Universal description of a Neutron Star’s Surface and it’s key global properties using machine learning

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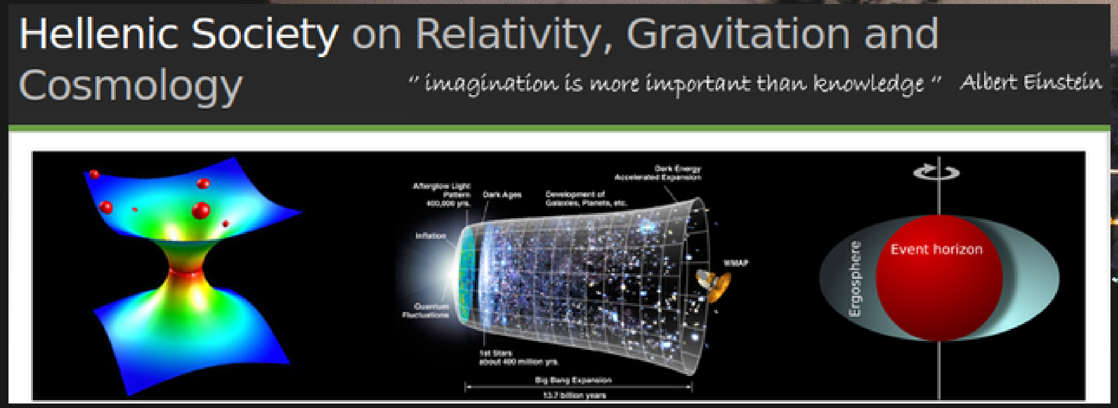

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Plan of the presentation

- Rotating Neutron Stars
- *Equation of state models*
- *RNS code-Numerical setup*

Universal relations

- *Analytical fits*
- *ANN models*

Main target: To describe some of the star's key surface properties in a way that does not depend on the EoS (universal description)

Rotating Neutron Star & EoS models

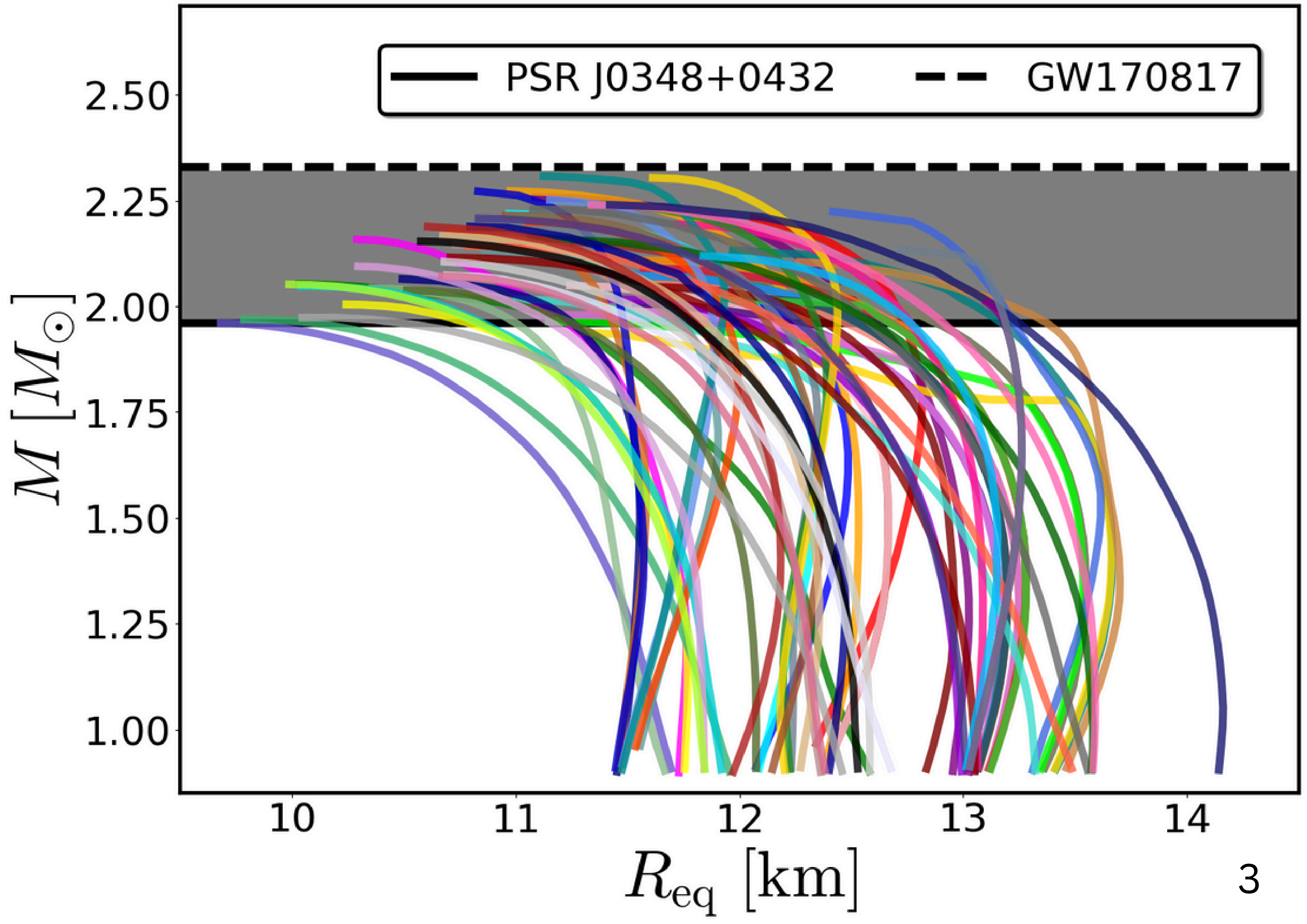
- *A rotating compact object is characterized by its mass M and its angular momentum J.*
- *Stationary and axisymmetric spacetime in equilibrium*:

$$ds^2 = -e^{(\gamma+\rho)}dt^2 + e^{(\gamma-\rho)}r^2 \sin^2\theta(d\phi - \omega dt)^2 + e^{2a}(dr^2 + r^2 d\theta^2)$$

- *Interior: perfect fluid matter in hydrostationary equilibrium*
- *Barotropic EoS ϵ = ϵ(P) that correlates the thermodynamic variables ϵ(r) and P(r)*
- *We have used 70 tabulated EoSs of cold, dense matter from **comPOSE** database*

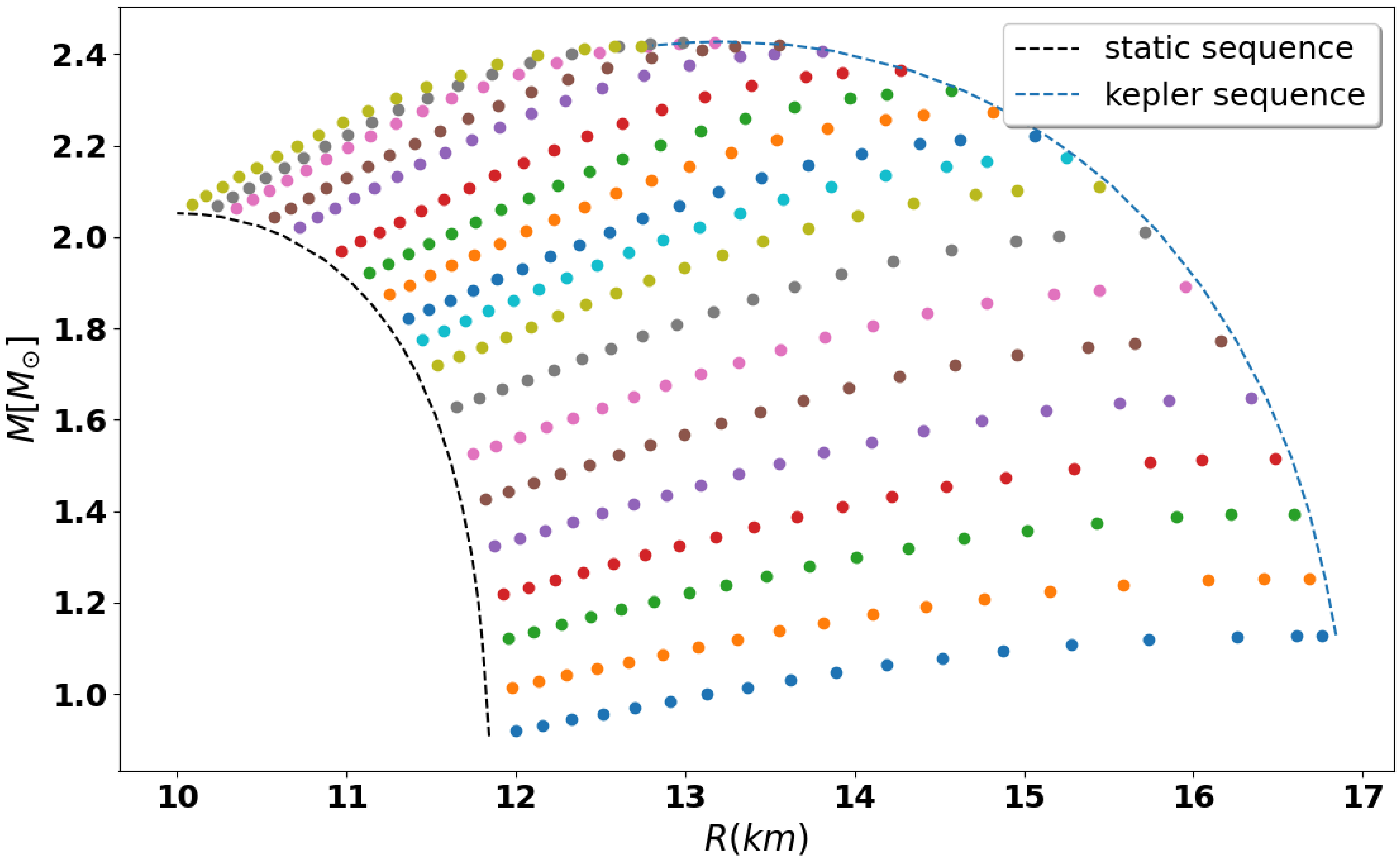
Hadronic ([n, p, e⁻, μ⁻]), Hyperonic (n, p, e⁻, H) and Hybrid: Quark+Hadron+H (n, p, e⁻,H, q) models

- *Constraints based on physical acceptability conditions*
- *Multimessenger constraints*



RNS code - numerical setup

- *RNS code: Is used to construct NS equilibrium model sequences* https://github.com/cgca/rns
- *Indicative stellar model-sequences representation for EoS SLy4*

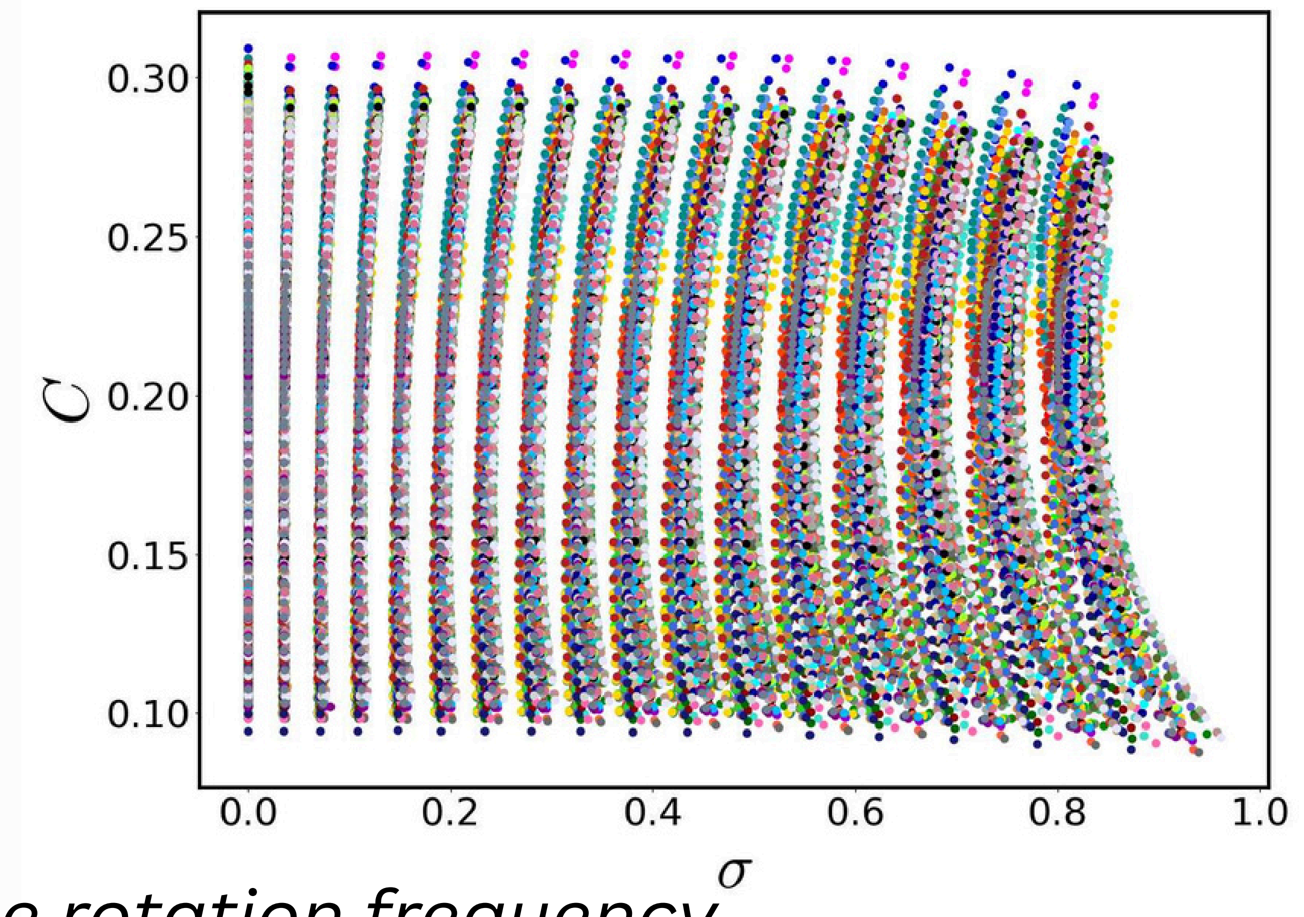


- *Extended ensemble of 42694 NS models static and rotating with ϵ_c $(3.928 \times 10^{14} - 3.029 \times 10^{15})$ gr $/cm^3$ and masses starting from $0.9M_\odot$ and up to the star's M_{max} .*

- *Uniformly rotating NSs: Ω = const, $f \in [0.190, 1.871]$ kHz*

- *Stellar parameters:*

$$C = M/R_{eq}\,, \quad \sigma = \Omega^2 R^3_{eq}/GM$$



Locating the star's surface

- *The oblate shape of the star depends on the EoS and the rotation frequency*
- *Enthalpy method: numerical solution of the* ***H(P) = 0*** *→ star's surface $R(\mu)$* **, $\mu \in [0,1]$**
- *Additional parameters: $R_* = R_{pole}/R_{eq}$, eccentricity $e = (1-R_*^2)^{1/2}$*
- *Deviation from sphericity of the star's surface*

$$\frac{d\log R(\mu)}{d\theta} = -(1-\mu^2)^{1/2}\frac{1}{R(\mu)}\frac{dR(\mu)}{d\mu}$$

$$\left[\frac{d\log R(\mu)}{d\theta}\right]_{\mu=0} = \left[\frac{d\log R(\mu)}{d\theta}\right]_{\mu=1} = 0$$

crucial role *in computing the beaming angle, α_e, for a photon emitted at the surface of the NS*

Effective gravity at surface

- *3-velocity of a fluid element as measured by a ZAMO:* $$V = (\Omega - \omega) r \sin(\theta) e^{-\rho}$$

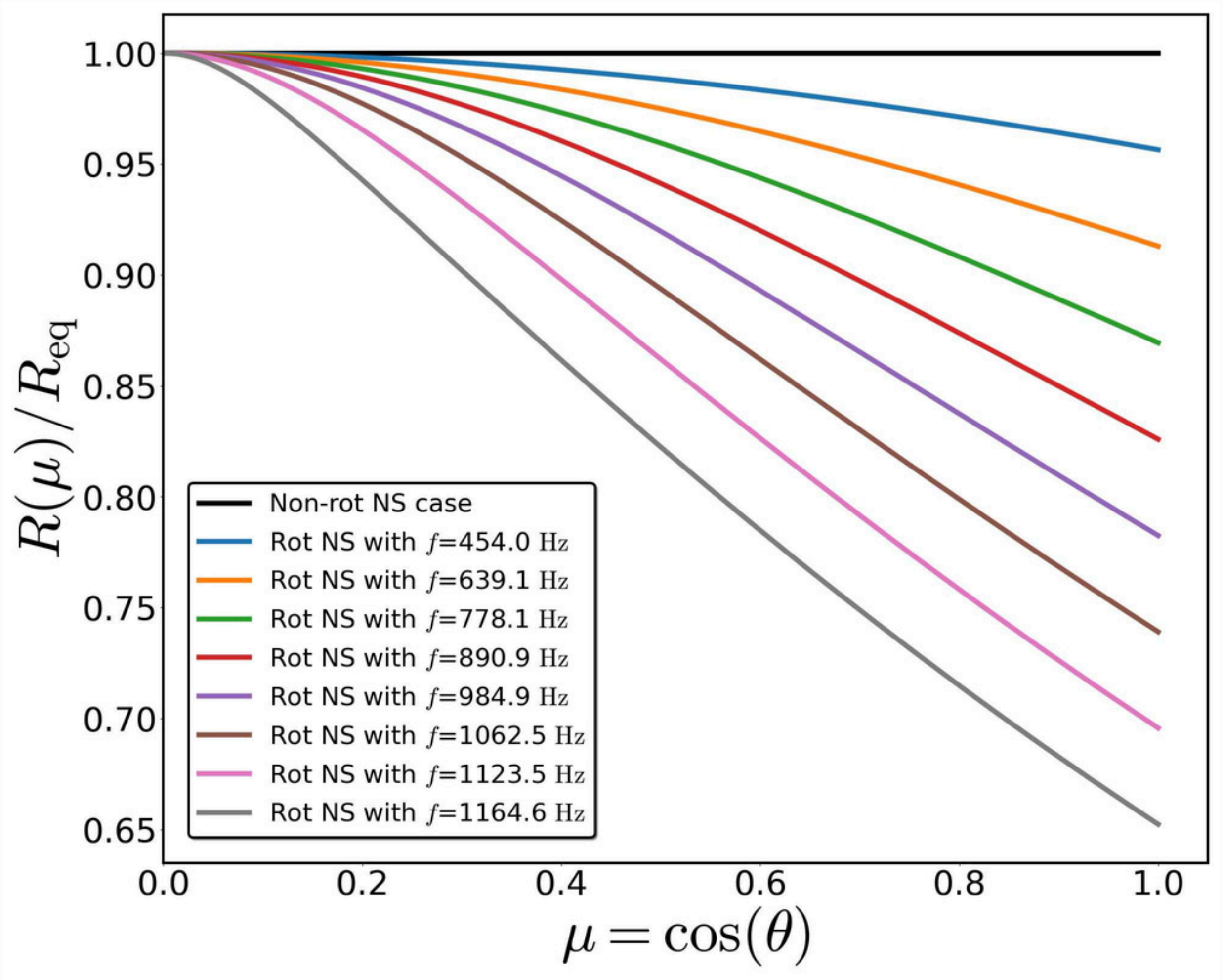
- *For the utilized metric:* $$a_\alpha = \frac{1}{2}\frac{\partial(\rho + \gamma)}{\partial x^\alpha} - \frac{V}{1 - V^2}\frac{\partial V}{\partial x^\alpha}.$$

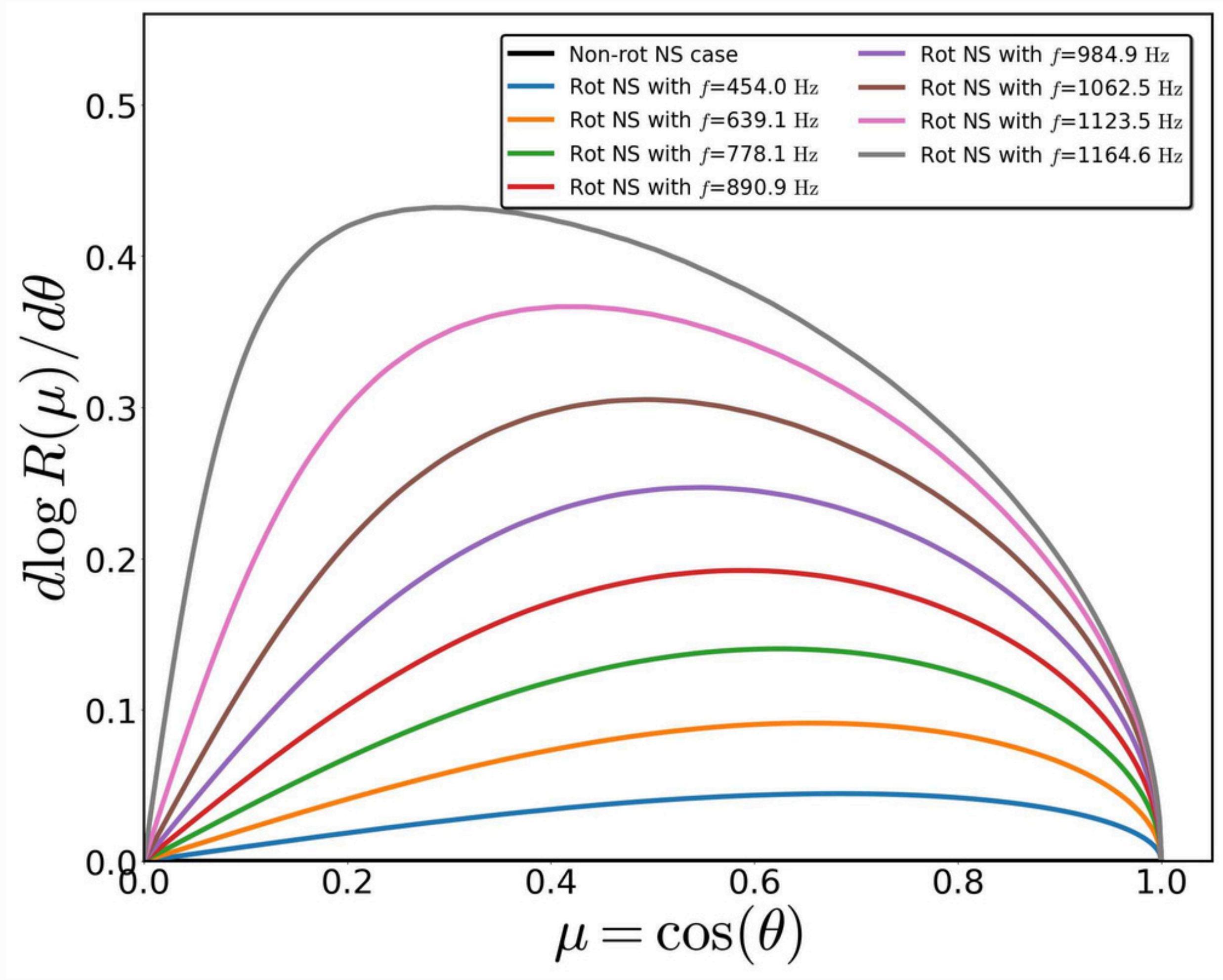
- *Effective acceleration due to gravity:* $$g = a = \left(g^{\alpha\beta} a_\alpha a_\beta\right)^{1/2} \longrightarrow g = e^{-a}\left(\alpha_r^2 + \left(\frac{\alpha_\theta}{r}\right)^2\right)^{1/2}$$

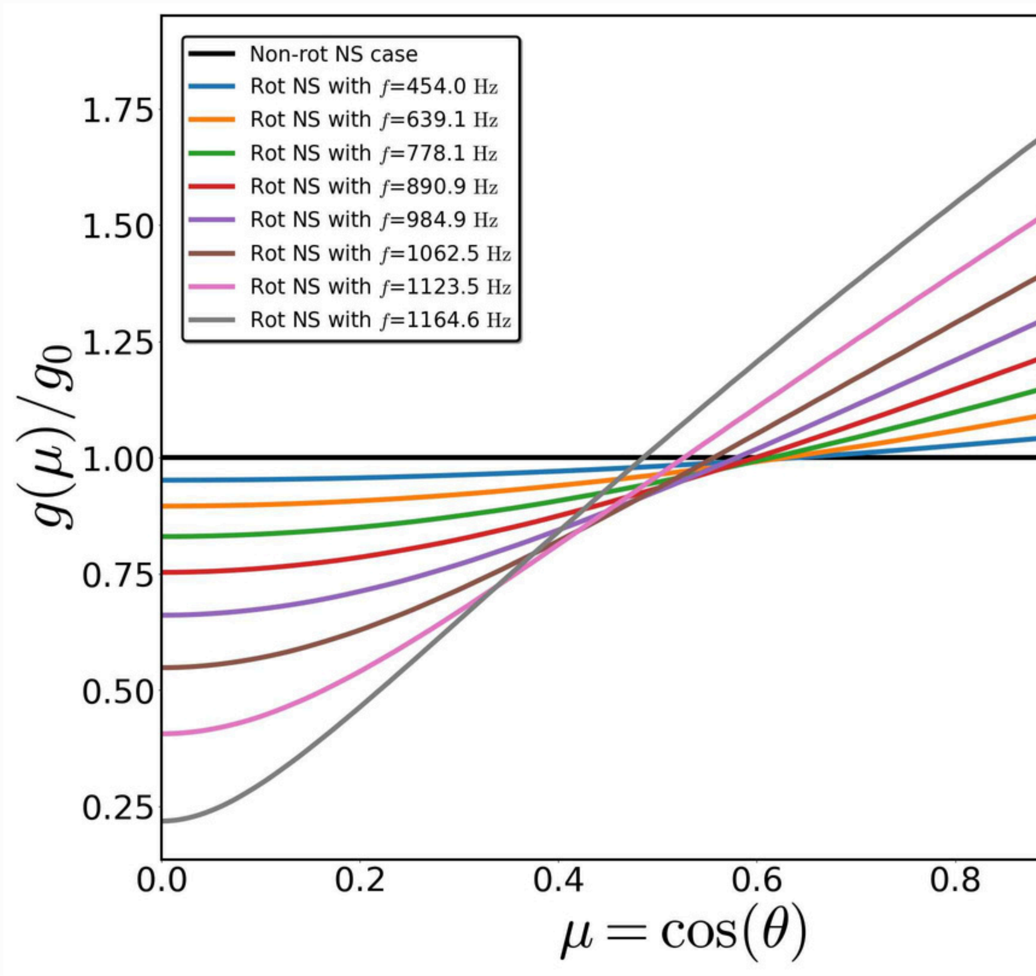
- *scaling factor:* $$g_0 = \frac{M}{R^2}\left(1 - \frac{2M}{R}\right)^{-1/2}$$

Indicative NS models and their properties

Model	$M\ (M_\odot)$	$R_{\rm eq}$ (km)	$R_{\rm pole}$ (km)	$C\ [-]$	$\tilde{r}_{\rm pole}/\tilde{r}_{\rm eq}$	f (Hz)	$\sigma\ [-]$	$e\ [-]$	$g_{\rm eq}/g_0$	$g_{\rm pole}/g_0$
1	1.404	11.688	11.688	0.1773	1.000	0.0	0.000	0.000	1.000	1.000
2	1.439	11.963	11.442	0.1774	0.950	454.0	0.073	0.292	0.952	1.065
3	1.476	12.269	11.201	0.1775	0.900	639.1	0.152	0.408	0.896	1.136
4	1.518	12.613	10.965	0.1775	0.850	778.1	0.238	0.494	0.830	1.215
5	1.563	13.002	10.738	0.1773	0.800	890.9	0.332	0.564	0.753	1.303
6	1.612	13.449	10.524	0.1768	0.750	984.9	0.436	0.623	0.661	1.405
7	1.663	13.973	10.327	0.1756	0.700	1062.5	0.551	0.674	0.548	1.525
8	1.713	14.601	10.158	0.1731	0.650	1123.5	0.683	0.718	0.406	1.672
9	1.753	15.386	10.036	0.1681	0.600	1164.6	0.839	0.758	0.218	1.868







Surface Universal Relations I: Analytical fits

Universal relations for the global properties of the star's surfce

- *Description of parameters in a way that does not depend on the internal structure ⟶ EoS*

% Fract Difference ⟶ Better than:

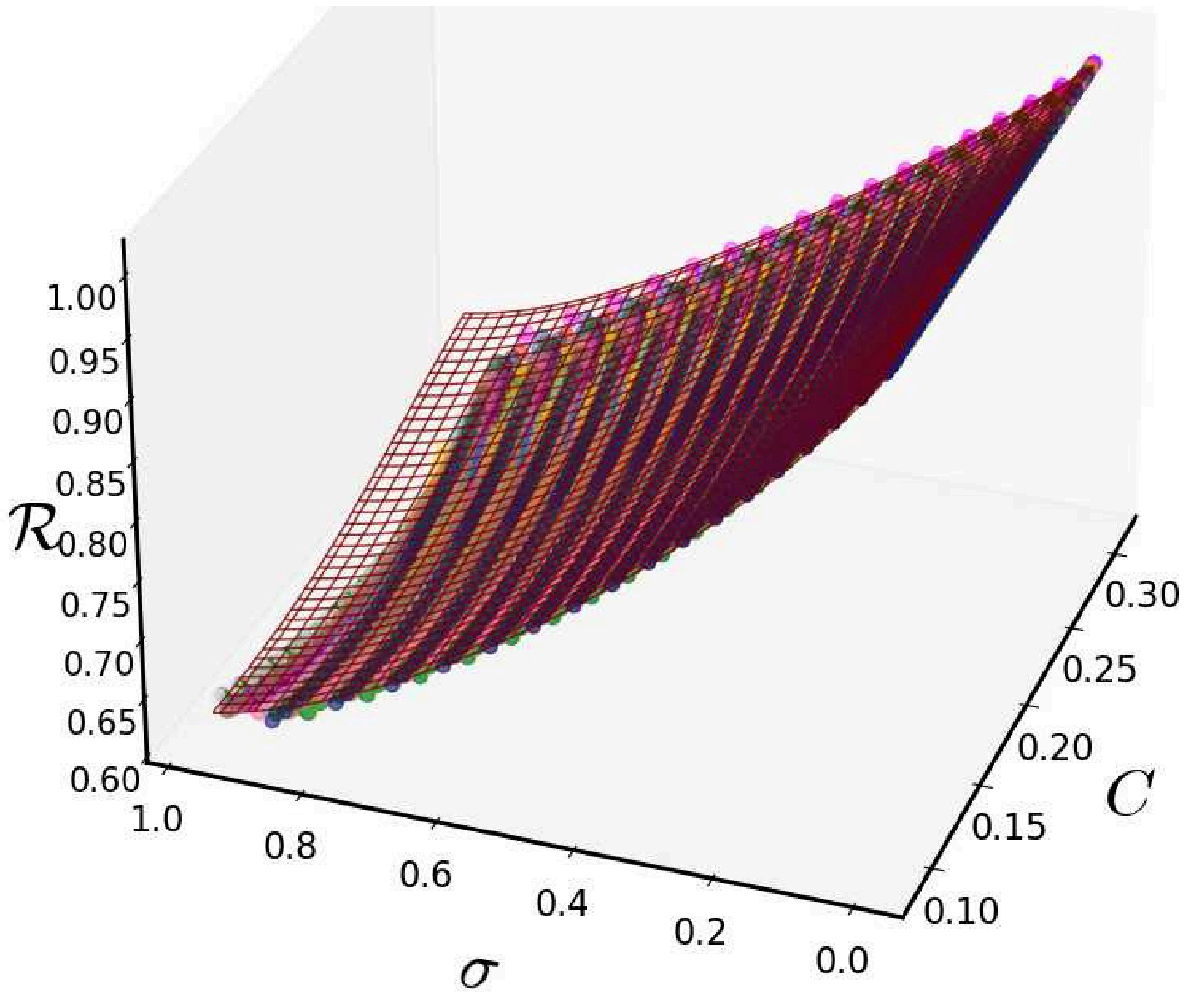
$$\mathcal{R}(C,\sigma) = \sum_{n=0}^{4}\sum_{m=0}^{4-n} \hat{\mathcal{A}}_{nm} C^n \sigma^m$$ ~ 2.79 %

$$e(C,\sigma) = \sum_{n=0}^{5}\sum_{m=0}^{5-n} \hat{\mathcal{B}}_{nm} C^n \sigma^m$$ ~ 4.57 %

$$\left(\frac{d\log R(\mu)}{d\theta}\right)_{\max} = \sum_{n=0}^{3}\sum_{m=0}^{3-n}\sum_{q=0}^{3-(n+m)} \hat{\mathcal{C}}_{nmq} C^n \sigma^m \mathcal{R}^q .$$ ~ 3.21 %

$$g_{\text{pole}}(C,\sigma) = g_0 \sum_{n=0}^{4}\sum_{m=0}^{4-n} \hat{\mathcal{D}}_{nm} C^n \sigma^m$$ ~ 3.07 %

$$g_{\text{eq}}(C,\sigma,e) = g_0 \sum_{n=0}^{3}\sum_{m=0}^{3-n}\sum_{q=0}^{3-(n+m)} \hat{\mathcal{E}}_{nmq} C^n \sigma^m e^q$$ ~ 4.26 %



Leave-One-Out validation process is applied to identify the best-fitting function used to describe the data.

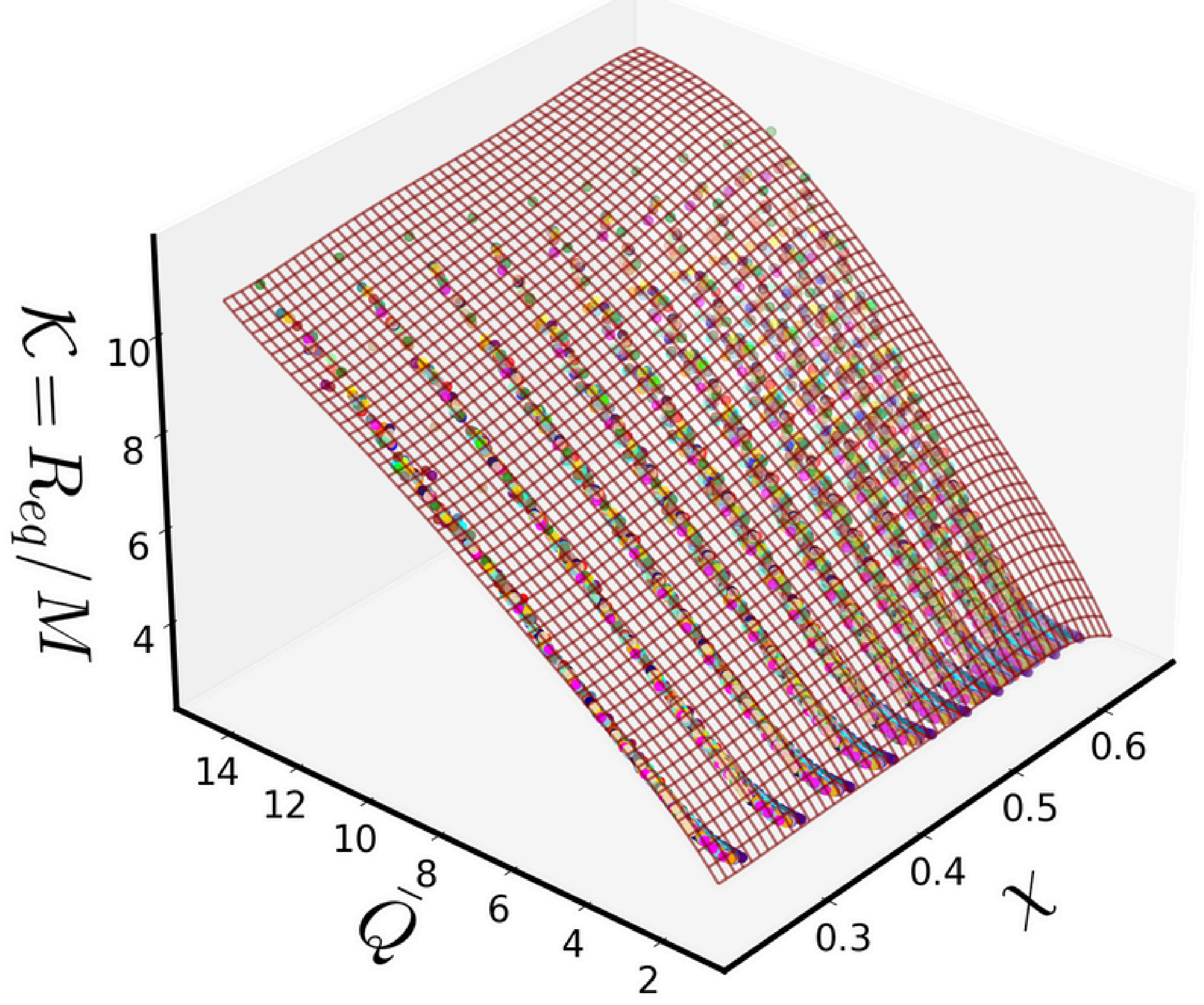
Universal relations for R_{eq}

% Fract Difference → *Better than:*

$$\frac{R_{\rm eq}}{M} = \sum_{n=0}^{5}\sum_{m=0}^{5-n} \hat{b}_{nm}\chi^n \bar{Q}^m$$

~ 6.44 %

Phys. Rev. D 107, 103050,(2023)



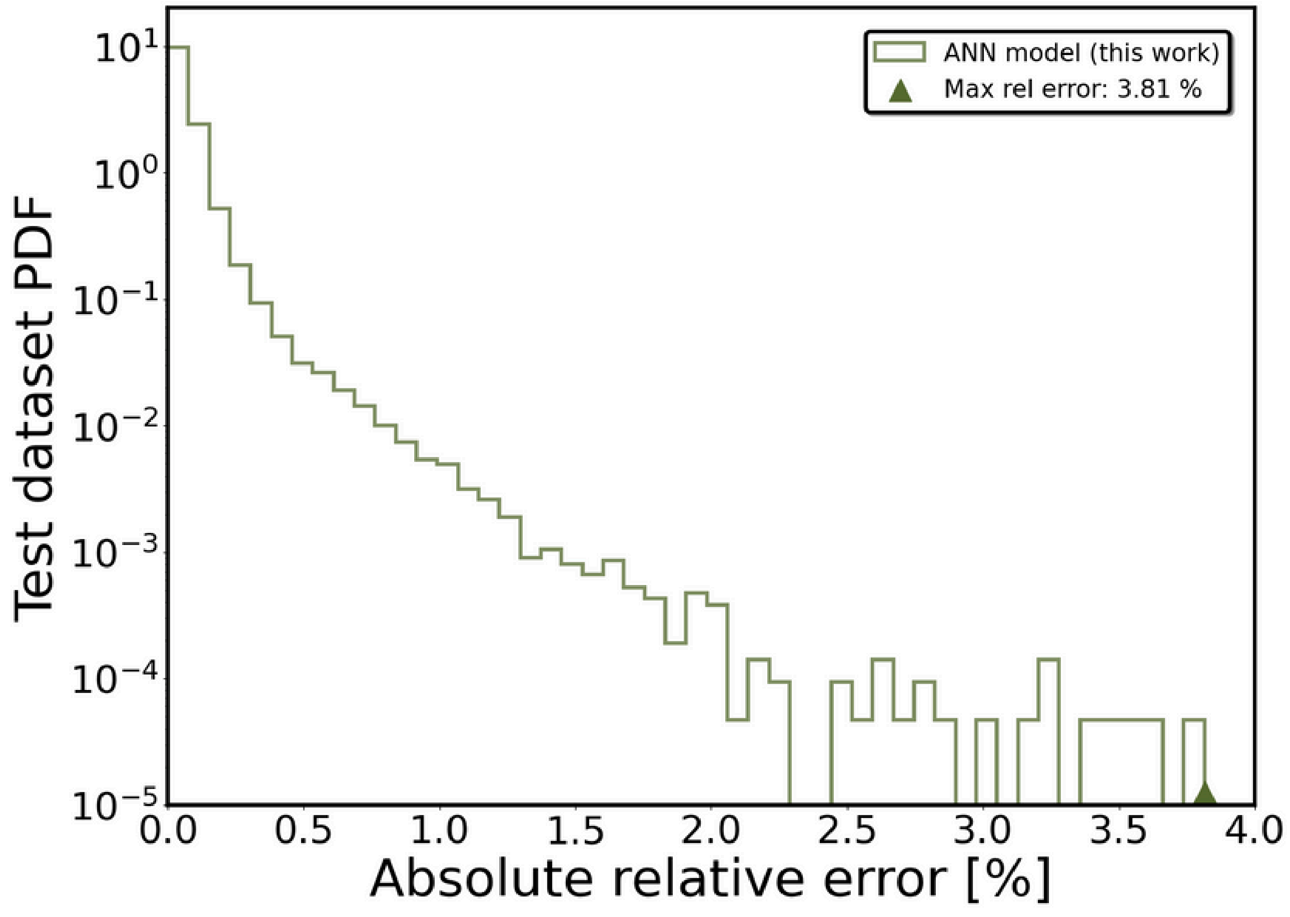
$$\bar{\mathcal{R}}_{\rm model} = \hat{\bar{\mathcal{R}}}_{\theta\star}(\bar{M}, \chi, \bar{Q}, \bar{S}_3)$$

~ 3.81 %

Only 0.15% of the test data exhibit relative deviations > 1%

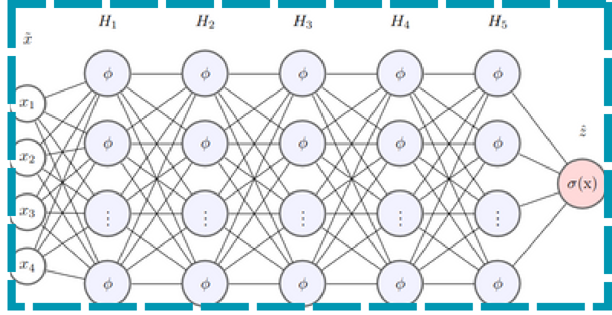
arXiv:2508.05850

submitted, under review



Surface Universal Relations II: ANN models

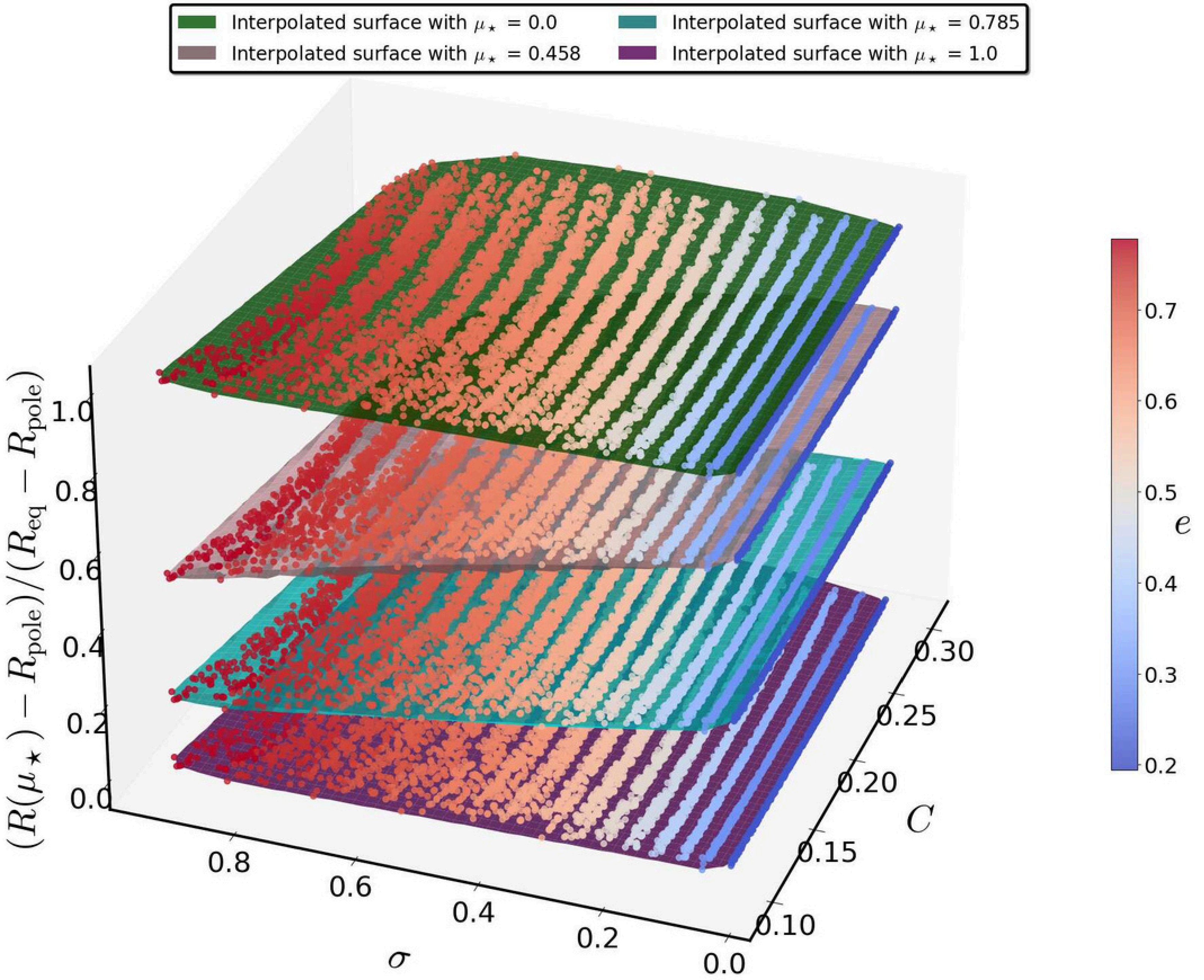
Global inference of the star's surface using an ANN



- *Along a given sequence of data points associated with the star's surface*

$$\hat{z}_1 = \begin{cases} \frac{R(\mu)-R_{\text{pole}}}{R_{\text{eq}}-R_{\text{pole}}}, & \sigma \neq 0 \\ \frac{R(\mu)}{R_{\text{eq}}}, & \sigma = 0. \end{cases}$$

- *Universal plane for each specific value of the colatitude θ*



ANN training properties

- *Goal: To accurately predict the universal hyperstructure*
- *Model: A feed forward neural network (ANN)*

Hidden layer	No. neurons	Activation function
H_1	200	$\phi = \text{LeakyReLU}(x;\beta)$
H_2	100	$\phi = \text{LeakyReLU}(x;\beta)$
H_3	50	$\phi = \text{LeakyReLU}(x;\beta)$
H_4	25	$\phi = \text{LeakyReLU}(x;\beta)$
H_5	10	$\phi = \text{LeakyReLU}(x;\beta)$

- *Model's input layer parameters:* *|μ|, C, σ, e*

min-max scaling was employed to map the values of eachinput feature within the interval [0, 1]

- *80:20 train/test ratio*

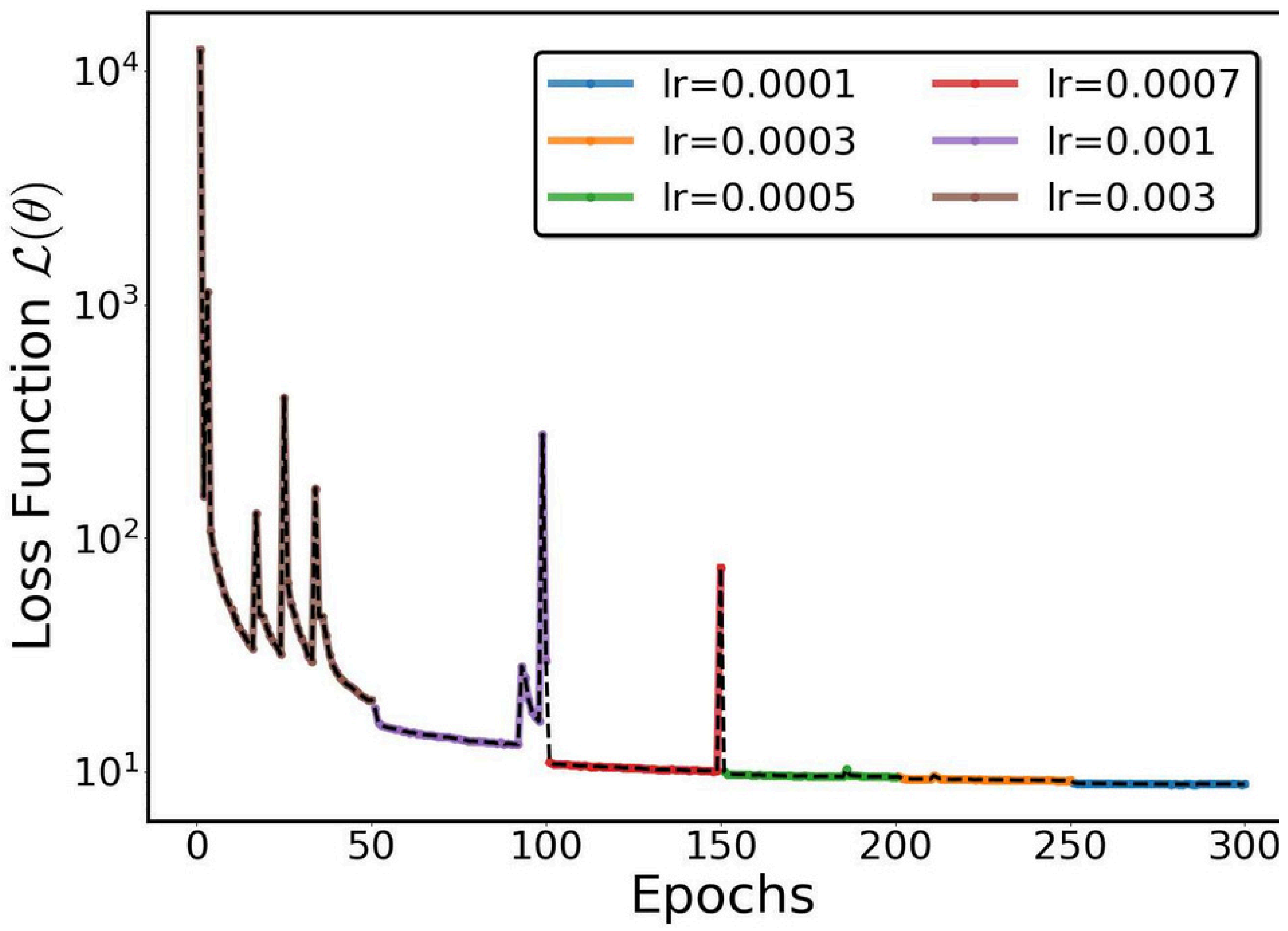
- *Final layer: Seigmoid Activation function*

Optimization process

- *Typical MSE loss as the objective function*

- *Optimizer: Adamax*

- *Effective learning rate strategy* →



Training epochs	Learning rate
1–50	$\eta_1 = 3 \times 10^{-3}$
51–100	$\eta_2 = 1 \times 10^{-3}$
101–150	$\eta_3 = 7 \times 10^{-4}$
151–200	$\eta_4 = 5 \times 10^{-4}$
201–250	$\eta_5 = 3 \times 10^{-4}$
251–300	$\eta_6 = 1 \times 10^{-4}$

- ***Proposed Surface Regression model***

$$R(\mu) = R_{\rm pole} + (R_{\rm eq} - R_{\rm pole})\hat{F}_{\theta^\star}(|\mu|, C, \sigma, e).$$

- *Two extra parameters (R_{pole},e) compared to the already established analytical methods:*

Morsink et al. fit

$$\frac{R(\mu)}{R_{eq}} = 1 + \sum_{n=0}^{2} a_{2n}(C,\sigma)P_{2n}(\mu)$$

Astrophys. J. 663, 1244, (2007)

AlGendy and Morsnik fit

$$\frac{R(\mu)}{R_{eq}} = 1 + a_2(C,\sigma)\mu^2$$

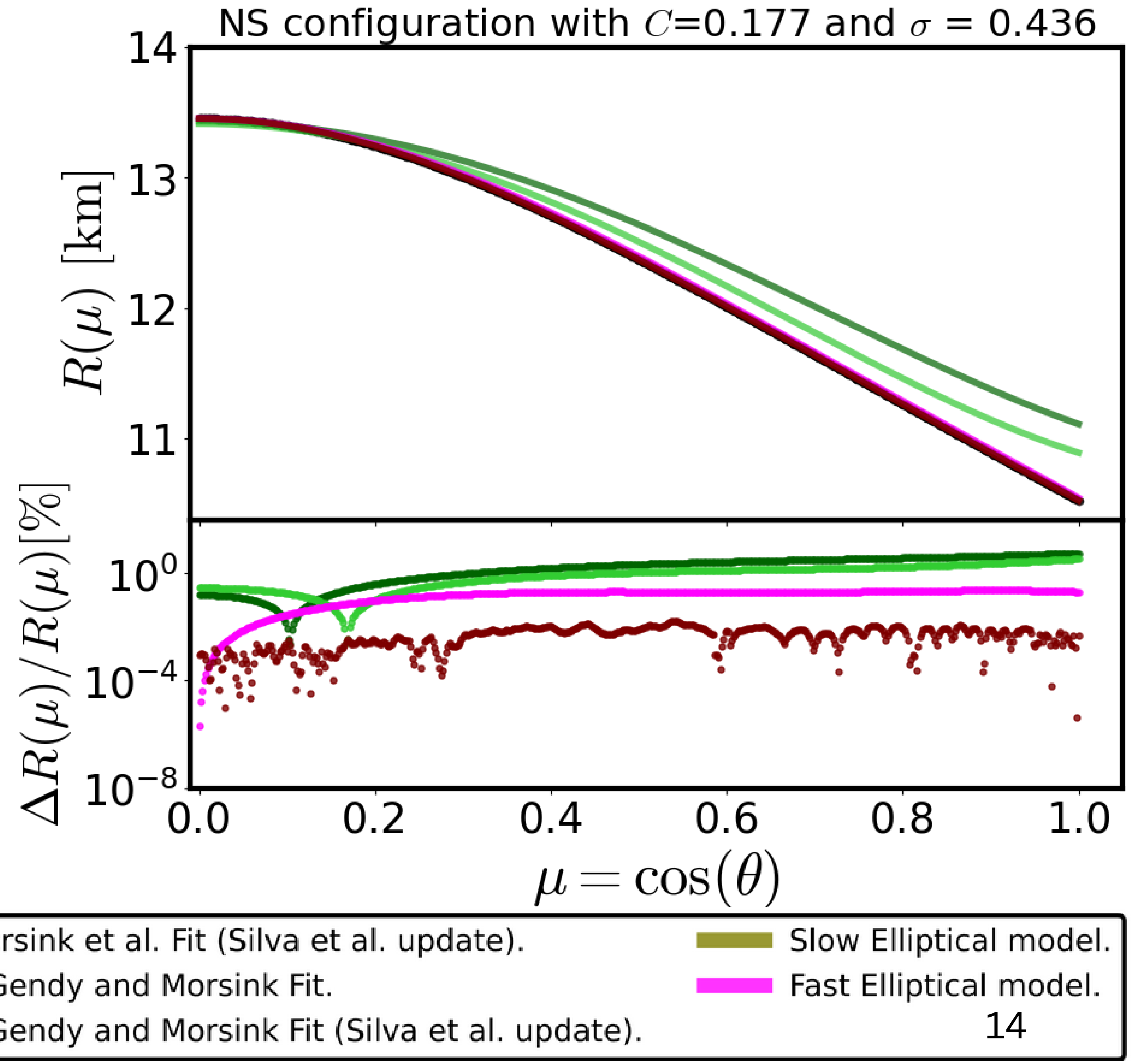
Astrophys. J. 791, 78 (2014)

Elliptical formula (Silva et al. fit)

slow (σ < 0.25) & fast (σ>0.20)

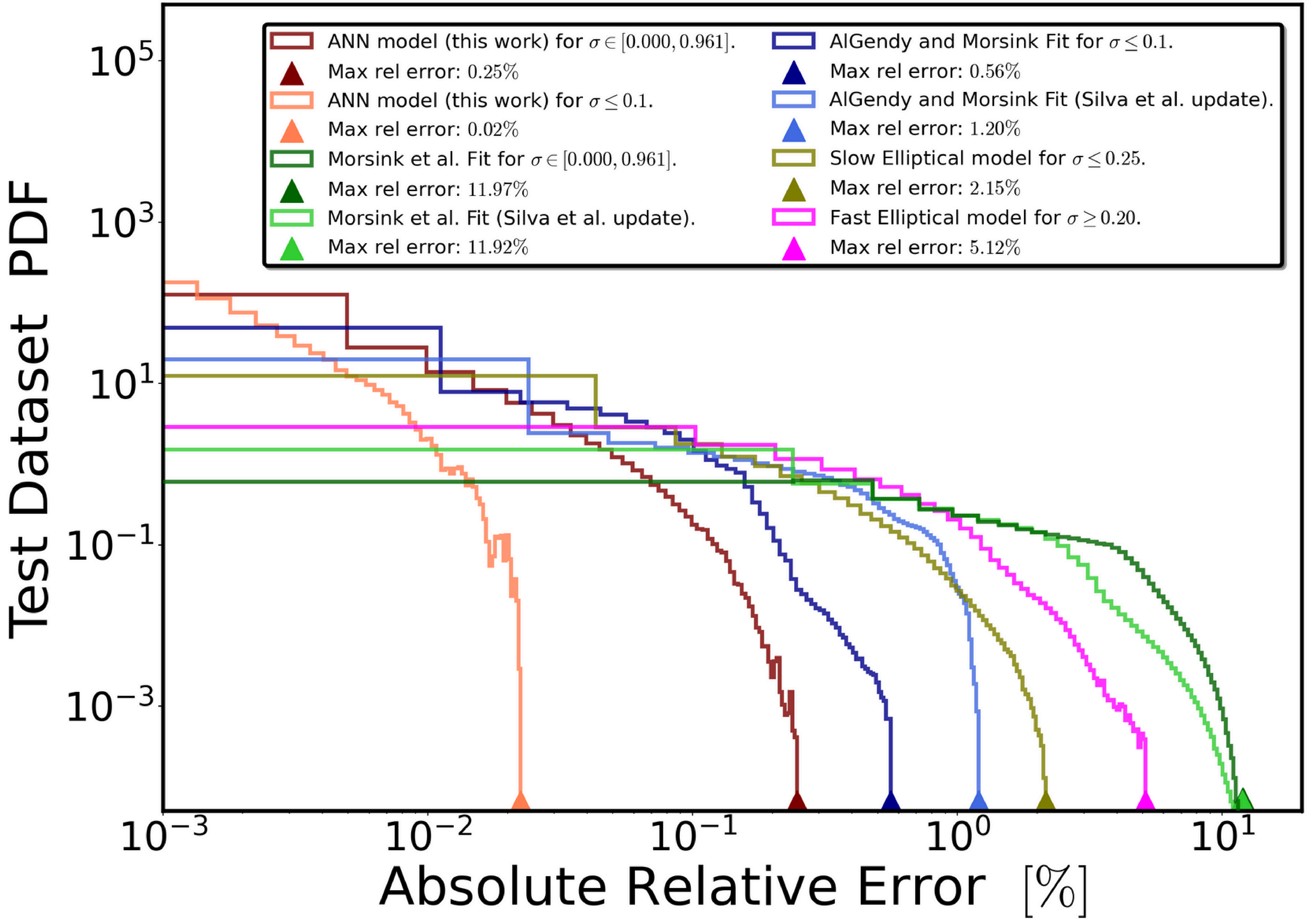
$$\frac{R(\mu)}{R_{eq}} = \sqrt{\frac{1-e^2}{1-e^2 g(\mu,C,\sigma)}}$$

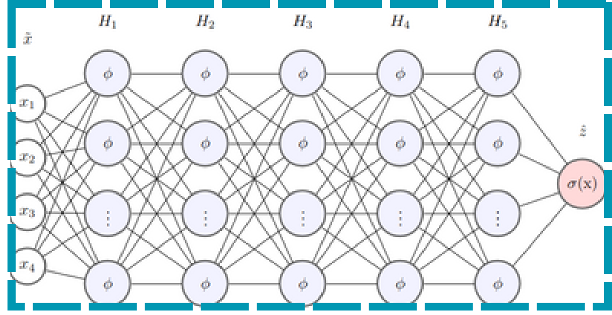
Phys. Rev. D 103, 063038 (2021)

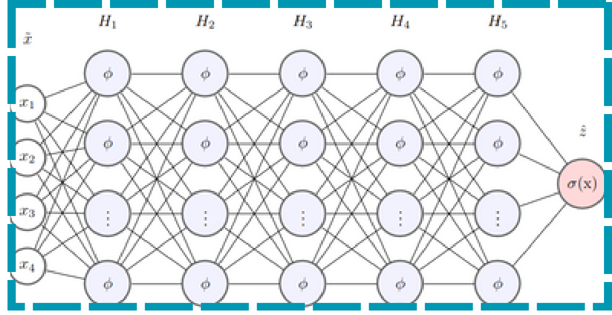


Note: Silva et al. also gave updated coefficients for the Morsink et al. and AlGendy and Morsink analytical fits

Distribution of model's relative errors







Logarithmic Derivative and Effective gravity fits

$$\left(\frac{d\log R(\mu)}{d\theta}\right) = \left(\frac{d\log R(\mu)}{d\theta}\right)_{\max} \hat{\mathcal{F}}_{\theta^\star}(\mu, C, \sigma, \mathcal{R}).$$

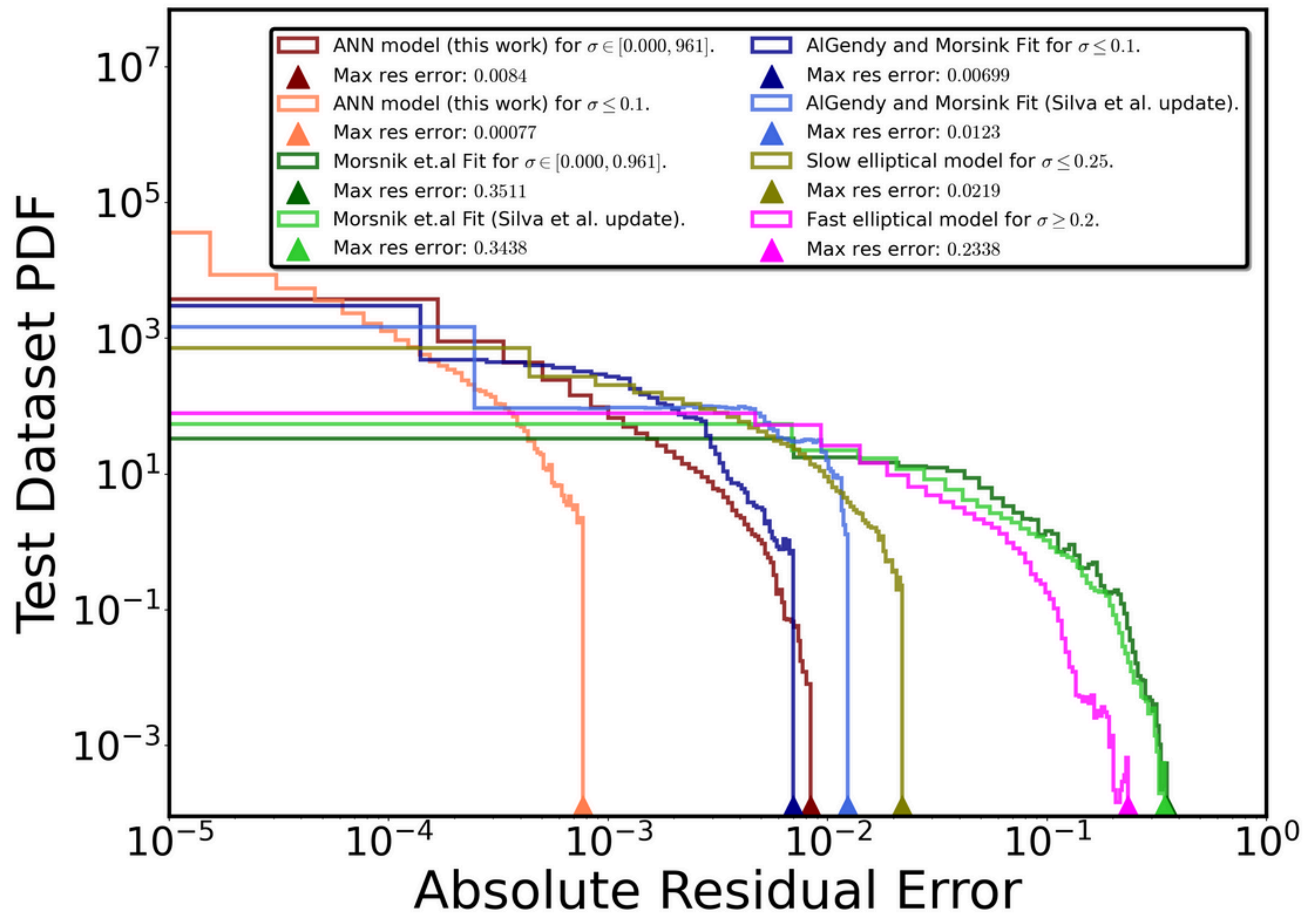
~ *data verification: better than 8.360 x 10^{-3}*

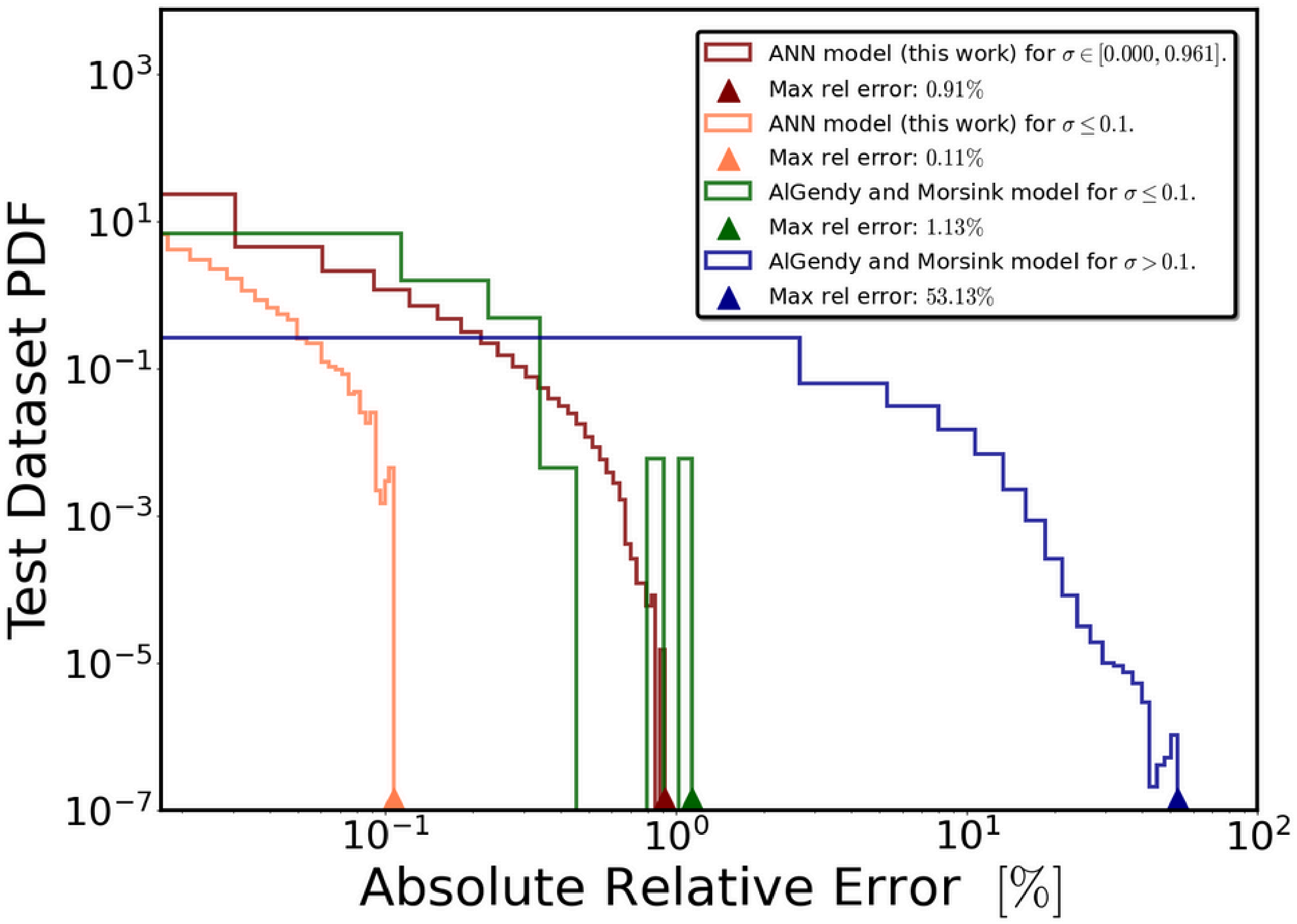
- *crucial to model X-ray pulsations that are emitted from the star's surface*

$$g(\mu) = g_{\text{pole}} + (g_{\text{eq}} - g_{\text{pole}})\hat{\mathbb{F}}_{\theta^\star}(|\mu|, C, \sigma, e).$$

~ *data verification: better than 0.91 %*

- *crucial to better model Hydrogen atmospheres* which *depend on the local effective gravity*

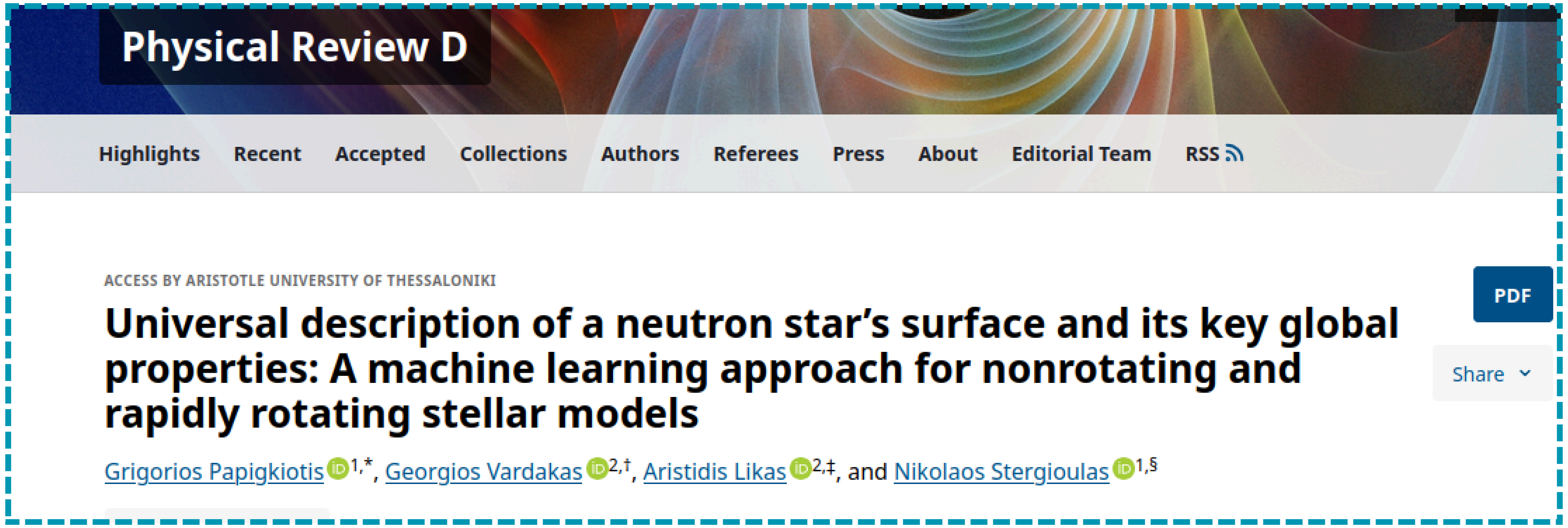




Recap: EoS-insensitive Relations suggested

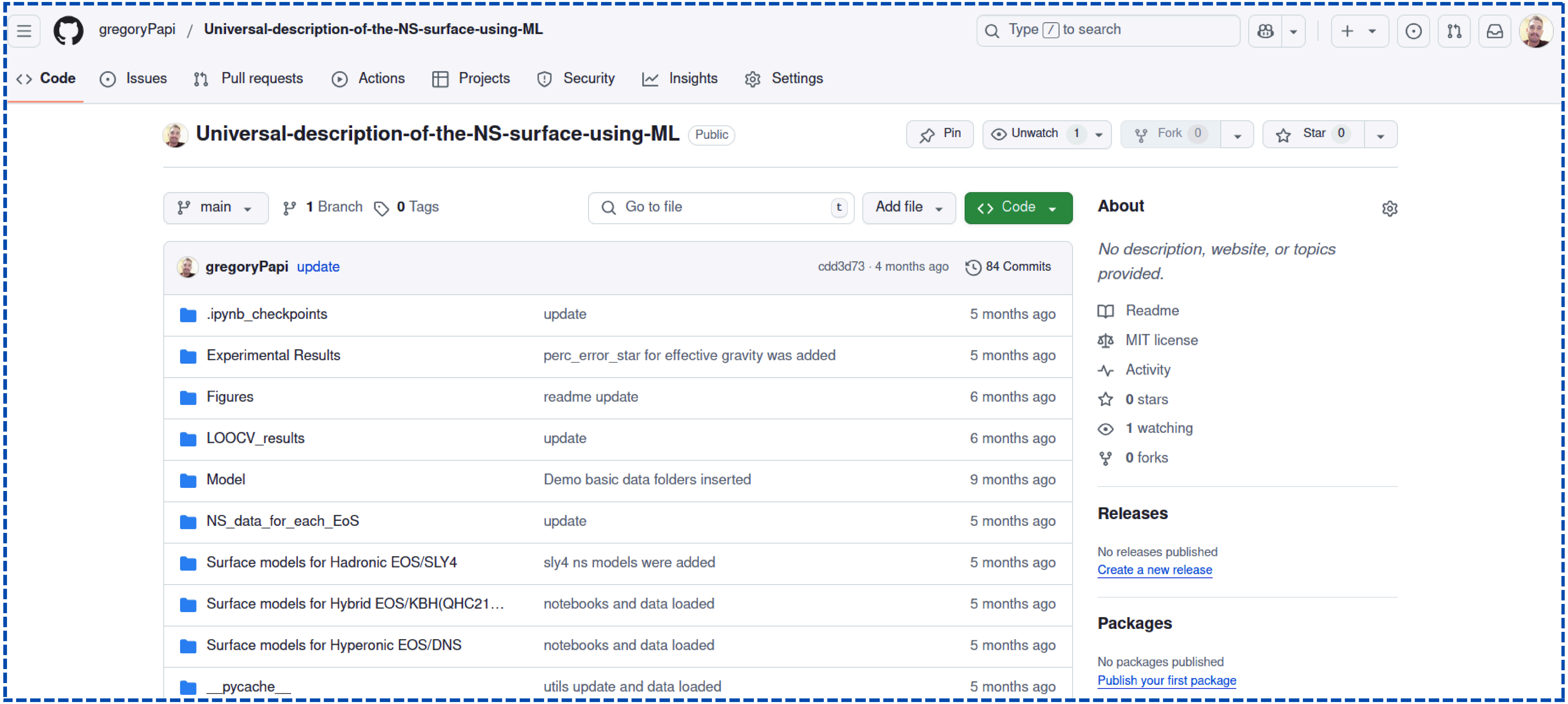
Universal relation	Parameters and their respective ranges	Equation	Max % error
$e(C,\sigma)$	$C\in[0.0876,0.3075], \sigma\in[0.0328,0.9612]$	Improved Fit Eq. (25)	4.57
$g_{\rm pole}(C,\sigma)$	$C\in[0.0876,0.3095], \sigma\in[0.0000,0.9612]$	Improved Fit Eq. (27)	3.07
$\mathcal{R}(C,\sigma)$	$C\in[0.0876,0.3095], \sigma\in[0.0000,0.9612]$	New Fit Eq. (24)	2.79
$(d\log R(\mu)/d\theta)_{\rm max}(C,\sigma,\mathcal{R})$	$C\in[0.0876,0.3075],\ \sigma\in[0.0328,0.9612],\ \mathcal{R}\in[0.626,0.981]$	New Fit Eq. (26)	3.21
$g_{\rm eq}(C,\sigma,e)$	$C\in[0.0876,0.3095],\ \sigma\in[0.0000,0.9612],\ e\in[0.000,0.780]$	New Fit Eq. (28)	4.26
$R(\mu;R_{\rm pole},R_{\rm eq},C,\sigma,e)$	$R_{\rm pole}\in[8.618,14.161]$ km, $R_{\rm eq}\in[9.683,19.413]$ km, $C\in[0.0876,0.3095],\ \sigma\in[0.0000,0.9612],\ e\in[0.000,0.780]$	New Fit Eq. (30)	0.25
$g(\mu;g_{\rm pole},g_{\rm eq},C,\sigma,e)$	$g_{\rm pole}/g_0\in[0.987,2.107], g_{\rm eq}/g_0\in[0.069,1.000]$, $C\in[0.0876,0.3095], \sigma\in[0.0000,0.9612],\ e\in[0.000,0.780]$	New Fit Eq. (35)	0.91
$(\frac{d\log R(\mu)}{d\theta})\,(\mu;(d\log R(\mu)/d\theta)_{\rm max},C,\sigma,\mathcal{R})$	$(d\log R(\mu)/d\theta)_{\rm max}\in[0.019,0.503], C\in[0.0876,0.3075]$, $\sigma\in[0.0328,0.9612], \mathcal{R}\in[0.626,0.981]$	New Fit Eq. (33)	8.36×10^{-3}

- ***For more information:*** ***DOI: https://doi.org/10.1103/PhysRevD.111.083056***



Github repository

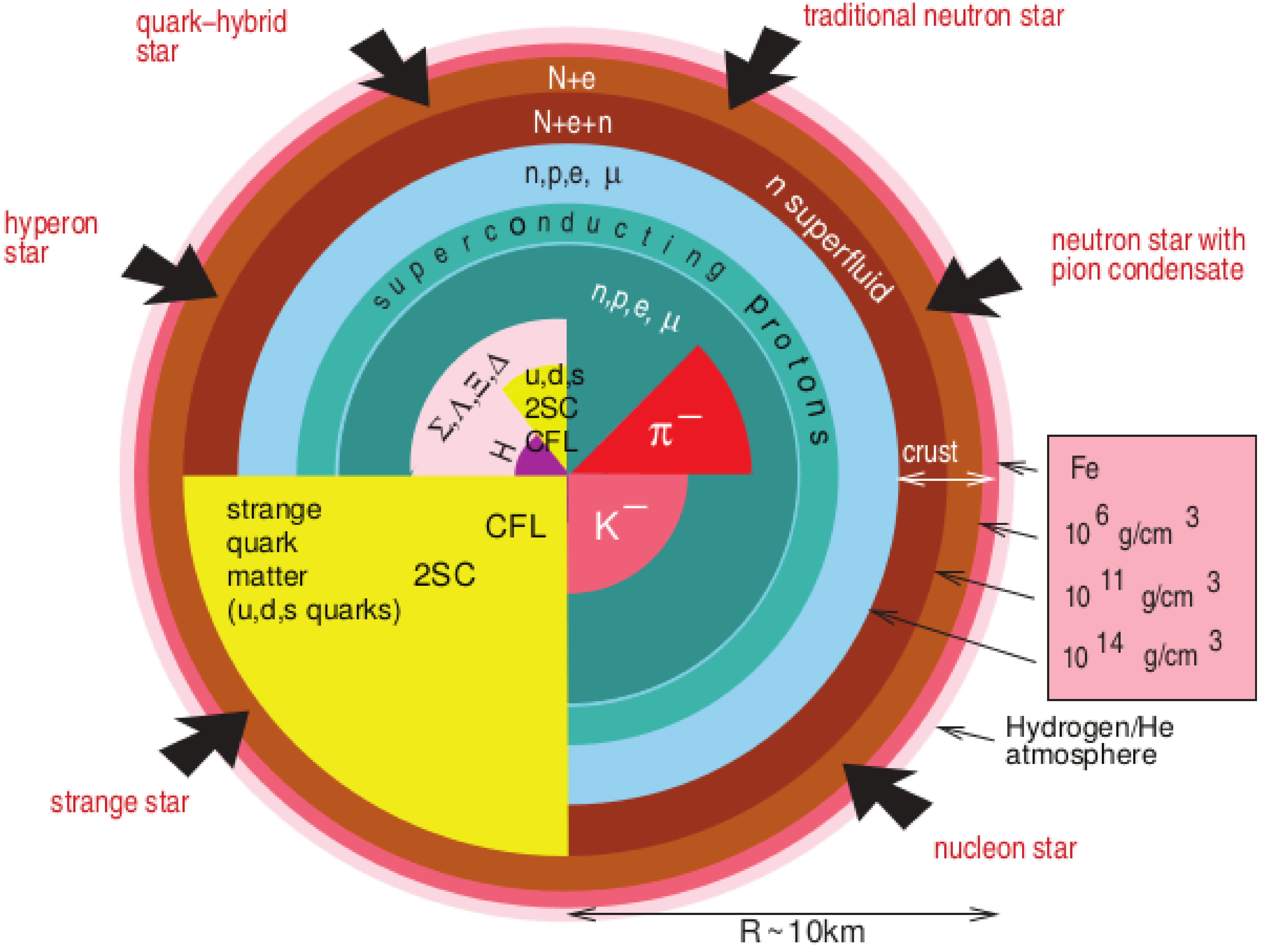
- ***github.com/gregoryPapi/Universal-description-of-the-NS-surface-using-ML***



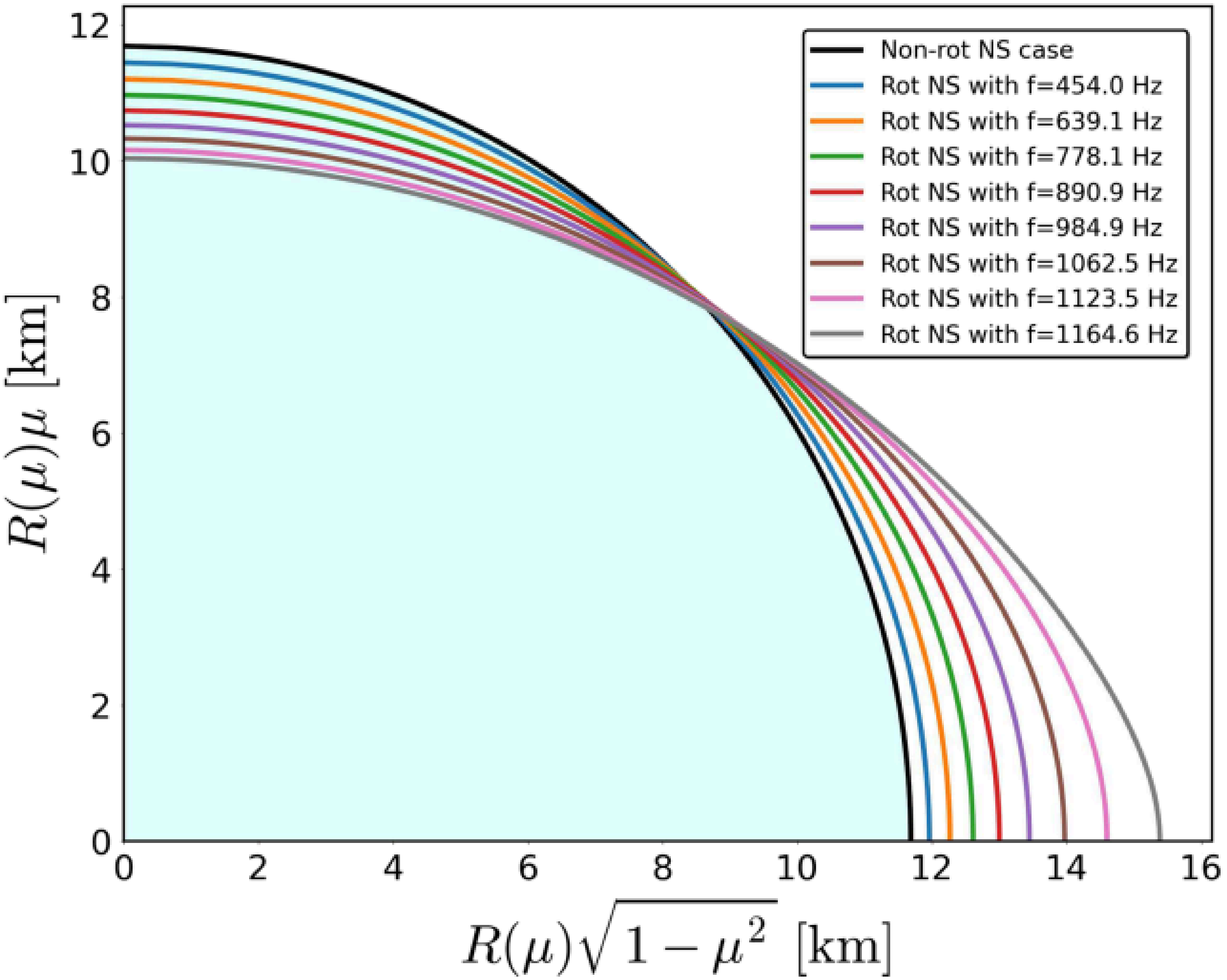

Thank you for your attention!

Backup Slides

Star's Cross Section



EoS SLy4: Benchmark models and their surface representations



Physical acceptability conditions

Each EoS satisfies physical acceptability conditions, which ensure β- equilibrium

- *first law of thermodynamics* $d\epsilon/d\rho = (\epsilon+P)/\rho$, *where* ρ *is the baryon mass density,*
- *dominant energy condition* $\epsilon c^2 > P$
- *microscopic stability* $c_s^2 = dP/d\epsilon \geq 0$ *and causality* $c_s^2 = dP/d\epsilon \leq c^2$, *which ensures that the speed of sound* c_s *in the dense matter should not exceed the speed of light*
- *Harrison-Zeldovich-Novikov stability condition* $dM/d\epsilon_c \geq 0$, *i.e., considering the* $M - \epsilon_c$ *curve, stars with* $\epsilon_c > \epsilon_c\,(M_{max})$ *have* $dM/d\epsilon_c < 0$ *and are unstable, thus not astrophysically relevant. Therefore, a NS with the maximum possible mass should have the maximum possible central energy density* ϵ_c.

Constraints based on observational (E/M signals)

- *Radio pulsar: PSR J0348+0432:* $M = 2.01 \pm 0.04\ M_\odot$

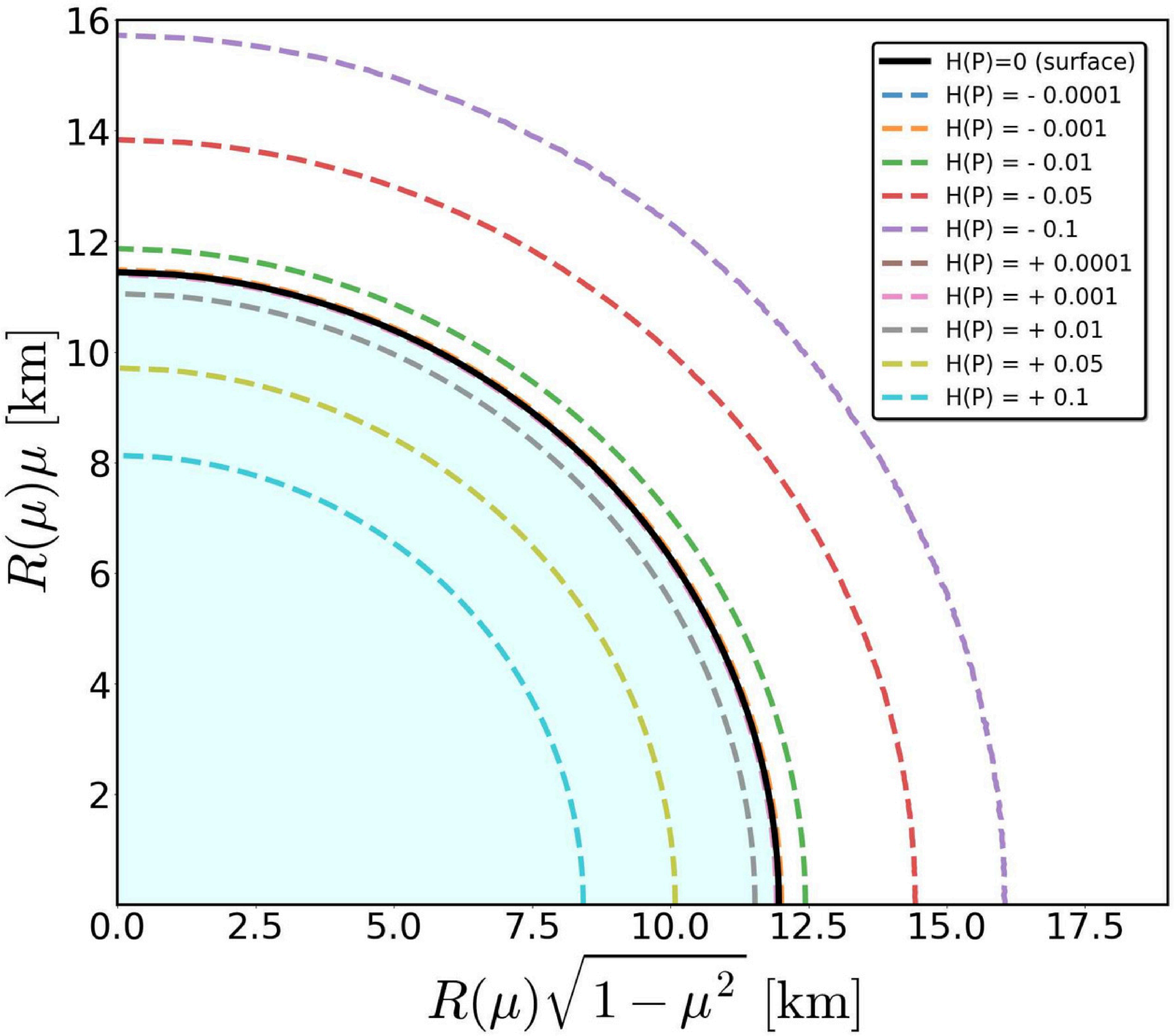
Constraints based on GWs:

⋆ *GW170817:NS-NS merger analysis:* $R_{M_{max}} \geqslant 9.60^{+0.14}_{-0.03}$ km

⋆ *GW170817:* $M_{max} = 2.32\ M_\odot$, *(2σ) bound, assuming that the final remnant was a BH.*

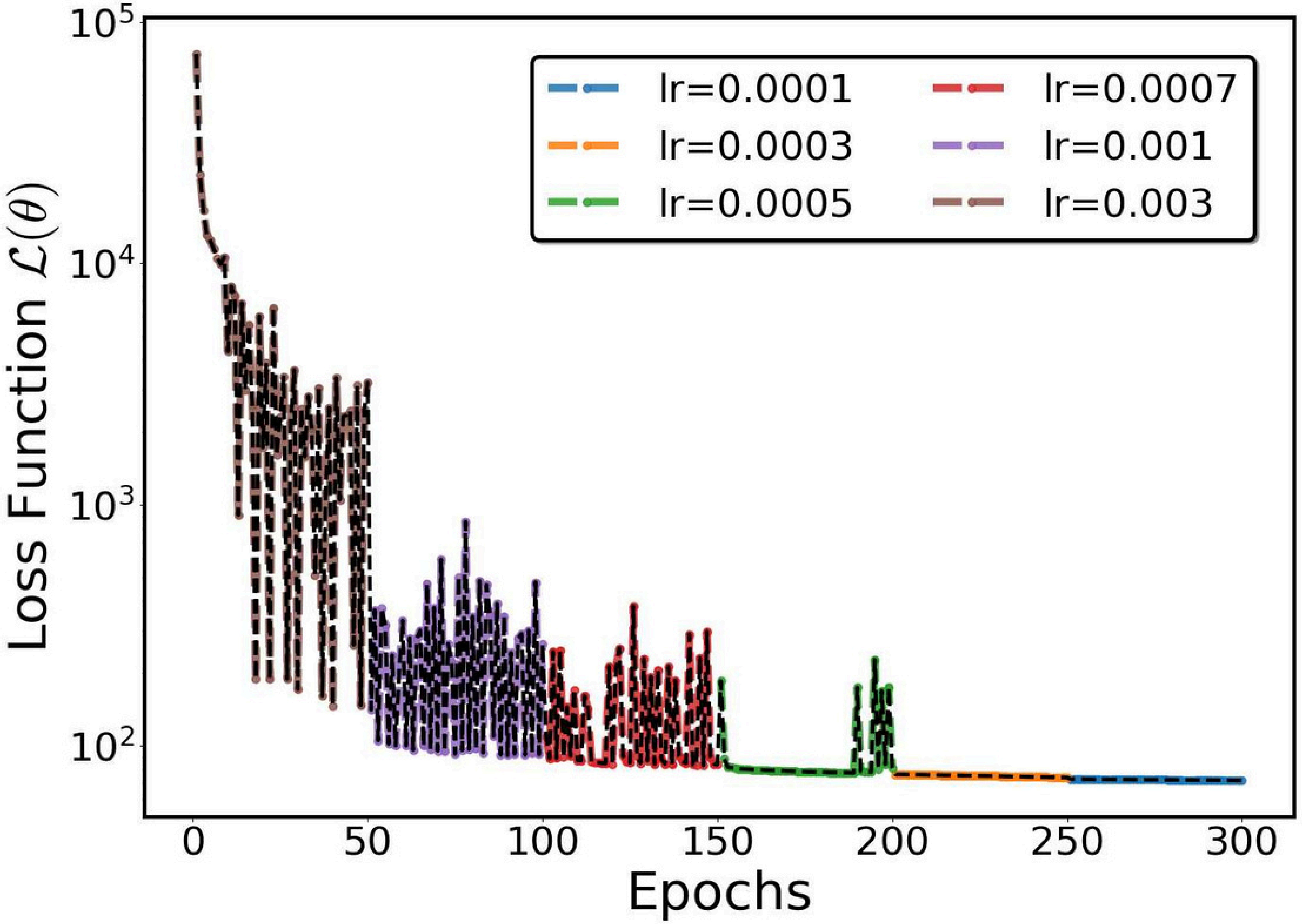
Enthalpy contours

- *Star's interior: H(p)>0*
- *Star's exterior: H(p)<0*
- *Star's surface: H(p)=0*

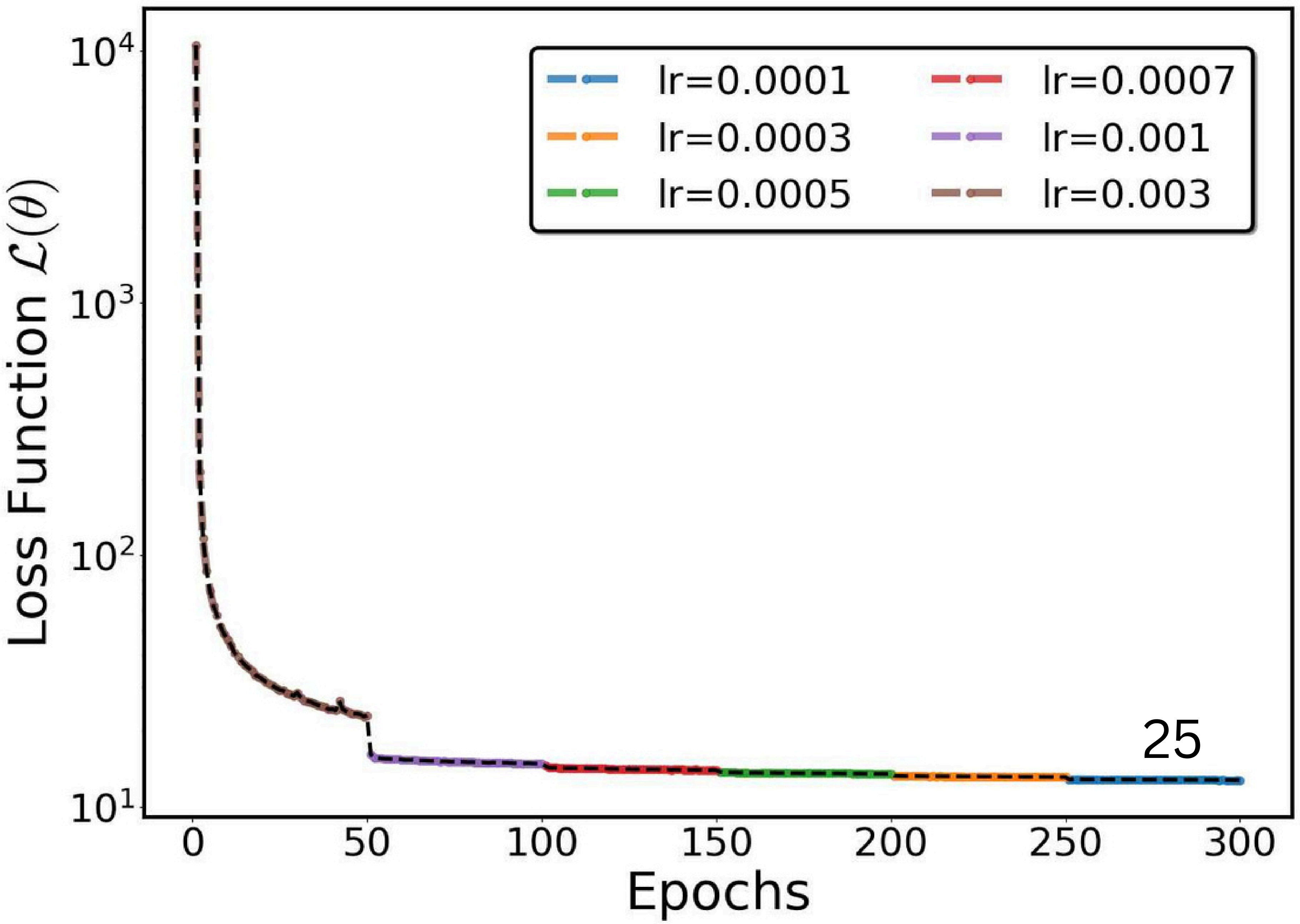


ANNs optimization process

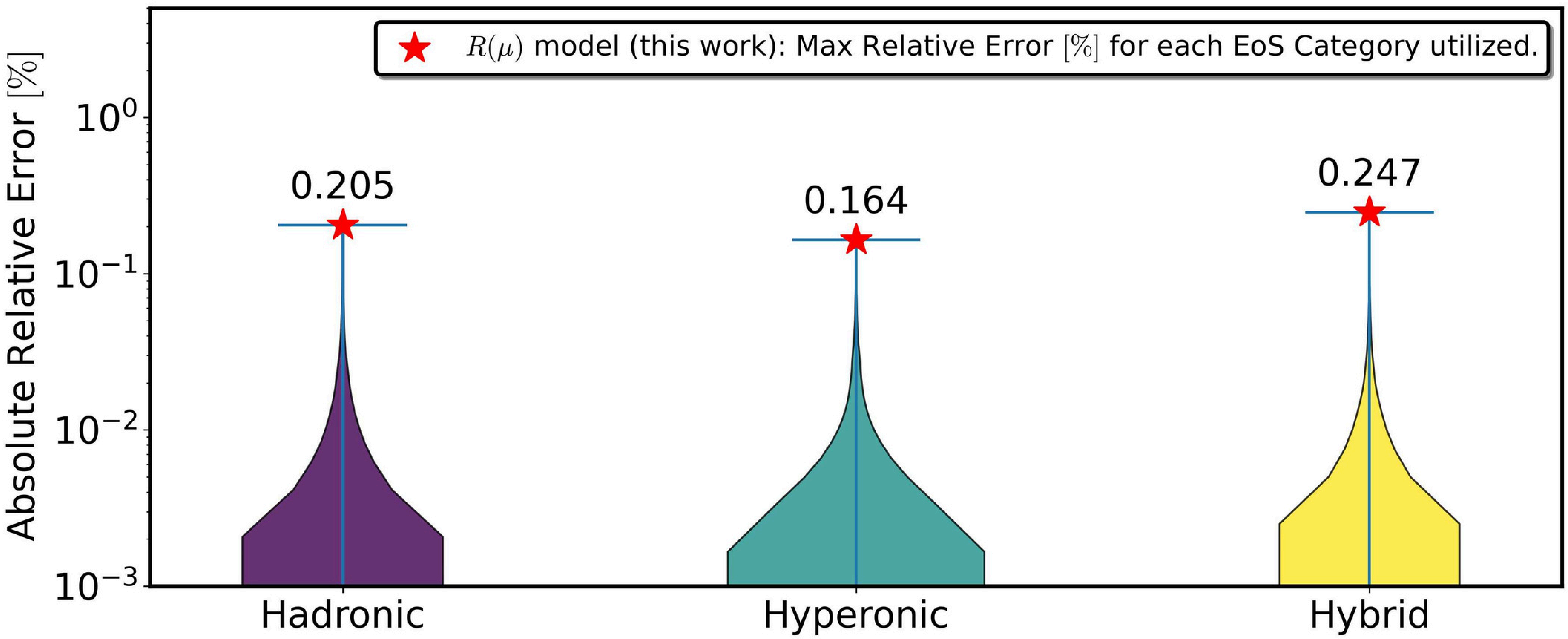
Logarithmic Derivative



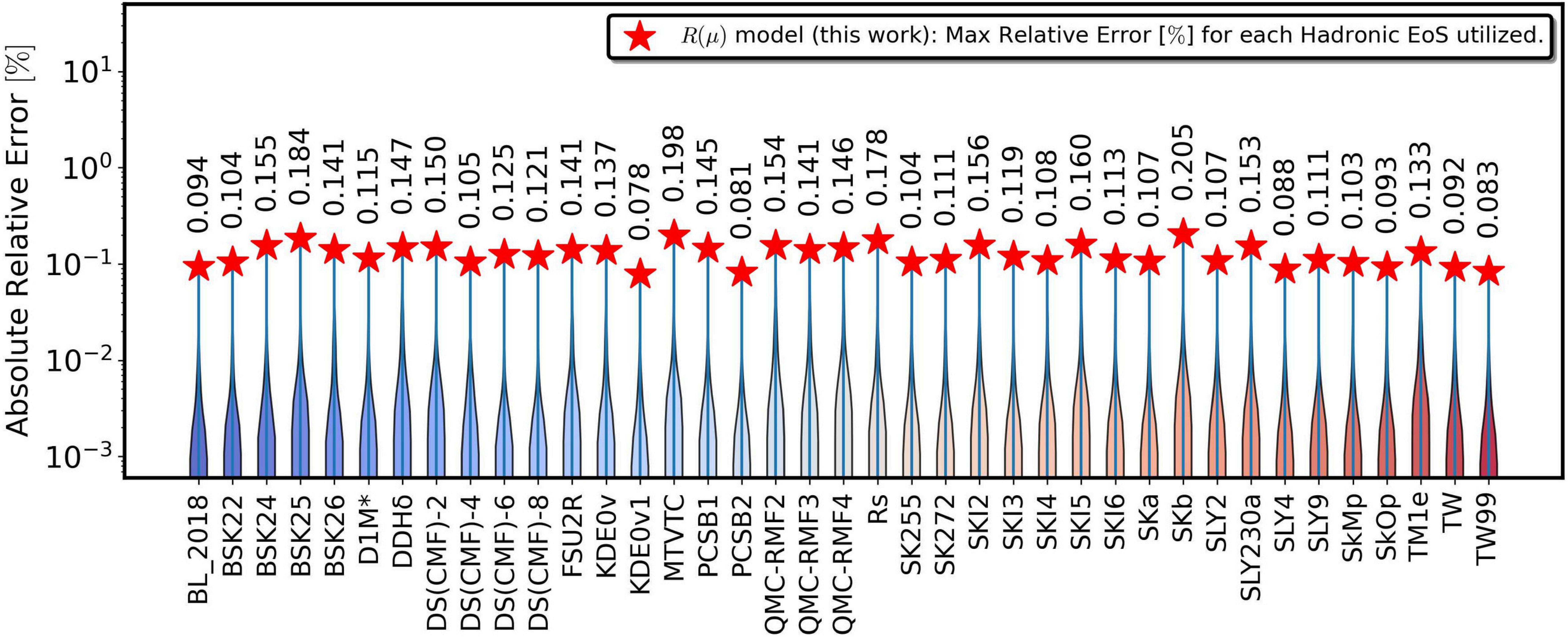
g(μ): Effective gravity



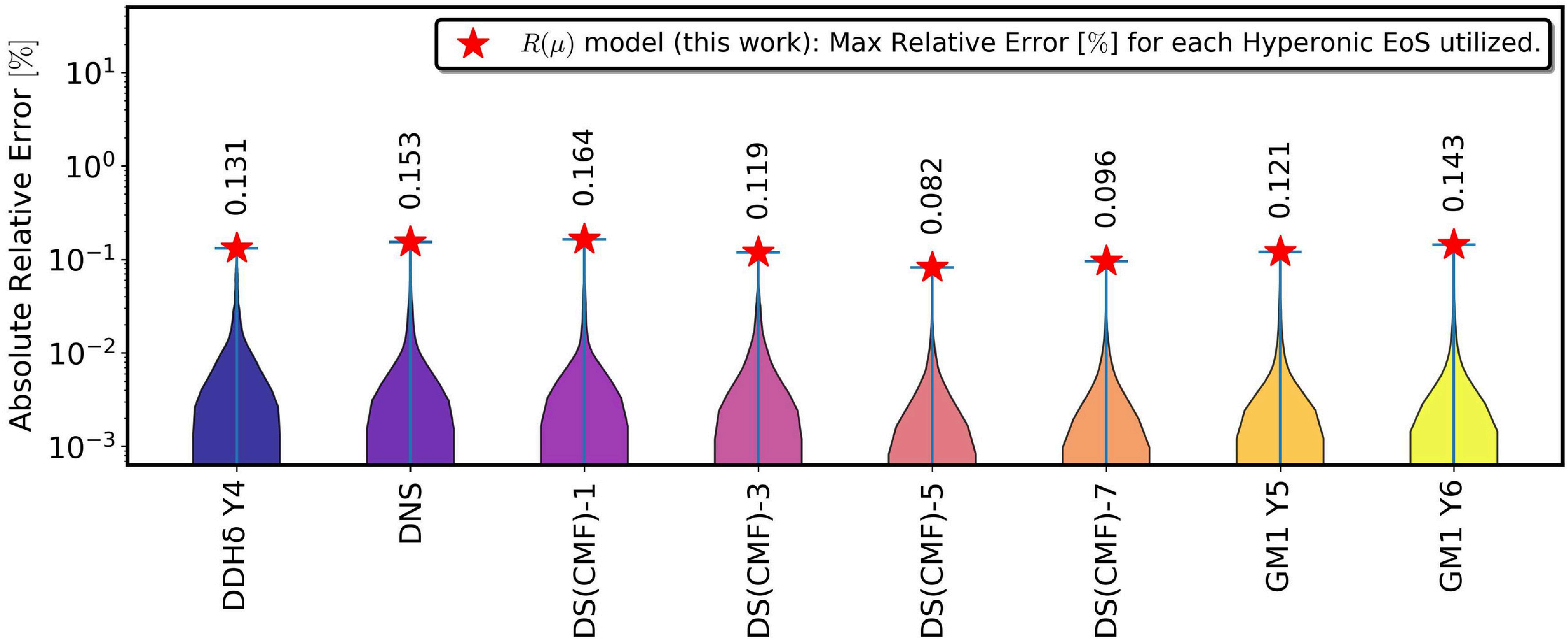
R(μ): Variance of relative errors across EoS categories



R(μ): relative errors for the hadronic EoSs



R(μ): relative errors for the hyperonic EoSs



R(μ): relative errors for the hybrid EoSs