Non-linear dynamics during kination

based on arxiv2507.19166 with C. Cheng, L. Heurtier and E. Lim

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Outlook

- Why kination?
- Linear perturbations during kination
- Non-linear dynamics with Numerical Relativity

FLRW Evolution

FLRW metric: $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$

Standard tale: Inflation $w \approx -1\,, \rho \approx constant$

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Not strong constraints ⟵ **?**

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radiation domination $w = 1/3\,, \rho \sim a^{-4}$

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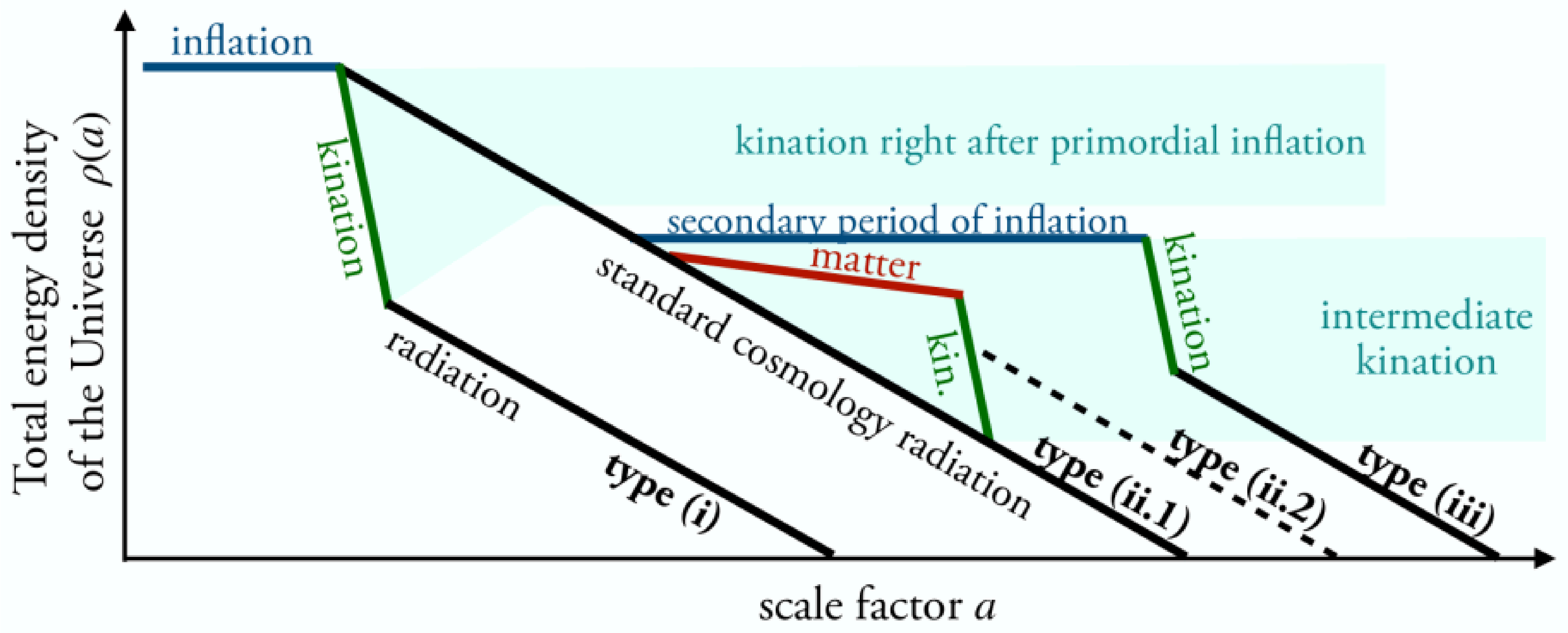
matter domination $w = 0\,, \rho \sim a^{-3}$

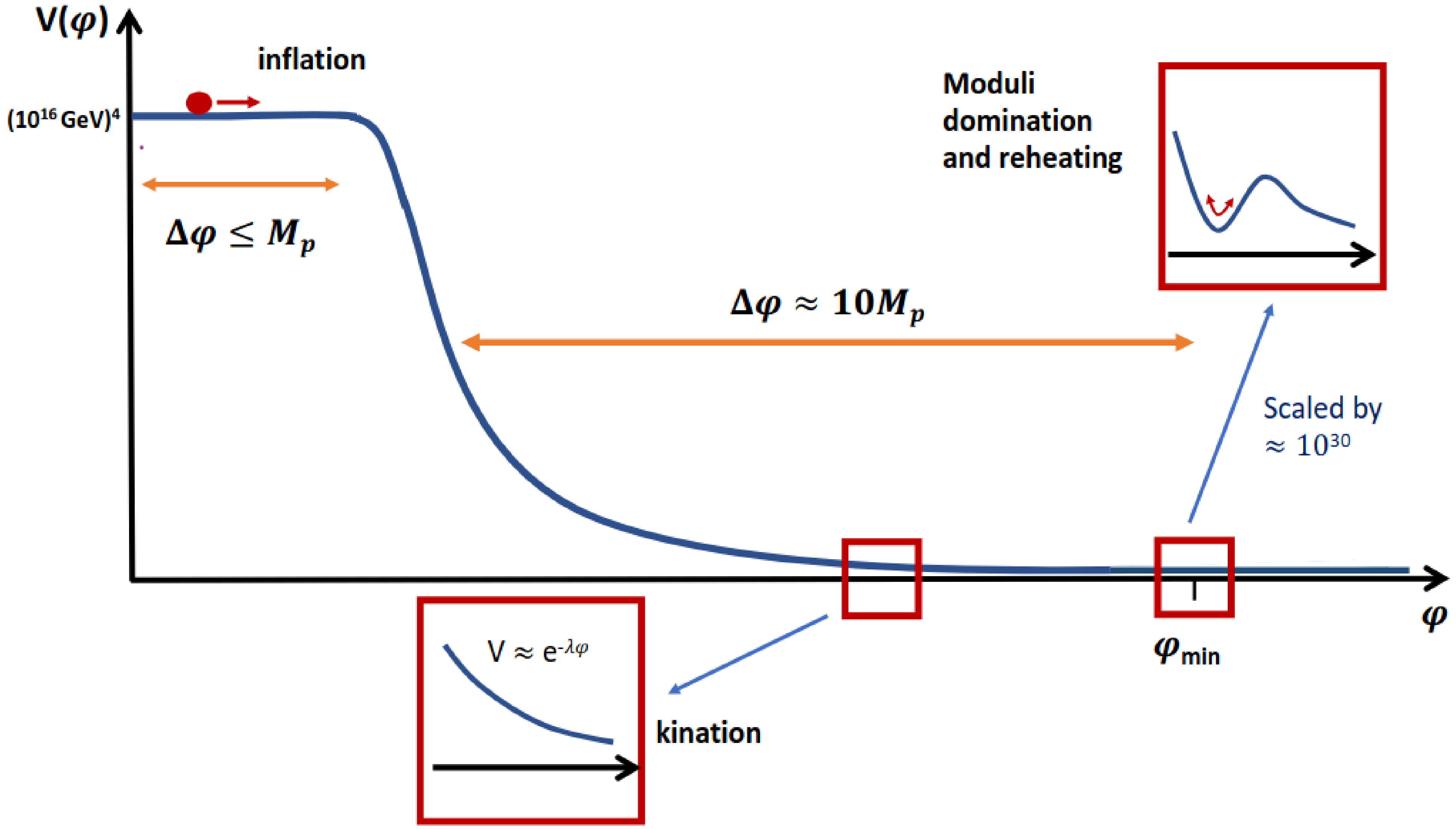
FLRW Evolution



2111.01150 Gouttenoire et Al.

Kination: epoch where dynamics are dominated by the kinetic energy of a rolling scalar field

Realisitic string inspired scenario (LVS)



2401.04064 Apers, Conlon et Al.

Kination

GR + minimally coupled scalar field

$$S = \int d^4x \sqrt{-g} \left(\frac{m_p^2}{16\pi} R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right)$$

Kination: $\rho_{kin} \equiv \frac{1}{2}\dot{\phi}^2 \gg V(\phi)$

$$\rho_{kin} \gg \rho_\nabla$$

$$w = \frac{\rho}{p} = \frac{\frac{1}{2}\dot{\phi}^2 + \cancel{\frac{1}{2a^2}(\nabla\phi)^2} + \cancel{V(\phi)}}{\frac{1}{2}\dot{\phi}^2 + \cancel{\frac{1}{2a^2}(\nabla\phi)^2} - \cancel{V(\phi)}} \approx 1$$

EoM: $\ddot{\phi} - \frac{1}{a^2}\nabla^2\phi + 3H\dot{\phi} + \cancel{V'(\phi)} = 0$

$$\rho \sim a^{-6}$$

Linear Perturbations during kination

Assume linear perturbations in FLRW background

$$\phi(t) \to \phi(t, \vec{x}) = \bar{\phi}(t) + \delta\phi(t, \vec{x})$$

$$\delta\phi = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \delta\phi_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}}$$

EoM:

$$\ddot{\bar{\phi}} + 3H\dot{\bar{\phi}} + V'(\bar{\phi}) = 0$$

$$\delta\ddot{\phi}_{\mathbf{k}} + 3H\delta\dot{\phi}_{\mathbf{k}} + \left(\frac{\mathbf{k}^2}{a^2} + V''(\bar{\phi})\right)\delta\phi_{\mathbf{k}} + \cancel{\mathcal{O}(\delta\phi^2)} = 0$$

Solution:

$$\bar{\phi} \sim \ln t$$

$$\delta\phi_{\mathbf{k}} = At^{-\frac{3c-1}{2}} J_\alpha\left(\frac{ka^{-1}}{1-c}t\right) + Bt^{-\frac{3c-1}{2}} J_{-\alpha}\left(\frac{ka^{-1}}{1-c}t\right)$$

kination

$$\alpha = 0\,, c = 1/3$$

Linear Perturbations during kination

EoM: $\ddot{\bar{\phi}} + 3H\dot{\bar{\phi}} + V'(\bar{\phi}) = 0$

$$\ddot{\delta\phi}_{\mathbf{k}} + 3H\dot{\delta\phi}_{\mathbf{k}} + \left(\frac{\mathbf{k}^2}{a^2} + V''(\bar{\phi})\right)\delta\phi_{\mathbf{k}} + \cancel{\mathcal{O}(\delta\phi^2)} = 0$$

density contrast pert.: $\delta = \frac{\rho_{\delta\phi}}{\bar{\rho}}\,, \bar{\rho} \sim a^{-6}$

subhorizon pert.: $k \gg aH \Rightarrow \quad \delta\phi_{\mathbf{k}} \sim \frac{\cos\left(\frac{3ka^{-1}}{2}\right)}{a} \sim a^{-1} \Rightarrow \rho_{\delta\phi} \sim \frac{\delta\phi^2}{a^2} \sim a^{-4} \Rightarrow \delta \sim a^2$

superhorizon pert.: $k \ll aH \Rightarrow \delta\phi_{\mathbf{k}} \sim \frac{\delta\phi_{\mathbf{k}\,0}}{a} \Rightarrow \rho_{\delta\phi} \sim a^{-2} \Rightarrow \delta \sim a^4$

End state of those perturbations: perturbations take over as **new radiation** background, possible tracker solutions 2507.04161 Mosney, Conlon, Copeland

Non-linear Perturbations during kination

- What is the evolution of large perturbations?
- Strong Gravitational backreaction and collapse?
- Primordial Black holes?

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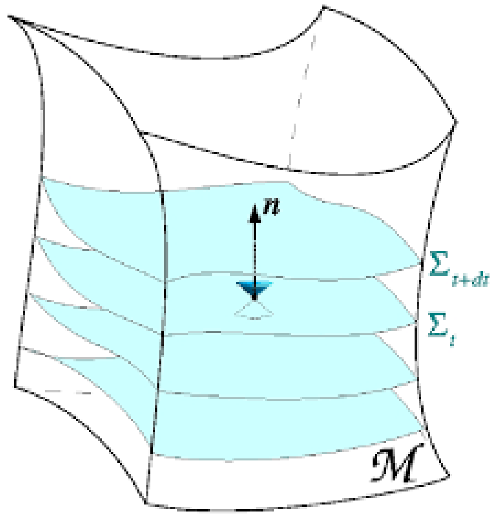
Numerical Relativity Simulations

3+1 Decomposition

Foliate the spacetime into 3D slices evolving in a time coordinate

Gravity sector:



3d metric γ_{ij}

Extrinsic curvature $K_{ij} \sim \partial_t \gamma_{ij}$

$$K_{ij} = A_{ij} + \frac{1}{3}\gamma_{ij}K$$

traceless: tensor modes

trace: expansion/contraction

Matter sector: $\phi(\vec{x}), \dot{\phi}(\vec{x})$

$K = -3H$ for FLRW

For IC specify matter , solve constraints for geometry, fix gauge dofs

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How to simulate inhomogeneous kination

- Assume for simplicity **pure kination** $V(\phi) = 0$
- Specify initial field configuration with harmonic perturbations in 3 directions, in a pseudo-isotropic set up and put **only one** perturbation mode

Kinetic backround + "Scalar" inhomogeneities:

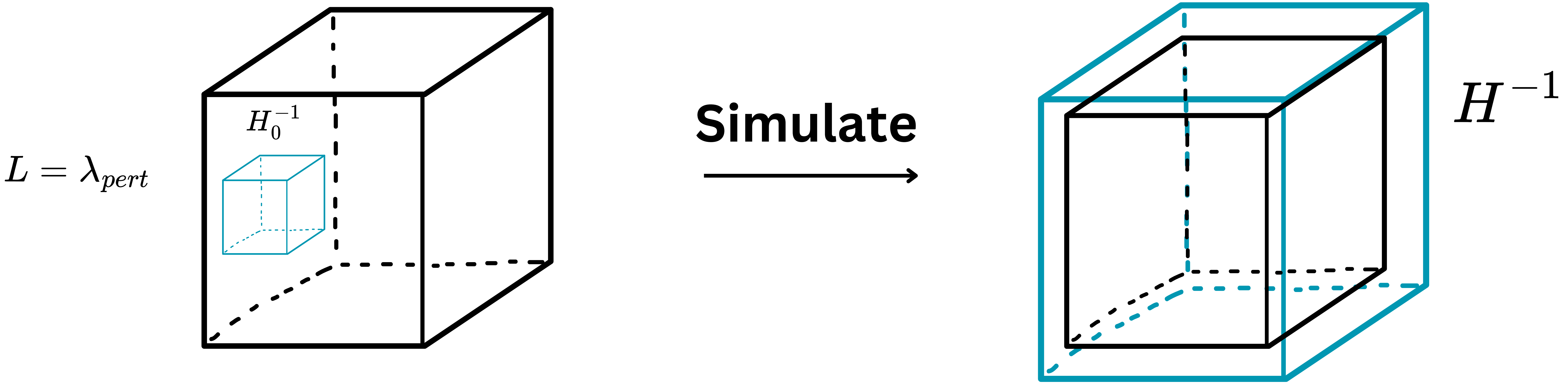
$$\phi_{init}(\vec{x}) = \cancel{\phi_0} + \sum_{i}^{3} \frac{\Delta\phi}{3} \cos\left(\frac{2\pi N x_i}{L}\right)$$

$$\dot{\phi}_{init} = \dot{\bar{\phi}}_0 + \cancel{\Delta\Pi(t, \mathbf{x})} = \sqrt{\frac{3}{4\pi}} m_P H_0$$

- Choose simulation domain size such that there is one wavelength per box initially

$$L = N H_0^{-1} = N \left(\sqrt{\frac{8\pi}{3 m_p^2} \rho_{kin\,0}} \right)^{-1}$$

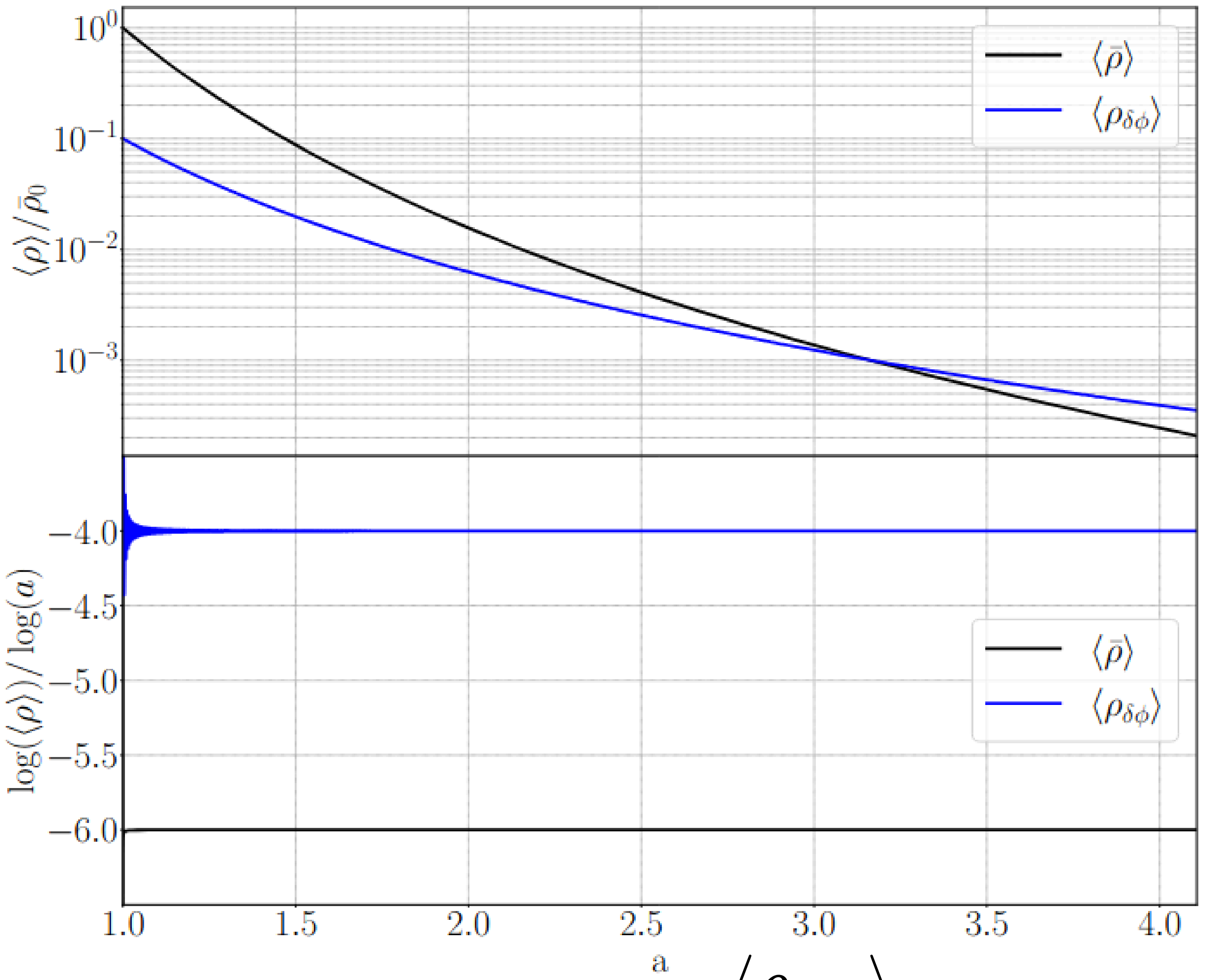
- Solve constraints for the rest of geometric dofs



diagnostic: $\langle \rho_{\delta\phi} \rangle \sim \langle \rho_{\nabla} \rangle + \frac{1}{2}\langle \dot{\phi}^2 \rangle - \frac{1}{2}\langle \dot{\phi} \rangle^2$

Any Lessons??

Lesson 1: subhorizon modes do not strongly backreact



$$\lambda_0 = 0.01 H_0^{-1}\,,\ \frac{\langle \rho_{\nabla\,0} \rangle}{\bar{\rho}_0} = 0.1$$

$$\langle \bar{\rho} \rangle \sim a^{-6}$$

$$\langle \rho_{\delta\phi} \rangle \sim a^{-4}$$

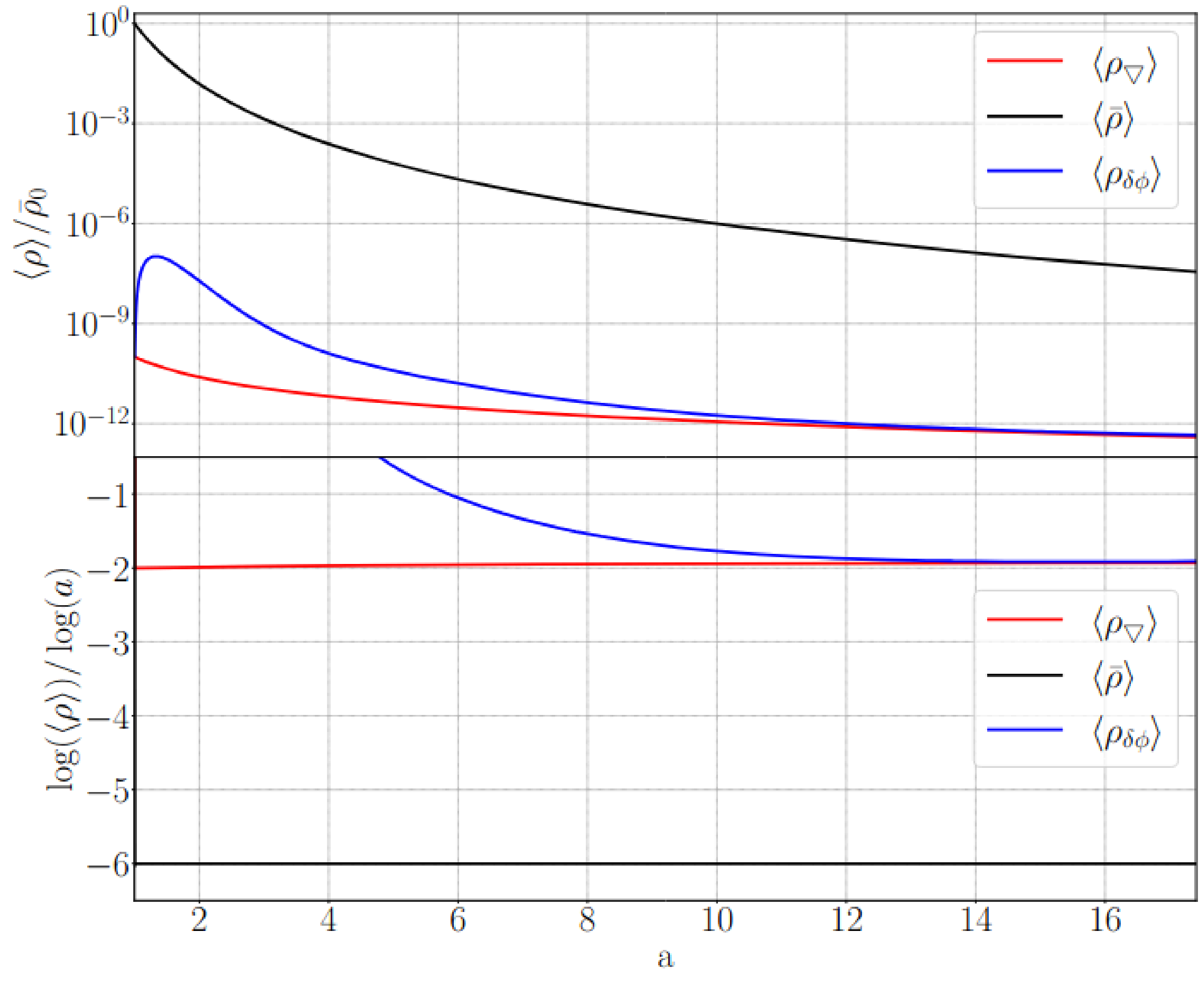
perturbations take over and become the new radiation background no strong backreaction

gravitational collapse for

$$\lambda_0 = \mathcal{O}(10^{-1}) H_0^{-1}\,,\ \frac{\langle \rho_{\nabla\,0} \rangle}{\bar{\rho}_0} = \mathcal{O}(10^2)$$

$$\lambda_0 = H_0^{-1}\,,\ \frac{\langle \rho_{\nabla\,0} \rangle}{\bar{\rho}_0} \approx 2$$

Lesson 2: superhorizon modes can strongly backreact at horizon re-entry



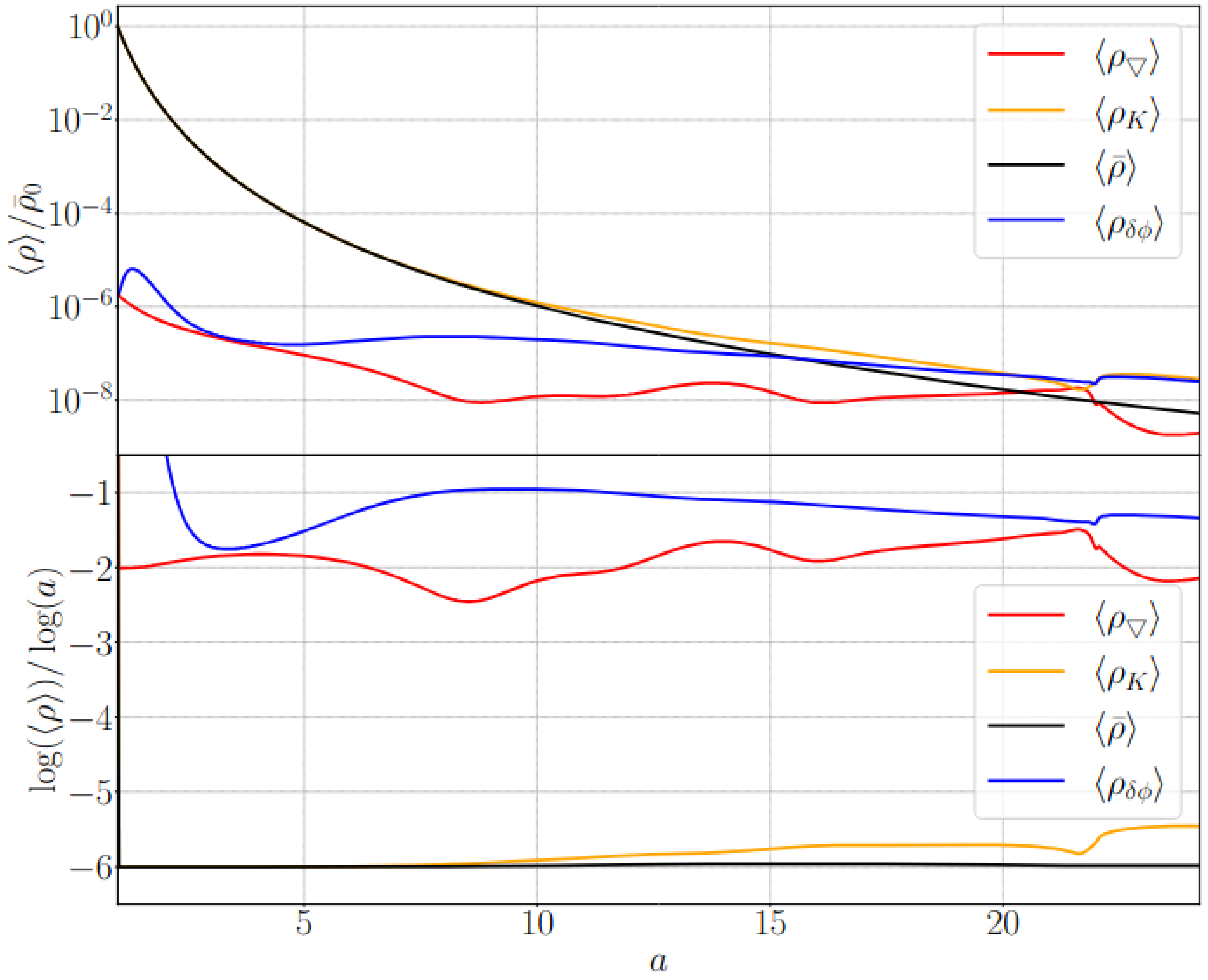
$$\lambda_0 = 10^4 H_0^{-1}, \frac{\langle\rho_{\nabla\,0}\rangle}{\bar{\rho}_0} = 10^{-10}$$

For small superhorizon perturbations

$$\langle\bar{\rho}\rangle \sim a^{-6}$$

$$\langle\rho_{\delta\phi}\rangle \sim a^{-2}$$

Lesson 2: superhorizon modes can strongly backreact at horizon re-entry

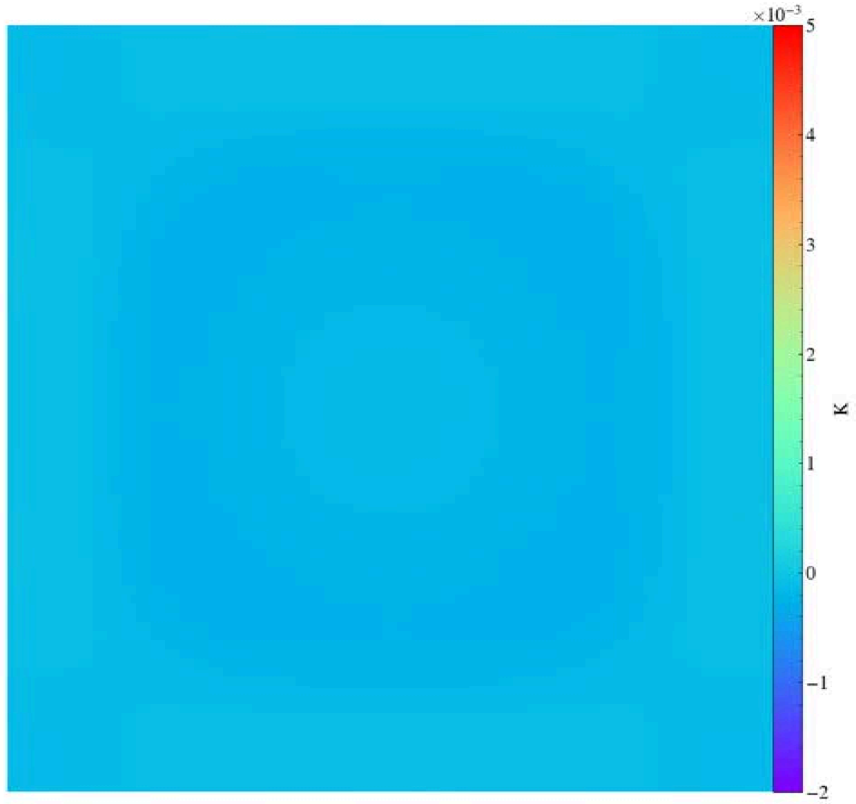


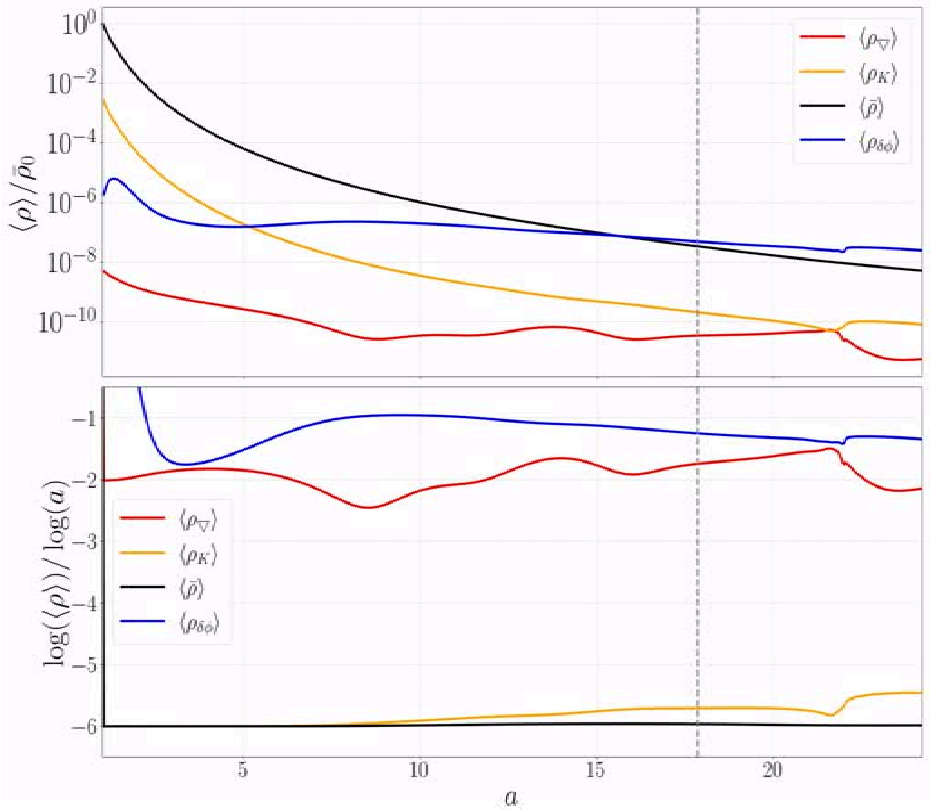
$$\lambda_0 = 200 H_0^{-1}\,,\ \frac{\langle\rho_\nabla\rangle}{\bar{\rho}} = 1.8\times10^{-6}$$

$$a_{re-entry} \approx \sqrt{\lambda_0 H_0}$$

PBH formation 0.5 efolds after re-entry

end state: BH matter domination





Lesson 3: Critical density contrast to form PBHs

Perturbative density contrast estimate for spherical collapse based on Press-Schechter formalism:

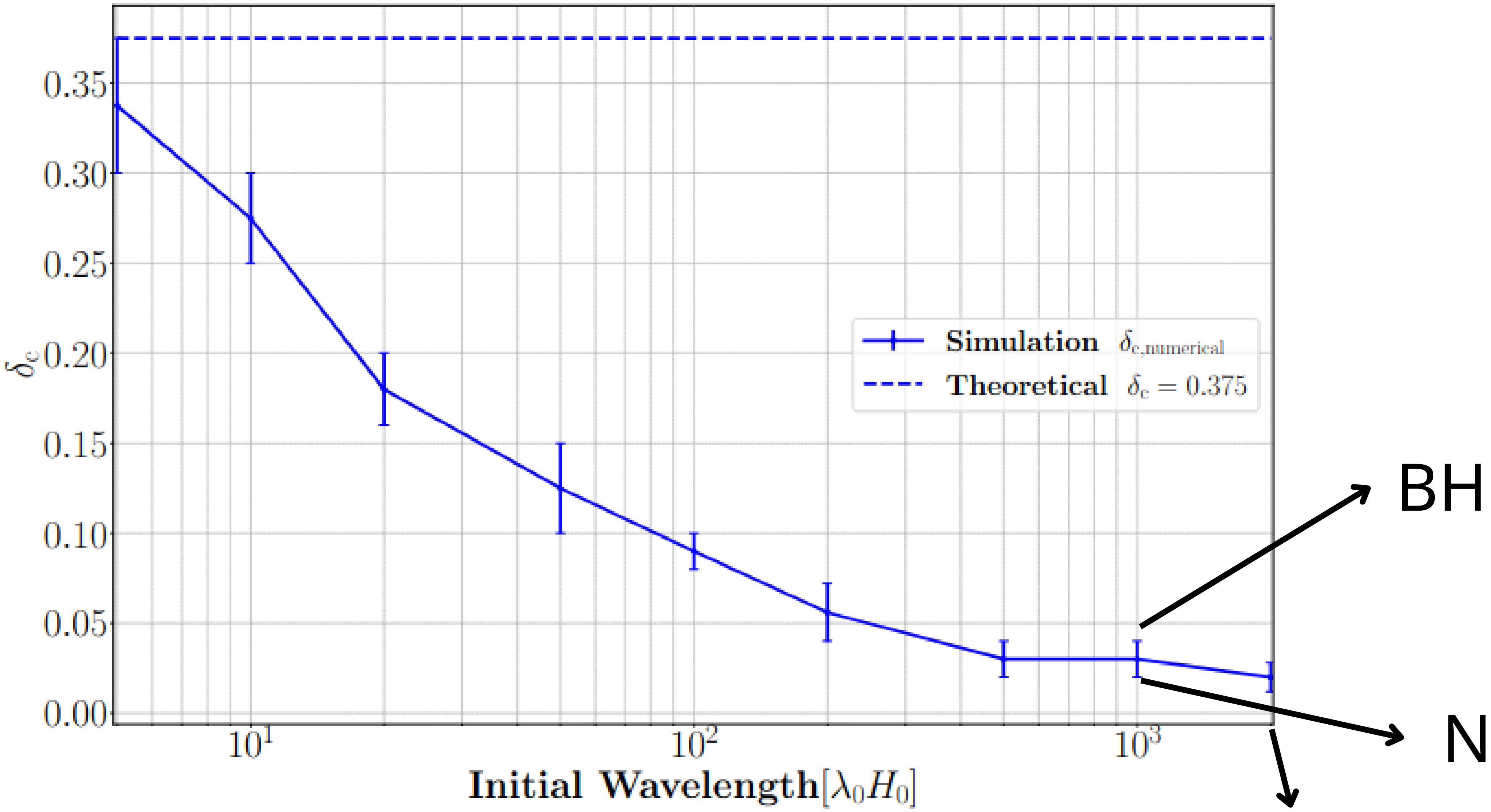
$$\delta_c = \frac{3(1+w)}{5+3w}\sin^2\left(\frac{\pi\sqrt{w}}{1+3w}\right) \approx 0.375$$

Numericall estimate for collapse assuming kinetic background dominates up to horizon re-entry:

$$\delta_c = a^4_{re-entry}\frac{\langle\rho_{\nabla\,0}\rangle}{\bar{\rho}_0}$$

Check for various wavelengths for what δ_c black holes form

Lesson 3: Critical density contrast to form PBHs



3 efolds of kination max 10

Conclusion: $\delta_{c\,numerical} < \delta_{c\,theoretical}$

Lesson 4: Implications for cosmology

To form PBHs we typically need a quasi-monochromatic enhancement in the primordial power spectrum

$$\Delta^2_{R\,peak} \sim 10^{-2} \gg \Delta^2_{R\,CMB} \sim 10^{-9}$$

In the context of kination we are able to constrain the value of $\Delta^2_{R\,peak}$ via

$$\beta = \frac{\rho(M_{PBH})}{\rho_{total}} \sim \frac{\Delta_{\mathcal{R},\text{peak}}}{\sqrt{2\pi}\delta_c}\exp(-\frac{\delta_c^2}{2\Delta^2_{\mathcal{R},\text{peak}}})$$

$$\delta_c \sim 0.03 \Rightarrow \Delta^2_{R\,peak} \sim \mathcal{O}(10^{-5})$$

Conclusions and future directions

- Linear perturbations during kination eventually take over as new radiation background
- Large subhorizon perturbations don't collapse
- Large superhorizon perturbation can collapse to PBHs
- Less fine tuning from a smaller peak in the power spectrum
- Future directions: include potential and assume inhomogeneities following a gaussian power spectrum

Thank you!!