Emergent symmetries in open EFTs

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Fundamental physics is developed for phenomena in equilibrium

At the **classical** level we write a Lagrangian and study the system

At the **quantum** level we can use the Lagrangian to derive **in-out** correlators

There exist many other interesting out of equilibrium processes, including

- Turbulence
- Vorticity
- Non adiabatic evolution

Interactions with an environment lead to energy transfer, particle number exchange

or more generally to failure of some conservation law

For simple systems openess can be identified from energy or charge non-conservation. However for gravity or gauge theory situation is more tricky

We can extend the notion of **open systems**:

- Hydrodynamics: energy transfer from large scales to small scale → "energy cascade"

- Stochastic inflation: system (superhorizon modes) - environment (subhorizon modes)

continuous inflow of modes as they cross their horizon. Backreaction of short modes on classical field

Real time correlators are calculated in the in-in formalism. Double contours naturally arise in non-equilibrium processes

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- **Incorporating non conservation**
- Symmetries of closed/open EFTs
- Superfluid and Maxwell
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Incorporating non-conservation

How the Lagrangian description fails for non-conservative systems?

How dissipation arises when focusing on degrees of freedom of a subsystem?

Elementary example*:* Friedman equations for a scalar field

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$

$$\dot{H} = -\frac{1}{2}\dot{\phi}^2 \qquad\qquad H \equiv \dot{a}/a$$

$$3H^2 = \frac{1}{2}\dot{\phi}^2 + V$$

This set of equations can be derived from the minisuperspace Lagrangian

$$L = -M_{\rm pl}^2 \frac{3a\dot{a}^2}{\mathcal{N}} + a^3\left(\frac{1}{2\mathcal{N}}\dot{\phi}^2 - \mathcal{N}V\right)$$

Being a Hamiltonian system it can not exhibit dissipation [G. N. Remmen, S. M. Carroll 1309.2611]

The would-be energy vanishes (Hamiltonian constraint)

We can eliminate the Hubble parameter to obtain a reduced system

$$\ddot{\phi} + \dot{\phi}\sqrt{3}\sqrt{\frac{1}{2}\dot{\phi}^2 + V} + V_{,\phi} = 0$$

1) This EOM **can not be derived** from a typical Lagrangian $L(\phi, \dot{\phi})$

2) The EOM implies

$$\ddot{\phi} + V_{,\phi} = -\dot{\phi}\sqrt{3}\sqrt{\frac{1}{2}\dot{\phi}^2 + V} \Rightarrow \frac{d}{dt}(K + V) = -\dot{\phi}^2\sqrt{3}\sqrt{K + V}$$

Energy decreases ⇒ dissipation

Incorporating dissipation:

At the level of **EOM:** generalised forces

$$\frac{\partial L}{\partial q} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}}\right) = F(q, \dot{q})$$

At the level of the **action:**

- Overall time-dependent factor. E.g. in **fixed** FLRW background

$$S = \int \mathrm{d}^4x\, \bar{a}(t)^3 \left(\frac{1}{2}\dot{\phi}^2 - V\right) \qquad \text{yields EOM} \qquad \ddot{\phi} + 3\bar{H}\dot{\phi} + V_{,\phi} = 0$$

- Introduce Lagrange multipliers to modify dynamics (not systematic)

The most **systematic** method: double all dynamical fields $\Phi \equiv \{\phi, A_\mu, g_{\mu\nu}, \ldots\}$ and write an extended action with 2N variables [C. R. Galley 1210.2745]

$$S_{\text{extended}}[\Phi_1, \Phi_2] = S[\Phi_1] - S[\Phi_2]$$

Require

1. Equality at the final time $\Phi_1(t_\text{f}) = \Phi_2(t_\text{f}), \dot{\Phi}_1(t_\text{f}) = \dot{\Phi}_2(t_\text{f}), \ldots$

2. specify the fields and their derivatives at the initial time

This would give two sets of EOM, with identical solutions. At this stage the two fields are unphysical. We define the **physical solution** as the average of the two $\Phi = \frac{1}{2}(\Phi_1 + \Phi_2)$

This is a trivial extension of the action principle. Interesting case: add a piece

that mixes the two fields that cannot be written as a difference of two identical actions

$$S_{\text{extended}}[\Phi_1, \Phi_2] = S[\Phi_1] - S[\Phi_2] + S_{\text{mix}}[\Phi_1, \Phi_2]$$

Now the EOM for Φ_1, Φ_2 will not be equal in general!

It is instructive to change variables to $\Phi_r \equiv \frac{\Phi_1 + \Phi_2}{2}$, $\Phi_a \equiv \Phi_1 - \Phi_2$ (Keldysh basis/center of mass variables)

Φ_r satisfies an initial value problem and for the difference field $\Phi_a(t_f) = 0, \dot{\Phi}_a(t_f) = 0$,.. so this field **always** satisfies the **same** conditions at the final time

In the path integral we only impose equality of the fields at the final time.

It also needs to satisfy the non-equilibrium constraints [P. Glorioso H. Liu 1805.09331]

In the **bottom-up** approach (EFT): assume symmetries and write all possible terms associated with them

$$S_{\text{low-energy}}[\phi_1, \phi_2] = \underbrace{S[\phi_1] - S[\phi_2]}_{\text{conservative}}$$

Two copies of same action ⇒ $\phi_{1,2}$ have the same transformation rule. In a-r basis:

$$S_{\text{low-energy}}[\phi_r, \phi_a] = \underbrace{\phi_a \mathscr{D}_1 \phi_r + \phi_a^2 \mathscr{D}_2 \phi_r + \dots}_{\text{conservative}}$$

Add extra terms

$$S_{\text{low-energy}}[\phi_r, \phi_a] = \underbrace{S_c[\phi_r, \phi_a]}_{\text{conservative}} + \underbrace{S_{\text{mix}}[\phi_r, \phi_a]}_{\text{non-conservative}}$$

e.g. $S_{\text{mix}}[\phi_r, \phi_a] = \phi_a \mathscr{N} \phi_a + \dots$

with $\mathscr{N}$ be such that the closed symmetries are satisfied

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Symmetries of closed/open EFTs

Given an SK EFT the advanced symmetries determine conservation laws of physical fields

[L. M. Sieberer, M. Buchhold, S. Diehl 1512.00637]

Term proportional to $\phi_{\rm a}$ (*drop r subscripts for physical fields*) yields EOM

$\mathscr{L} = \phi_{\rm a} E(\phi) + \ldots$ similar to the variation $\delta \mathscr{L}_{\rm usual} = \delta\phi$ EOM(ϕ)

This shows that symmetry transformations of advanced fields yield conservation laws

Example: shift symmetry $\delta\phi = c$. Imposing this symmetry on both r/a fields

$$\mathscr{L} = \dot{\phi}_{\rm a}\dot{\phi} - \nabla\phi_{\rm a}\nabla\phi + \ldots \qquad c_{\rm s}^2 = 1$$

Break the symmetry: write non-conservative terms to describe interaction with an environment

Open system = non-trivial (or non-existent) advanced symmetry

Introducing dissipation: direction $u_\mu = (-1,0,0,0)$. Typical dissipative term: $-\Gamma \phi_a \dot{\phi}_r$

[L. V. Delacrétaz, B. Goutéraux, V. Ziogas 2111.13459]

The EOM becomes $\ddot{\phi} - \nabla^2 \phi + \Gamma \dot{\phi} = 0$

Noise terms also typically break physical conservation laws $i\beta\phi_a^2$

EOM: before was $\partial_\mu(\partial^\mu \phi) = 0$ and after $\partial_\mu(\partial^\mu \phi) = -\Gamma u^\mu \partial_\mu \phi$

$$\partial_\mu \left(\partial^\mu \phi + \Gamma u^\mu \phi\right) = 0$$

A new conservation law emerges!!

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Superfluid and Maxwell

The superfluid action: spontaneously broken U(1) symmetry

$$S_{\mathrm{SK}} = \int \partial_\mu \phi_{\mathrm{a}} \partial^\mu \phi + \dots$$

Add dissipative term $\Gamma \phi_{\mathrm{a}} u_\mu \partial^\mu \phi$ and obtain

$$S_{\mathrm{open}} = \int \left(\partial_\mu \phi_{\mathrm{a}} + \Gamma u_\mu \phi_{\mathrm{a}} \right) \partial^\mu \phi + \dots$$

Global $U(1)_{\mathrm{a}}$ symmetry is broken. However, another emerges

$$\delta \phi_{\mathrm{a}} = \Lambda(t) \,, \quad \text{with} \quad \dot{\Lambda} - \Gamma \Lambda = 0$$

The deformed global $U(1)_{\mathrm{a}}$ symmetry yields a conserved current. Interestingly, gauging the symmetry yields the deformed gauge symmetry found in [S. A. Salcedo, T. Colas, E. Pajer 2412.12299]

Non-dissipative Maxwell SK action

$$S_{\rm SK} = \int \mathrm{d}\phi_{\rm a} \wedge \star \mathrm{d}\phi + \ldots \qquad \text{(equivalent notation)} \qquad \int F_{{\rm a}\mu\nu} F^{\mu\nu}$$

The action has two sets of gauge symmetries: $\delta\phi = \mathrm{d}\Lambda\,, \quad \delta\phi_{\rm a} = \mathrm{d}\Lambda_{\rm a}$

Add dissipative terms (introduce dissipation covector u: $\mathrm{d}u = 0\,,\ \iota_u u = -1$)

$$S_{\rm dis} = \int \Gamma u \wedge \phi_{\rm a} \wedge \star \mathrm{d}\phi + \ldots \Rightarrow S_{\rm open} = \int \left(\mathrm{d}\phi_{\rm a} + \Gamma u \wedge \phi_{\rm a}\right) \wedge \star \mathrm{d}\phi + \ldots$$

Introduce $f_{\rm a} \equiv \mathrm{d}\phi_{\rm a} + \Gamma u \wedge \phi_{\rm a}$ and observe it remains invariant under

$$\delta\phi_{\rm a} = \mathrm{d}\Lambda_{\rm a} + \Gamma u \Lambda_{\rm a} \qquad \text{deformed gauge invariance}$$

Defining $\mathrm{D} \equiv \mathrm{d} + u\,\wedge$ the action is neatly written in terms of $\mathrm{D}\phi_{\rm a}\,, \mathrm{d}\phi\,, u$. The operator satisfies $\mathrm{D}^2 = 0$, so for the modified field-strength tensor $\delta f_{\rm a} = \mathrm{D}^2\Lambda = 0$. The symmetry implies Noether identities

$$\mathrm{D}^\dagger \mathrm{EOM} = 0$$

An external current

$$\int \phi_{\mathrm{a}} \wedge \star j \qquad \text{(equivalent notation)} \qquad \int A_{\mathrm{a}\mu} j^{\mu}$$

for a closed system satisfies (EOM + gauge invariance)

$$\mathrm{d} \star j = 0 \qquad \text{(or)} \qquad \partial_\mu j^\mu = 0$$

For the open action, current deformation

$$\mathrm{D}^\dagger \star j = 0 \qquad \text{(or)} \qquad (\partial_\mu - \Gamma u_\mu) j^\mu = 0$$

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Gravity

The simplest action for gravity: EH term $S = \int \sqrt{-g} R$. Simple SK action?

Many obstacles

- Gravity is non-linear not guaranteed to obtain simple expressions
- Not clear how to transform covariant derivatives of average and difference
- Physical fields should live on one spacetime. If more, then bimetric theory?

Work in the semi-classical limit: treat advanced metric as small fluctuation

$$S_{\mathrm{SK}} = \int \sqrt{-g} G_{\mu\nu} g_{\mathrm{a}}^{\mu\nu}$$

From this we can read the form of advanced diffeomorphisms

$$\delta S = \int \sqrt{-g} G_{\mu\nu} \delta g^{\mu\nu} \qquad \sim \qquad \delta g_{\mathrm{a}}^{\mu\nu} = \mathscr{L}_{\xi} g^{\mu\nu}$$

Advance shifts are a symmetry because $\nabla^{\nu} G_{\mu\nu} = 0$ (identities between EOM)

Symmetries of SK action: (physical) 4D diffs + (advanced) shifts by Lie derivative

[P. Glorioso H. Liu 1805.09331]

Open gravitational system: break explicit advanced diff symmetry

$$E_{\mu\nu} = G_{\mu\nu} + \Delta_{\mu\nu} \qquad \text{with} \qquad \nabla^\mu \Delta_{\mu\nu} \neq 0$$

Simplest choice: $\Delta_{\mu\nu} = \Gamma R g_{\mu\nu}$

A new (advanced) deformed diffeomorphism emerges

$$\delta g^{\mu\nu}_{\rm a} = -\,2\,\nabla^{(\mu}\xi^{\nu)} + g^{\mu\nu}\frac{\Gamma}{4\Gamma - 1}\nabla_\alpha \xi^\alpha$$

that provides a set of 4 identities between the EOM

These identities cut 4 DOF. The remaining 4 constraint equations (Hamiltonian + momentum cut another 4 resulting in 2 propagating DOF (gravitational waves)

More general choices that yield explicit expressions for deformed diffs in the presence of preferred frames

$$\Delta_{\mu\nu} = A g_{\mu\nu} + B n_\mu n_\nu + \gamma_3 n^\kappa G_{\kappa(\mu} n_{\nu)} \quad \text{with}$$

$$A = \Gamma_1 R + \Gamma_2 G_{\rm nn}\,, \qquad B = \gamma_1 R + \gamma_2 G_{\rm nn}$$

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Final remarks

- **Open systems** are abundant in nature - necessary to develop non-conservative EFTs
- The **Schwinger-Keldysh** formalism provides a systematic framework for non-conservative systems
- For open systems advanced symmetries are **broken.** When physical symmetries are kept intact advanced symmetries get **deformed**
- Key examples: **superfluid**, **Maxwell** and **gravity**