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Secondary Gravitational Waves as probes of the early Universe and Gravity

Based on :

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Tzerefos *2025 (in prep.)*, ***Tzerefos***, *PBSM 2025, BNP* ***Tzerefos*** *2023, BNP* ***Tzerefos*** *2023 and P* ***Tzerefos*** *BS 2022.*

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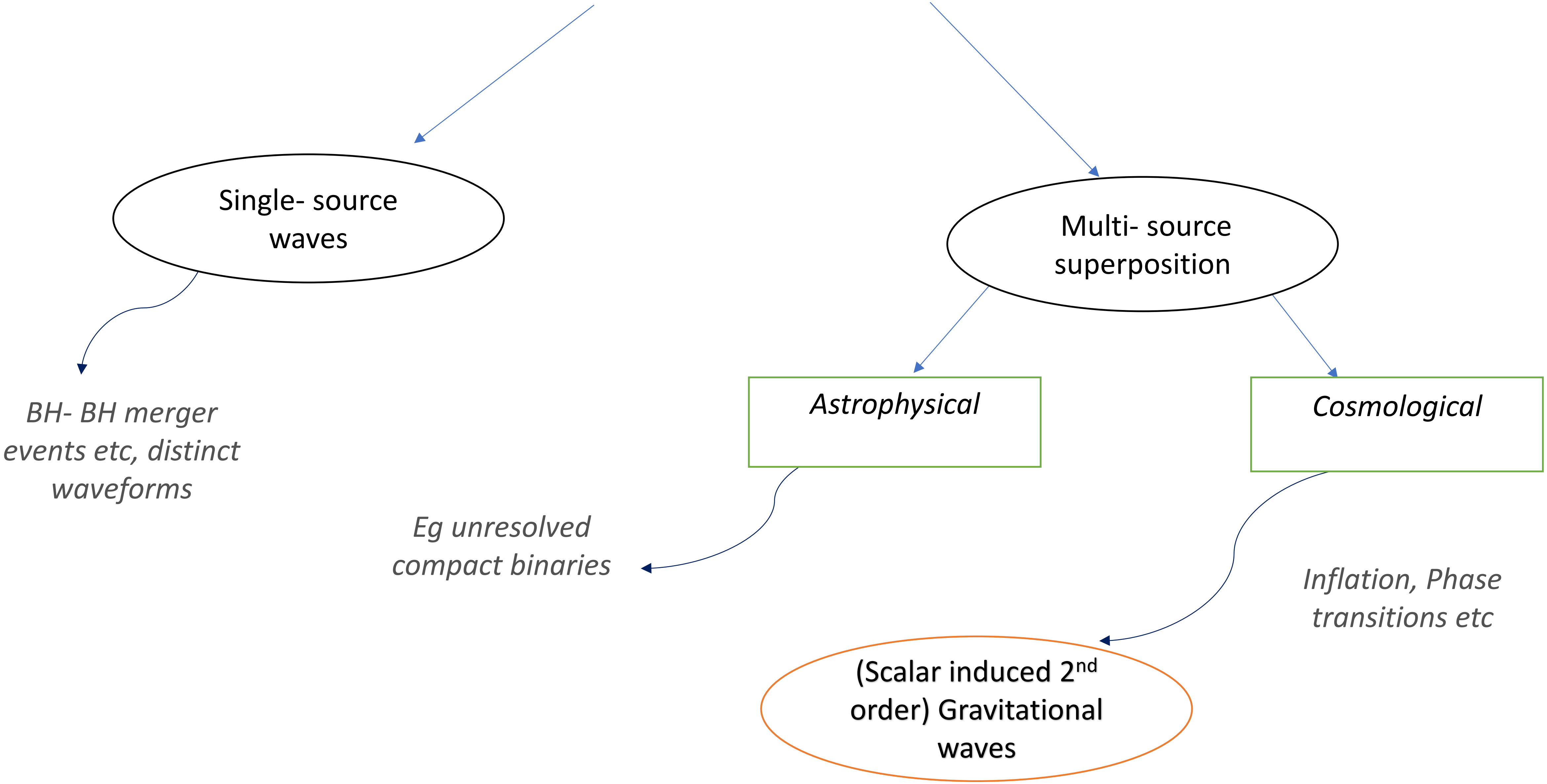
Gravitational Waves



What are these secondary GWs?

Cosmological Principle: The metric is *homogeneous* and *isotropic*

Need to describe structure:

$$For\ spacetime: g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$$

$$For\ matter: T_{\mu\nu} = \bar{T}_{\mu\nu} + \delta T_{\mu\nu}$$

$$\delta g_{\mu\nu} = \sum_{n=1} \frac{\varepsilon^n}{n!} \times \begin{Bmatrix} Scalars: \Phi^{(n)}, \Psi^{(n)} \ \ldots \\ Vectors: \mathrm{V}_\mu^{(n)} \ \ldots \\ Tensors: h_{\mu\nu}^{(n)} \ \ldots . \end{Bmatrix}$$

Same for $\delta T_{\mu\nu}$

Gravitational Action (aka combinations of *geometric tensors* like R)

Variation wrt $g_{\mu\nu}$

Field Equations (typically consist of $g_{\mu\nu}$ *and its derivs)*

Expansion to 1st order

$SVT\ decomp$

- $Scalars$: Diff eqs of $\Phi^{(1)}, \Psi^{(1)} \ \rightarrow Describe\ structure$
- $Vectors$: Diff eqt of $\mathrm{V}_\mu^{(1)} \rightarrow Typically\ not\ sourced$
- $Tensors$: Diff eqt of $h_{\mu\nu}^{(1)} \left(eg\ \triangle h_{\mu\nu}^{(1)} = \Pi_{\mu\nu}^{(1)} \right) \rightarrow$ $GWs\ at\ linear\ order$

Expansion to 2nd order, focusing only on $h_{\mu\nu}^{(2)}$

Diff eqt of $h_{\mu\nu}^{(2)}$ $= \underbrace{Function\ of\ 1^{st}\ and\ 2^{nd} order\ quantities}_{Source\ term}$

Focusing only on 1st order scalars

Scalar Induced Gravitational Waves : SIGW $\left(eg\ \triangle h_{\mu\nu}^{(2)} = \mathrm{S}_{\mu\nu}^{(2)}(\Phi^{(1)}, \Psi^{(1)}) \right)$

Intuition about SIGWs

- General form of SIGW produced by a scalar field φ [Domenech 2021]

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \partial_k\partial^k h_{ij} = \mathcal{P}^{ab}{}_{ij}\{T_{ab}\} \approx \mathcal{P}^{ab}{}_{ij}\{\partial_a\delta\varphi\partial_b\delta\varphi\} \longrightarrow \mathcal{P}_h \equiv \sum_\lambda \mathcal{P}_{h,\lambda} \propto \mathcal{P}^2_{\delta\varphi}$$

- In an inflationary setting $\mathcal{P}_{\delta\varphi} \sim \mathcal{P}_\mathcal{R}$ and assuming only RD we get $\Omega_{\rm GW}^{\rm induced}h^2 \sim \frac{1}{12}\Omega_{r,0}h^2 \times \mathcal{P}^2_\mathcal{R}$
  $\longrightarrow$ $\Omega_{\rm GW}^{\rm induced}h^2 \sim 10^{-6}\mathcal{P}^2_\mathcal{R}$
- Approximate sensitivity of future detectors $\Omega_{\rm GW}^{\rm induced}h^2 \sim 10^{-16}$
- On CMB scales $k \sim 10^{-4} - 0.5\ {\rm Mpc}^{-1}$ $\mathcal{P}_\mathcal{R} \sim 10^{-9}$ $\longrightarrow$ too small
- In general, there is a need of an *enhancement* of $\delta\varphi$ for SIGW to be observable
- Even non detection gives meaningful constrains $\mathcal{P}_\mathcal{R} \sim 10^{-4} - 10^{-5}$
- *Main advantage* : SIGWs provide *unique access to sub-CMB scales* $k \sim 10^{7} - 10^{18}\ {\rm Mpc}^{-1}$

Essentials of SIGWs in GR

- Working in the Newtonian gauge, the 2nd order tensor perturbations are described as follows

$$ds^2 = a^2(\eta)\left\{-(1+2\Phi)d\eta^2 + \left[(1-2\Phi)\delta_{ij} + \frac{h_{ij}}{2}\right]dx^i dx^j\right\}$$

- Their equation of motion in fourier space is $h_{\boldsymbol{k}}^{s,\prime\prime} + 2\mathcal{H}h_{\boldsymbol{k}}^{s,\prime} + k^2 h_{\boldsymbol{k}}^s = 4S_{\boldsymbol{k}}^s$

- The source term is $S_{\boldsymbol{k}}^s = \int \frac{d^3\boldsymbol{q}}{(2\pi)^{3/2}} e_{ij}^s(\boldsymbol{k}) q_i q_j \left[2\Phi_{\boldsymbol{q}}\Phi_{\boldsymbol{k}-\boldsymbol{q}} + \frac{4}{3(1+w_{\rm tot})}(\mathcal{H}^{-1}\Phi_{\boldsymbol{q}}' + \Phi_{\boldsymbol{q}})(\mathcal{H}^{-1}\Phi_{\boldsymbol{k}-\boldsymbol{q}}' + \Phi_{\boldsymbol{k}-\boldsymbol{q}})\right]$

- At the end, the spectral abundance of GWs can be given by

$\Omega_{\rm GW}(\eta,k) \equiv \frac{1}{\bar{\rho}_{\rm tot}} \frac{d\rho_{\rm GW}(\eta,k)}{d\ln k} = \frac{1}{24}\left(\frac{k}{\mathcal{H}(\eta)}\right)^2 \overline{\mathcal{P}_h^{(s)}(\eta,k)}$ and $\mathcal{P}_h^{(s)}(\eta,k) \equiv \frac{k^3|h_k|^2}{2\pi^2} \propto \int dv \int du I^2(u,v,x)\mathcal{P}_\Phi(kv)\mathcal{P}_\Phi(ku)$

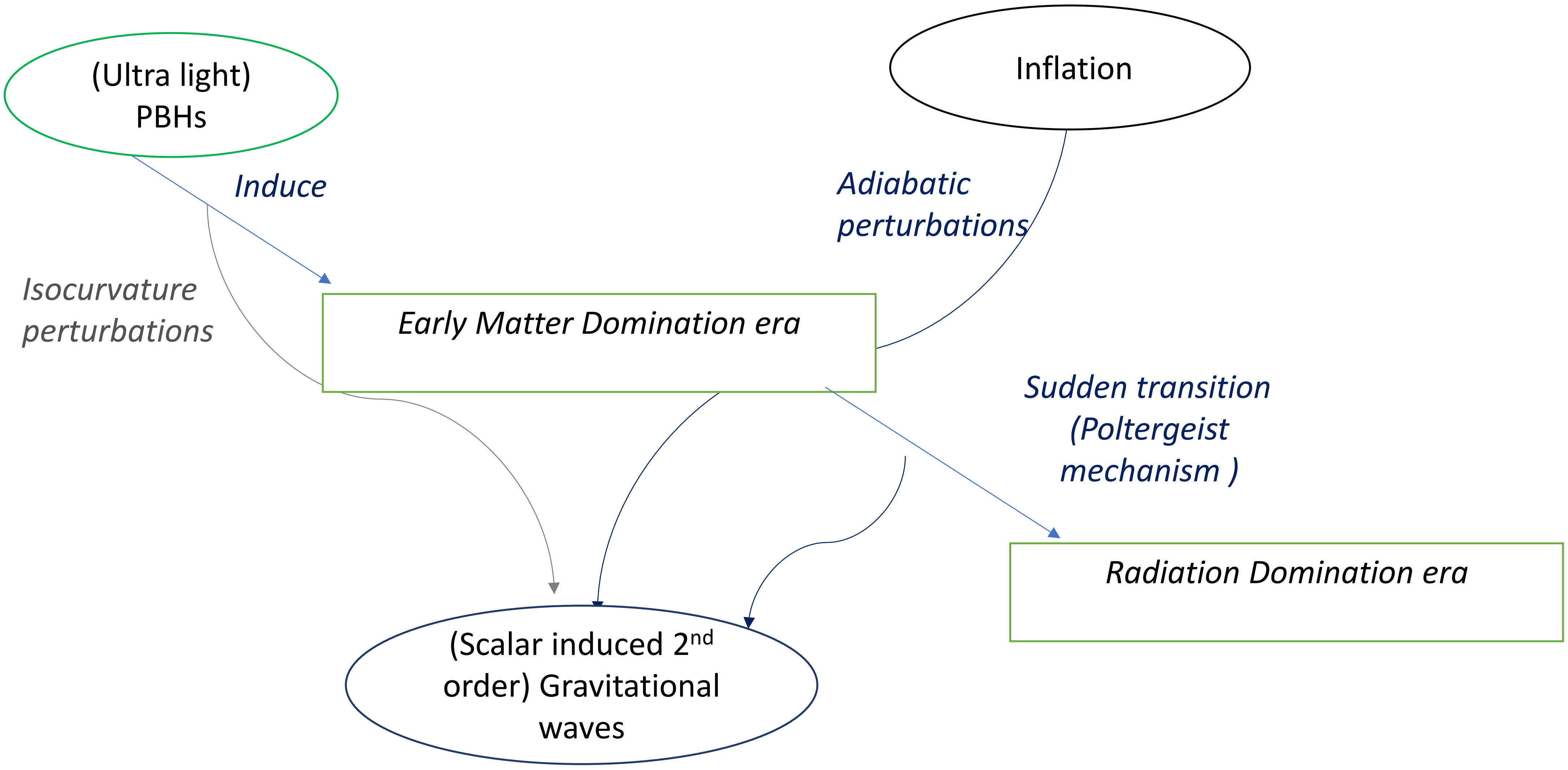
The kernel $I(u,v,x)$ is a complicated function which describes the time evolution of the potentials (via transfer functions).

Common sources of SIGWs

- Enhanced curvature perturbations during inflation
  → Large scalar fluctuations at small scales (eg via inflection points)→ strong SIGWs
- Preheating and resonances
  → Nonlinear dynamics of inflaton and spectator fields
- Primordial black holes (PBHs)
  → Poisson/isocurvature fluctuations in PBH distributions
- Cosmological phase transitions
  → Strong transitions (e.g. GUT, EW scale) amplify scalar inhomogeneities
- Sudden transitions between eras
  → e.g. Early Matter Domination → Radiation Domination ("poltergeist mechanism")
- Isocurvature modes from spectator fields
  → Generate additional SIGW channels

This talk!

General concept



Sources of eMDs we considered

- The source with which we were primarily preoccupied is a gas of ultra- light PBH.

➢ It was first studied in [Papanikolaou, Vennin, Langlois, JCAP, 2021] and we applied it to various works, among which the most notable are:

➢ Its pioneering extension to an *f(R) gravity theory* in [T. Papanikolaou, **C. Tzerefos**, S. Basilakos, E.N. Saridakis, JCAP, 2022] and then [Papanikolaou, 2025]. Currently, I am refining that approach in [**C. Tzerefos**, in prep].

➢ Its production in a *no-scale SUGRA model* in [S. Basilakos , D.V. Nanopoulos, T. Papanikolaou, E.N. Saridakis, **C. Tzerefos,** PLB, 2024]

- We also considered BSM cosmological models with *particular reheating processes,* which induce an eMD.

➢ *Flipped SU(5) theory* in [S. Basilakos , D.V. Nanopoulos, T. Papanikolaou, E.N. Saridakis, **C. Tzerefos ,**PLB, 2023]

➢ String – inspired *Axion- Chern- Simmons – Running- Vacuum cosmology* [**C. Tzerefos,** T. Papanikolaou, S. Basilakos , E.N. Saridakis, N.E. Mavromatos PRD, 2025].

What are ultra-light PBHs

Primordial black holes (PBH) form in the early universe out of the collapse of enhanced energy density perturbations upon horizon reentry of the typical size of the collapsing overdensity region $\delta \equiv \frac{\delta\rho}{\rho_{\rm b}} > \delta_{\rm c}$, $m_{\rm PBH} = \gamma M_{\rm H} \propto H^{-1}$

We will consider **ultra-light PBHs** for which $m_{\rm PBH} < 10^9 {\rm g}$. Some of their advantages:

✓They can induce an early matter dominated era (eMD) since $\Omega_{\rm PBH} = \rho_{\rm PBH}/\rho_{\rm tot} \propto a^{-3}/a^{-4} \propto a$ and evaporate before BBN. Their evaporation could **drive the reheating process** (e.g. [Martin++, JCAP, 2019])

✓This eMD era enhances the magnitude of the curvature perturbation and consequently gives rise to **scalar induced gravitational waves (SIGWs)** with very interesting phenomenology. For instance, one can constrain the underlying gravity theory (see later)

✓Their Hawking evaporation can alleviate the **Hubble tension** by injecting to the primordial plasma dark radiation degrees of freedom which can increase N_{eff} (e. g. [Nesseris++, 2019])

SIGWs from Poisson fluctuations of a gas of PBHs in GR

- Random distribution of PBHs + same mass → they follow *Poisson* statistics :

$$P_{\delta_{\rm PBH}}(k) \equiv \langle |\delta_k^{\rm PBH}|^2 \rangle = \frac{4\pi}{3}\left(\frac{\bar{r}}{a}\right)^3 = \frac{4\pi}{3k_{\rm UV}^3}, \quad \text{where} \quad k < k_{\rm UV} = \frac{a}{\bar{r}}$$

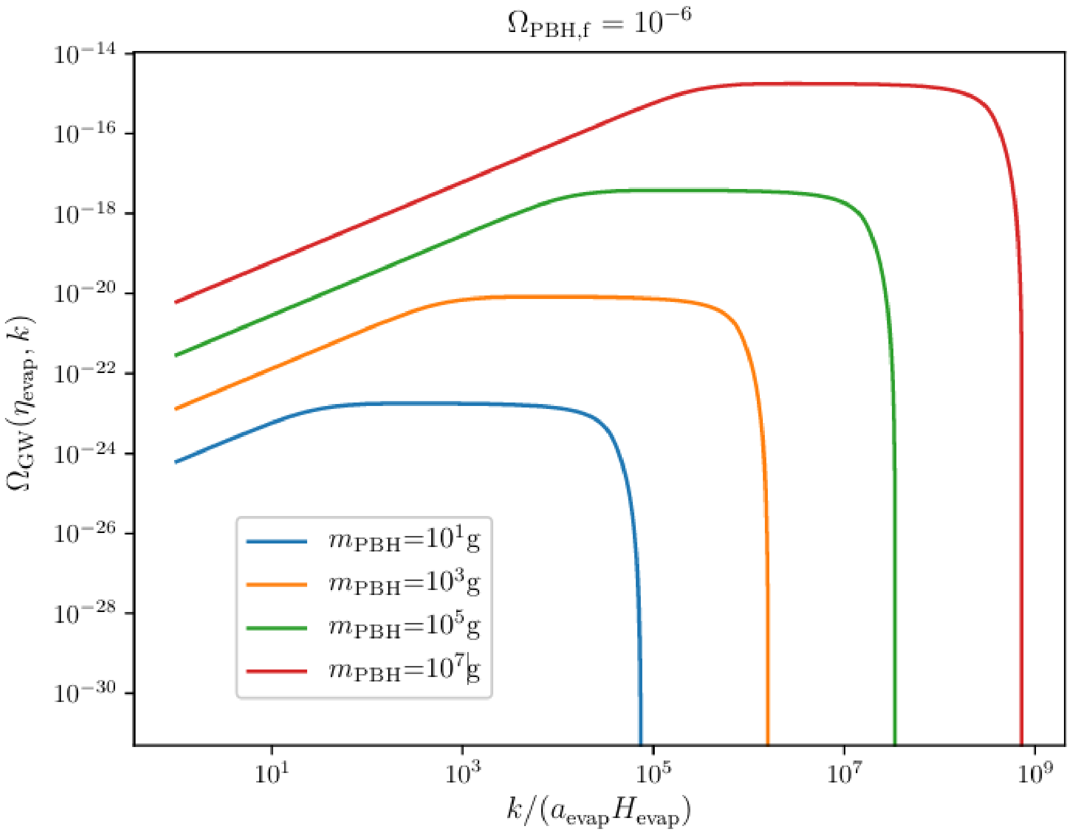
- Since ρ_{PBH} is inhomogeneous and ρ_{tot} is homogenous → $\delta_{\rm PBH}$ is an **isocurvature perturbation**

- $\delta_{\rm PBH}$ generated in the eRD era will be converted in an eMD era to a curvature perturbation ζ_{PBH} associated with the scalar potential [Papanikolaou, Vennin, Langlois, JCAP, 2021]

$$\mathcal{P}_\Phi(k) = \frac{2}{3\pi}\left(\frac{k}{k_{\rm UV}}\right)^3\left(5+\frac{4}{9}\frac{k^2}{k_{\rm d}^2}\right)^{-2} \longrightarrow \mathcal{P}_h^{(s)}(\eta,k) \equiv \frac{k^3|h_k|^2}{2\pi^2} \propto \int \mathrm{d}v \int \mathrm{d}u I^2(u,v,x)\mathcal{P}_\Phi(kv)\mathcal{P}_\Phi(ku)$$

$$\Omega_{\rm GW}(\eta,k) \equiv \frac{1}{\bar{\rho}_{\rm tot}}\frac{\mathrm{d}\rho_{\rm GW}(\eta,k)}{\mathrm{d}\ln k} = \frac{1}{24}\left(\frac{k}{\mathcal{H}(\eta)}\right)^2\overline{\mathcal{P}_h^{(s)}(\eta,k)}$$

SIGWs from Poisson fluctuations of a gas of PBHs in GR

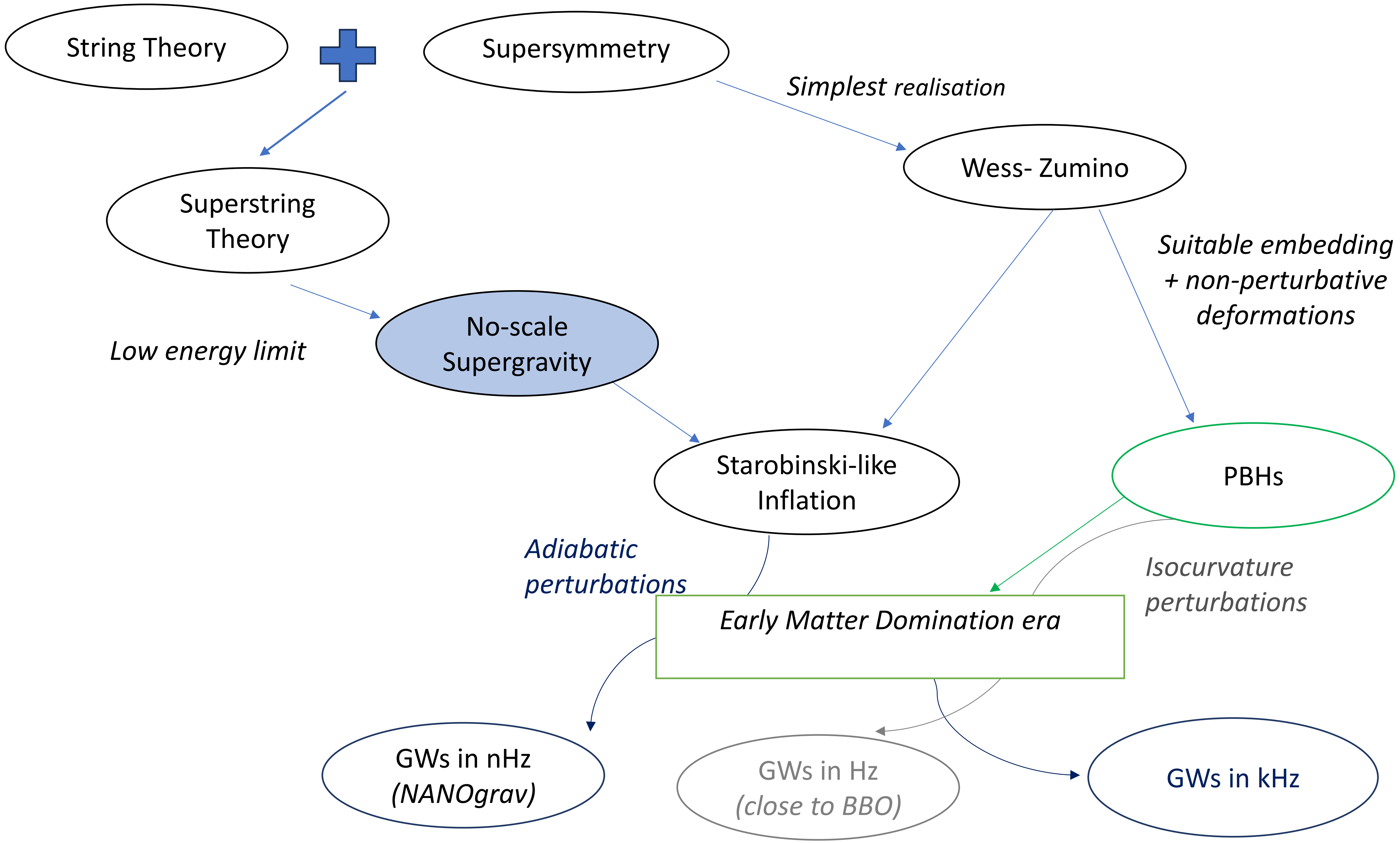


$$\Omega_{\rm GW}(\eta_{\rm evap}, k) \propto \left(\frac{m_{\rm PBH}}{10^9 {\rm g}}\right)^{4/3} \Omega_{\rm PBH,f}^{16/3} \times \begin{cases} \frac{k}{k_{\rm d}} & \text{for} \quad k \ll \mathscr{H}_{\rm d} \\ 8 & \text{for} \quad k \gg \mathscr{H}_{\rm d} \end{cases}$$

A) First application

We first implemented all these ideas in a model of no-scale SUGRA

[S. Basilakos , D.V. Nanopoulos, T. Papanikolaou, E.N. Saridakis, **C. Tzerefos ,**PLB, 2023]


String Theory
Supersymmetry
Simplest realisation
Superstring Theory
Wess- Zumino
Suitable embedding + non-perturbative deformations
Low energy limit
No-scale Supergravity
Starobinski-like Inflation
PBHs
Adiabatic perturbations
Isocurvature perturbations
Early Matter Domination era
GWs in nHz (NANOgrav)
GWs in Hz (close to BBO)
GWs in kHz

Theoretical introduction

- The most general (N=1) SUGRA is characterized by two functions: The Kahler potential K, which is a Hermitian function of the matter scalar field and quantifies its geometry, and a holomorphic function of the fields called superpotential W. V is the scalar potential :

$$\mathcal{S} = \int d^4x\sqrt{-g}\left(K_{i\bar{j}}\partial_\mu\Phi^i\partial^\mu\bar{\Phi}^{\bar{j}} - V\right)$$ with $$V = e^K\left(\mathcal{D}_{\bar{i}}\bar{W}K^{\bar{i}j}\mathcal{D}_jW - 3|W|^2\right) + \frac{\tilde{g}^2}{2}(K^iT^a\Phi_i)^2$$

and $K_{i\bar{j}}(\Phi,\bar{\Phi}) = \dfrac{\partial^2K}{\partial\Phi^i\partial\bar{\Phi}^{\bar{j}}}$, $\mathcal{D}_iW \equiv \partial_iW + K_iW$ and $i = \{\phi, T\}$ which are chiral superfields.

- The simplest globally supersymmetric model is the Wess-Zumino one, which is characterized by one single chiral superfield φ and the following superpotential: $W = \dfrac{\hat{\mu}}{2}\varphi^2 - \dfrac{\lambda}{3}\varphi^3$,

with a mass term $\hat{\mu}$ and a trilinear coupling term λ

No-scale Wess-Zumino (NSWZ) SUGRA

- In order to facilitate early universe inflationary scenarios, we shall embed this model in the context of $SU(2,1)/SU(2)\times U(1)$ no-scale supergravity by matching the T field to the modulus field and the φ field to the inflaton. The corresponding Kahler potential for this construction is

$$K = -3\ln\left(T+\bar{T}-\frac{\varphi\bar{\varphi}}{3}\right)$$

- Remarkably, by setting $T=\bar{T}=\frac{c}{2}$, $\mathrm{Im}\varphi=0$ and making a transformation of φ in order to obtain a canonical kinetic term, one obtains Starobinski inflation for $\lambda/\mu=1/3$ and $\hat{\mu}=\mu\sqrt{c/3}$

[Ellis, Nanopoulos, Olive, PRL, 2013],

[Nanopoulos, Spanos, Stamou, PRD, 2020]

$$V(\chi)=\frac{\mu^2}{4}\left(1-e^{-\sqrt{\frac{2}{3}}\chi}\right)^2$$

$$\varphi=\sqrt{3c}\,\tanh\left(\frac{\chi}{\sqrt{3}}\right)$$

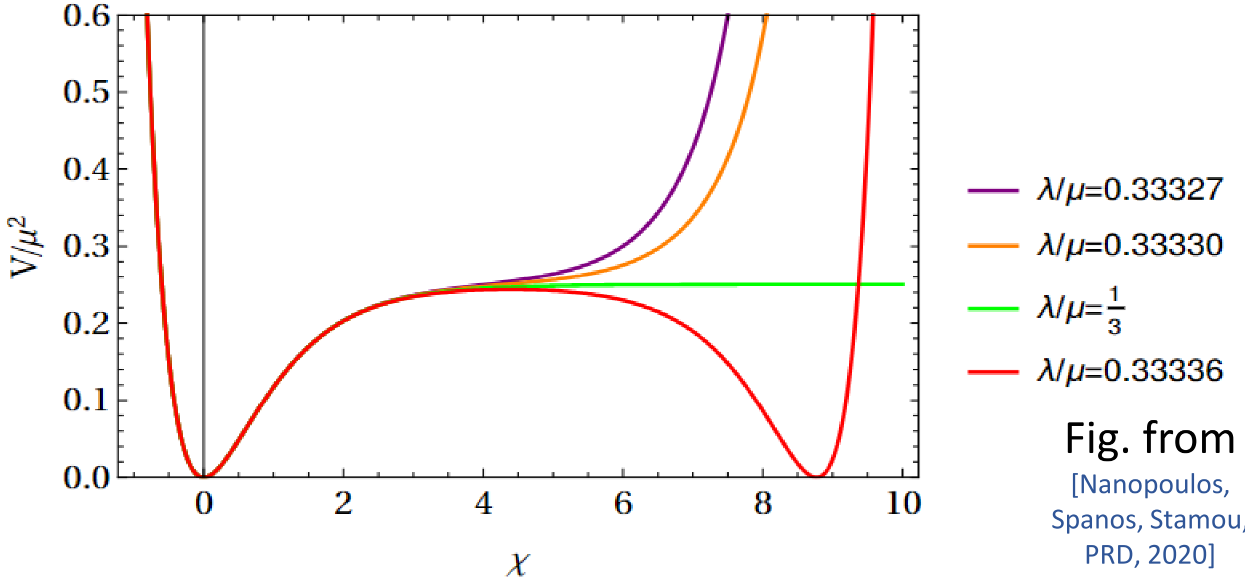


Fig. from [Nanopoulos, Spanos, Stamou, PRD, 2020]

NSWZ SUGRA inflection point inflation

- A common mechanism to produce PBHs is via the use of inflationary potentials with inflection points aka points where $V''(\chi_{\text{inflection}}) = V'(\chi_{\text{inflection}}) \simeq 0$ which induce the so-called **ultra slow roll inflation** (USR)

- To realize such set-ups, one can introduce the following **non-perturbative deformations** to the Kahler potential first introduced in [Nanopoulos, Spanos, Stamou, PRD, 2020]:

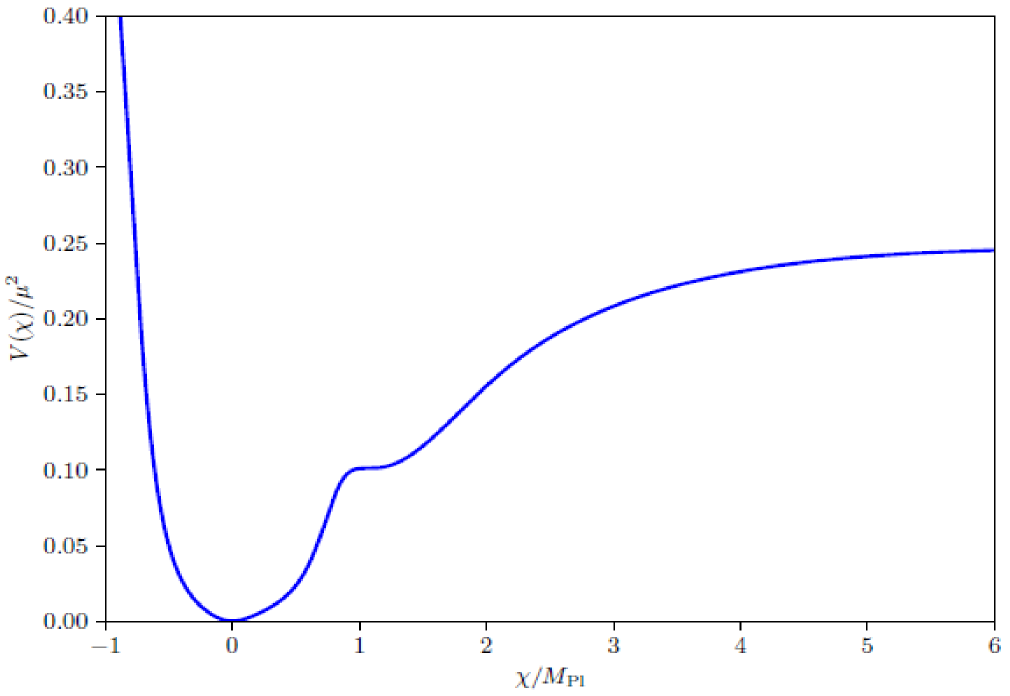
$$K = -3\ln\left[T + \bar{T} - \frac{\varphi\bar{\varphi}}{3} + a\, e^{-b(\varphi+\bar{\varphi})^2}(\varphi + \bar{\varphi})^4\right]$$ with a and b real constants

- At the end, one obtains the following potential

$$V(\phi) = \frac{3e^{12b\phi^2}\phi^2(c\mu^2 - 2\sqrt{3c}\,\lambda\,\mu\,\phi + 3\lambda^2\,\phi^2)}{\left[-48a\phi^4 + e^{4b\phi^2}(-3c + \phi^2)\right]^2\,\left[e^{4b\phi^2} - 24\,a\,\phi^2(6 + 4b\,\phi^2(-9 + 8b\,\phi^2))\right]}$$

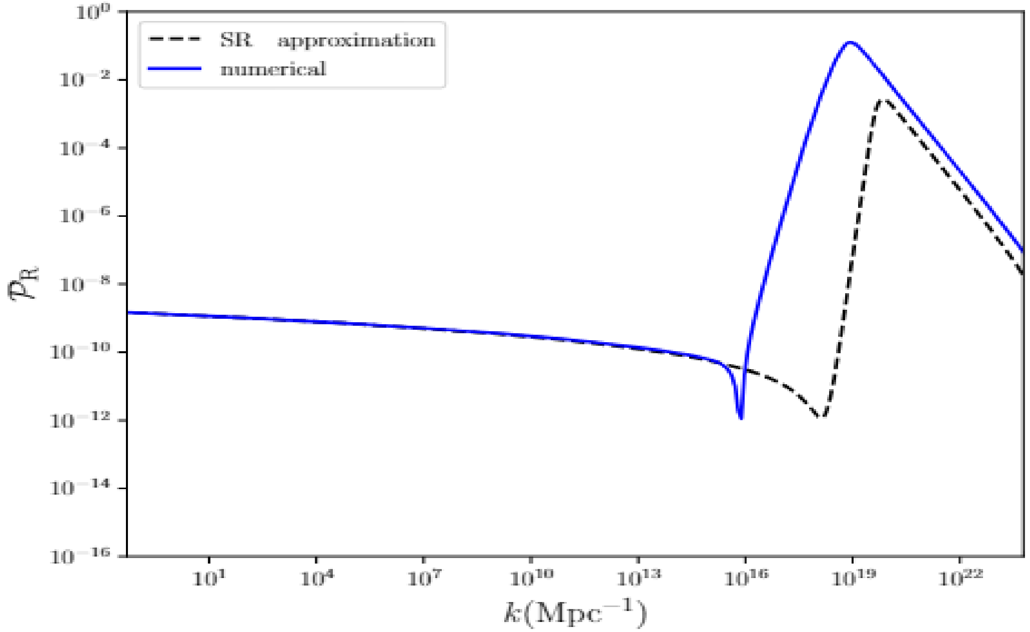
Ultra light PBHs in no-scale SUGRA

Our modified potential gives rise to the following power spectrum given our choice of fiducial parameters:



$$a = -1,\ b = 22.35,\ c = 0.065,\ \mu = 2 \times 10^{-5}$$

$$\lambda/\mu = 0.3333449 \text{ and } \phi_0 = 0.4295$$

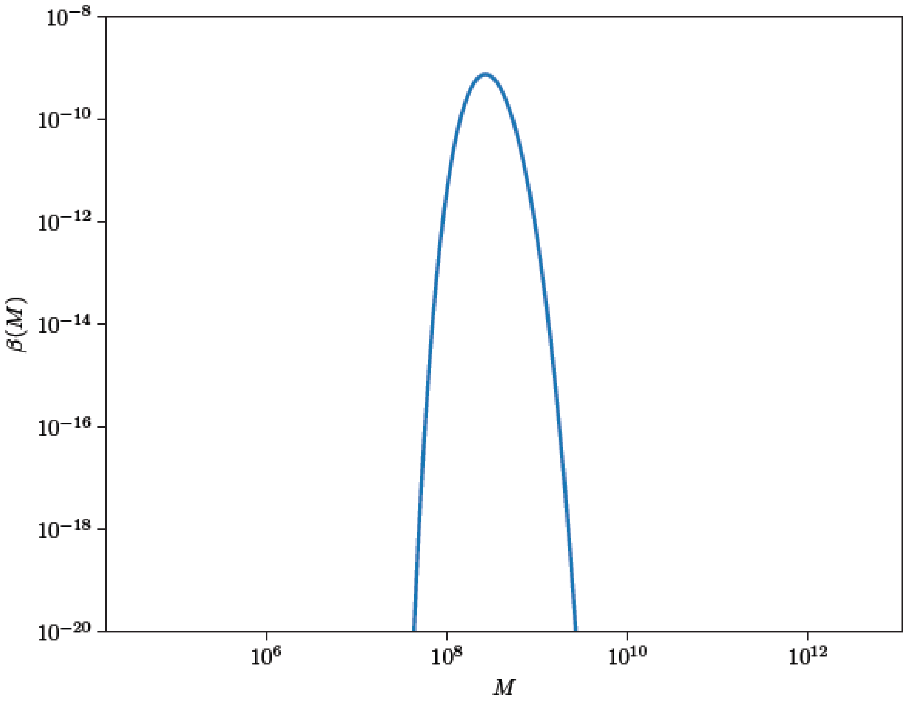


$$M_{\rm PBH} = 17 M_\odot \left(\frac{k}{10^6 \mathrm{Mpc}^{-1}} \right)^{-2} \sim 10^8 \mathrm{g}$$

eMD driven by PBHs

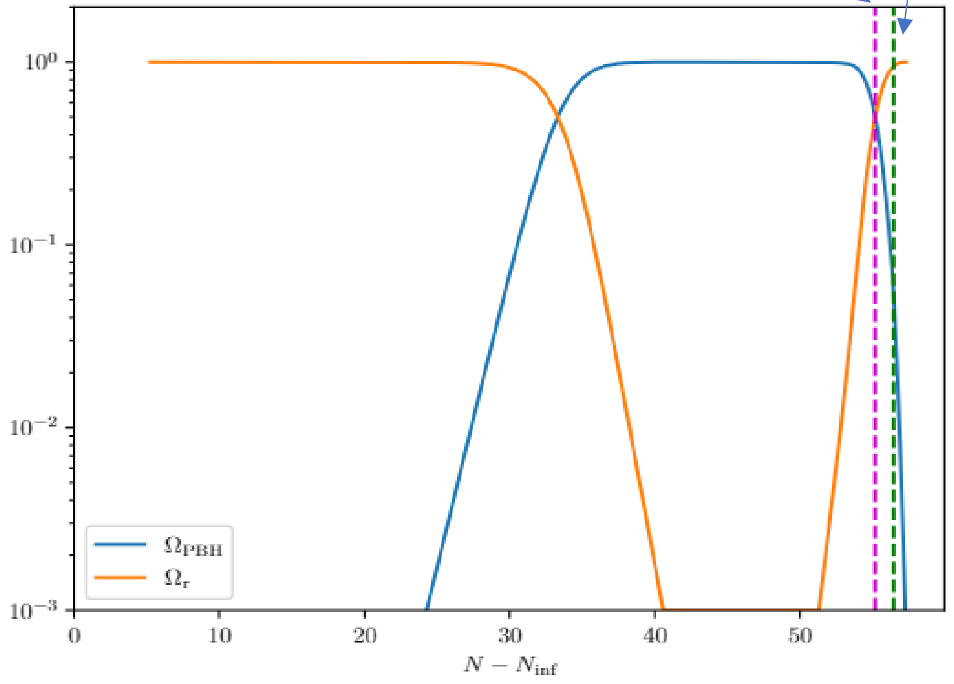
- Since $\Omega_{\rm PBH} = \rho_{\rm PBH}/\rho_{\rm tot} \propto a^{-3}/a^{-4} \propto a$ an eMD era driven by them arises

- To find their mass function $\beta(M) \equiv \frac{1}{\rho_{\rm tot}} \frac{{\rm d}\rho_{\rm PBH}}{{\rm d}\ln M}$ we use peak theory and obtain:



Taking into account their Hawking evaporation

$$M(t) = M_{\rm f}\left\{1 - \frac{t-t_{\rm f}}{\Delta t_{\rm evap}(M_{\rm f})}\right\}^{1/3}$$



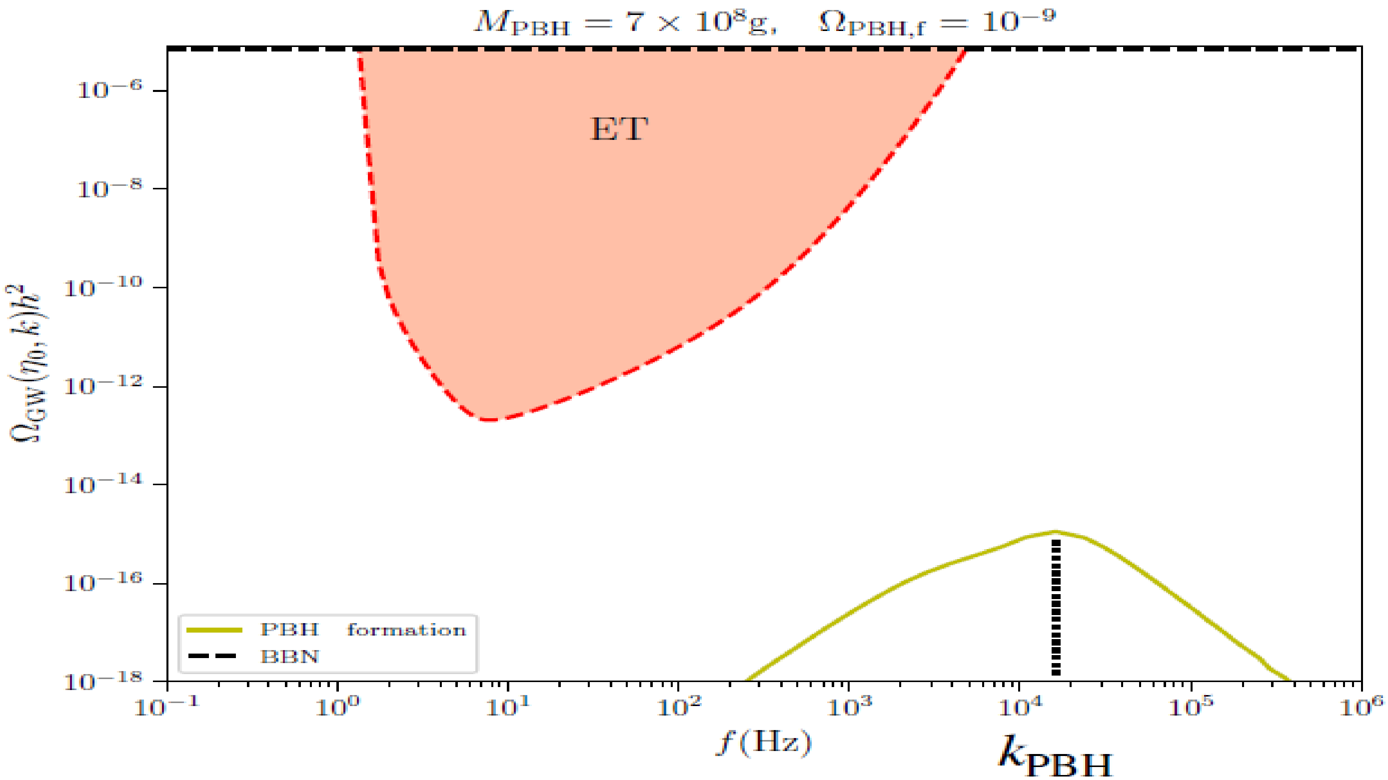
Note: We treat mass function as monochromatic → eMD to lRD sudden

The relevant GW sources and their spectrum

A) Inflationary adiabatic perturbations → GWs with two peaks

i) GWs are produced by the enhancement of $\mathcal{P}_{\mathcal{R}}(k)$ (peaked at $10^{19}\mathrm{Mpc}^{-1}$) at PBHs scales peaked at the **kHz range** [Kohri, Terada, PRD, 2018]

$$\Omega_{\mathrm{GW}}^{\mathrm{form}}(\eta_0,k)=\frac{a_{\mathrm{d}}}{a_{\mathrm{evap}}}c_g\Omega_{\mathrm{r}}^{(0)}\Omega_{\mathrm{GW}}(\eta_{\mathrm{f}},k),\text{ with } c_g=\frac{g_{*\rho,*}}{g_{*\rho,0}}\left(\frac{g_{*\mathrm{S},0}}{g_{*\mathrm{S},*}}\right)^{4/3}\sim 0.4 \text{ and } \Omega_{\mathrm{r}}^{(0)}\simeq 10^{-4}$$



Inflationary adiabatic perturbations

ii) It is related to the ***resonant amplification*** of curvature perturbations of scales entering the horizon during the eMD era. Specifically, since Φ' goes **quickly** from 0 (since Φ=*constant* in any MD) to $\Phi' \neq 0$ during lRD s, there is a resonant enhancement of GWs mainly sourced by the term $\mathcal{H}^{-2}\Phi'^2$

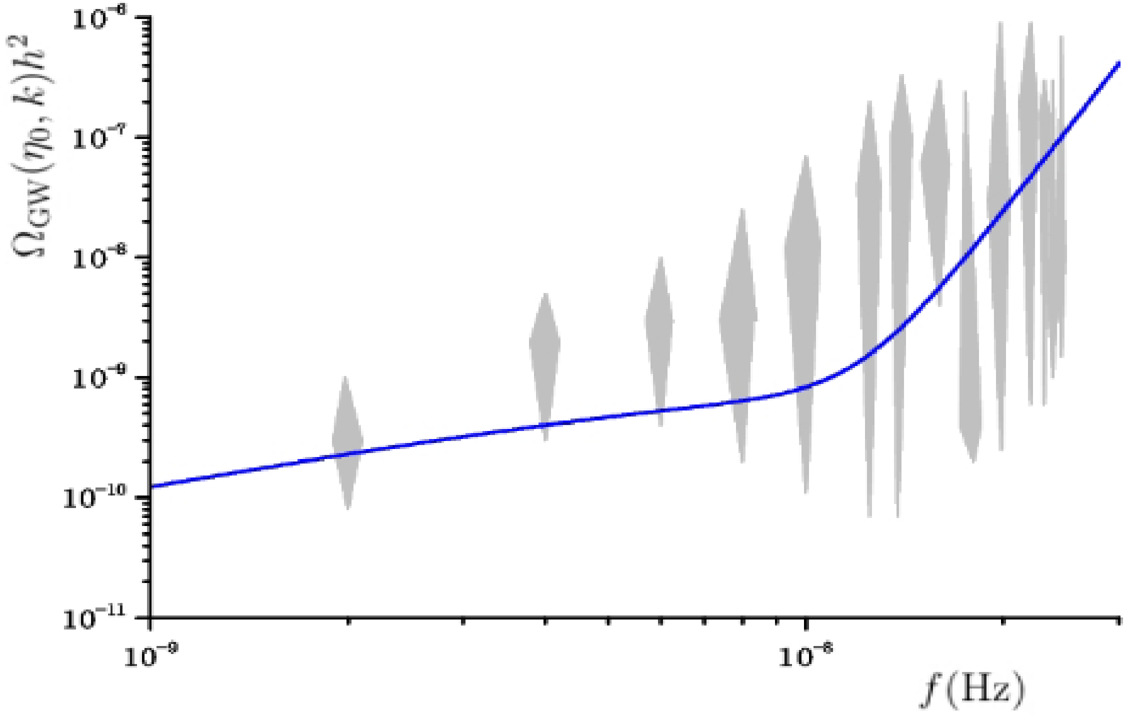
Also, since during a MD $\delta \sim \alpha$, we need to ensure that we are working in the perturbative regime → we introduce a nonlinear cut- off scale : $\delta_{k_{NL}}(\eta_r) = 1$

At the end, $f_{GW\,peak} = c\,\frac{k_{NL}}{2\pi a_0} \sim nHz$

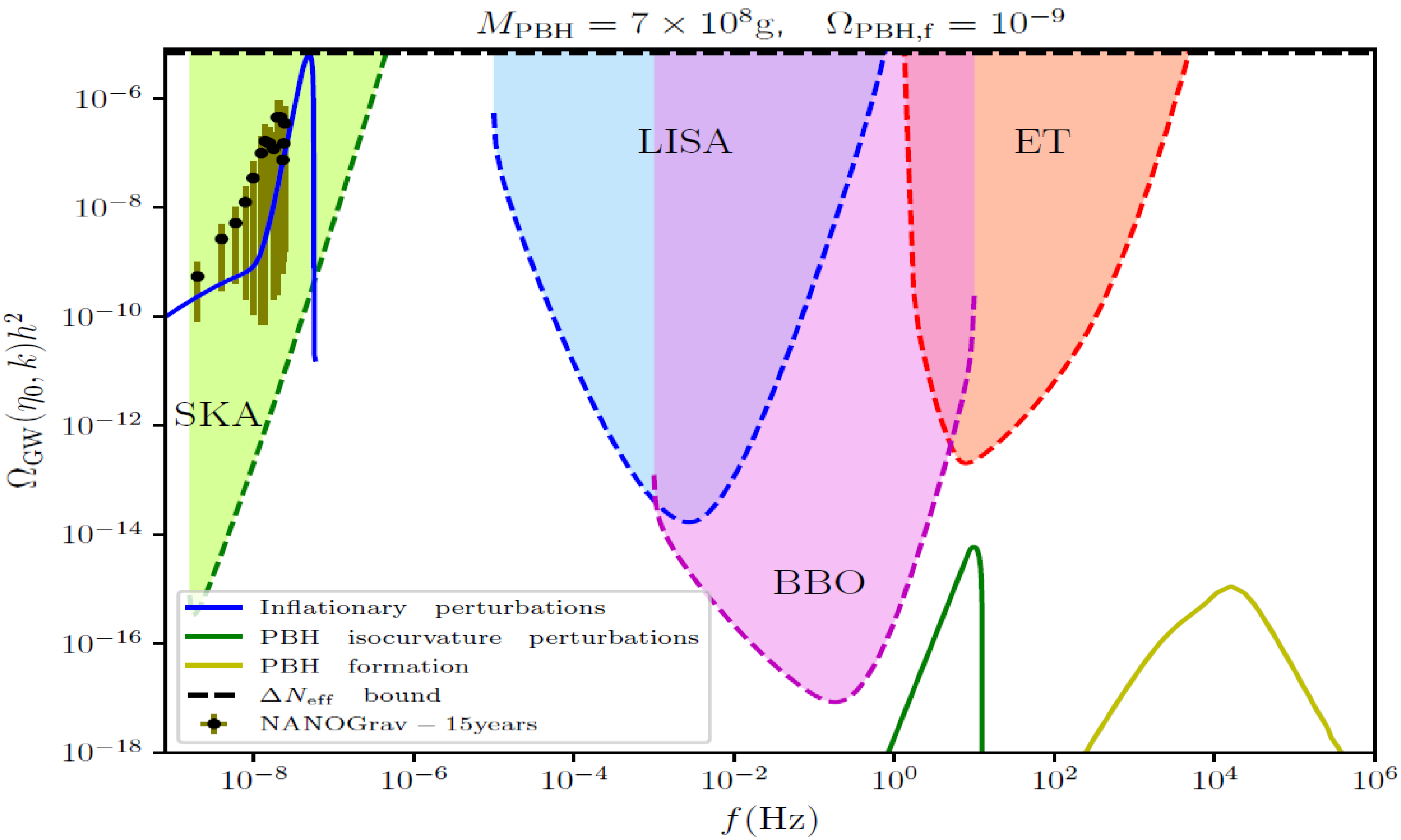
Inflationary adiabatic perturbations

This GW signal peaks at the **nHz frequency range** and is in agreement with **NANOGrav/PTA GW data**.

$$\Omega_{\rm GW}^{\rm res}(\eta_0, k) = c_g \Omega_{\rm r}^{(0)} \Omega_{\rm GW}(\eta_{\rm lRD}, k)$$



The complete three-peaked signal

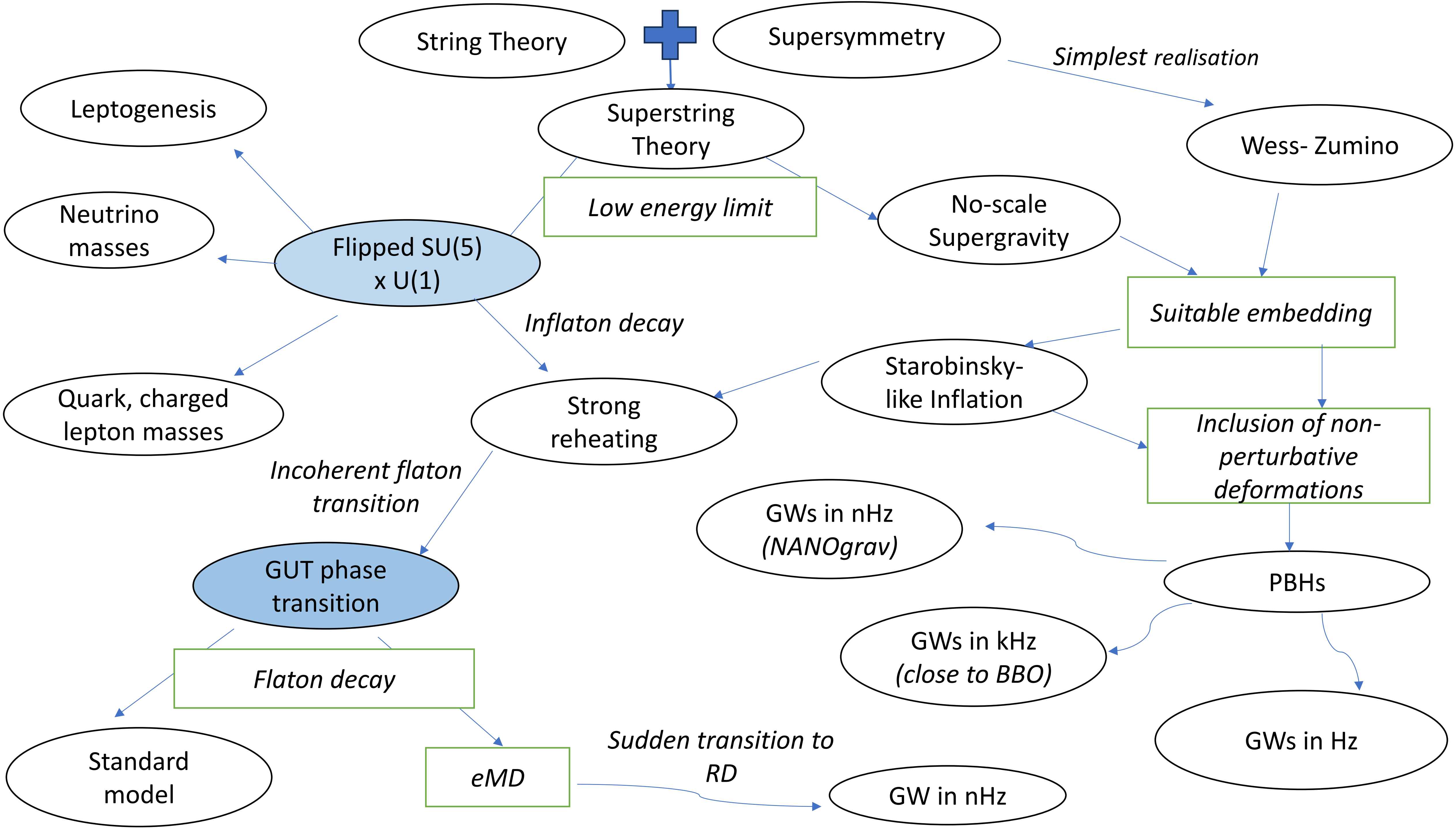


A simultaneous detection of all three peaks could be an indication in favor of no-scale SUGRA, or the presence of eMDs by PBHs in general

Bi) Further application

A similar set-up naturally exists in the for flipped SU(5)x U(1) theory

[S. Basilakos , D.V. Nanopoulos, T. Papanikolaou, E.N. Saridakis, **C. Tzerefos** ,PLB, 2023]


String Theory
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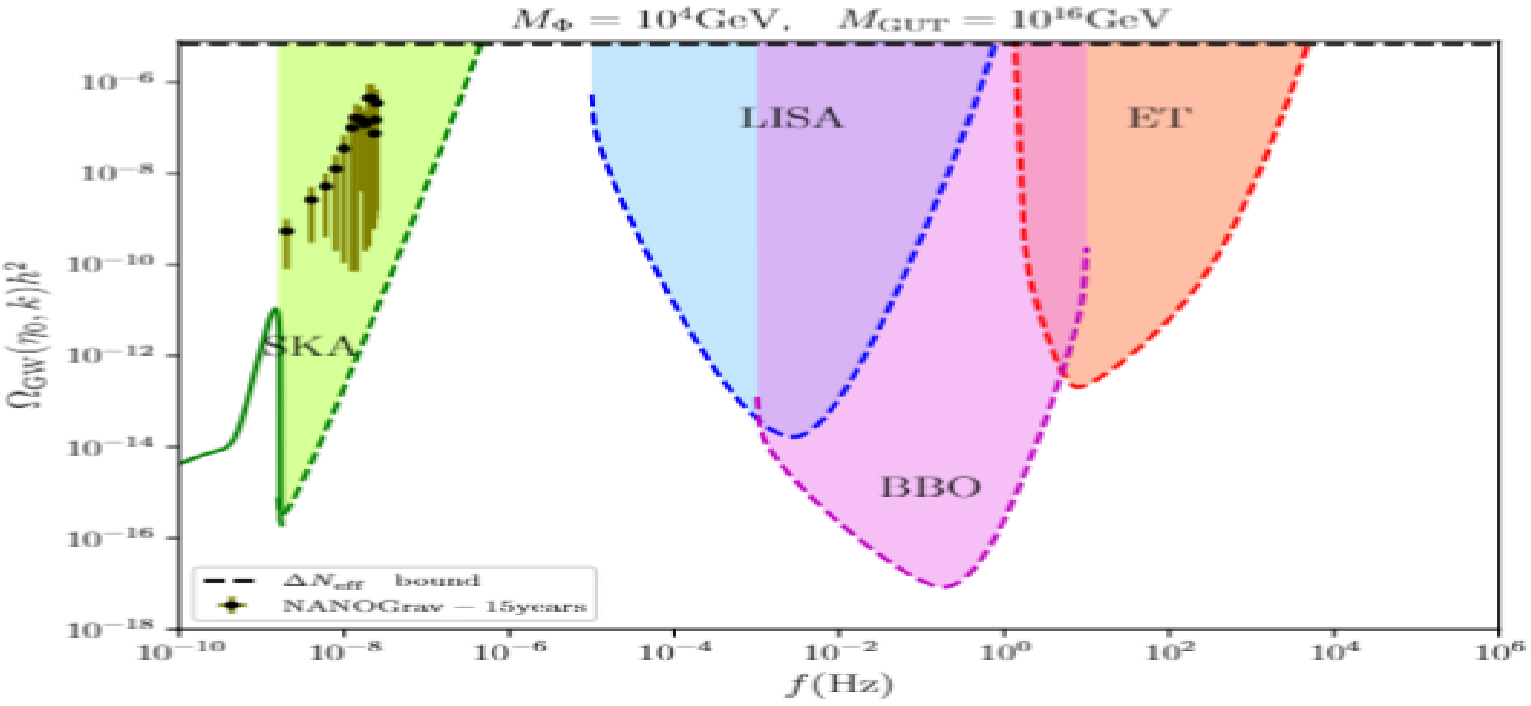
Flipped SU(5) x U(1)
No-scale Supergravity
Inflaton decay
Suitable embedding
Starobinsky-like Inflation
Quark, charged lepton masses
Strong reheating
Inclusion of non-perturbative deformations
Incoherent flaton transition
GWs in nHz (NANOgrav)
PBHs
GUT phase transition
GWs in kHz (close to BBO)
Flaton decay
GWs in Hz
Sudden transition to RD
Standard model
eMD
GW in nHz

GW signal from an eMD era in flipped SU(5)

This time, instead of the PBH, the eMD is induced by the flaton field, a field responsible for the GUT phase transition (eg [Ellis, Garcia, Nagata, Nanopoulos, Olive, JCAP, 2019])

Again, the transition from eMD to lRD is *sudden* ⟶ there is the same resonant amplification, with frequency:

$$f_{\rm GW,peak} = \frac{k_{\rm NL}}{2\pi a_0} = \frac{k}{2\pi a_{\rm d\Phi}}\frac{a_{\rm d\Phi}}{a_{\rm eq}}\frac{a_{\rm eq}}{a_0} = \frac{k}{2\pi a_{\rm d\Phi}}\left(\frac{\rho_{\rm eq}}{\rho_{\rm d\Phi}}\right)^{1/4}\left(\frac{\rho_0}{\rho_{\rm eq}}\right)^{1/3}$$
$$= 1.5\times 10^{-9}\left(\frac{\lambda_{1,3,5,7}}{0.5}\right)^2\left(\frac{m_\Phi}{10^4{\rm GeV}}\right)^{3/2}\left(\frac{10^{16}{\rm GeV}}{M_{\rm GUT}}\right).$$



Bii) Further application

- A prolonged reheating which induces an eMD also occurs in string – inspired *Axion-Chern- Simmons – running- vacuum cosmology* [**C. Tzerefos,** T. Papanikolaou, S. Basilakos , E.N. Saridakis, N.E. Mavromatos PRD, 2025].

$$S_B^{\rm eff} = \int d^4x\,\sqrt{-g}\Big\{c_0 + R\Big[c_1 + c_2\log\Big(\frac{R}{R_0}\Big)\Big] + \mathcal{L}_m + \dots\Big\} \qquad \mathcal{L}_m = -\frac{1}{2}\,\partial_\mu b\,\partial^\mu b - \frac{1}{2}\,\partial_\mu a\,\partial^\mu a - V(b,a) + \dots$$

$$V(b,\,a) = {\Lambda_1}^4\Big(-1 + f_a^{-1}\,\tilde{\xi}_1\,a(x)\Big)\,\cos({f_a}^{-1}a(x)) + \frac{1}{f_a}\Big(f_b\,{\Lambda_0}^3 + \Lambda_2^4\Big)\,a(x) + {\Lambda_0}^3\,b(x)$$

- Depending on the parameters, the reheating period can be prolonged and is driven by the *(sudden) decay of the axion field a(x).*

SIGWs in $f(R)$

$$f(R) = c_0 + R\left(c_1 + c_2 \log\left(\frac{R}{R_0}\right)\right)$$

$$FR_{\mu\nu} - \frac{1}{2}g_{\mu\nu}f + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)F = 8\pi G T^{\rm m}_{\mu\nu}$$

$$F \equiv \frac{{\rm d}f(R)}{{\rm d}R}$$

Extra polarization: Scalaron

$$\Box F(R) = m_{\rm sc}^2 F(R), \quad \text{with} \quad m_{\rm sc}^2 \equiv \frac{1}{3}\left(\frac{F}{F_{,R}} - R\right)$$

$$\phi_s \equiv \frac{{\rm d}f(R)}{{\rm d}R} = F(R)$$

$$\phi_s^{(1)\prime\prime} + 2\mathcal{H}\phi_s^{(1)\prime} - \left(\Delta + m_s^2\right)\phi_s^{(1)} = -\Box^{(1)}\phi_s^{(0)} + \frac{\kappa}{3}T^{M,(1)}$$

$$\phi_{s,\mathbf{k}}^{(1)}(\eta) = F_{\mathbf{k}}^{(1)}(\eta) = F_R^{(0)}(\eta)R_{\mathbf{k}}^{(1)}(\eta)$$

$$\mathcal{P}_{h,sc}^{(1)}(k) = T_s^2(x)\mathcal{P}_\zeta(k)$$

$${\rm d}s^2 = a^2(\eta)\left[-\left(1+2\Phi^{(1)}\right){\rm d}\eta^2 + \left(\left(1-2\Psi^{(1)}\right)\delta_{ij} + \frac{1}{2}h_{ij}^{(2)}\right){\rm d}x^i\,{\rm d}x^j\right]$$

$$h_{\mathbf{k}}^{\lambda,(2)\prime\prime}(\eta) + \frac{1}{F^{(0)}}\left(2\mathcal{H}F^{(0)} + F^{(0)\prime}\right)h_{\mathbf{k}}^{\lambda,(2)\prime}(\eta) + k^2 h_{\mathbf{k}}^{\lambda,(2)}(\eta) = \frac{4}{F^{(0)}}\left(\mathcal{S}_{\mathbf{k}}^{\lambda,(2)}(\eta) + \sigma_{\mathbf{k}}^{\lambda,(2)}(\eta)\right)$$

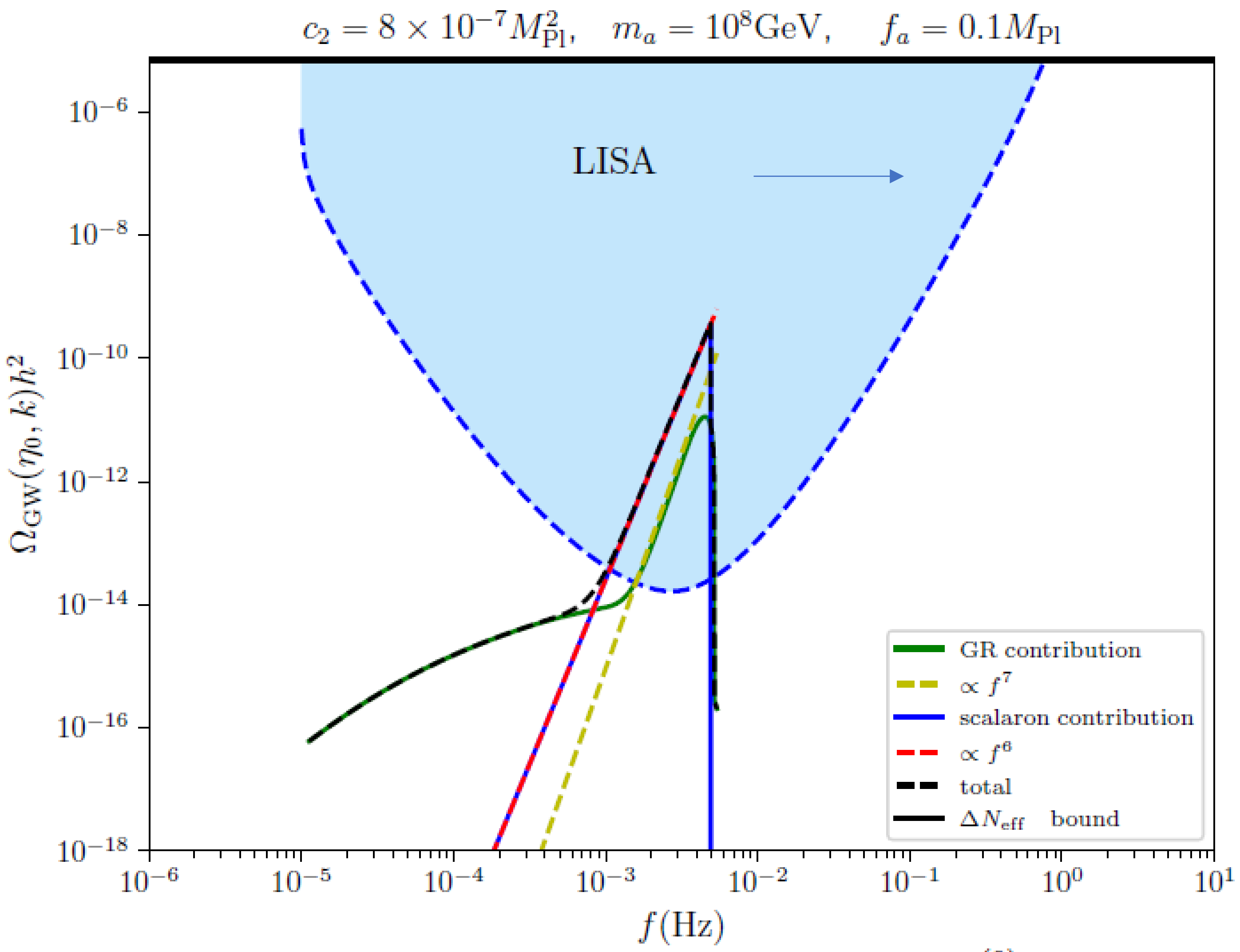
GR

$$\sigma_{\mathbf{k}}^{\lambda,(2)}(\eta) = \int \frac{d^3p}{(2\pi)^{3/2}}\varepsilon^{\lambda,lm}(\mathbf{k})p_l p_m \left(2\Psi_{\mathbf{k}-\mathbf{p}}^{(1)}(\eta)F_{\mathbf{p}}^{(1)}(\eta) + \left(F^{(0)} - 1\right)\left(3\Psi_{\mathbf{k}-\mathbf{p}}^{(1)}(\eta)\Psi_{\mathbf{p}}^{(1)}(\eta) - \Phi_{\mathbf{k}-\mathbf{p}}^{(1)}(\eta)\Phi_{\mathbf{p}}^{(1)}(\eta)\right)\right)$$

$$\Omega_{\rm GW}(\eta,k) = \frac{1}{6}\left(\frac{k}{\mathcal{H}(\eta)}\right)^2\left[\frac{\overline{\mathcal{P}_{h,s}^{(2)}(\eta,k)}}{4} + \mathcal{P}_{h,sc}^{(1)}(\eta,k)\right]$$

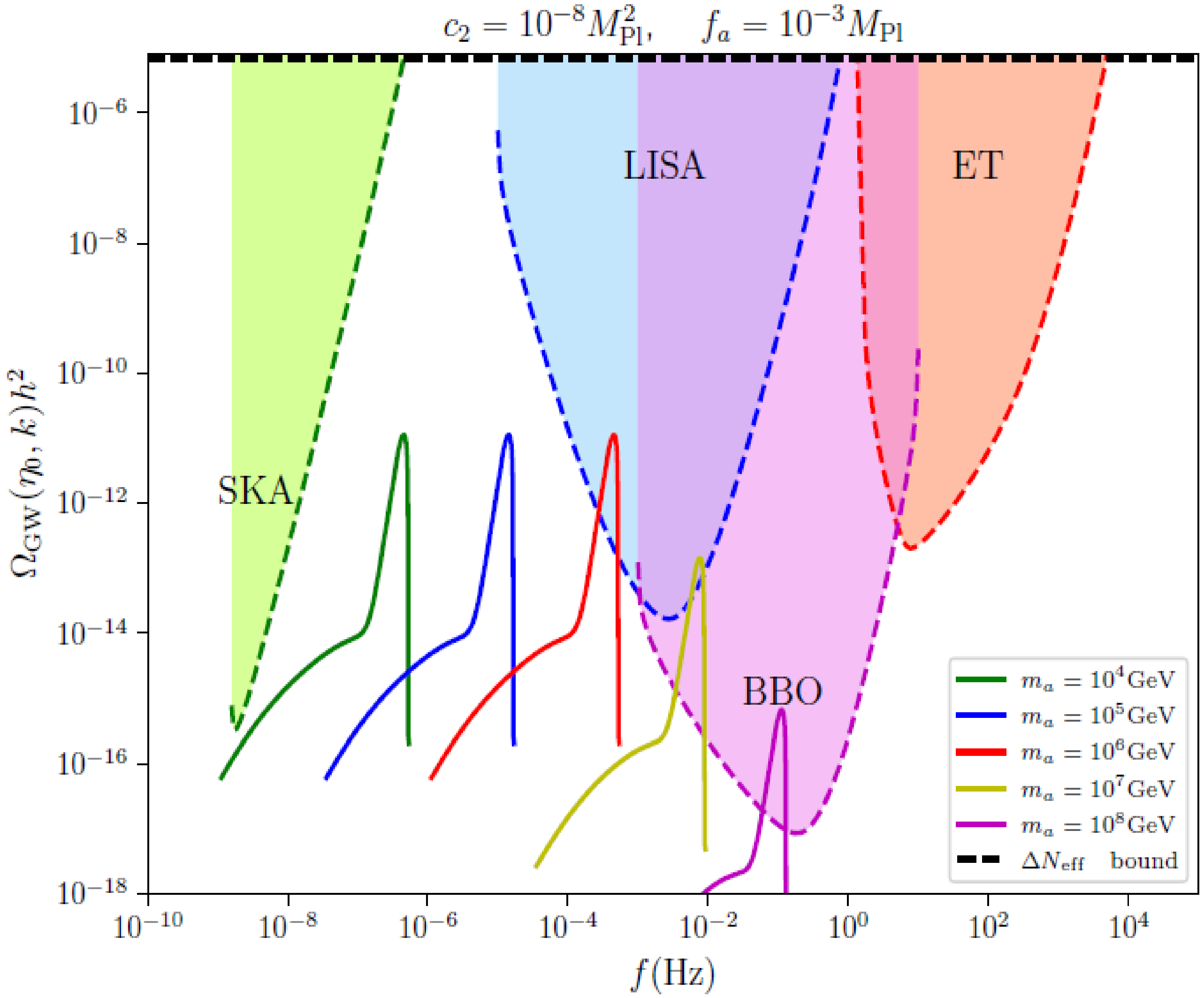
For this model, $\frac{k^2}{a^2}\frac{F_R^{(0)}}{F^{(0)}} \ll 1$ so the SIGW is GR

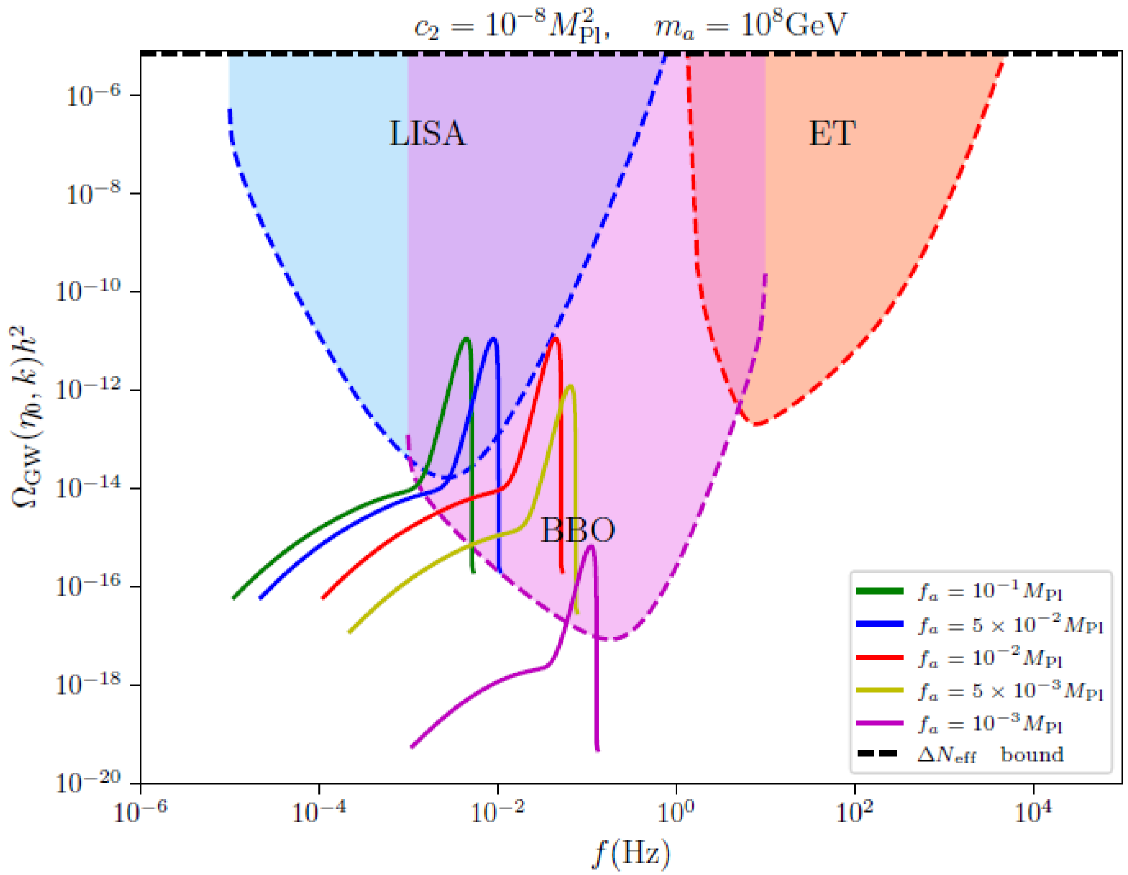
Choices for dominant Scalaron signal



Characteristic signal for any f(R) with $\frac{k^2}{a^2}\frac{F_R^{(0)}}{F^{(0)}} \ll 1$

Parameter combinations of comparable Ω_{res} and Ω_{scal} in aCSRVM





Conclusions

- We studied SIGWs in various contexts, mainly: eMD to lRD transitions and those coming from PBH themselves.
- We ve also worked to extend the usual GR formulation of SIGW and calculated them in f(R) gravity, thus allowing direct gravitational theory probing.
- In the cosmological models we studied, we uncovered interesting phenomenology which is potentially detectable by future observations.
- We also argued that even current observations can be explained by this portal, most notably the NANOgrav signal by this no-scale construction.
- In general, by harnessing this phenomenology we can learn new cosmological aspects as well as investigate the gravitational theory directly.

Thank you!

Theoretical introduction for no- scale SUGRA

- **Supergravity (SUGRA)** is a quantum field theory in which global supersymmetry has been promoted to a *local* symmetry. Therefore, its *gauging* describes *gravitation*.

- **No-scale supergravity** is a particular class of SUGRA which is characterized by the *absence of any external scales*, hence its name every relevant energy scale is a function of M_{pl} only. Its significant perks include:

✓ It has been explicitly demonstrated that it naturally arises as the *low energy limit of superstring theory* [Antoniadis, Ellis, Floratos, Nanopoulos, Tomaras, PLB, 1987]

✓ It cures the cosmological constant problem in the sense that it naturally providing *vanishing cosmological constant at the tree level* [Cremmer, Ferrara, Kounnas, Nanopoulos, PLB, 1983]

✓ Through its framework it can produce *Starobinsky-like inflation*, compatible with the Planck data [Ellis, Nanopoulos, Olive, JCAP, 2013]

✓ It can provide an efficient mechanism for *reheating*, the generation of *neutrino masses* and *leptogenesis* [Antoniadis, Nanopoulos, Rizos, JCAP ,2021]

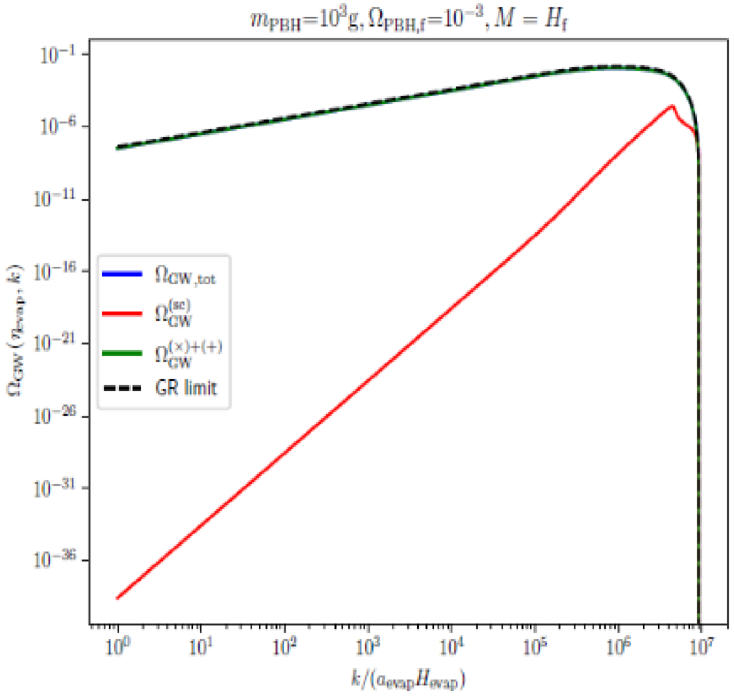
EXTRA application

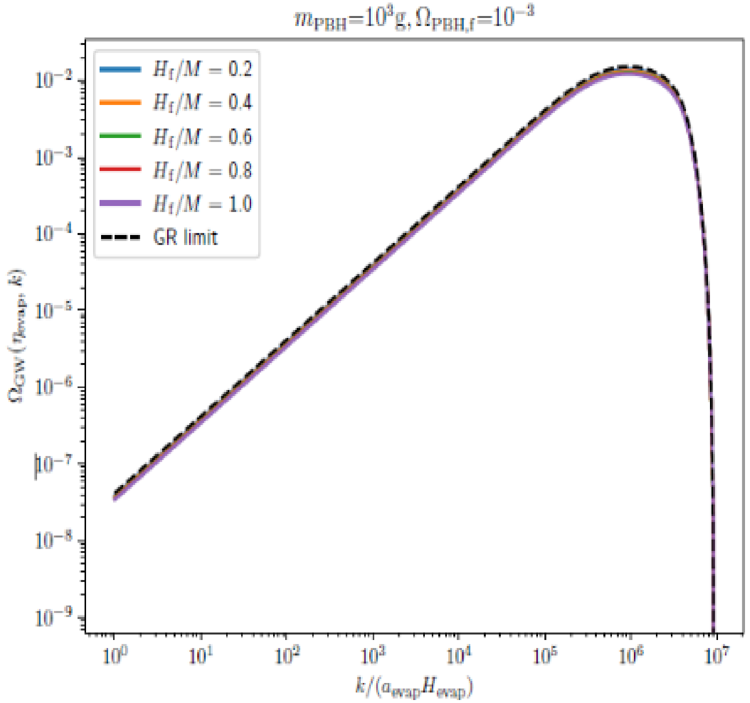
We assumed the existence of a similar gas of PBHs and parametrizing via its $\Omega_{\mathrm{PBH,f}}$ and m_{PBH} and studied these SIGWs in the framework of $f(R)$ gravity, with Starobinski $f(R) = R + R^2/6M$^2 as a case study

[T. Papanikolaou, **C. Tzerefos**, S. Basilakos, E.N. Saridakis, JCAP, 2022]

SIGWs in Starobinski

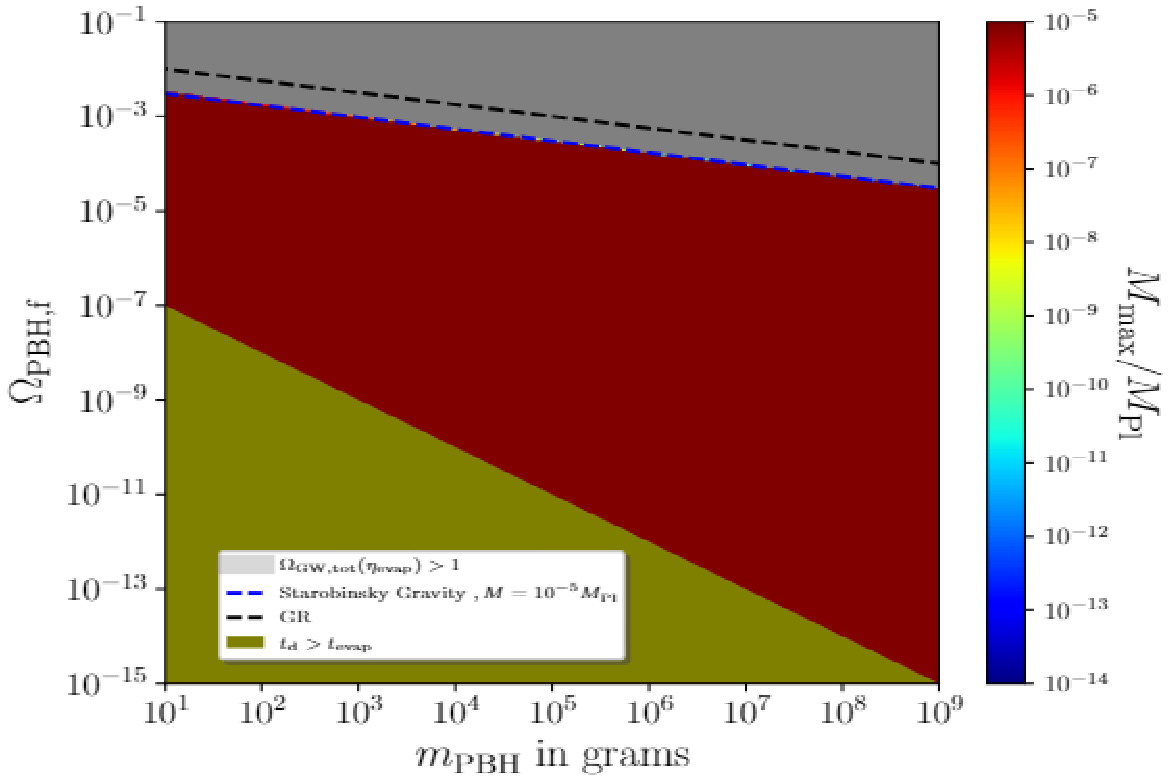
$$\Omega_{\rm GW}(\eta,k) \simeq \frac{1}{\bar{\rho}_{\rm tot}}\frac{{\rm d}\rho_{\rm GW,grad}(\eta,k)}{{\rm d}\ln k} = \frac{1}{96}\left(\frac{k}{\mathcal{H}(\eta)}\right)^2\left[2\overline{\mathcal{P}_h^{(\times)}}(\eta,k)+\overline{\mathcal{P}_h^{(\rm sc)}}(\eta,k)\right]$$





SIGWs in Starobinski

$$\Omega_{\rm GW,tot} \leq 1 \Rightarrow \Omega_{\rm PBH,f} \leq 5.5 \times 10^{-5} \left(\frac{10^9 \rm g}{m_{\rm PBH}}\right)^{1/4}$$ (45% tighter than GR!)



Appendix: SU(5) flipped outline