RE-EXAMINING THE FOUNDATIONS OF SUPERGRAVITY: THE SPIN-1/2 SECTOR

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Recent Developments in Gravity

Corfu

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MAIN IDEA

Simple Supergravity is the minimal extension of GR that implements supersymmetry

It requires the introduction a new fermion field that transforms as a vector and a spinor

$$\psi_{\mu}^{\alpha}$$

How many degrees of freedom are carried by this field.

Early analysis around late 70´s shown that it carries two degrees of freedom, of helicities 3/2 and -3/2.

We recently understood that these results relied on some assumption that does not generally hold… otherwise helicity 1/2 and -1/2 modes also propagate.

BASIC SUPERGRAVITY

It massless case is essential in the (basic) supergravity action:

$$S = \frac{1}{2}\int d^4x\, e\left(R - \bar{\psi}_\mu \gamma^{\mu\nu\rho} D_\nu \psi_\rho\right),$$

Gauge supersymmetry

$$\delta e_\mu{}^a = \frac{1}{2}\bar{\epsilon}\gamma^a\psi_\mu, \quad \delta\psi_\mu = D_\mu\epsilon$$

RARITA SCHWINGER SECTOR

Flat case RS action: $$S = \frac{i}{2}\int d^4x\ \bar{\psi}_\mu \gamma^{\mu\nu\rho}\partial_\nu \psi_\rho,$$

$$\gamma^{\mu\nu\lambda}\partial_\nu\psi_\lambda = 0 \quad \longrightarrow \quad \not{\partial}\psi_\mu - \partial_\mu\,\gamma\cdot\psi = 0\,, \qquad \partial\cdot\psi - \not{\partial}\gamma\cdot\psi = 0$$

Gauge symmetry $\delta\psi_\mu = \partial_\mu\epsilon$

$$\not{\partial}\psi_\mu = 0\,, \qquad \partial^\mu\psi_\mu = 0\,, \qquad \gamma^\mu\psi_\mu \overset{gf}{=} 0$$

UNCONVENTIONAL SUSY

$$S[\psi_\mu = \gamma_\mu \kappa] = i\frac{(D-1)(D-2)}{2}\int \bar{\kappa}\, \partial\!\!\!/\, \kappa\,,$$

Matter Ansatz: **Supersymmetry of a different kind,** P. Alvarez, M. V, and J. Zanelli, JHEP04(2012)058 arXiv:1109.3944.

Unconventional Supersymmetry

Reassessing the foundations of Metric-Affine Gravity #1

J. François (Masaryk U., Brno (main) and U. Graz (main) and U. Mons), L. Ravera (Polytech. Turin and INFN, Turin and Concepcion Catolica U.) (May 8, 2025)

e-Print: 2505.05349 [gr-qc]

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Thales Bittencourt (Niteroi, Fluminense U.), Rodrigo Sobreiro (Niteroi, Fluminense U.) (Mar 25, 2025)

e-Print: 2503.19803 [hep-th]

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J. François (Masaryk U., Brno and Graz U. and U. Mons), L. Ravera (Polytech. Turin and Turin U. and Concepcion Catolica U.) (Mar 24, 2025)

Published in: *Annalen Phys.* (2025) • e-Print: 2503.19077 [hep-th]

pdf DOI cite claim reference search 4 citations

D=4, N=2 Supergravity: An Unconventional Application #4

Laura Andrianopoli (Polytech. Turin and INFN, Turin) (Feb 24, 2025)

e-Print: 2502.17324 [hep-th]

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Three "layers" of graphene monolayer and their analog generalized uncertainty principles #21

Alfredo Iorio, Boris Ivetić, Salvatore Mignemi, Pablo Pais (Jul 29, 2022)

Published in: *Phys.Rev.D* 106 (2022) 11, 116011 • e-Print: 2208.02237 [gr-qc]

pdf DOI cite claim reference search 16 citations

Hunting Quantum Gravity with Analogs: The Case of Graphene + #22

Giovanni Acquaviva (Arquimea Research Center), Alfredo Iorio (Charles U.), Pablo Pais (Charles U. and Chile Austral U., Valdivia), Luca Smaldone (Warsaw U.) (Jul 8, 2022)

Published in: *Universe* 8 (2022) 9, 455 • e-Print: 2207.04097 [hep-th]

pdf DOI cite claim reference search 19 citations

On AdS_4 Holography - Towards applications to 2+1 dimensional graphene-like systems #24

Riccardo Matrecano (Turin Polytechnic) (Jun 7, 2022)

Graphene, Dirac equation and analogue gravity #26

Antonio Gallerati (Polytech. Turin and INFN, Turin) (May 18, 2022)

Published in: *Phys.Scripta* 97 (2022) 6, 064005, *Phys.Scripta* 97 (2022) 064005 • Contribution to: ICNFP 2021 • e-Print: 2205.08843 [hep-th]

pdf DOI cite claim reference search 10 citations

HELMOLTZ DECOMPOSITION OF THE MAXWELL FIELD

$$S[A] = -\frac{1}{4}\int F^{\mu\nu}F_{\mu\nu}$$

$$S[A+\partial\phi] = S[A], \qquad S[\partial\phi] \approx 0$$

Transverse and Longitudinal decomposition

$$(P^{\perp})_{\mu}{}^{\nu} + (P^{\parallel})_{\mu}{}^{\nu} = \delta_{\mu}{}^{\nu}$$

$$A_\mu = A_\mu^{\perp} + A_\mu^{\parallel}, \qquad A_\mu^{\perp} := \left(\delta_\mu^\nu - \frac{\partial_\mu\partial^\nu}{\Box}\right)A_\nu, \quad A_\mu^{\parallel} := \left(\frac{\partial_\mu\partial^\nu}{\Box}\right)A_\nu$$

$$S[A^{\perp} + A^{\parallel}] = -\frac{1}{4}\int A^{\perp\mu}\,\Box\,A_\mu^{\perp} \approx S[A^{\perp}]$$

OBSERVATIONS

- **The pure gauge modes** $(\partial_\mu \lambda)$ belong to the Kernel of the action functional

- $\dot{A}_0^T = \overrightarrow{\nabla} \cdot \overrightarrow{A}^T$ is not independent

- Using the gauge symmetry we can chose $A_3 = 0$, for example.

- **So the Maxwell action describe two degrees of freedom**

HELMOLTZ DECOMPOSITION OF THE RS FIELD: SPIN-BLOCK PROJECTORS

Helmholtz-like decomposition using **Behrends-Fronsdal projectors**

R.E. Behrends, C. Fronsdal. Fermi Decay of Higher Spin Particles. Phys.Rev. 106 (1957) 2, 345

V. I. Ogievetsky and E. S. Sokatchev. Equations of Motion for Superfields. In *4th International Conference on Nonlocal Quantum Field Theory*, pages 183–203, 1976.

P. van Nieuwenhuizen. Supergravity. *Phys. Rept.*, 68:189, 1981.

$$P^{(\perp\!\perp)}{}_{\mu\nu} = \theta_{\mu\nu} - \frac{1}{\mathtt{D}-1}\hat{\gamma}_\mu\hat{\gamma}_\nu\,, \qquad P^{(\gamma)}{}_{\mu\nu} = \frac{1}{\mathtt{D}-1}\hat{\gamma}_\mu\hat{\gamma}_\nu, \qquad P^{(\partial)}{}_{\mu\nu} = w_\mu w_\nu$$

$$\theta_{\mu\nu} := \eta_{\mu\nu} - \omega_\mu\omega_\nu\,, \qquad \hat{\gamma}_\mu := \gamma_\mu - \omega_\mu\,, \qquad \omega_\mu = \not{\partial}^{-1}\partial_\mu$$

$$\hat{\gamma}^\mu\hat{\gamma}_\mu = \mathbb{1}\,, \qquad \omega^\mu\omega_\mu = \mathbb{1}\,, \qquad \omega^\mu\hat{\gamma}_\mu = 0\,.$$

$$\xi_\mu := \psi_\mu^{\perp\!\perp}\,, \qquad \lambda := w^\mu\psi_\mu = \not{\partial}^{-1}\partial^\mu\psi_\mu\,, \qquad \zeta := (\mathtt{D}-1)^{-1}\hat{\gamma}^\mu\psi_\mu$$

$$\gamma^\mu\xi_\mu \equiv 0\,, \qquad \partial\cdot\xi \equiv 0$$

VECTOR-SPINOR DECOMPOSITION

$$\psi_\mu^\alpha = \xi_\mu^\alpha + (\hat{\gamma}_\mu \zeta)^\alpha + (\omega_\mu \lambda)^\alpha$$

Spin-decomposition (1+0) x 1/2 = 3/2 + 1/2 + 1/2

Gauge transformations $\delta\psi_\mu = \partial_\mu \epsilon \quad \Rightarrow \quad \delta\xi = 0 = \delta\zeta\,, \;\; \delta\lambda = \partial\!\!\!/\epsilon,$

$$\mathcal{L} := -\frac{i}{2}\bar{\psi}_\mu \gamma^{\mu\nu\lambda}\partial_\nu \psi_\lambda \quad \longrightarrow \quad \mathcal{L} = -\frac{i}{2}\Big(\bar{\xi}_\mu \partial\!\!\!/ \xi^\mu - (\mathrm{D}-1)(\mathrm{D}-2)\bar{\zeta}\partial\!\!\!/\zeta\Big)$$

$$\psi_\mu^\alpha = \xi_\mu^\alpha + (\hat{\gamma}_\mu \zeta)^\alpha + (\omega_\mu \lambda)^\alpha$$

- We can use the gauge symmetry to remove ψ_3.
  Hence ξ_μ, ζ, and λ, depend only on ψ_0, ψ_1, ψ_2
- λ Does not show up in the action or in the EoMs because it is the longitudinal mode.
- From $\gamma^\mu \xi_\mu \equiv 0\,,$ $\partial \cdot \xi \equiv 0$, two components of ξ_μ are not independent, so it carries only 4 off-shell dof.
- There remain 8 off-shell dof.

…. which satisfy the Dirac equation. Hence 4 helicities propagate, of values $\pm 1/2, \pm\, 3/2,$.

DEMONSTRATION

$$\partial\!\!\!/ \psi_\mu = 0\,, \qquad \partial^\mu \psi_\mu = 0\,, \qquad \gamma^\mu \psi_\mu \overset{gf}{=} 0$$

Using the projector operators:

$$\partial\!\!\!/ \xi_\mu \approx 0\,, \qquad \partial\!\!\!/ \zeta \approx 0\,, \qquad \partial \cdot \xi \equiv 0\,, \qquad \gamma \cdot \xi \equiv 0$$

SPACE-TIME SPLITTING

$$\mathcal{L} = -i\bar{\psi}_0\gamma^{0ij}\partial_i\psi_j + \frac{i}{2}\bar{\psi}_i\gamma^{0ij}\dot{\psi}_j - \frac{i}{2}\bar{\psi}_i\gamma^{ijk}\partial_j\psi_k$$

- We introduce spatial spin-block projectors

$$\psi_i = \xi_i + N_i\zeta + L_i\lambda$$

$$(P^N)_{ij} := \frac{1}{D-2}N_iN_j\,, \qquad (P^L)_{ij} := L_iL_j, \qquad P^{\perp\!\!\!\perp} = \mathbb{1} - P^N - P^L$$

$$L_i := \not{\nabla}^{-1}\partial_i \qquad N_i := \gamma_i - L_i \qquad \not{\nabla}^{-1} := (\gamma^i\partial_i)^{-1}$$

$$N_i N^i = D-2,\ L_i L^i = 1,\ N_i L^i = 0.$$

$$\xi_i = P^{\perp\!\!\!\perp}{}_i{}^j\psi_j\,, \quad \zeta = \frac{1}{D-2}N^i\psi_i\,, \quad \lambda = L^i\psi_i$$

$$\not{\partial}\psi_\mu = 0\,, \qquad \partial^\mu \psi_\mu = 0\,, \qquad \gamma^\mu \psi_\mu \stackrel{gf}{=} 0$$

The gauge fixed RS equations reduce to

$$\not{\partial}\xi_i = 0\,, \qquad \not{\partial}\tilde{\lambda} = 0\,, \qquad \dot{\zeta} = 0\,, \qquad \not{\nabla}\zeta = 0$$

$$\tilde{\lambda} = \gamma^0 \lambda \qquad\qquad \psi_0 = -\gamma_0 \gamma^i \psi_i = -\gamma_0((D-2)\zeta + \lambda)$$

HAMILTONIAN APPROACH

TOTAL HAMILTONIAN DYNAMICS

In order to obtain a variational principle in phase-space equivalent to a gauge invariant Lagrangian we need to add the "primary constraints" to the Hamiltonian action;

$$S_T = \int dt(\dot{q}^s p_s - H_T)$$

Total Hamiltonian $\quad H_T := H_C + c_{(0)} \cdot \mu_{(0)} \,,$

Equations of motion: $\quad \dot{f} = \{f, H_T\} \,, \qquad c_{(0)}(q,p) \approx 0$

The total Hamiltonian dynamics is equivalent to the Lagrangian one

SECONDARY CONSTRAINTS

$$c_1 := \dot{c}_0 = \{H_T, c_0\} \approx 0.$$

Suppress other phase space directions

Actually, we demand, $\frac{d^n c_{(0)}}{dt^n} = 0\,, \qquad n \geq 1$

Which yields a hierarchy of constraints,

$$c_{(n)}(q(t), p(t)) := \{H_T, c_{(n-1)}\} = 0$$

FIRST-CLASS AND SECOND-CLASS CONSTRAINTS

Constraints split in "first-class" and "second-class"

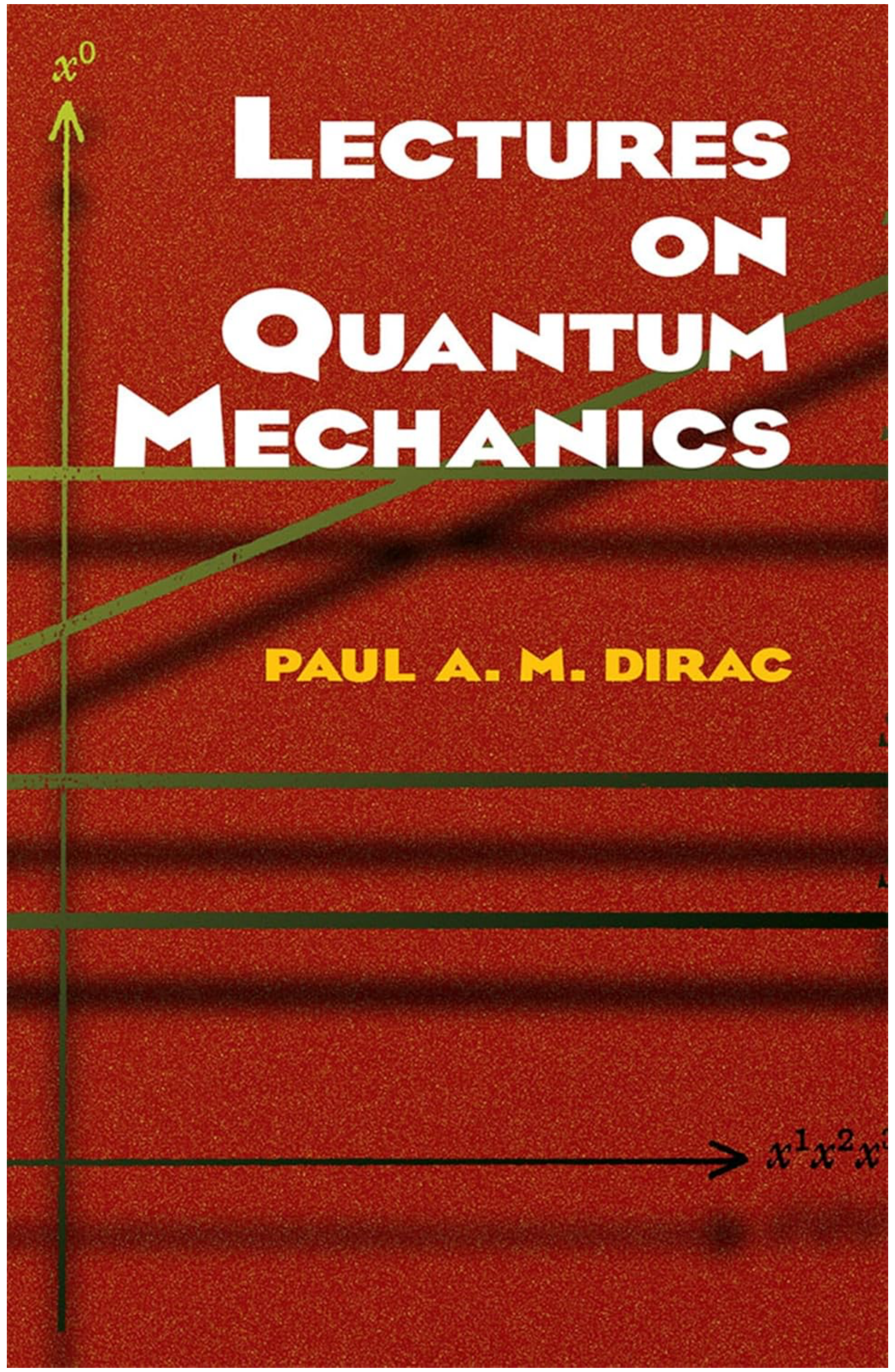
$$\{c_{\text{fc}}, c_{\text{fc}}\} \approx 0, \quad \{c_{\text{sc}}, c_{\text{sc}}\} \approx M, \quad \{c_{\text{fc}}, c_{\text{sc}}\} \approx 0$$

where M is an invertible matrix

$$H_T = H_C + \mu\, c_{\text{fc}} + \nu\, c_{\text{sc}}$$

The algebra structure is such that Lagrange multipliers associated to FC constraints are not determined by the EoMs, while those associated to SC are.

DIRAC HAMILTONIAN DYNAMICS



Dirac's conjecture

I believe that

first-class secondary constraints should be included among the transformations which don't change the physical state, but I haven't been able to prove it. Also, I haven't found any example for which there exist first-class secondary constraints which do generate a change in the physical state.

Then Dirac proposes to "modify" the (total Hamiltonian dynamics) by the one given by the "extended Hamiltonian"

$$H_E = H_T + c_{(1)}\mu_{(1)} + c_{(2)}\mu_{(2)} + \dots$$

RARITA-SCHWINGER HAMILTONIAN AND CONSTRAINT STRUCTURE

$$\mathcal{L} = -i\bar{\psi}_0\gamma^{0ij}\partial_i\psi_j + \frac{i}{2}\bar{\psi}_i\gamma^{0ij}\dot{\psi}_j - \frac{i}{2}\bar{\psi}_i\gamma^{ijk}\partial_j\psi_k$$

Primary constraint

$$\pi^0 \approx 0\,,$$

$$\chi^i := \pi^i - \frac{i}{2}\mathcal{C}^{ij}\psi_j \approx 0\,, \qquad \{\chi^i, \chi^j\} = i\mathcal{C}^{ij}$$

$$\mathcal{C}^{ij}_{\alpha\beta} := -(C\gamma^{0ij})_{\alpha\beta} = \mathcal{C}^{ji}_{\beta\alpha} \text{ is invertible, } \mathcal{C}^{ij}_{\alpha\beta}(\mathcal{C}^{-1})^{\beta\kappa}_{jm} := \delta^i_m\delta^\kappa_\alpha$$

$$H_T = \int d^{D-1}x\left(i\bar{\psi}_0\gamma^{0ij}\partial_i\psi_j + \frac{i}{2}\bar{\psi}_i\gamma^{ijk}\partial_j\psi_k + \chi^i_\alpha\mu^\alpha_i + \pi^0_\alpha\mu^\alpha_0\right)$$

Secondary constraint

$$\dot{\pi}^0 = -\{H_T, \pi^0\} = -\frac{\delta H}{\delta\psi_0} = -iC\gamma^{0ij}\partial_i\psi_j \approx 0 \quad \Leftrightarrow \quad \varphi := -i\mathcal{C}^{ij}\partial_i\psi_j \approx 0$$

On the surface of the second class constraints and using the Helmholtz decomposition

$$H_T|_{\chi\approx 0} = \int d^{D-1}x \left(i(D-2)\bar{\psi}_0\gamma^0 \nabla\!\!\!/ \zeta - \frac{i(D-2)(D-3)}{2}\bar{\zeta}\nabla\!\!\!/\zeta + \frac{i}{2}\bar{\xi}^i \nabla\!\!\!/ \xi_i + \pi^0_\alpha \mu_0^\alpha \right)$$

$$\varphi \approx 0 \text{ is equivalent to } \nabla\!\!\!/\zeta \approx 0$$

The Poisson bracket is replaced by the Dirac bracket

$$\begin{aligned}\{f\,,g\}_D \;=\; (-1)^f \int d^{D-1}z \Bigg[& -i\frac{\delta f}{\delta \xi_i} P^{\perp\!\!\perp}_{ij}\gamma_0 C^{-1}\frac{\delta g}{\delta \xi_j} - i\frac{D-3}{D-2}\frac{\delta f}{\delta \lambda}\gamma_0 C^{-1}\frac{\delta g}{\delta \lambda} \\ & + i\frac{1}{D-2}\left(\frac{\delta f}{\delta \lambda}\gamma_0 C^{-1}\frac{\delta g}{\delta \zeta} + \frac{\delta f}{\delta \zeta}\gamma_0 C^{-1}\frac{\delta g}{\delta \lambda}\right) + \left(\frac{\delta f}{\delta \psi_0^\alpha}\frac{\delta g}{\delta \pi^0_\alpha} + \frac{\delta f}{\delta \pi^0_\alpha}\frac{\delta g}{\delta \psi_0^\alpha}\right)\Bigg]\end{aligned}$$

An we obtain the EoM

Total-Hamiltonian EoM

$$\begin{aligned}\dot{\xi}_i &= -\gamma_0 \nabla\!\!\!/ \xi_i\,, \qquad \dot{\lambda} = -(D-3)\gamma_0\nabla\!\!\!/\zeta + \nabla\!\!\!/\psi_0 \\ \dot{\psi}_0 &= -\mu_0\,, \qquad \pi^0 = 0\,, \qquad \dot{\zeta} = 0, \qquad \nabla\!\!\!/\zeta = 0\,.\end{aligned}$$

SHOULD WE USE THE EXTENDED HAMILTONIAN INSTEAD?

Conservative approach

Do not assume the Dirac conjecture: Then gauge fix only for primary first class constraints.

If unphysical functions still remain in the system

Then: Assume the Dirac conjecture. Then add secondary 1st class constraint to the Total Hamiltonian and gauge fix for all first class constraint.

CONSERVATIVE APPROACH

We only gauge-fix for the primary first class constraint

$$\gamma^\mu \psi_\mu \approx 0 \qquad \longleftrightarrow \qquad \psi_0 + \gamma_0 \gamma^i \psi_i \approx 0$$

$$\not{\partial} \xi_i = 0\,, \qquad \not{\partial} \tilde{\lambda} = 0\,, \qquad \tilde{\lambda} = \gamma_0 \lambda$$

Explicit solution with spin 1/2 and 3/2

$$\psi_0 = \gamma^0 \lambda\,, \qquad \psi_i = \xi_i + \partial_i \not{\nabla}^{-1} \lambda$$

We do not observe the existence of unphysical degrees of freedom!

ASSUMING THE DIRAC-CONJECTURE

We add the secondary 1st class constraint back to the Hamiltonian with a Lagrange multiplier

$$H_T|_{\chi\approx 0} \longrightarrow H_T|_{\chi\approx 0} + \tau^\alpha \varphi_\alpha \qquad \varphi = \not\nabla\zeta \approx 0$$

$$\begin{aligned} \dot{\xi}_i &= -\gamma_0 \not\nabla \xi_i\,, \qquad \dot{\lambda} = -(D-3)\gamma_0 \not\nabla\zeta + \not\nabla\psi_0 + \not\nabla\tau\,, \\ \dot{\psi}_0 &= -\mu_0\,, \qquad \pi^0 = 0\,, \qquad \dot{\zeta} = 0, \qquad \not\nabla\zeta = 0\,. \end{aligned}$$

Since $\{\not\nabla\zeta\,,\lambda\} \approx \not\nabla$, the constraint's conjugate variable is lambda.

We have to gauge fix, lambda, as eg. $\lambda = 0$. Hence $\longrightarrow$ $\not\partial\xi_i = 0$

And the spin 1/2 mode is lost.

Hamiltonian analysis of RS

G. Senjanovic. Hamiltonian Formulation and Quantization of the Spin 3/2 Field. *Phys. Rev. D*, 16:307, 1977.

M. Pilati. The Canonical Formulation of Supergravity. *Nucl. Phys. B*, 132:138–154, 1978.

S. Deser, J. H. Kay, and K. S. Stelle. Hamiltonian Formulation of Supergravity. *Phys. Rev. D*, 16:2448, 1977.

E. S. Fradkin and Mikhail A. Vasiliev. Hamiltonian Formalism, Quantization and S Matrix for Supergravity. *Phys. Lett. B*, 72:70–74, 1977.

POTENTIAL APPLICATIONS

The total Hamiltonian and the Lagrangian principles suggest that the spin-half mode propagates.

These findings may lead to new supergravity phenomenology, *eg*., in cosmology and particle physics.

$$\mathcal{L}_{\texttt{sugra}}[e^a_\mu, \omega^{ab}_\mu, \psi_\mu] = \mathcal{L}_{\texttt{RS}} + \frac{1}{2} e\, R^{ab}_{\mu\nu}\, e^\mu_a e^\nu_b$$

$$\mathcal{L}_{\texttt{U-sugra}}[e^a_\mu, \omega^{ab}_\mu, \gamma_\mu \kappa] = \frac{i(D-1)(D-2)}{2} e\, \bar{\kappa} \not{D} \kappa + \frac{i(D-1)}{2} \bar{\kappa}\, \gamma^{\mu\nu} T^a_{\mu\nu} \gamma_a\, \kappa + \frac{1}{2} e\, R^{ab}_{\mu\nu}\, e^\mu_a e^\nu_b$$

CONCLUSIONS

- With the Dirac conjecture the resulting Hamiltonian is not equivalent to the Lagrangian dynamics and it implies the elimination of the spin 1/2 mode.
- Without the Dirac conjecture the both Lagrangian and Hamiltonian description are equivalent and the spin 1/2 mode propagates.
- The Dirac conjecture explains why in unconventional supersymmetry there is a spin 1/2 propagating mode
- The implication for supergravity may be important. So further investigations are needed.

$$\partial\!\!\!/ \xi_\mu \approx 0$$

1977

$$\partial\!\!\!/ \xi_\mu \approx 0\,, \qquad \partial\!\!\!/ \zeta \approx 0$$

Today

$$\partial\!\!\!/ \xi_\mu \approx 0\,, \qquad \partial\!\!\!/ \zeta \approx 0$$

$$\partial\!\!\!/ \xi_\mu \approx 0$$

Red pill: accept potentially life-changing truth …

Blue pill: remaining in the ordinary reality.

Thank you!!!

Supersymmetry of a different kind,
P. Alvarez, M. V., and J. Zanelli, JHEP04(2012)058, ArXiv:1109.3944.

On the spin content of the classical massless Rarita--Schwinger system,
M. V., and J. Zanelli, SciPost Phys. Proc. 14 (2023) 047. Group 34th. ArXiv: 2207.03009.

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