In search of a higher-dimensional Kerr-Newman solution

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Motivation

- Broad motivation: Mathematical curiosity to explore exact solutions in higher dimensional General Relativity.
- $D = 1$ time $+ (D-1)$ space.
- Field Equations: $R_{ab} - \frac{1}{2}g_{ab}R + \Lambda g_{ab} = \kappa T_{ab}$.

Motivation

- Specific motivation:

4D	Higher dimensions
Schwarzschild, RN	Schwarzschild-Tangherlini
Kerr	Myers-Perry
Kerr-Newman	**???**

Table: Exact solutions in HD vs 4D GR.

- Exact solution of charged rotating black hole in HD **Einstein-Maxwell** theory has been elusive.
- Our Goal: carry out a systematic study to understand why the solution has been elusive.
- For the systematic study we consider **Kerr-Schild** metrics.

Overview

Kerr-Schild spacetimes

- Spacetimes whose metric can be cast as

$$\mathbf{g} = \eta - 2H\mathbf{k} \otimes \mathbf{k}$$

for some null (co)-vector field $\mathbf{k}$, scalar function H and η maximally symmetric spacetime.

Examples: Kerr and Kerr-Newman

- Kerr-Schild form of Kerr and Kerr-Newman

$$\begin{aligned}\eta =& - du^2 + 2dr(du + a\sin^2\theta d\phi) \\ &+ (r^2 + a^2\cos^2\theta)d\theta^2 + (r^2 + a^2)\sin^2\theta d\phi^2, \\ \mathbf{k} =& du + a\sin^2\theta d\phi,\end{aligned}$$

- The Kerr metric is a vacuum solution with

$$2H_{\text{Kerr}} = -\frac{2mr}{r^2 + a^2\cos^2\theta}.$$

Examples: Kerr and Kerr-Newman

- The Kerr-Newman metric is a solution to the Einstein-Maxwell equations with

$$\begin{aligned}\mathbf{A} &= -\frac{er}{r^2 + a^2\cos^2\theta}\mathbf{k},\\ 2H_{\mathsf{KN}} &= -\frac{2mr - e^2}{r^2 + a^2\cos^2\theta}\\ &= 2H_{\mathsf{Kerr}} + \frac{e^2}{r^2 + a^2\cos^2\theta}.\end{aligned}$$

- Explicitly, $\mathbf{g}_{\mathsf{KN}} = \mathbf{g}_{\mathsf{Kerr}} - \underbrace{\frac{e^2}{r^2 + a^2\cos^2\theta}\mathbf{k}\otimes\mathbf{k}}_{\text{Kerr-Schild transformation}}$.

The setup for our study

- Einstein-Maxwell theory (with arbitrary Λ) in spacetime dimensions $D \geq 4$:

1. $$\mathbf{g}_{\text{Charged}} = \underbrace{\mathbf{g}_{\text{Vacuum}}}_{\text{arbitrary Einstein-KS}} - \overbrace{2\mathcal{H}\mathbf{k} \otimes \mathbf{k}}^{\text{Kerr-Schild transformation}}.$$
2. $\mathbf{A} = \alpha \mathbf{k}$.

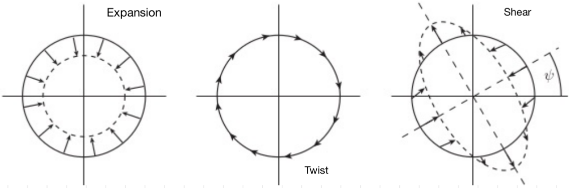
Some final definitions before the results

- Optical scalars for null vector $\mathbf{k}$:

$$\theta = \nabla_a k^a, \quad \omega^2 = \nabla_{[a} k_{b]} \nabla^{[a} k^{b]},$$
$$\sigma^2 = \nabla_{(a} k_{b)} \nabla^{(a} k^{b)} - \frac{1}{n-2} (\nabla_a k^a)^2.$$

- **Example:** For Kerr(-Newman)

$$\theta = \frac{2r}{r^2 + a^2 \cos^2\theta}, \quad \omega^2 = \frac{2a^2 \cos^2\theta}{r^2 + a^2 \cos^2\theta}, \quad \sigma^2 = 0.$$



Results

- Rotating BH: $\theta \neq 0 \neq \omega$ for $\mathbf{k}$.

Theorem
(Ortaggio and Srinivasan 2023, arXiv: 2309.02900 (gr-qc);)

For $D \geq 4$ and our assumed setup, a charged rotating BH in the Einstein-Maxwell theory $\implies$ $\mathbf{k}$ is shearfree.

- **Remark:** Kerr-Newman is consistent with our result since it has $\sigma = 0$.

Implications of the Theorem: Can Myers-Perry be charged?

- Myers-Perry black holes are HD generalizations of 4D Kerr black holes: Myers and Perry 1986;
- Generalized to include Λ: Gibbons, Lu, Page, and Pope 2004, arXiv: `hep-th/0409155`;
- In 4D, Kerr black holes have only one plane of rotation.
- In HD, there are multiple spatial directions and hence rotations along multiple planes.
- The most general one has $N = [\frac{D-1}{2}]$ rotations, in D spacetime dimensions.

Implications of the Theorem: Can Myers-Perry be charged?

- Myers-Perry belong to the KS class.
- $\mathbf{k}$ has $\theta \neq 0 \neq \omega$.
- Most importantly, Myers-Perry BHs have $\sigma \neq 0$ for $\mathbf{k}$.
- Hence, the straightforward charging method of Kerr-Newman

$$\mathbf{g}_{\text{vacuum}} \xrightarrow{\mathbf{A}=\alpha\mathbf{k}} \mathbf{g}_{\text{charged}} = \mathbf{g}_{\text{vacuum}} - H_{\text{charge}} \mathbf{k} \otimes \mathbf{k},$$

has no solution in the case of Myers-Perry.

Take home message

1. Myers-Perry cannot be charged with $\mathbf{A} = \alpha \mathbf{k}$.
2. We identify shear of $\mathbf{k}$ as the "obstruction" to charging.
3. We also identify the full class of solutions consistent with our theorem as: Charged Taub-NUT $\cap$ KS..

Thanks for listening.