Explaining Dark Matter through Primordial Black Holes (PBHs) in Horndeski Gravity

Despina Totolou

Aristotle University of Thessaloniki & National Observatory of Athens

September 2, 2025

Modified Gravity

- GR's spacetime singularities $\rightarrow$ can be avoided
- non-renormalizability $\rightarrow$ can be improved
- Dark Matter $\rightarrow$ can be explained directly (gravity itself)
  or indirectly (its origin)
- Dark Energy $\rightarrow$ can be explained directly (gravity)
  or equivalently as an effective fluid or field

Horndeski framework

Most general scalar-tensor theory in 4-D with one scalar field, leading to 2nd-order field equations and no ghosts (Ostrogradsky)

$$\mathcal{L} = \sum_{i=2}^{5} \mathcal{L}_i$$

$$\begin{aligned}
\mathcal{L}_2 &= K(\phi, X) \\
\mathcal{L}_3 &= -G_3(\phi, X)\Box\phi \\
\mathcal{L}_4 &= G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)\right] \\
\mathcal{L}_5 &= G_5(\phi, X)G_{\mu\nu}(\nabla^\mu\nabla^\nu\phi) \\
&\quad - \frac{1}{6}G_{5,X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)\right. \\
&\qquad \left. + 2(\nabla^\mu\nabla_\alpha\phi)(\nabla^\alpha\nabla_\beta\phi)(\nabla^\beta\nabla_\mu\phi)\right] \\
X &\equiv -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi
\end{aligned}$$

~~$G_{4,X}$, G_5~~ (GW170817)

PBH concept

Black holes formed in the early universe
from collapse of overdensity regions

$\downarrow$

Inflation: enhancement of curvature power spectrum on small scales $\rightarrow$ collapse at horizon re-entry to form PBHs

$\downarrow$

Ulta-Slow Roll (USR), $V(\phi)$ features (step, bump, dip), hybrid/waterfall inflation, etc

$\downarrow$

$V(\phi)$ inflection point/plateau, modified friction, non-canonical kinetic terms

- Mass range: $\sim 10^5 g - 10^5 M_\odot$
- $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$, Standard USR $\rightarrow \ddot{\phi} \approx -3H\dot{\phi}$ $\rightarrow \dot{\phi} \propto e^{-3Ht}$

PBH abundance

(arXiv:1707.09578v3)

- $$f_{\rm PBH}(M) = \frac{\beta(M)}{3.94\times10^{-9}}\left(\frac{\gamma}{0.2}\right)^{1/2}\left(\frac{g_*}{10.75}\right)^{-1/4}\left(\frac{0.12}{\Omega_{\rm DM}h^2}\right)\left(\frac{M}{M_\odot}\right)^{-1/2}$$
- $$\beta(M) \approx \sqrt{\frac{2}{\pi}}\frac{\sqrt{P_\zeta}}{\mu_c}\exp\left(-\frac{\mu_c^2}{2P_\zeta}\right)$$
  $\mu_c = 9\delta_c/4$
- $$M(k) = 3.68\left(\frac{\gamma}{0.2}\right)\left(\frac{g_*}{10.75}\right)^{-1/6}\left(\frac{k}{10^6\ {\rm Mpc}^{-1}}\right)^{-2}M_\odot$$
- $\gamma = 0.2, g_* = 107.5$ for $T > 300 GeV, \delta_c = 0.4,\ \Omega_{\rm DM}h^2 = 0.12$

Model

- $$S = \int d^4x\sqrt{-g}\left[K(\phi, X) - \underline{G_3(\phi, X)}\Box\phi + \underline{G_4(\phi)}R\right]$$
  $\downarrow$
  derivative coupl. to $g_{\mu\nu}$
  - $G_4 = M_{Pl}^2/2$
  - $K = X - V(\phi)$

- $M_{Pl}^2 = 1,$

$$3H^2 = V + \frac{\dot\phi^2}{2} + \dot\phi^2(3H\dot\phi G_{3X} - G_{3\phi})$$

$$2\dot H + 3H^2 = V - \frac{\dot\phi^2}{2}(1 - 2[\ddot\phi G_{3X} + G_{3\phi}])$$

$$\ddot\phi + 3H\dot\phi\frac{1 + H\dot\phi G_{3X}[3-\epsilon] - 2G_{3\phi} + \dot\phi^2 G_{3\phi X}}{1 + 6H\dot\phi G_{3X} - 2G_{3\phi} + 3H\dot\phi^3 G_{3XX} - \dot\phi^2 G_{3\phi X}} + \frac{V_\phi - \dot\phi^2 G_{3\phi\phi}}{1 + 6H\dot\phi G_{3X} - 2G_{3\phi} + 3H\dot\phi^3 G_{3XX} - \dot\phi^2 G_{3\phi X}} = 0$$

$$\epsilon = -\frac{\dot H}{H^2} = \frac{\dot\phi^2}{2H^2}(1 - G_{3X}[\ddot\phi - 3H\dot\phi] - 2G_{3\phi})$$

Slow-roll inflation

- $G_3(\phi, X) = f(\phi)g(X)$ • $g(X) = Ae^{-nX}$, $A, n = const. > 0$
- $|\epsilon|, |\eta|, |\beta| \ll 1$ where $\eta = -\ddot{\phi}/(H\dot{\phi})$, $\beta = \dot{\phi}^2 G_{3\phi\phi}/V_\phi$

$\rightarrow$ SR

$$3H^2 \simeq V$$
$$3H\dot{\phi}(1 + D_1 + D_2) + V_\phi \simeq 0$$
$$\epsilon \simeq \frac{\dot{\phi}^2}{2H^2}(1 + D_1 + D_2)$$

$$D_1(\phi, X) = -3nAH\dot{\phi}fe^{-nX} = 3H\dot{\phi}G_{3X}$$
$$D_2(\phi, X) = -2Af_\phi e^{-nX} = -2G_{3\phi}$$
$$D_1 \ll D_2 \quad \rightarrow$$

$$3H^2 \simeq V$$
$$3H\dot{\phi}(1 + D_2) + V_\phi \simeq 0$$
$$\epsilon \simeq \frac{\dot{\phi}^2}{2H^2}(1 + D_2)$$

Perturbations

- 1st order power spectra

$$P_\zeta \simeq \frac{H^4}{4\pi^2\dot{\phi}^2(1+D_2)} \simeq \frac{V^3}{12\pi^2 V_\phi^2}(1+D_2)$$

$$P_T = \frac{2H^2}{\pi^2} \simeq \frac{2V}{3\pi^2}$$

$$n_S - 1 \simeq \frac{2}{1+D_2}\left(\eta_V - 3\epsilon_V + \frac{D_{2\phi}}{1+D_2}\sqrt{\frac{\epsilon_V}{2}}\right)$$

$$r \simeq 16\frac{\epsilon_V}{1+D_2}$$

$$\epsilon_V = (V'/V)^2/2$$

$$\eta_V = V''/V$$

- $f(\phi) = \frac{B}{\sqrt{1+\frac{(\phi-\phi_c)^2}{c^2}}}, \qquad B, c = const.$
- $V(\phi) = V_0 \ln(\alpha + \gamma\phi^\delta), \qquad \alpha, \delta, \gamma = const. > 0$

Parameters

(in prep)

- $\alpha = \gamma = \delta = 1$,
  $V_0 = 0.654 \times 10^{-9}$
  $n = 1, A = 10^3, c = 1.52 \times 10^{-10}, B = 7.69 \times 10^5, \phi_c = 2.7$

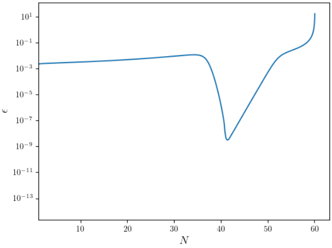
$$\downarrow$$

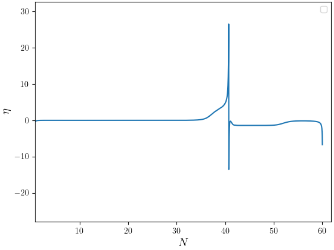
$$n_S = 0.9673, r = 0.038$$

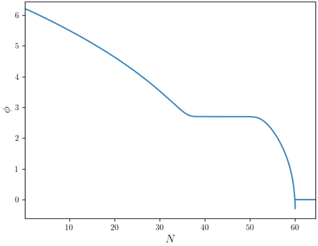
&

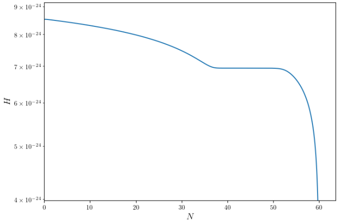
$$k_p = 3.0211 \times 10^{15}, P_{\zeta,p} = 1.157 \times 10^{-2},$$
$$M/M_\odot = 1.0987 \times 10^{-16}, f_{PBH} = 0.815$$

Slow-roll parameters ϵ. η, field ϕ, Hubble parameter H

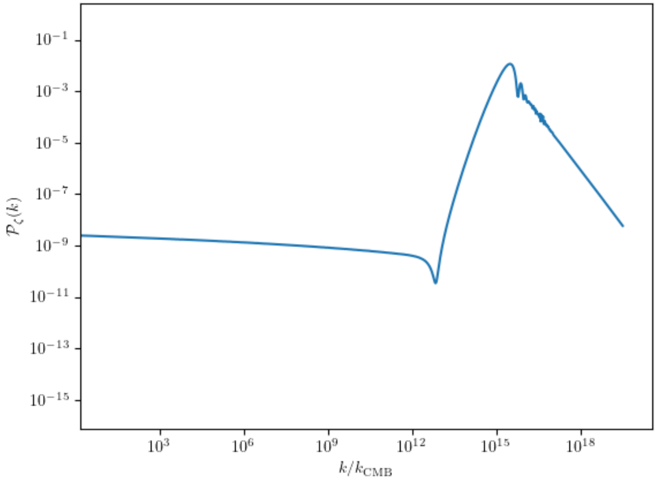




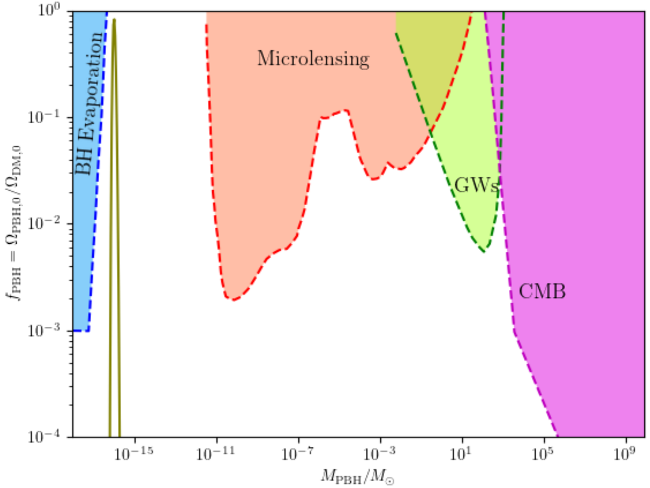




Curvature power spectrum



PBH abundance



Conclusions

- Successful inflation & PBH production, too
- Consistency with inflationary constraints, potentials like $\ln\phi$ could become favorable again & satisfies PBH formation requirements
- Could work for many types of potentials
- $g(X) \sim e^{-X}$-type dependence boosts $f(\phi)$'s contribution, too, giving the correct N, and appropriate enhancement for PBH production
- PBH formation's SIGW counterpart enhanced, too $\rightarrow$ potentially detectable

Thank you