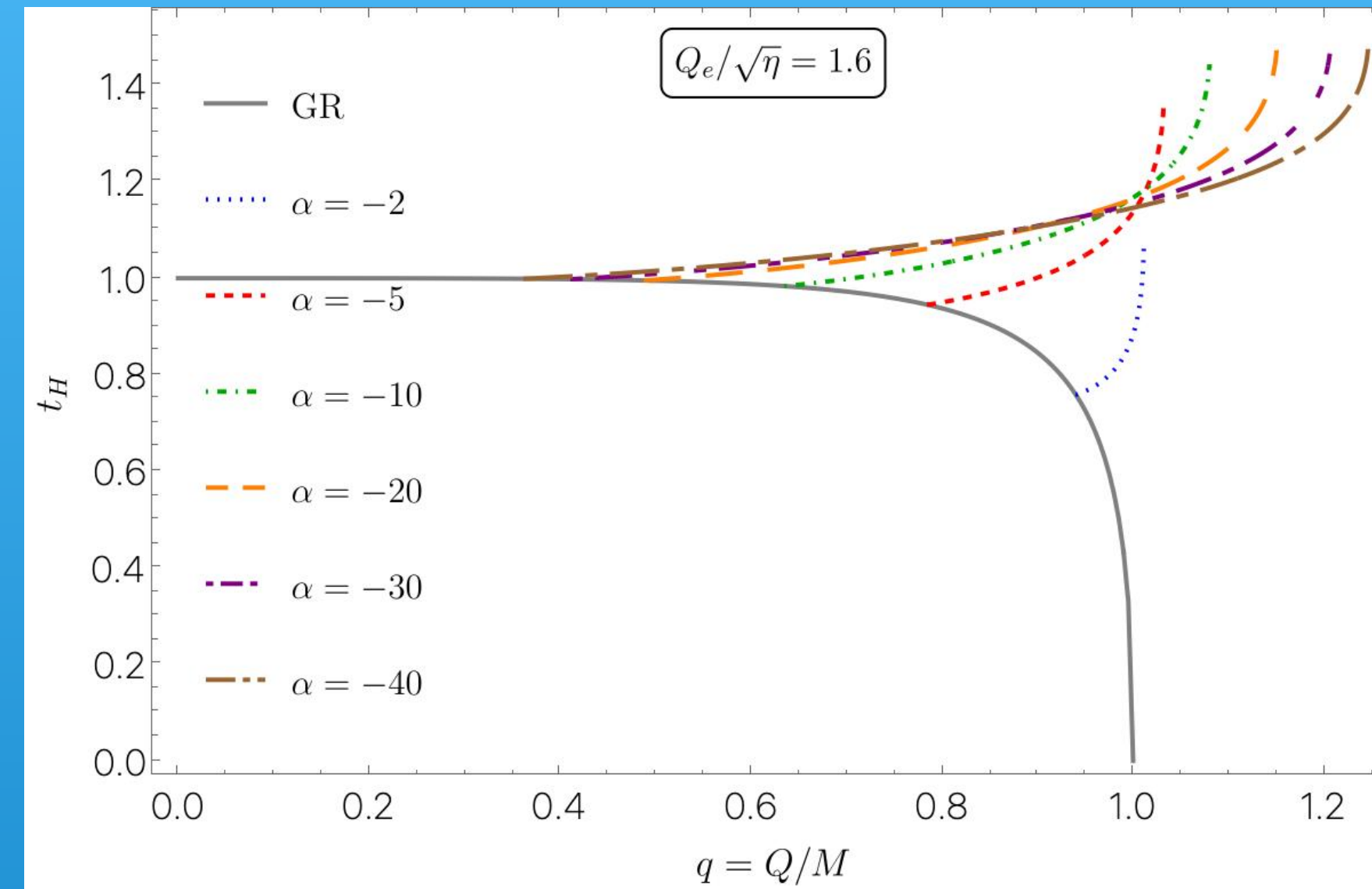
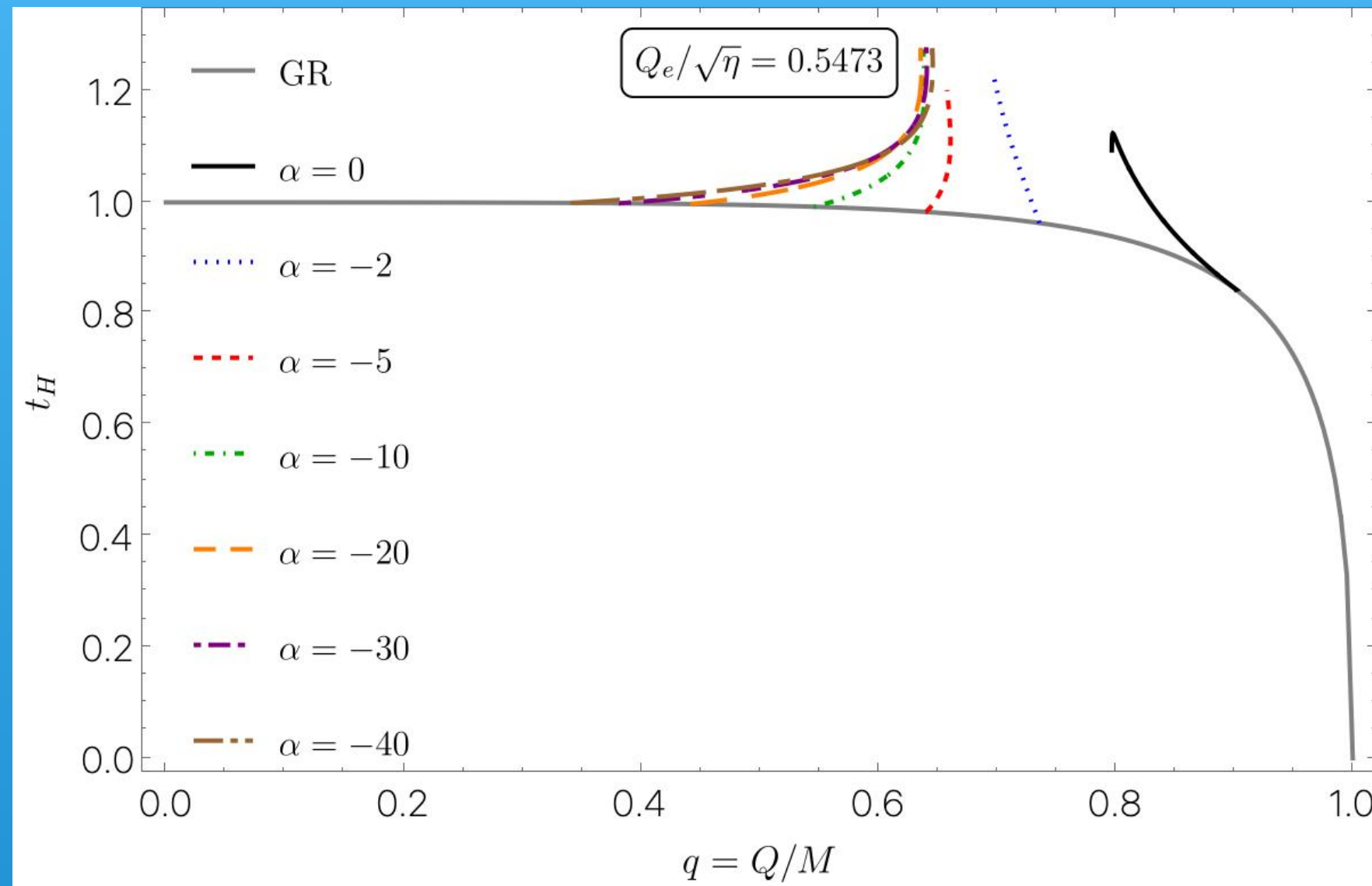


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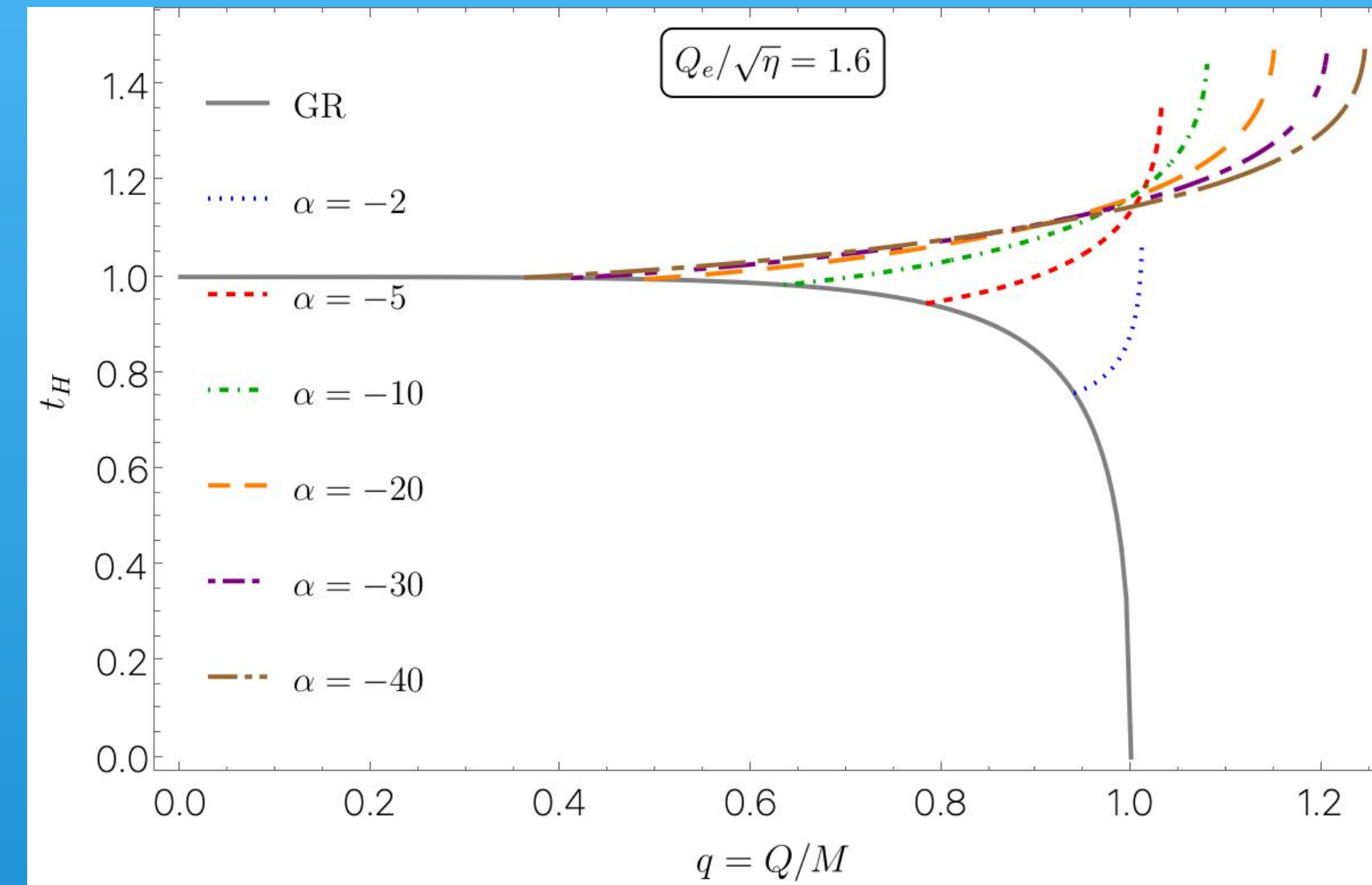
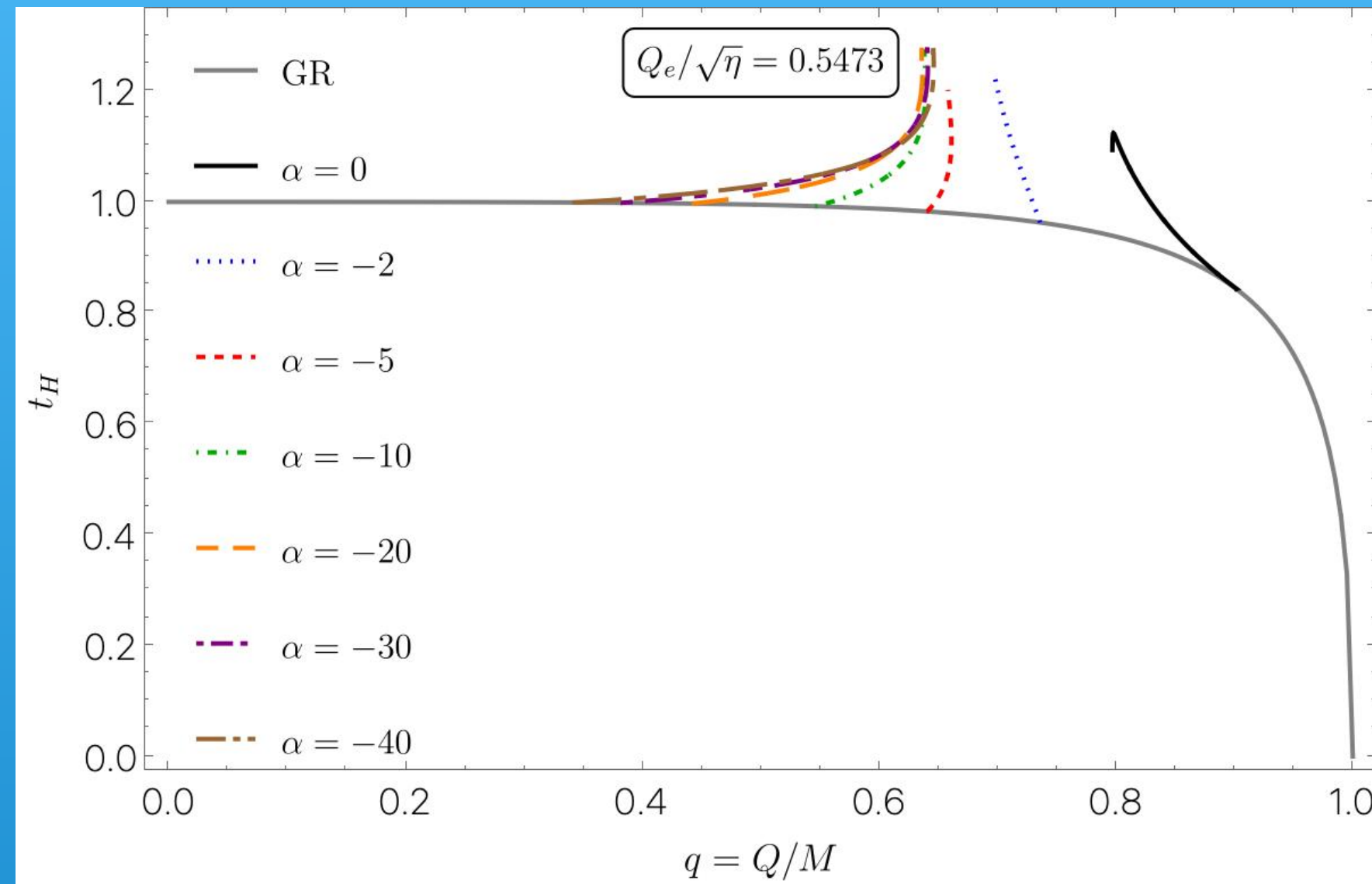


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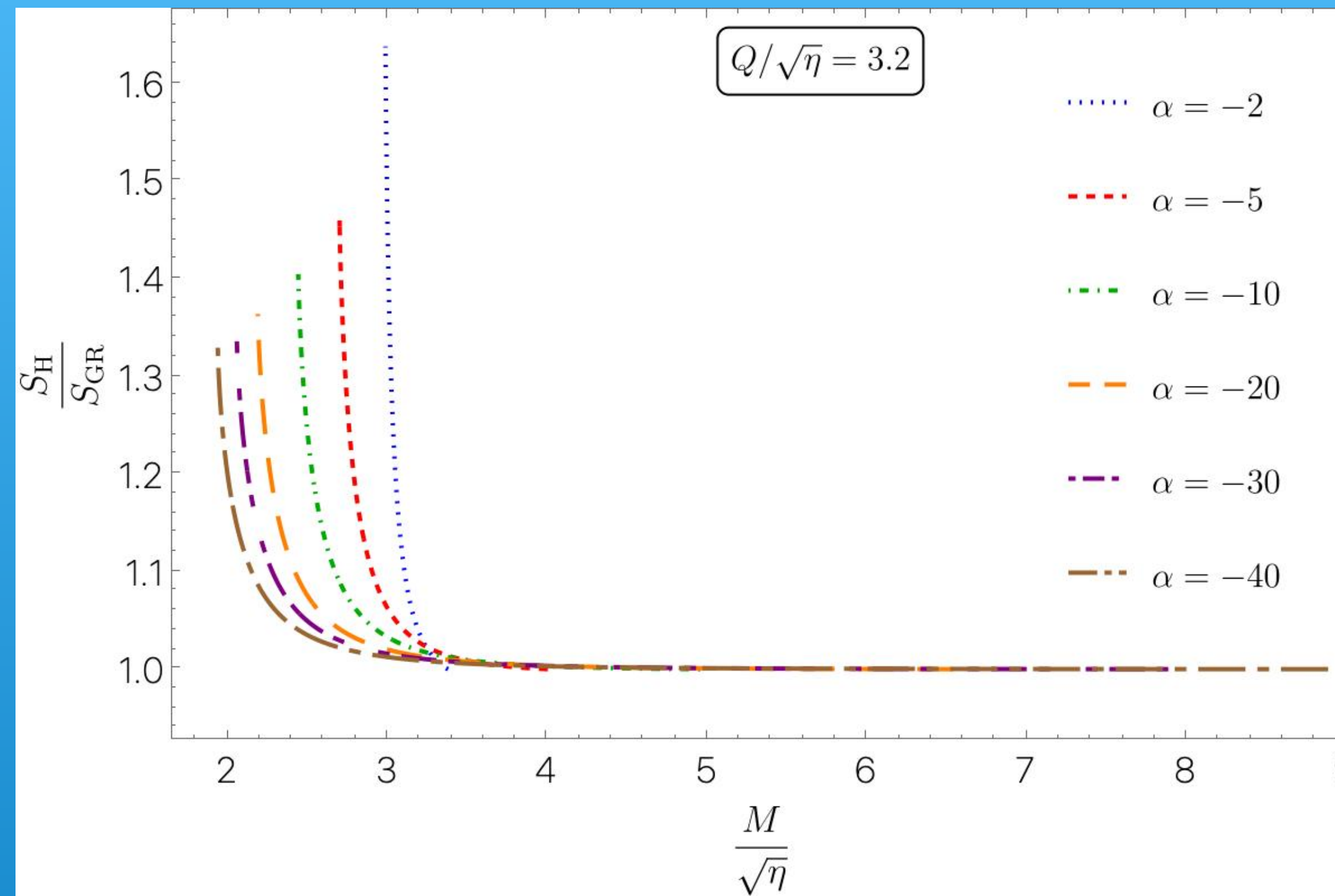
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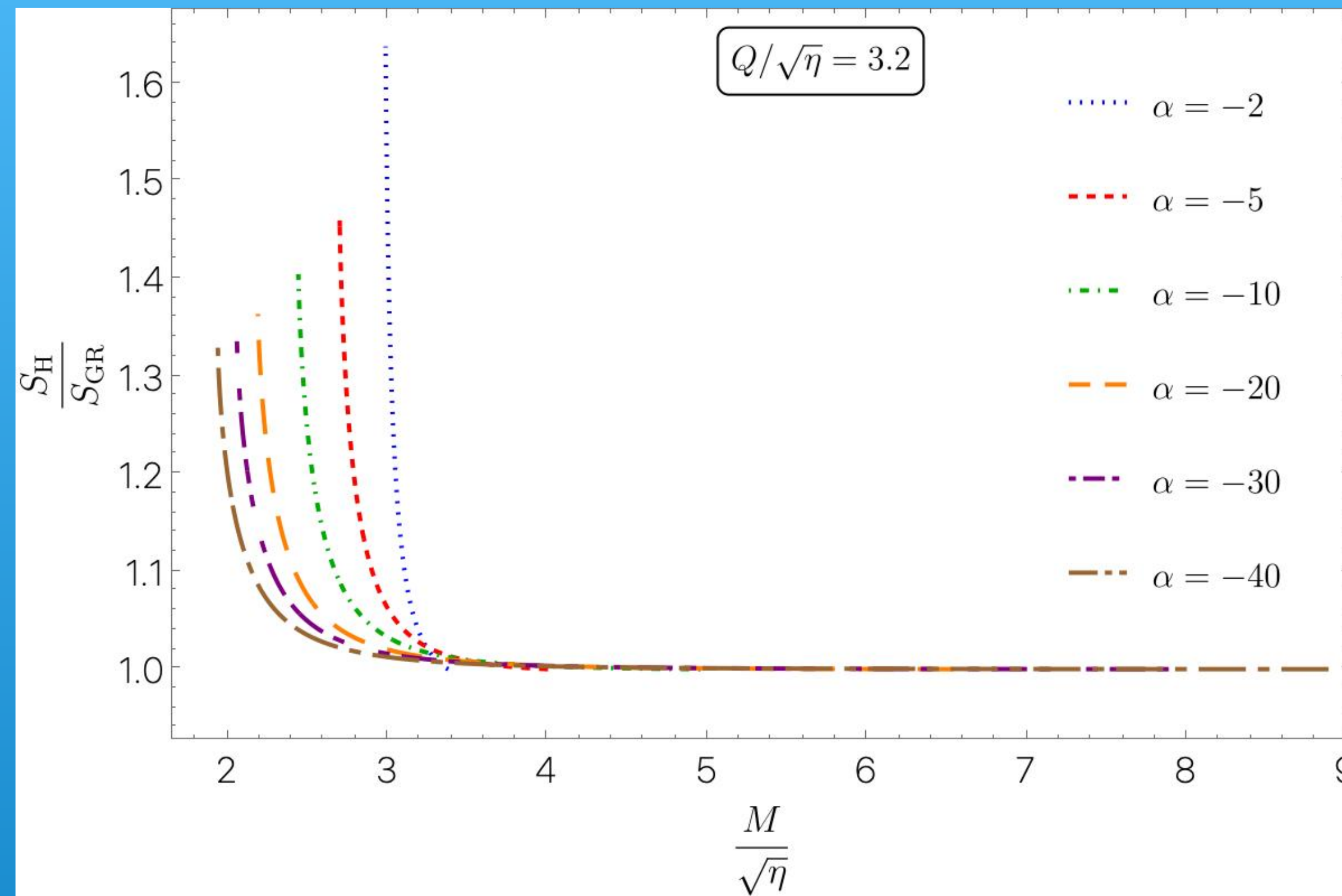


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- Another important fact is that they never reach extremality ($t_H \neq 0$).
- Starting from $Q/\sqrt{\eta} = 1.6$ the scalarized black hole can become overcharged ($q > 1$) after some limit, which was not observed for GB^+ ($\eta > 0$) scalarization before, but only in the case of GB^- ($\eta < 0$) as pointed by

Thermodynamic behaviour : temperature



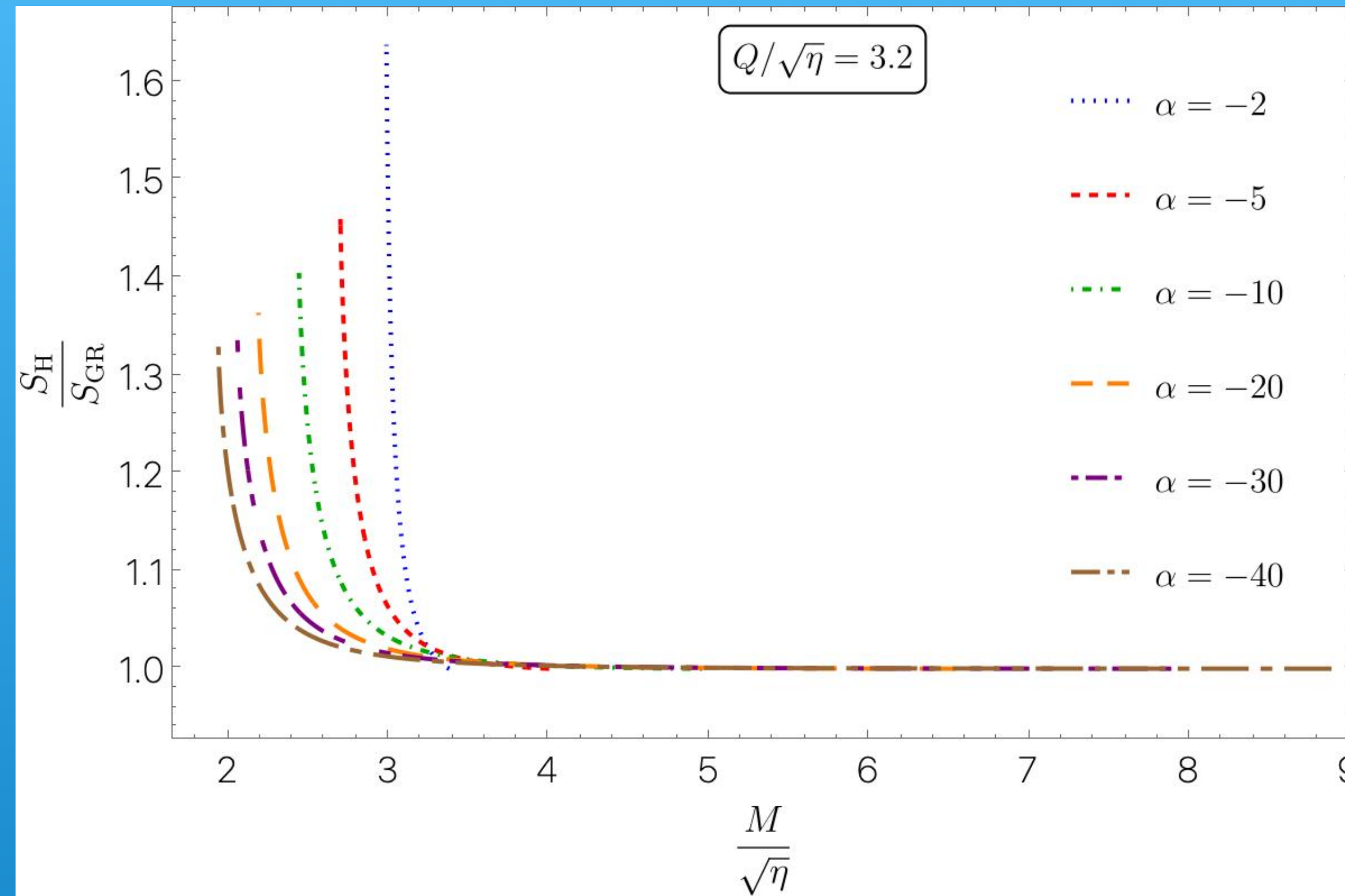
Thermodynamic behaviour : temperature



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Thermodynamic behaviour : temperature

$$S_H = \frac{1}{4}A_H + 4\pi f(\phi_0)$$



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