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Toward Testing Strong Gravity: Higher Post-Newtonian Corrections in Tidal Response

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Based on the ongoing work

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Today's Topic

Dynamical tidal Love number ; $\mathcal{O}(\omega^2)$ **for non-rotating**

→ **non-zero & logarithmically scaling by v**
even in 4d Schwarzschild BH within GR

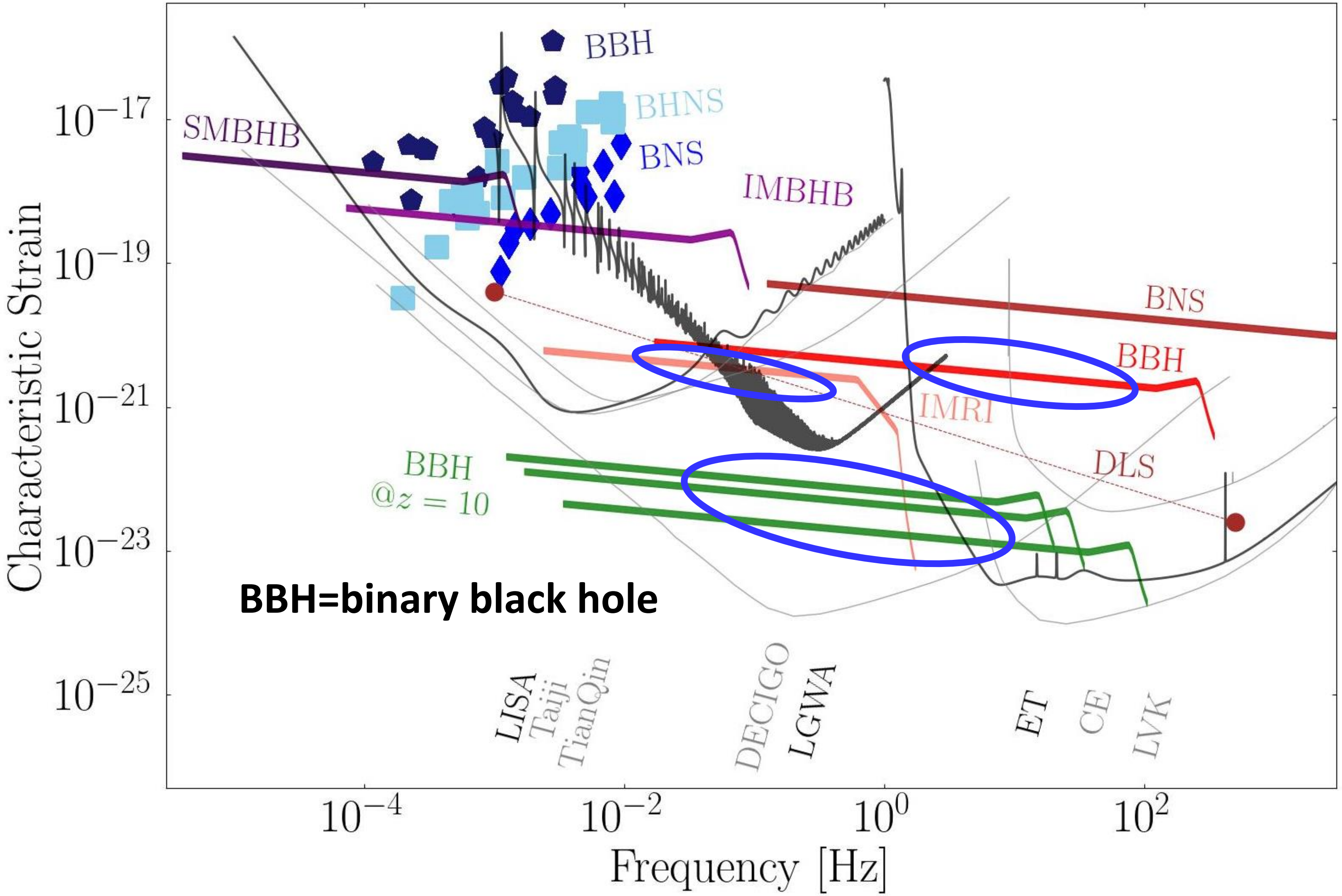
similar results in [Chakraborty+ (2025)]

Especially, I will discuss from the viewpoint of
Mano-Suzuki-Takasugi (MST) formalism and **worldline EFT**

→ **(8PN)** $\times \log v$ **effects**
in Post-Newtonian waveform for inspiral

1. Introduction

Gravitational Wave Landscape



[Einstein Telescope blue book, arXIv:2503.12263]

Is it possible to detect

- **small deviations from General Relativity (GR)**

especially, inspiral of BBH

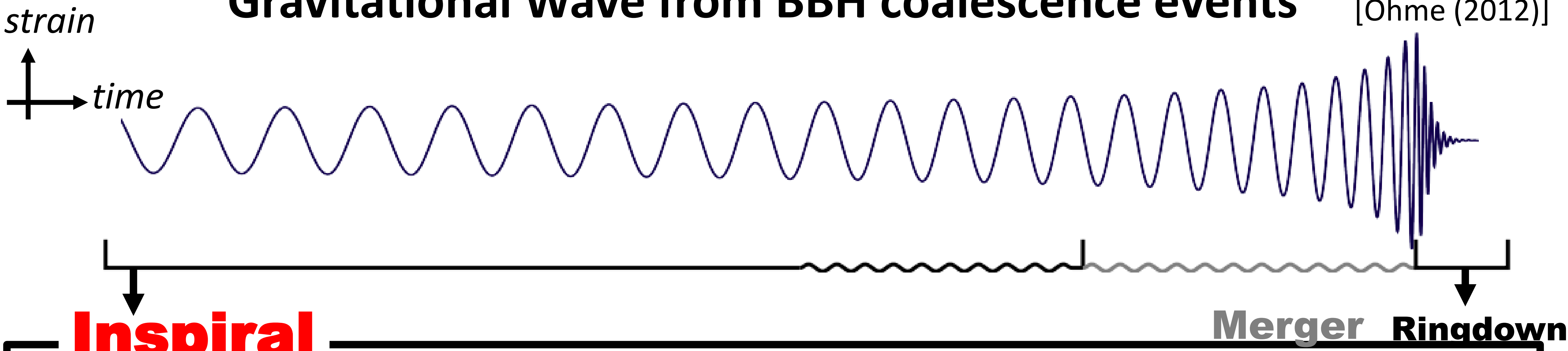
⇩

What can we learn about **the fundamental physics**

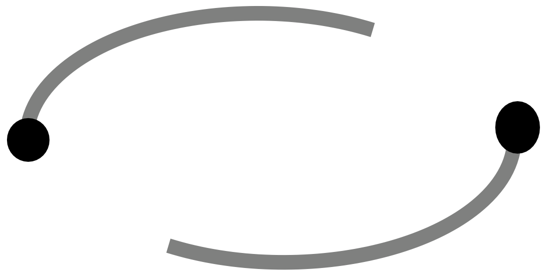
Inspiral Test of Strong Gravity

Gravitational Wave from BBH coalescence events

[Ohme (2012)]



Inspiral

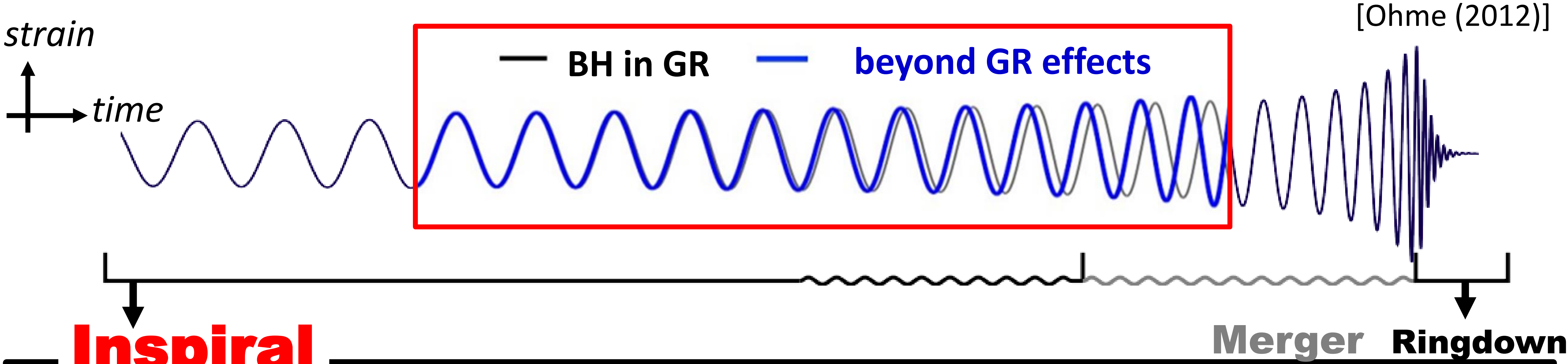


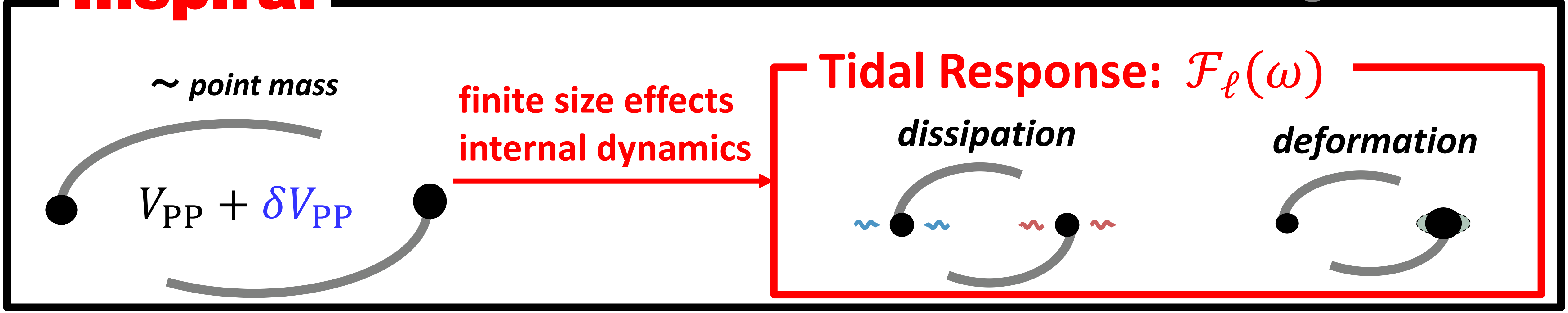
$$h(v) = \mathcal{A}(v)e^{-i(\varphi(v)+\delta\varphi(v))}$$

Post-Newton (PN) expansion = v^{2n}-expansion

- consistent with GR up to 3.5 PN order [LIGO-Virgo, 2021]

Inspiral Test of Strong Gravity





Waveform Modeling

should include higher PN terms

agnostic tests: $$h(v) = \mathcal{A}(v) e^{-i(\varphi(v)+\delta\varphi(v))}$$

Q. how to connect each order deviations to new physics

theory-specific tests:

Q. may miss some deviations

- even for GR, hard without hierarchy of mass

4.5PN for point-particles, 3PN for spins

Our formalism:

BH perturbation (Fundamental Theory) ⟷ **Matching** ⟷ **worldline EFT (Observational Parameter)**

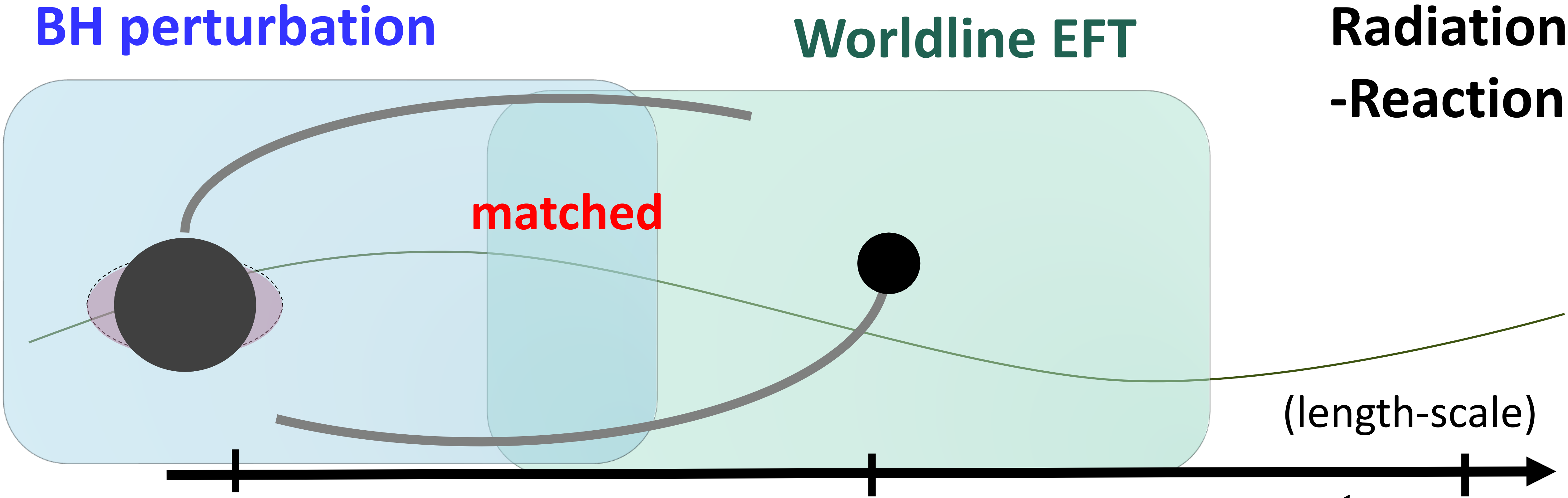
2. Worldline EFT

See e.g. [Goldberger (2006)], [Porto (2016)], [Levi (2020)]

Separations of Scales

$v \ll 1$: non-relativistic motion

$r_g \omega \sim M\omega \sim v^3$



$(\ell_{\rm bGR} <)\ r_g \ll r \ll r_{\rm orb} \ll \omega^{-1} \sim \lambda_{\rm GW}$

horizon (Schwarzschild) radius

$r_{\rm orb} \sim r_g/v^2$

$\omega^{-1} \sim \lambda_{\rm GW} \sim r_{\rm orb}/v \sim r_g/v^3$

Worldline Effective Field Theory

See e.g., [Goldberger (2006)], [Porto (2016)], [Levi (2020)]

- a compact object (BH) is represented as composite particle, i.e., **a point particle** with **internal degrees of freedom (DoFs)**
- Bulk DoF: 4-dim spacetime metric → $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$S_{\mathrm{pp}} + S_{\mathrm{EH}} = -\int \mathrm{d}\tau\, M + \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\,R$$

τ : proper time for worldline

- internal multipole coupled to tidal fields [Goldberger & Rothstein (2006)]

$$S_{\mathrm{int}} = -\int \mathrm{d}\tau\,\left(Q_{\mathrm{E}}^{L}(X)\mathcal{E}_L(\tau) + Q_{\mathrm{B}}^{L}(X)\mathcal{B}_L(\tau)\right)$$

Integrating Out Hidden DoF(s)

(In-In) effective action $\Gamma_{\rm eff}\left[h_{\mu\nu}, x^{\mu}_{\rm CM}(\tau)\right] = \Gamma^{\rm grav}_{\rm eff} + \Gamma^{\rm pp}_{\rm eff} + \Gamma^{\rm fin}_{\rm eff}$

$$\Gamma^{\rm fin}_{\rm eff} = \frac{1}{2}\int {\rm d}\tau \int {\rm d}\tau' \left\langle Q_{\rm E}^{L\,A}(\tau) Q_{\rm E}^{L'\,B}(\tau') \right\rangle \mathcal{E}_{L\,A}(\tau)\mathcal{E}_{L'\,B}(\tau') + ({\rm mag})$$

unknown function

$A, B \in \{{\rm cl}, {\rm q}\}$

contracted by $c_{AB} = c^{AB} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Under the classical tidal background $\bar{\mathcal{E}}^{L'}$

retarded Green's function for Q **non-linear terms**

$$\left\langle Q_L^{\rm E}(\omega) \right\rangle = r_g^{2\ell+1} \mathcal{F}_\ell^{\rm E}(\omega) \delta_{L,L'} \bar{\mathcal{E}}^{L'}(\omega) + \cdots$$

expanded by $r_g\omega \ll 1 \rightarrow (r_g\omega)^2 = v^6$: **3PN correction to the leading tidal deformations**

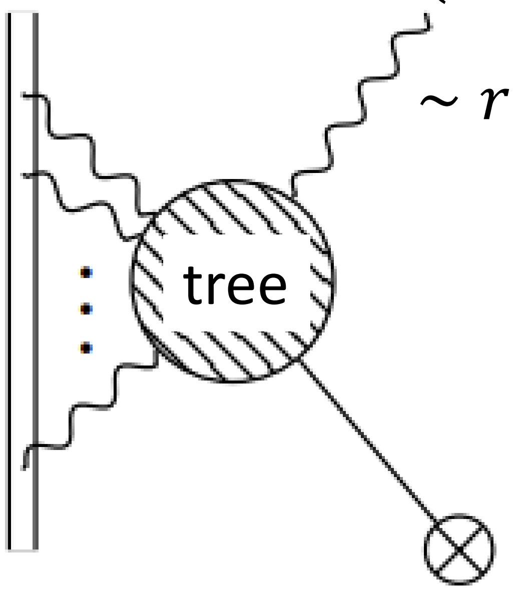
Tidal Perturbations to Worldline

sourcing **2-body binding potential** $p_0 \sim \omega \sim \frac{v}{r}$ & $|\vec{p}| \sim \frac{1}{r}$

- ✓ Up to $\mathcal{O}(\omega)$ [Hui+ (2019)], [Ivanov & Zhou (2021)], $v^{2m} \sim (\omega r)^{2m}$ terms are new
- ✓ We use the dimensional regularization; $\ell \to \ell + \delta\ell = \hat{\ell}$

time

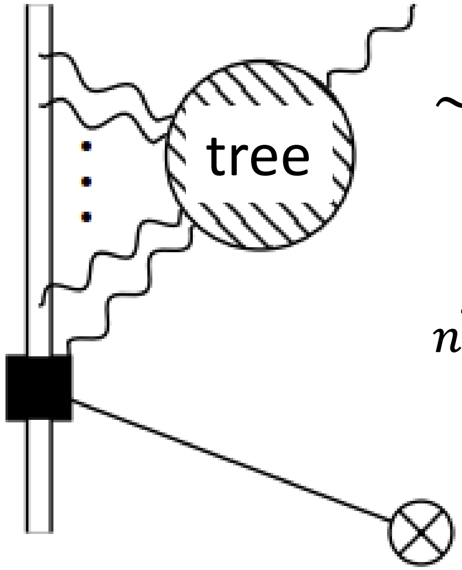
minimal coupling



$$\left(\delta h^{\mathrm{pp}}_{\mu\nu}(r;\omega)\right)_{\ell m} \sim r^{\hat{\ell}} \sum_{n\geq 1, m\geq 0} \# \left(\frac{r_g}{r}\right)^n (\omega r)^{2m}$$

$$\left(h^{\mathrm{tide}}_{\mu\nu}\right)_{\ell m} = j_\ell(\omega r)$$

$\mathcal{F}^{\mathrm{E/B}}_{\hat{\ell}}$



$$\left(\delta h^{\mathrm{fin}}_{\mu\nu}(r;\omega)\right)_{\ell m} \sim \mathcal{F}^{\mathrm{E/B}}_{\hat{\ell}} r^{\hat{\ell}} \left(\frac{r_g}{r}\right)^{2\hat{\ell}+1} \sum_{n,m\geq 0} \# \left(\frac{r_g}{r}\right)^n (\omega r)^{2m}$$

$$\left(h^{\mathrm{tide}}_{\mu\nu}\right)_{\ell m}$$

3. Matching to BH perturbation

Matching to the MST solutions

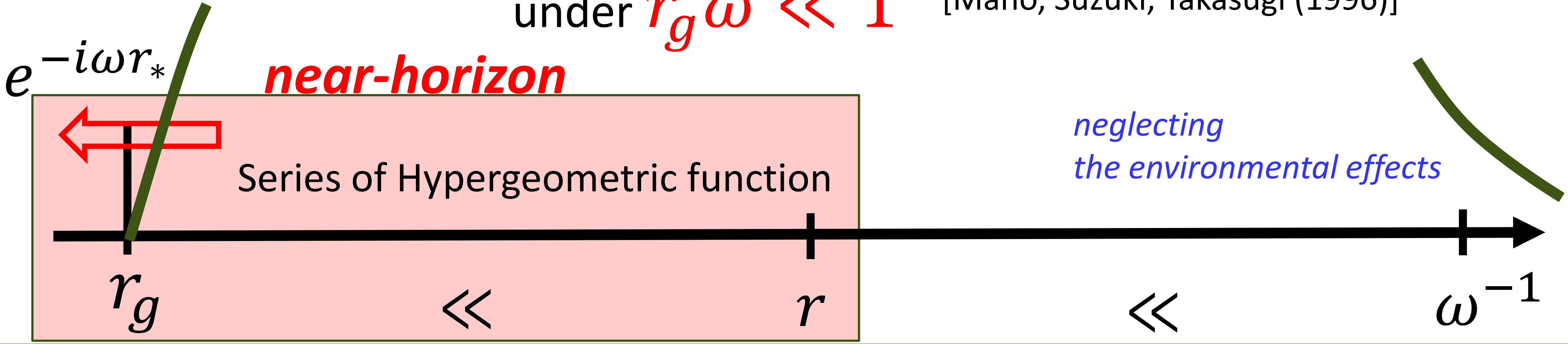
perturbation around BH sourced by $\Gamma^{\mathrm{pp}}_{\mathrm{in-in}}$ $\Gamma^{\mathrm{fin}}_{\mathrm{in-in}}$

$$\left(\delta g_{\mu\nu}\right)_{\ell m}(r;\omega) = \left(h^{\mathrm{tide}}_{\mu\nu}\right)_{\ell m} + \left(\delta h^{\mathrm{pp}}_{\mu\nu}\right)_{\ell m} + \left(\delta h^{\mathrm{fin}}_{\mu\nu}\right)_{\ell m}$$

matching gauge-inv. master variable, e.g., **Regge-Wheeler Ψ_{RW}**

→ analytically solved by **Mano-Suzuki-Takasugi (MST) methods**

under $r_g\omega \ll 1$ [Mano, Suzuki, Takasugi (1996)]



Matching to the MST solutions

$$\Psi_{\mathrm{RW}} \sim r^{\nu+1}\left[1+\mathcal{O}\left(\frac{r_g}{r},(\omega r)^2\right)+\bar{\mathcal{F}}_{\nu}^{\mathrm{B}}(\omega)\left(\frac{r_g}{r}\right)^{-2\nu-1}\left(1+\mathcal{O}\left(\frac{r_g}{r},(\omega r)^2\right)\right)\right]$$

horizon ingoing solution

in $r_g \ll r \ll \omega^{-1}$

✓ $\nu = \hat{\ell}$ →matched to the **worldline EFT calculation within dim-reg**

The **bare quantities of the tidal response function**

$$\bar{\mathcal{F}}_{\ell+\delta\ell}^{\mathrm{B}}(\omega) = \underline{\mathcal{N}_\ell}\left[i r_g \omega + \underline{\frac{(r_g\omega)^2+\cdots}{(-2\delta\ell)}} + (r_g\omega)^2\mathcal{K}_\ell^{(2)} + \frac{\delta\ell}{2} + \cdots \right]$$

tidal dissipation numbers

Counter terms → source the renormalization flow

Renormalized Quantities

What is ν?
- In MST, it assures **the convergence of series expansion**
- In EFT, ***β*-function** of **the absorptive multipole moments**

[Ivanov+ (2025)]

$$\nu = 2 - \frac{107}{210}(2GM\omega)^2 + \cdots \text{ for } \ell = 2$$

Minimal Subtraction for $\bar{\mathcal{F}}^{\mathrm{B}}_{\ell+\delta\ell}(\omega)$ respecting the separations in MST

$$\bar{\mathcal{F}}^{\mathrm{B,ren}}_{\ell=2}(\omega) = \mathcal{N}_{\ell=2}\left[i r_g \omega + \left(\frac{71}{35} + \log\frac{r_{\mathrm{orb}}}{r_g}\right)(r_g\omega)^2 + \cdots \right]$$

4. Summary & Outlooks

Summary

Dynamical tidal Love number ; $\mathcal{O}(\omega^2)$ **for non-rotating**

→ **non-zero & logarithmically scaling by v**
even in 4d Schwarzschild BH within GR

*Effects from the potential barrier in BH perturbation
→ **renormalized angular momentum**

*Renormalization of the composite **Wilsonian coeff**. in worldline EFT

→ **(8PN)** $\times \log v$ **effects**
in Post-Newtonian waveform for inspiral

*already known **2.5 PN log** terms for **EMRI in Kerr** [Huges (2001)]

Outlooks: Waveform Modeling

$$\varphi(v) \sim \# v^{-5}\{1 + \cdots + \# v^5 + \cdots + \# v^8 + \# v^8 \log v + \cdots\}$$

Coefficients mixing the V_{PP} and $\bar{\mathcal{F}}_\ell^{\mathrm{E(B),ren}}$ ⟶ **scheme dependent**

$$\varphi(v)$$

tidal dissipation

$$\sim \# v^{-5-2\delta\ell}\left\{1 + \sum_{n\geq 1} \varphi_{\mathrm{PP}}^{(n)} v^n + \sum_{\ell\geq 2} \# \mathrm{Im}\bar{\mathcal{F}}_{\hat{\ell}}^{\mathrm{E}}(v) v^{4\hat{\ell}} + \# \mathrm{Im}\bar{\mathcal{F}}_{\hat{\ell}}^{\mathrm{B}}(v) v^{4\hat{\ell}+2}\right.$$

$$\left. + \# \mathrm{Re}\bar{\mathcal{F}}_{\hat{\ell}}^{\mathrm{E}}(v) v^{4\hat{\ell}+2} + \# \mathrm{Re}\bar{\mathcal{F}}_{\hat{\ell}}^{\mathrm{B}}(v) v^{4\hat{\ell}+6} + \cdots\right\}$$

tidal deformation

Theoretical Extensions:

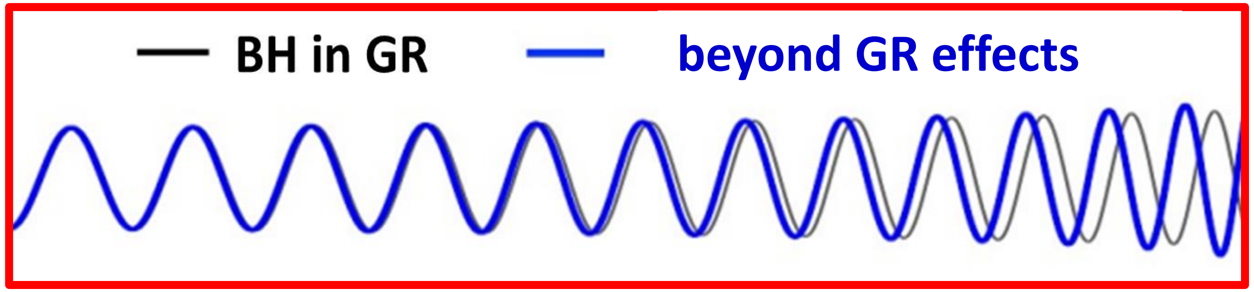
- **Non-linear response**
- **beyond GR vacuum**

  →Parametrized BH

for Dissipation Numbers see [HK+ (2025)]

Backup

PN-Orders of Tidal Response



Post-Newtonian analysis → $v \ll 1$

n-PN→ v^{2n}

$r_g \omega = v^3$ → 1.5 PN

The leading PN corrections to phase

$\mathcal{F}^{\mathrm{E}}_{\ell=2}(\omega)$	$(r_g \omega)^0$	$(r_g \omega)^1$	$(r_g \omega)^2$	$(r_g \omega)^3$
Conservative (real part)	5PN		**8PN**	
Dissipative (imaginary part)		4PN		7PN

Coupling to Internal DoFs

gravitational coupling to internal DoFs

originally, [Goldberger & Rothstein, 2006]

$$S_{\mathrm{int}} = -\int \mathrm{d}t \left(Q_{\mathrm{E}}^{L}(X)\mathcal{E}_L(t) + Q_{\mathrm{B}}^{L}(X)\mathcal{B}_L(t) \right)$$

$\mathcal{E}_L$ & $\mathcal{B}_L$: **ele/mag tidal tensor** (in the limit of center of mass of objects)

expanded by the ℓ-th rank symmetric trace-free tensor

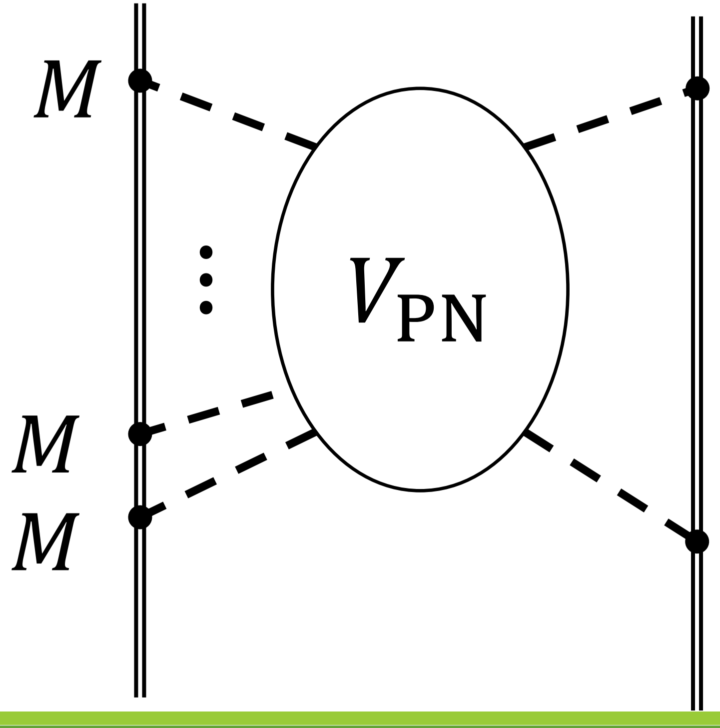
$Q_{\mathrm{E/B}}^{L}(X)$: 2^{ℓ}-th multipole moments operator

$X(t)$ denotes the internal DoF described by the action $S_X = \int \mathrm{d}t\, L_X$

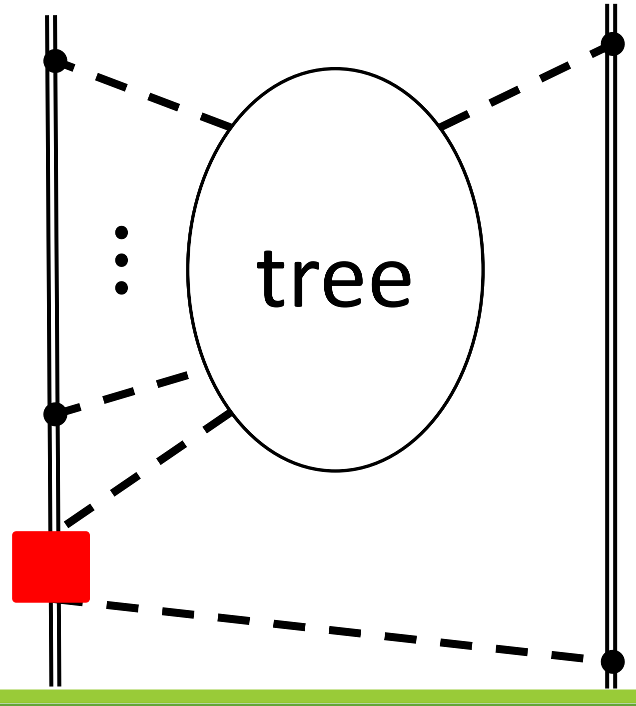
like a harmonic oscillator

Worldline Effective Field Theory

- a compact object (BH) is represented as composite particle
  a point particle with internal degrees of freedom (DoFs)
- Bulk DoF: 4-dim spacetime metric → $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}^{\mathrm{pot}}$



$\mathcal{F}_\ell^{\mathrm{E/B}}(\omega)$
$\sim \langle Q_\ell Q_\ell \rangle$



Retarded Green's Function

$$\langle Q_\ell^{\mathrm{B}}(X)\rangle = -\int \mathrm{d}t\, G_{\ell,\ell'}^{\mathrm{ret}\,\mathrm{B}}(t-t')\overline{\mathcal{B}}^{\ell'}(t')$$

$$G_{\ell,\ell'}^{\mathrm{ret}\,\mathrm{B}}(t-t') \coloneqq i\langle[Q_\ell^{\mathrm{B}}(t), Q_{\ell'}^{\mathrm{B}}(t')]\rangle\Theta(t-t')$$

$\Theta(x)$ is Heaviside's step function

Frequency domain

(induced multipole moments) = (linear response function) × (background curvature)

$$G_{\ell,\ell'}^{\mathrm{ret}\,\mathrm{E/B}}(\omega) = -r_g^{2\ell+1}\mathcal{F}_\ell^{\mathrm{E/B}}(\omega)\delta_{\ell,\ell'}$$

from bottom-up
→unknown function

(dim-less) tidal response function

Schwinger-Keldysh Effective Action

integrate out internal modes X → outside of BH becomes open system

$$e^{i\Gamma_{\rm eff}[x_{\rm CM}, g_{\mu\nu}]} = \int \mathcal{D}X_1 \mathcal{D}X_2 e^{i\left(S_1\left[x_{{\rm CM}_1}, g_{\mu\nu_1}, X_1\right] - S_2\left[x_{{\rm CM}_2}, g_{\mu\nu_2}, X_2\right]\right)}$$

extremize effective action

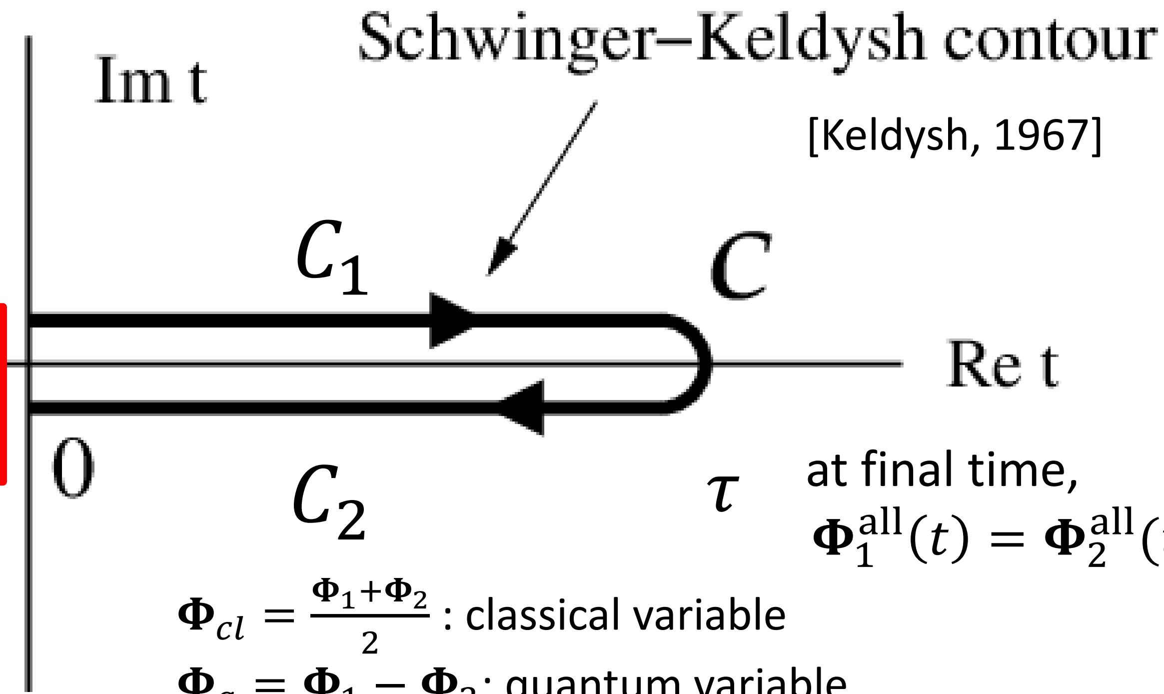
EoM including tidal effects

especially dissipative reaction force

[Goldberger & Rothstein, 2020]

initial state
$\rho_0 = |0\rangle\langle 0| \otimes \rho_{\rm BH}$

*$|0\rangle$ is Minkowski vacuum
* $\rho_{\rm BH}$: BH states



[Keldysh, 1967]

at final time,
$\mathbf{\Phi}_1^{\rm all}(t) = \mathbf{\Phi}_2^{\rm all}(t)$

$\mathbf{\Phi}_{cl} = \frac{\mathbf{\Phi}_1 + \mathbf{\Phi}_2}{2}$: classical variable
$\mathbf{\Phi}_q = \mathbf{\Phi}_1 - \mathbf{\Phi}_2$: quantum variable

Renormalized Quantities

$$\bar{\mathcal{F}}^{\mathrm{B}}_{\ell+\delta\ell}(\omega) = R^{2\delta\ell}\left[\bar{\mathcal{F}}^{\mathrm{B,ren}}_{\ell}(\omega;R) + \bar{\mathcal{F}}^{\mathrm{B,ct}}_{\ell}(\omega)\right]$$

R: renormalization "length" scale

$$R\frac{\partial}{\partial R}\bar{\mathcal{F}}^{\mathrm{B,ren}}_{\ell}(\omega;R) = \mathcal{N}_{\ell}\left\{(r_g\omega)^2 + \mathcal{O}\left((r_g\omega)^4\right)\right\} + \mathcal{O}(\delta\ell)$$

$$\bar{\mathcal{F}}^{\mathrm{B,ren}}_{\ell}(\omega;R) = \bar{\mathcal{F}}^{\mathrm{B,ren}}_{\ell}(\omega;r_g) + \mathcal{N}_{\ell}\left\{(r_g\omega)^2 + \mathcal{O}\left((r_g\omega)^4\right)\right\}\log\left(\frac{R}{r_g}\right)$$

finite part is scheme-dep.

How to compute the GW

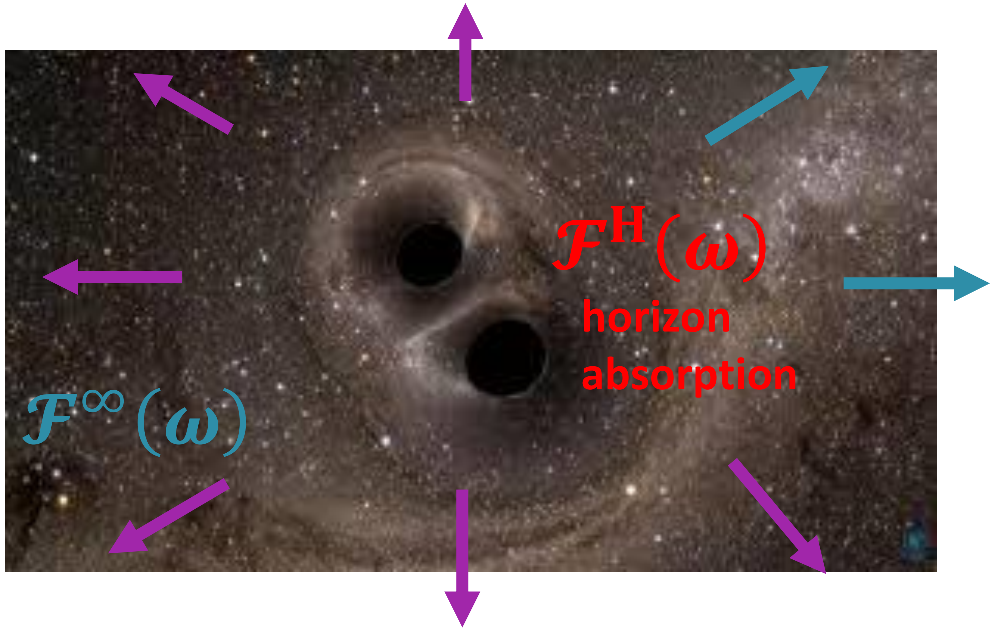
$$\frac{dE(\omega)}{dt} = -\mathcal{F}^{\infty}(\omega) - \mathcal{F}^{H}(\omega)$$

orbital energy

radiated power

flux absorbed by the event horizon (or internal states)

$\omega \sim v/r_{\rm orb}$: orbital frequency



calculate the orbital phase ϕ
which is proportional to the GW phase Ψ

$$E'(\omega) := \frac{dE}{d\omega}$$

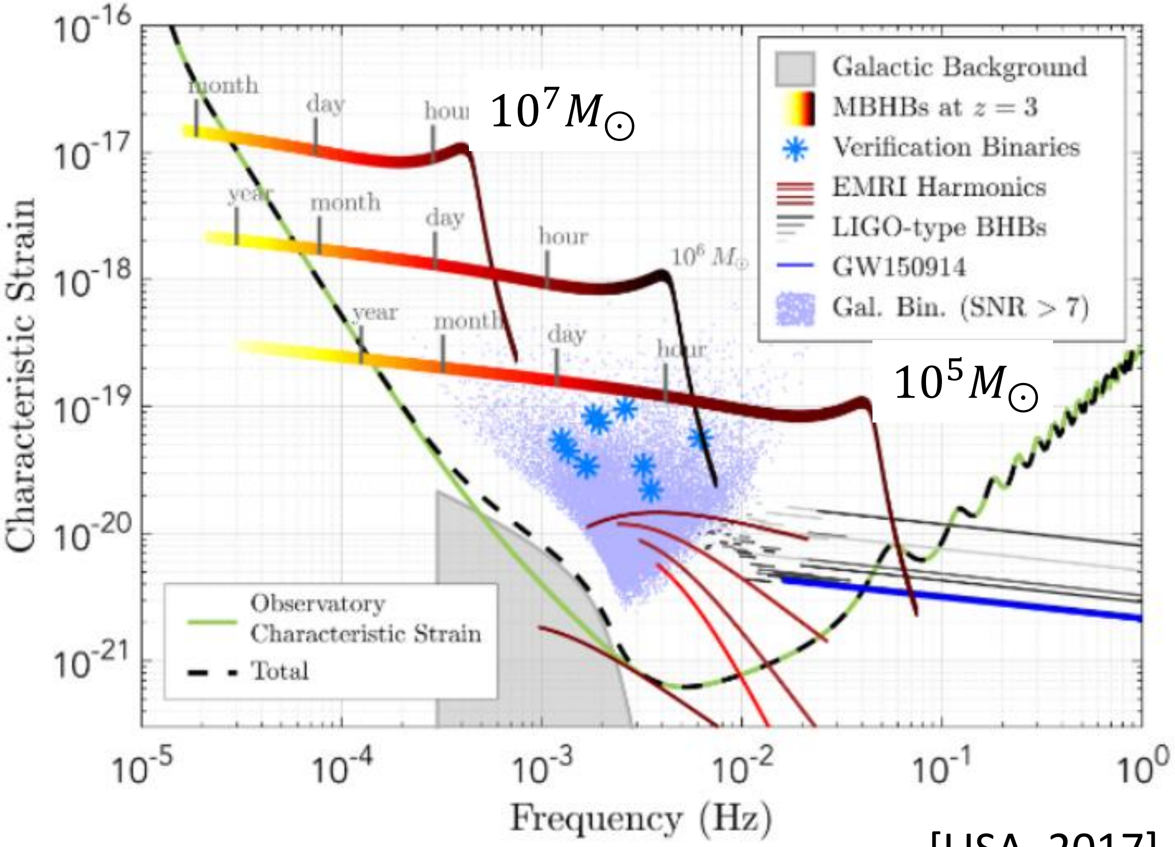
$$\dot{E}(\omega) := \frac{dE}{dt}$$

$$\Psi \propto \phi(\omega) = \int \omega\, dt = \int \omega \frac{E'(\omega)}{\dot{E}(\omega)}\, d\omega$$

Gravitational Wave Observations

- Next-gen ('30s) : LISA, Einstein Telescope (ET), Cosmic Explorer (CE)

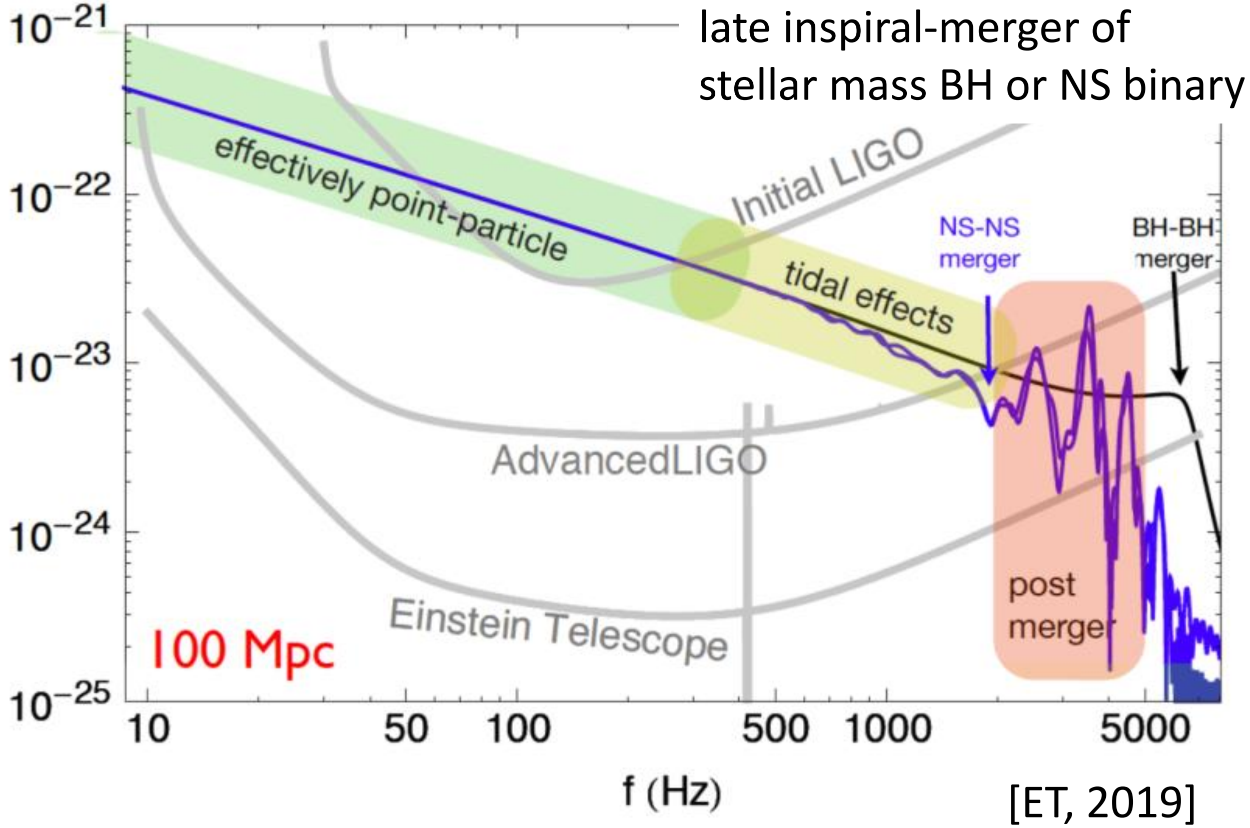
LISA



[LISA, 2017]

Einstein Telescope

late inspiral-merger of stellar mass BH or NS binary



[ET, 2019]