

Addressing the H_0 and S_8 Tensions in $f(Q)$ Cosmology

Maria Petronikolou

IAASARS, National Observatory of Athens

NEB-21: Recent Developments in Gravity
Hellenic Society for Relativity, Gravitation and Cosmology



Corfu, September 2, 2025



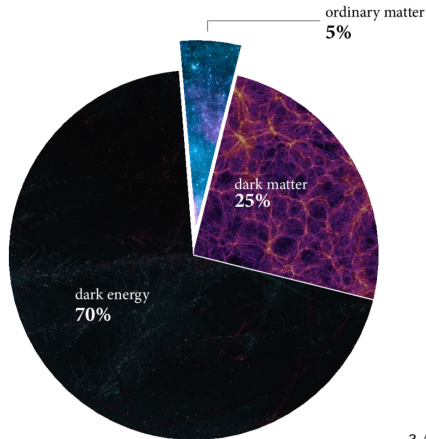
Overview

1. Current issues in Cosmology
2. The H_0 and S_8 tensions and Approaches to Resolve them
3. Addressing the H_0 and S_8 tensions in $f(Q)$ cosmology
4. Conclusions

Current issues in Cosmology

Λ CDM model: Cosmological constant Λ + **C**old **D**ark **M**atter

- **Open Issues:** The Cosmological constant problem, origin and properties of dark energy and dark matter, exact mechanism for inflation, non-renormalizability of General Relativity, H_0 and S_8 tensions



The H_0 tension

- **Planck Collaboration:** Indirect measurements from the Cosmic Microwave Background (assuming the Λ CDM)
- **SH0ES Collaboration (R19):** Direct measurements using Hubble Space Telescope observations of 70 long period Cepheids in the Large Magellanic Cloud.

The H_0 tension

- **Planck Collaboration:** Indirect measurements from the Cosmic Microwave Background (assuming the Λ CDM)
- **SH0ES Collaboration (R19):** Direct measurements using Hubble Space Telescope observations of 70 long period Cepheids in the Large Magellanic Cloud.

Planck Collaboration

$$H_0 = 67.27 \pm 0.6 \text{ km/s/Mpc}$$

SH0ES Collaboration (R19)

$$H_0 = 74.03 \pm 1.42 \text{ km/s/Mpc}$$

The H_0 tension

- **Planck Collaboration:** Indirect measurements from the Cosmic Microwave Background (assuming the Λ CDM)
- **SH0ES Collaboration (R19):** Direct measurements using Hubble Space Telescope observations of 70 long period Cepheids in the Large Magellanic Cloud.

Planck Collaboration
 $H_0 = 67.27 \pm 0.6 \text{ km/s/Mpc}$

SH0ES Collaboration (R19)
 $H_0 = 74.03 \pm 1.42 \text{ km/s/Mpc}$

Currently in tension at $\sim 5\sigma$!

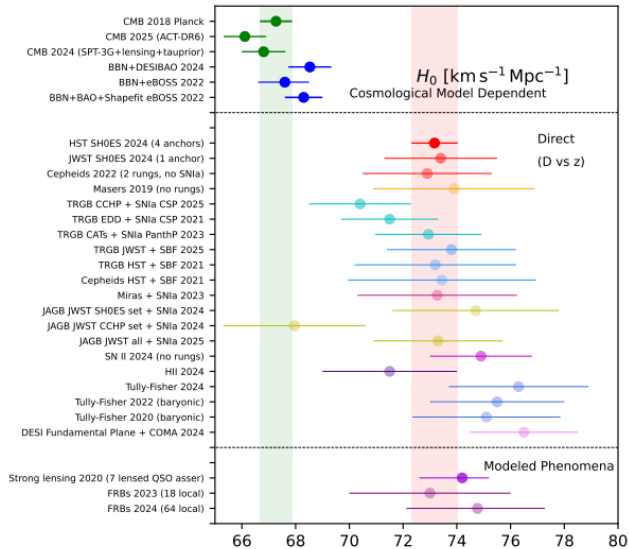
Hubble Parameter: $H \equiv \dot{a}/a$, a : the scale factor

The H_0 tension

The discrepancy in the Hubble constant measurements by early observations (CMB, assuming Λ CDM) and late time model-independent methods



CosmoVerse White Paper 2025



The S_8 tension

- The distribution of galaxies and matter in the late Universe's evolution, appears smoother than anticipated based on the evolution of fluctuations observed in the CMB.

Planck Collaboration

$$S_8 = 0.834 \pm 0.016$$

LSS Surveys: KiDS-450, DES

$$S_8 = 0.745 \pm 0.039 \text{ and } S_8 = 0.783^{+0.021}_{-0.025}$$

The S_8 tension

- The distribution of galaxies and matter in the late Universe's evolution, appears smoother than anticipated based on the evolution of fluctuations observed in the CMB.

Planck Collaboration

$$S_8 = 0.834 \pm 0.016$$

LSS Surveys: KiDS-450, DES

$$S_8 = 0.745 \pm 0.039 \text{ and } S_8 = 0.783^{+0.021}_{-0.025}$$

In tension at $\sim 2 - 3\sigma$!

$S_8 = \sigma_8(\Omega_{m0}/0.3)^{0.5}$: amplitude of matter clustering

σ_8 : root mean square of the amplitude of matter perturbations flattened over $8h^{-1}$ Mpc

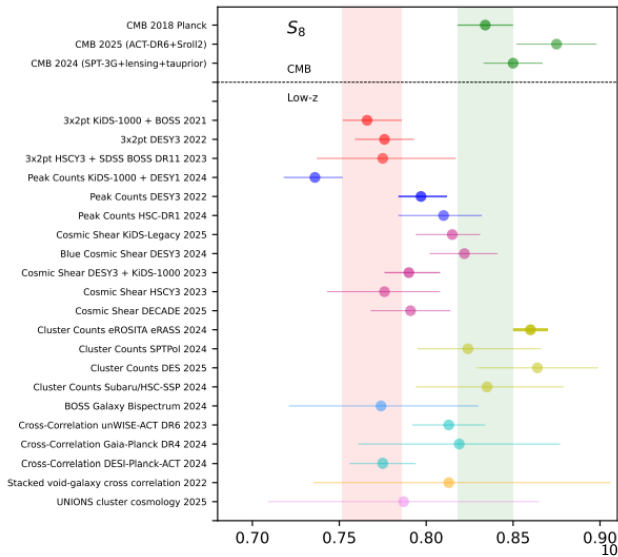
h : Hubble constant in units of 100 km/s/Mpc

The S_8 tension

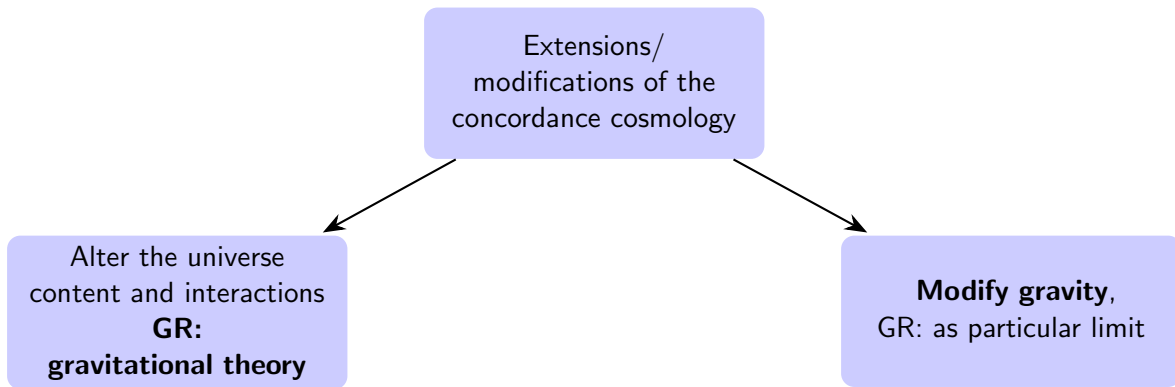
The discrepancy in the S_8 measurements by early and late Universe observations



CosmoVerse White Paper 2025



Approaches for Addressing the H_0 and S_8 tensions



$f(Q)$ gravity

The total action of the theory is

$$S = -\frac{1}{2} \int d^4x \sqrt{-g} f(Q) \quad (1)$$

- $Q = -\frac{1}{4} Q_{\alpha\beta\gamma} Q^{\alpha\beta\gamma} + \frac{1}{2} Q_{\alpha\beta\gamma} Q^{\gamma\beta\alpha} + \frac{1}{4} Q_{\alpha} Q^{\alpha} - \frac{1}{2} Q_{\alpha} \tilde{Q}^{\alpha}$
- where $Q_{\alpha} \equiv Q_{\alpha}^{\mu}{}_{\mu}$, and $\tilde{Q}^{\alpha} \equiv Q_{\mu}^{\mu\alpha}$ are contractions of the non-metricity tensor:
 $Q_{\alpha\mu\nu} \equiv \nabla_{\alpha} g_{\mu\nu}$

(STEGR recovered: $f = Q/8\pi G$)

J. Beltrán Jiménez et al, Phys. Rev. D 101 (2020) no.10, 103507, arXiv:1906.10027

$f(Q)$ Cosmology

We consider an expanding Universe, described by a flat homogeneous and isotropic **Friedmann-Lemaître-Robertson-Walker (FLRW)** geometry and given the field equations we obtain the modified Friedmann equations:

$$6f_Q H^2 - \frac{1}{2}f = \rho_m \quad (2)$$

$$(12H^2 f_{QQ} + f_Q)\dot{H} = -\frac{1}{2}(\rho_m + p_m) \quad (3)$$

where: $Q = 6H^2$, $H \equiv \dot{a}/a$.

For $f(Q) = Q - 2\Lambda$, we recover Λ CDM

f(Q) Cosmology

At the perturbation level for a general $f(Q)$ form we obtain

$$\delta'' + \mathcal{H}\delta' = \frac{4\pi G\rho_m}{f_Q}\delta \quad (4)$$

where

$\delta = \delta\rho_m/\rho_m$: matter overdensity
 $\mathcal{H} = a'/a = aH$ and $f_Q = \partial f/\partial Q$

The effective Newton's constant is given by

$$G_{\text{eff}} \equiv \frac{G}{f_Q} \quad (5)$$

Addressing H_0 and S_8 tensions

C. G. Boiza, M. Petronikolou, M. Bouhmadi-Lopez, E.N. Saridakis, arXiv:2505.18264

We explore the observational implications of 3 specific $f(Q)$ forms

$$f_1(Q) = Q \exp\{(\lambda Q_0/Q)\} \quad (6)$$

$$f_2(Q) = Q + Q_0 \exp\{(-\lambda Q_0/Q)\} \quad (7)$$

$$f_3(Q) = Q + \lambda Q_0 [1 - \exp\{(-Q_0/Q)\}] \quad (8)$$

Setting $z = 0$, ($H = H_0$), in the Friedmann eqs., the λ parameter for each model is given by:

- $\lambda = 0.5 + \mathcal{W}_0\left(-\frac{\Omega_{m0}}{2\sqrt{e}}\right)$
- $\lambda = 0.5 - \mathcal{W}_0\left[-\frac{\sqrt{e}}{2}(\Omega_{m0} - 1)\right]$
- $\lambda = \frac{e}{1+e}(1 - \Omega_{m0})$

where \mathcal{W}_0 is the principal branch of the Lambert function and Ω_{m0} the present matter density parameter

Data and Methodology

We perform likelihood analysis using Markov Chain Monte Carlo (MCMC)

Data sets:

- **Combination I**: Cosmic chronometers (CC), supernovae (SNIa) and Gamma-ray bursts (GRB), with $\theta = \{H_0, \Omega_{m0}\}$
- **Combination II**: Baryon acoustic oscillations (BAO) and Redshift-space distortions (RSD), with $\theta = \{\Omega_{m0}, S_8\}$
- **Combination III**: CC + SN + GRB + BAO + RSD (Full combination), with $\theta = \{H_0, \Omega_{m0}, r_d, S_8\}$

where $r_d = \int_{z_d}^{\infty} \frac{c_s(z)}{H(z)} dz$

Results: Combinations I (CC + SN + GRB) & II (BAO + RSD)

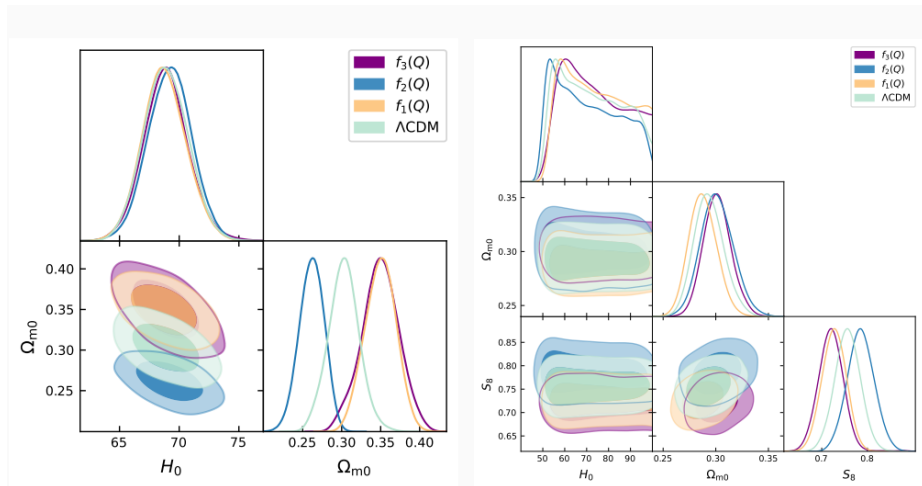


Figure: Two-dimensional posterior distributions for the $f(Q)$ models and Λ CDM scenario. The contours correspond to the 68% and 95% confidence levels (C.L.) in the Ω_{m0} - H_0 plane

Results: Combination III (CC + SN + GRB + BAO + RSD)

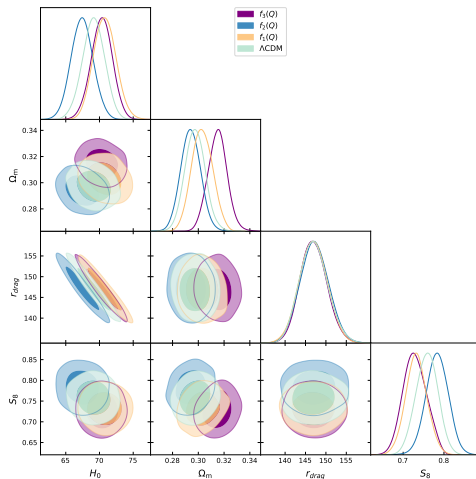


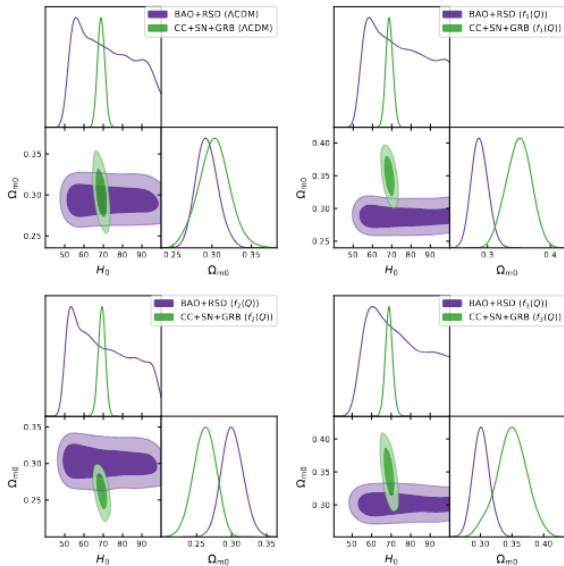
Figure: Two-dimensional posterior distributions for the $f(Q)$ models and ΛCDM scenario, using Combination III (CC + SN + GRB + BAO + RSD). The contours correspond to the 68% and 95% confidence levels (C.L.).

Parameter Estimation

Model	H_0	Ω_{m0}	r_d	S_8	ΔAIC
CC + SN + GRB					
$f_3(Q)$	68.91 ± 1.90	0.3495 ± 0.0241	—	—	-0.23
$f_2(Q)$	69.20 ± 1.84	0.2616 ± 0.0160	—	—	1.67
$f_1(Q)$	68.76 ± 1.85	0.3497 ± 0.0200	—	—	0.17
ΛCDM	68.89 ± 1.86	0.3027 ± 0.0198	—	—	—
BAO + RSD					
$f_3(Q)$	—	0.3015 ± 0.0133	—	0.7206 ± 0.0285	2.63
$f_2(Q)$	—	0.3013 ± 0.0156	—	0.7856 ± 0.0294	0.51
$f_1(Q)$	—	0.2877 ± 0.0132	—	0.7270 ± 0.0263	2.30
ΛCDM	—	0.2937 ± 0.0142	—	0.7567 ± 0.0279	—
CC + SN + GRB + BAO + RSD					
$f_3(Q)$	70.31 ± 1.71	0.3163 ± 0.0117	147.09 ± 3.49	0.7280 ± 0.0270	6.08
$f_2(Q)$	68.01 ± 1.67	0.2827 ± 0.0109	147.62 ± 3.46	0.7773 ± 0.0280	5.19
$f_1(Q)$	70.56 ± 1.69	0.3080 ± 0.0113	146.98 ± 3.43	0.7361 ± 0.0264	8.90
ΛCDM	69.15 ± 1.73	0.2958 ± 0.0115	147.33 ± 3.57	0.7580 ± 0.0271	—

$\text{AIC} = -2 \ln L_{\max} + 2k$, where L_{\max} is the maximum likelihood of the model
 k is the number of free parameters

Combination I (CC + SN + GRB) and II (BAO + RSD)



Conclusions

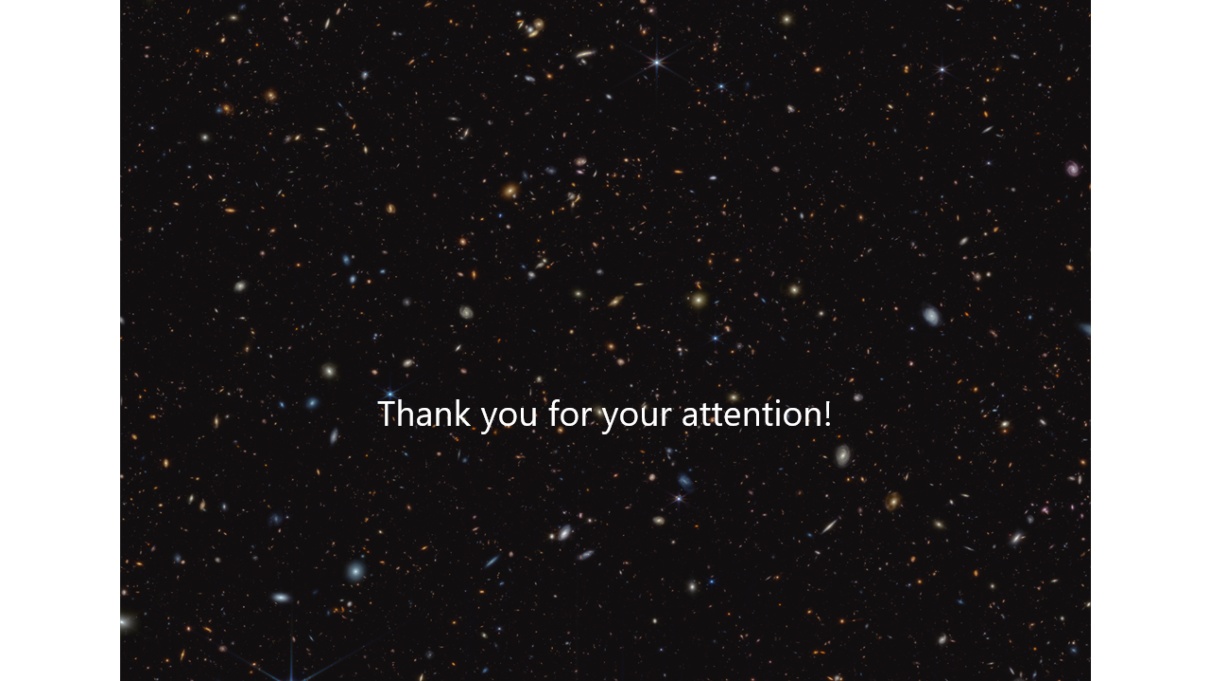
- In this work, we explored $f(Q)$ gravity, introducing late-time modifications that not only reproduce the observed cosmological behavior, but also show potential in alleviating the H_0 and S_8 tensions.

Conclusions

- In this work, we explored $f(Q)$ gravity, introducing late-time modifications that not only reproduce the observed cosmological behavior, but also show potential in alleviating the H_0 and S_8 tensions.
- Key findings from dataset analysis:
 - Models 1 & 3 favor higher Ω_{m0} in background probes, H_0 increases when all datasets are combined.
 - Model 2 favors lower Ω_{m0} , shows suppressed growth and $G_{eff} < G$, potentially addressing S_8 tension.

Conclusions

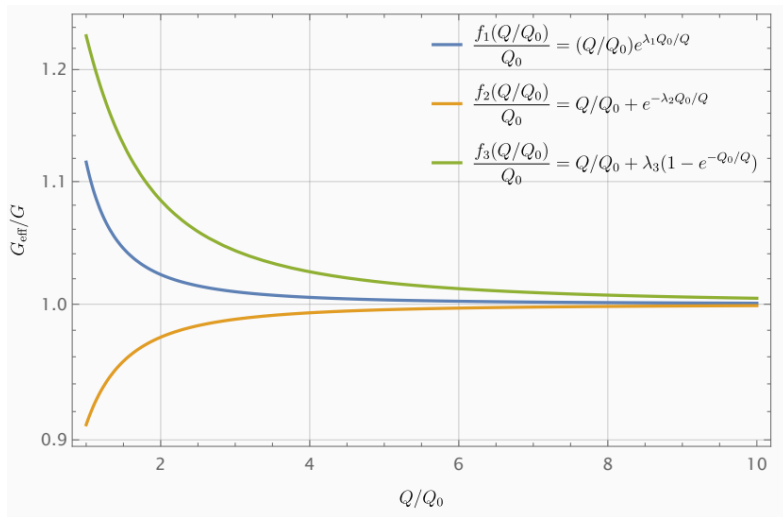
- In this work, we explored $f(Q)$ gravity, introducing late-time modifications that not only reproduce the observed cosmological behavior, but also show potential in alleviating the H_0 and S_8 tensions.
- Key findings from dataset analysis:
 - Models 1 & 3 favor higher Ω_{m0} in background probes, H_0 increases when all datasets are combined.
 - Model 2 favors lower Ω_{m0} , shows suppressed growth and $G_{eff} < G$, potentially addressing S_8 tension.
- Future work: Incorporate additional probes (e.g. CMB) and explore more flexible $f(Q)$ functional forms.



Thank you for your attention!

Appendix

Evolution of G_{eff}/G



AIC (Akaike Information Criterion)

Interpretation of AIC values based on Jeffreys' scale.

ΔAIC	Interpretation
> 10	Desively disfavoured
$5 \sim 10$	Strongly disfavoured
$2 \sim 5$	Moderately disfavoured
$-2 \sim 2$	Compatible
$-5 \sim -2$	Moderately favoured
$-10 \sim -5$	Strongly favoured
< -10	Decisively favoured

Field Equations

$$\begin{aligned} & \frac{2}{\sqrt{-g}} \nabla_{\alpha} \left\{ \sqrt{-g} g_{\beta\nu} f_Q \left[-\frac{1}{2} L^{\alpha\mu\beta} + \frac{1}{4} g^{\mu\beta} (Q^{\alpha} - \tilde{Q}^{\alpha}) \right. \right. \\ & \quad \left. \left. - \frac{1}{8} (g^{\alpha\mu} Q^{\beta} + g^{\alpha\beta} Q^{\mu}) \right] \right\} \\ & + f_Q \left[-\frac{1}{2} L^{\mu\alpha\beta} - \frac{1}{8} (g^{\mu\alpha} Q^{\beta} + g^{\mu\beta} Q^{\alpha}) \right. \\ & \quad \left. + \frac{1}{4} g^{\alpha\beta} (Q^{\mu} - \tilde{Q}^{\mu}) \right] Q_{\nu\alpha\beta} + \frac{1}{2} \delta_{\nu}^{\mu} f = T^{\mu}_{\nu}, \end{aligned} \tag{9}$$

where

$L^{\alpha}_{\mu\nu} \equiv \frac{1}{2} Q^{\alpha}_{\mu\nu} - Q^{\alpha}_{(\mu\nu)}$ is the disformation tensor, and T^{μ}_{ν} is the standard energy-momentum tensor.