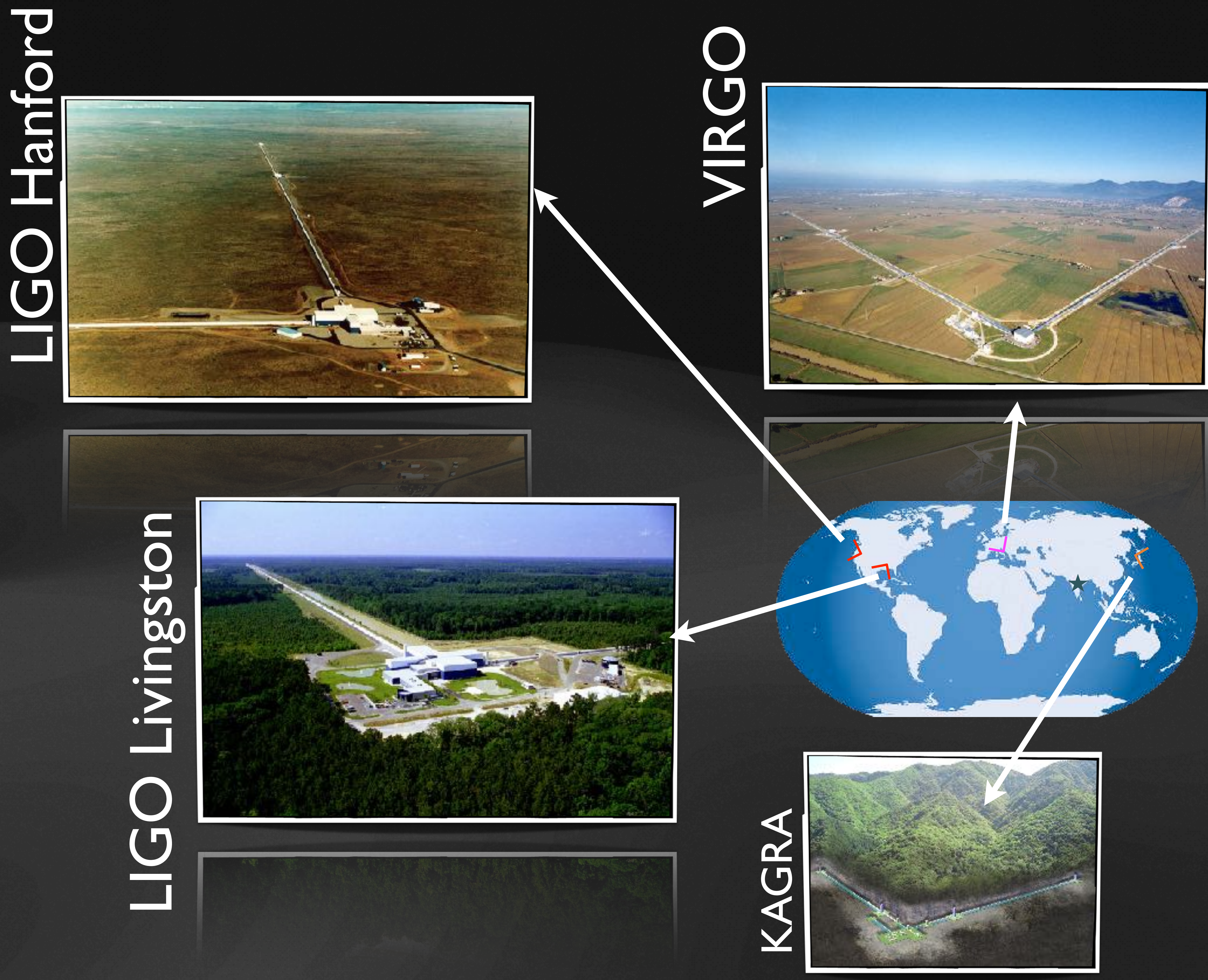
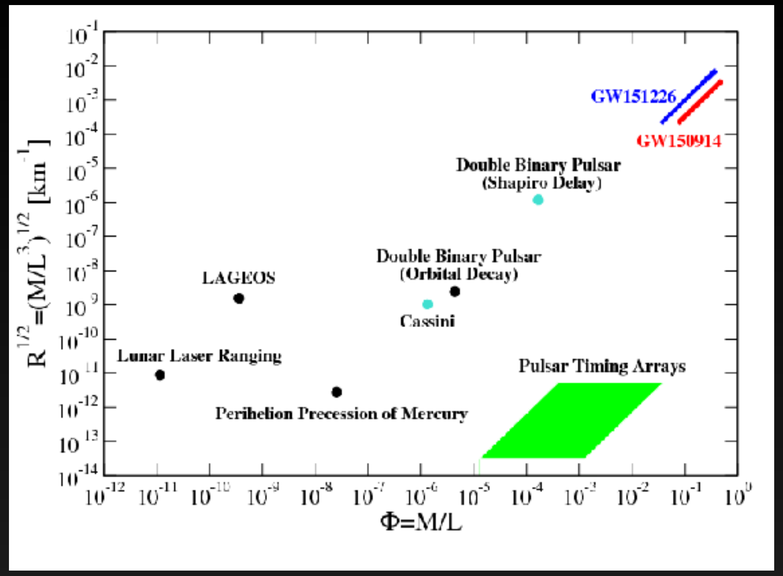
10 Years of Gravitational Wave Tests of GR

Michalis Agathos

NEB21
Corfu - 04/09/2025


LIGO Hanford
VIRGO
LIGO Livingston
KAGRA

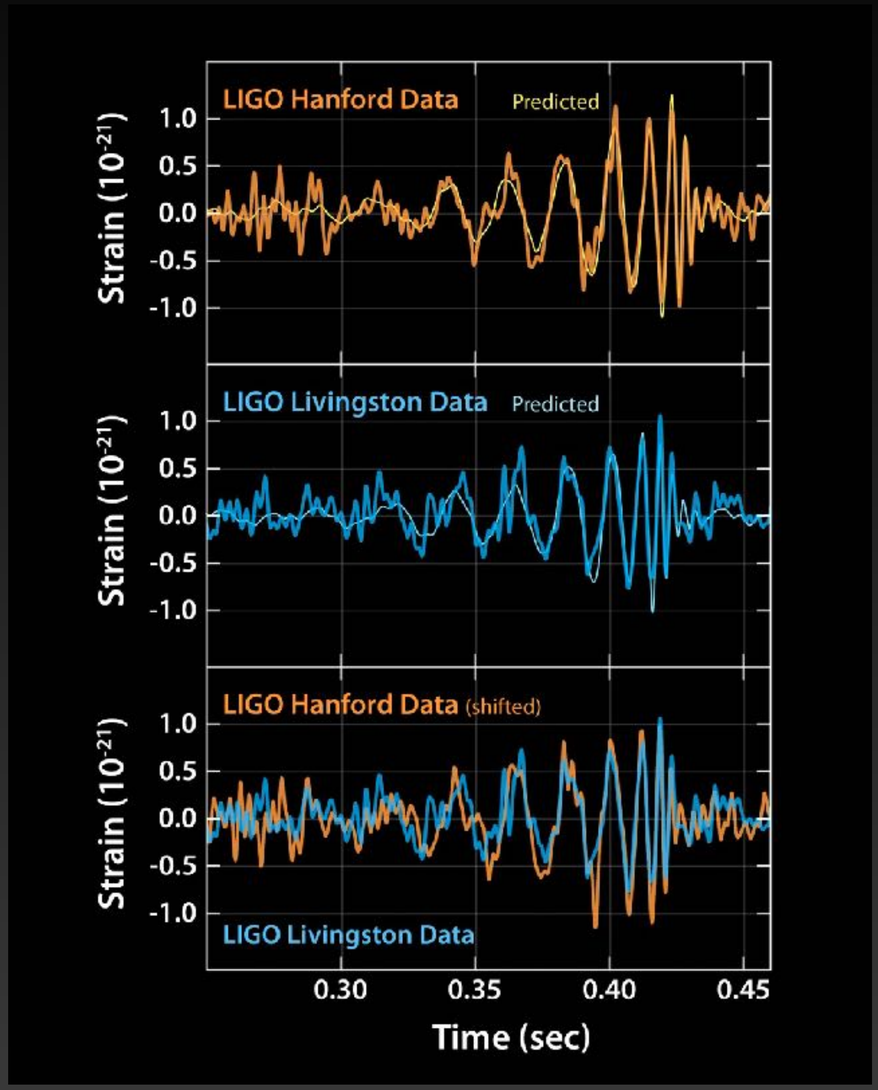
[Yunes+ PRD 94 084002 (2016)]

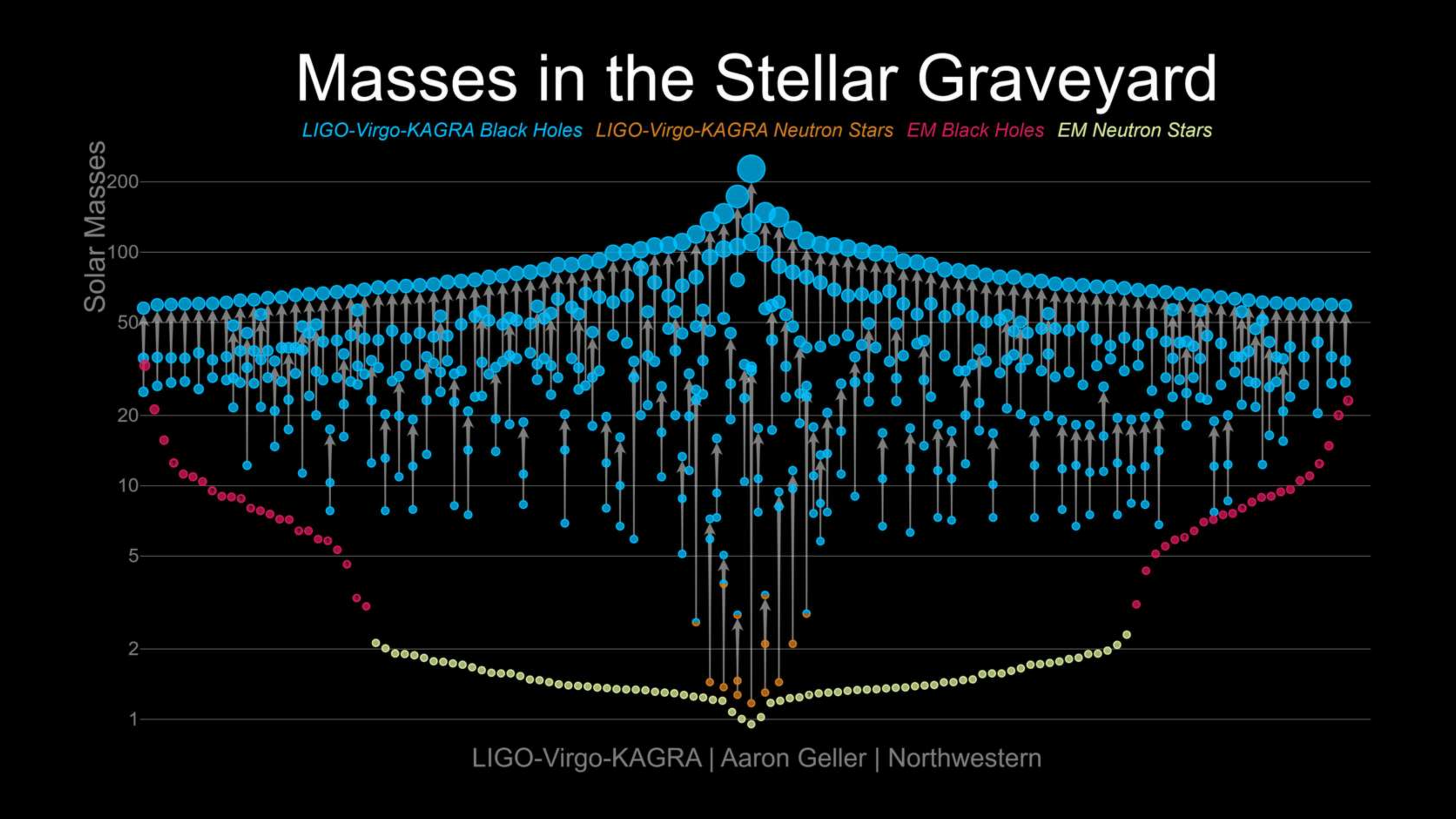


GW150914 🎂

- f_{gw}: 30 → 132 → 250 Hz within < 0.2s
- v/c: 0.03 → 0.13 → 0.5

- ☑ Strong-field
- ☑ Highly relativistic
- ☑ Dynamical evolution




Masses in the Stellar Graveyard
LIGO-Virgo-KAGRA Black Holes
LIGO-Virgo-KAGRA Neutron Stars
EM Black Holes
EM Neutron Stars
Solar Masses
200
100
50
20
10
5
2
1
LIGO-Virgo-KAGRA | Aaron Geller | Northwestern

“

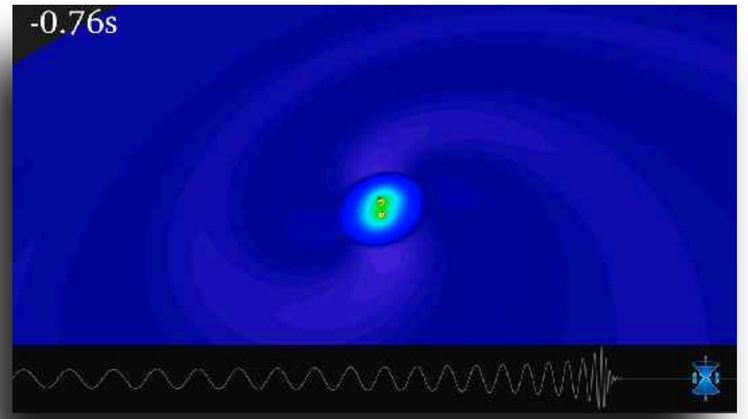
You may wanna try to test GR with those

-Captain Obvious

GRAVITATIONAL WAVE DATA ANALYSIS

- I) Direct problem (source modelling):

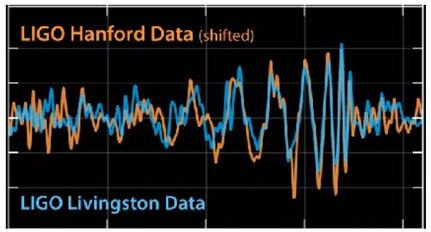
  "Given a GW source of known properties (e.g. BBH of known masses, spins, etc), what is the emitted gravitational wave signal?"

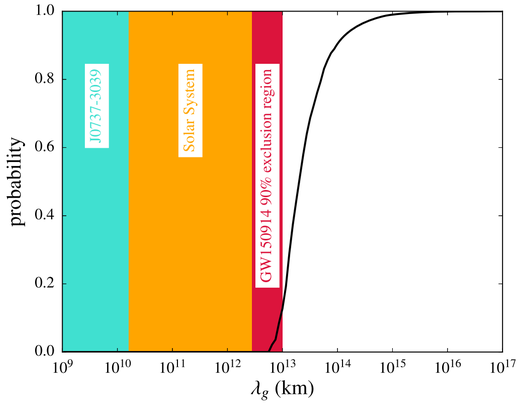


https://www.ligo.caltech.edu/video/ligo20160211v10

- II) Inverse problem (data analysis):

  "Given a stretch of noisy data and the stochastic properties of noise, can you reconstruct a signal and estimate the properties of the source?"



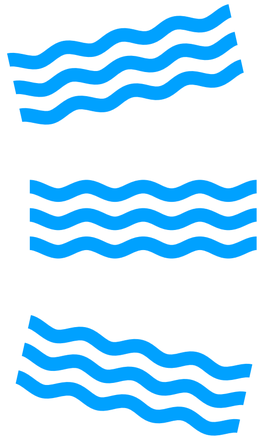


[Abbott+ (LVC) PRL 116, 221101 (2016)]

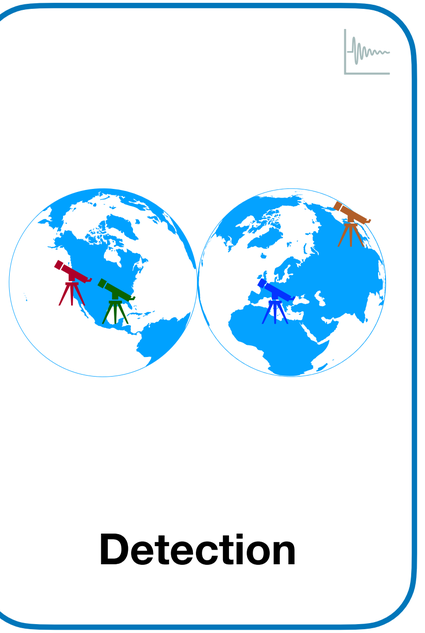
Testing GR with Gravitational Waves

What are we testing?

Generation



Propagation



Detection

+ *waveform systematics*
+ *noise model & calibration*

Consistency tests of GR model

GW150914: Analysis on residuals

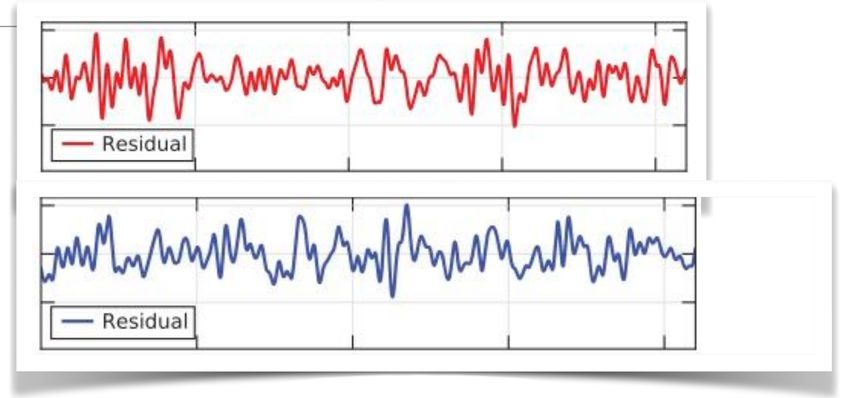
- Subtract best-fit waveform from data
- “Is residual consistent with pure LIGO noise?”
- BayesWave (unmodelled-burst) analysis says “YES”

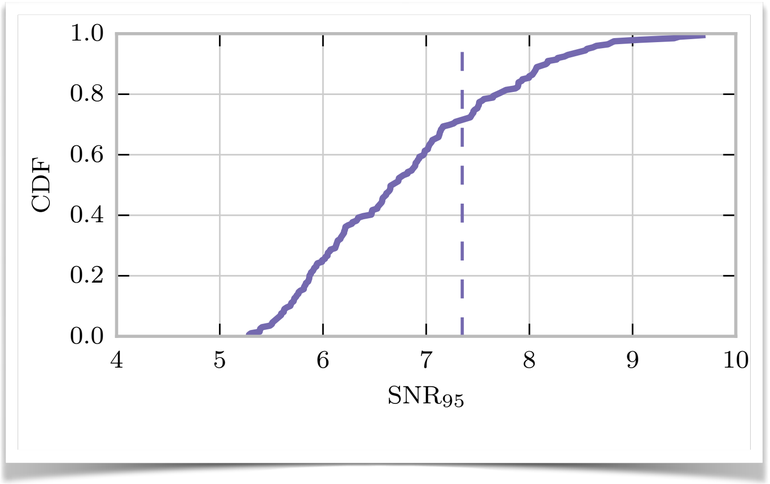
$$\mathrm{SNR}_{\mathrm{res}} \leq 7.3$$

- Assuming entire residual is due to mismatch:

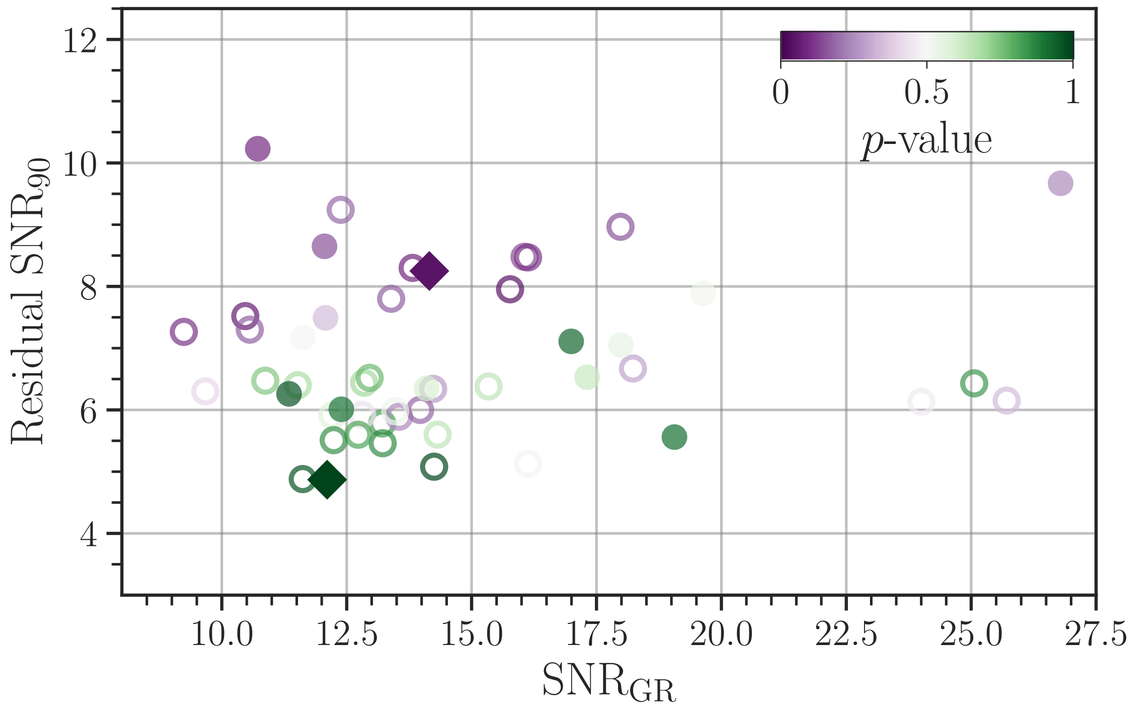
$$\mathrm{FF} \geq 96\%$$

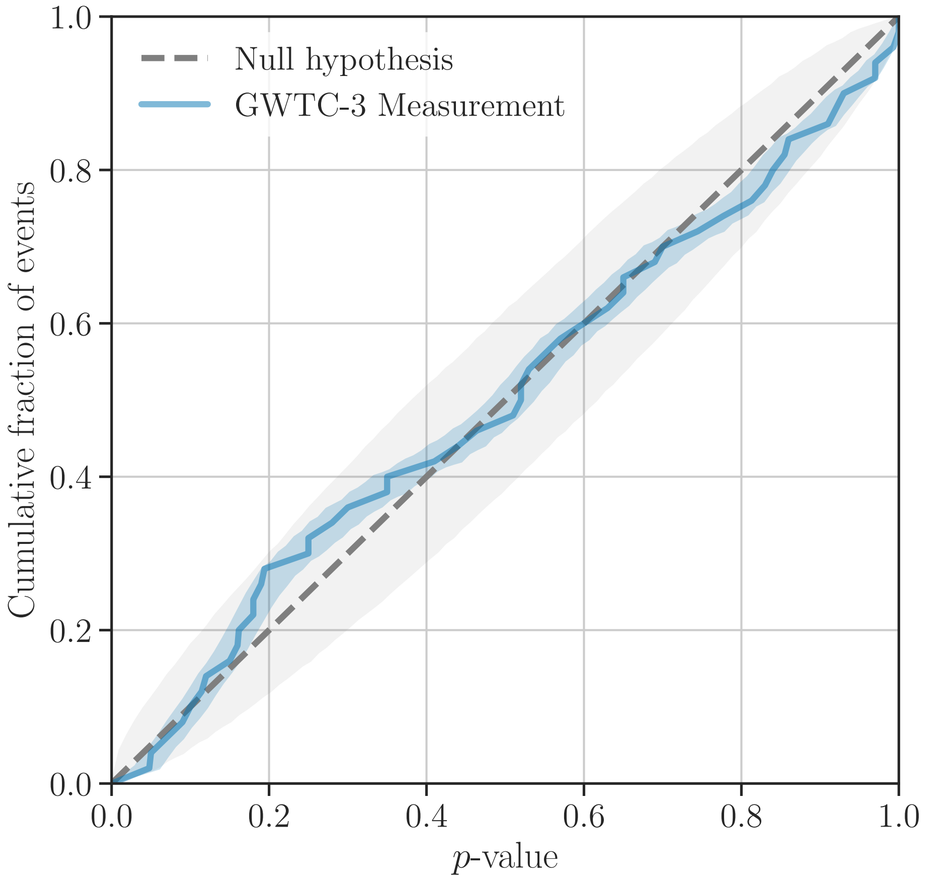
between GR and true signal





GWTC-3: Analysis on residuals



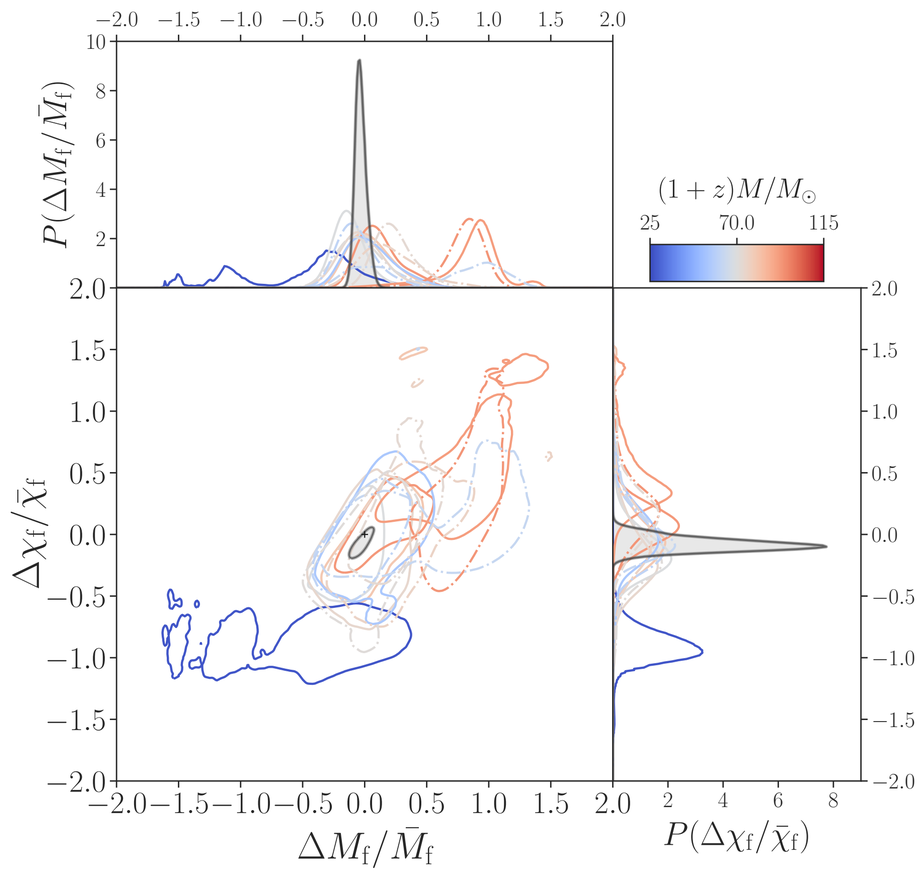


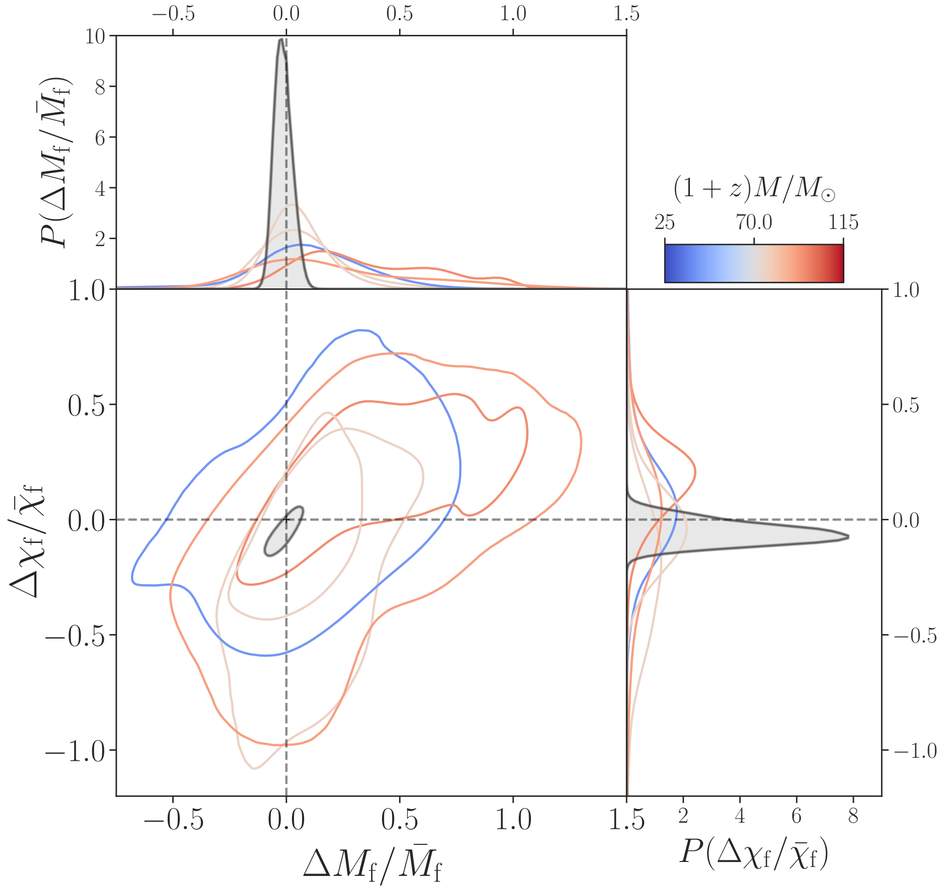
[LVK (2021) arXiv:2112.06861]

Inspiral - Merger-Ringdown consistency

[LVK (2021) arXiv:2112.06861]

- Posteriors show consistency with GR predictions





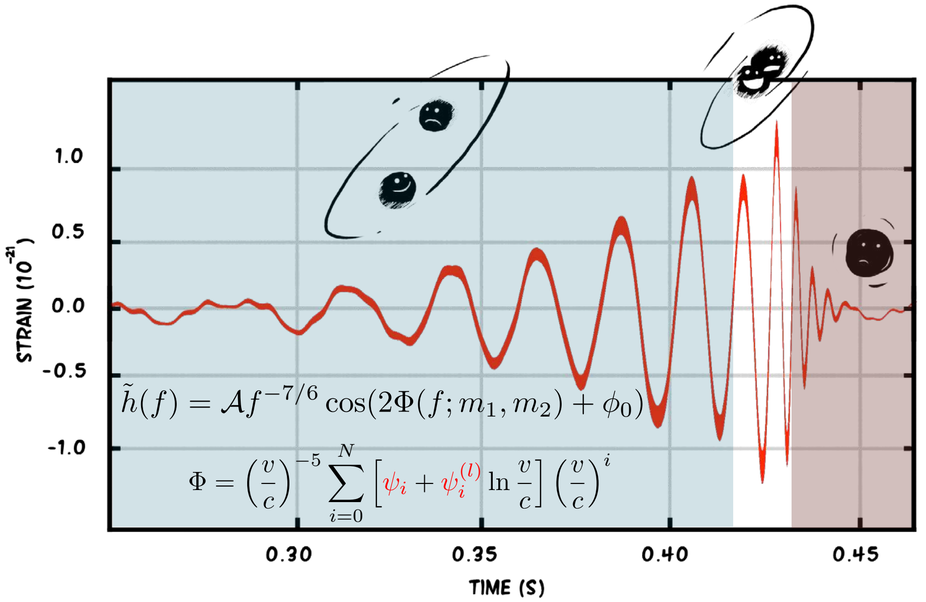
Tests of binary dynamics

Generation of GWs in Compact Binaries

Arun+ [PRD 74, 024006 (2006)]
Yunes+ [PRD 80, 122003 (2009)]
Li, MA+ [PRD 85, 082003 (2012)]
MA+ [PRD 89, 082001 (2014)]
Meidam, MA+ [PRD 97, 044033 (2018)

- Dynamics are dictated by theory
- Look for dephasing in the inspiral

$$\varphi_i \to \varphi_i(1 + \delta\hat{\varphi}_i)$$



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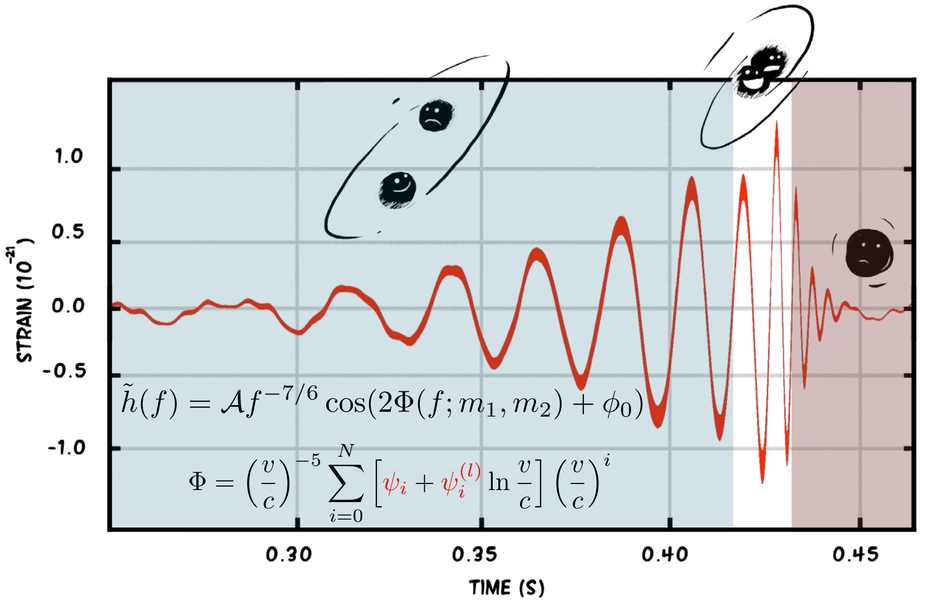
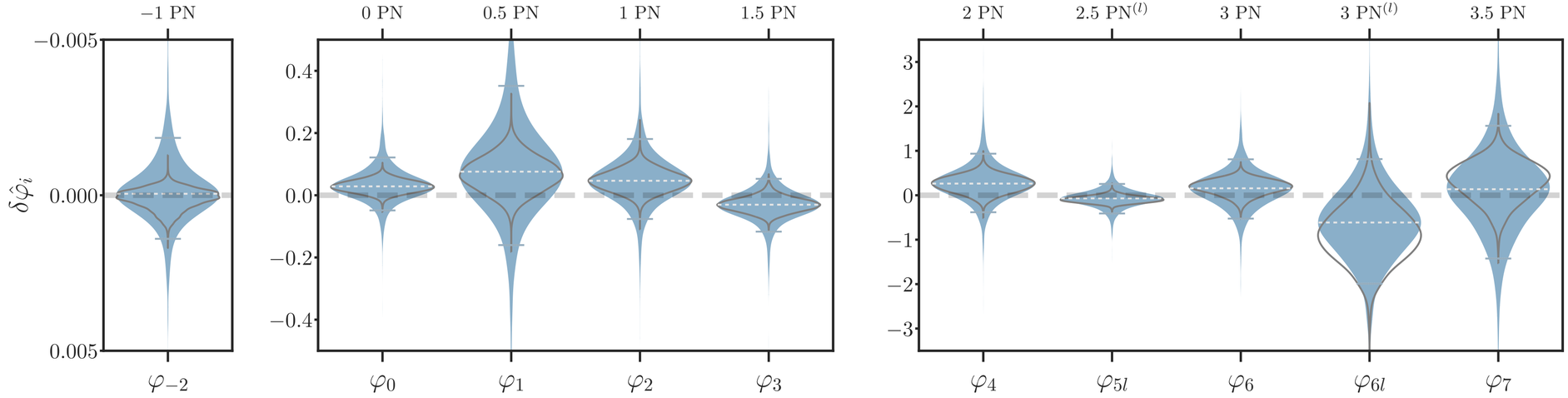
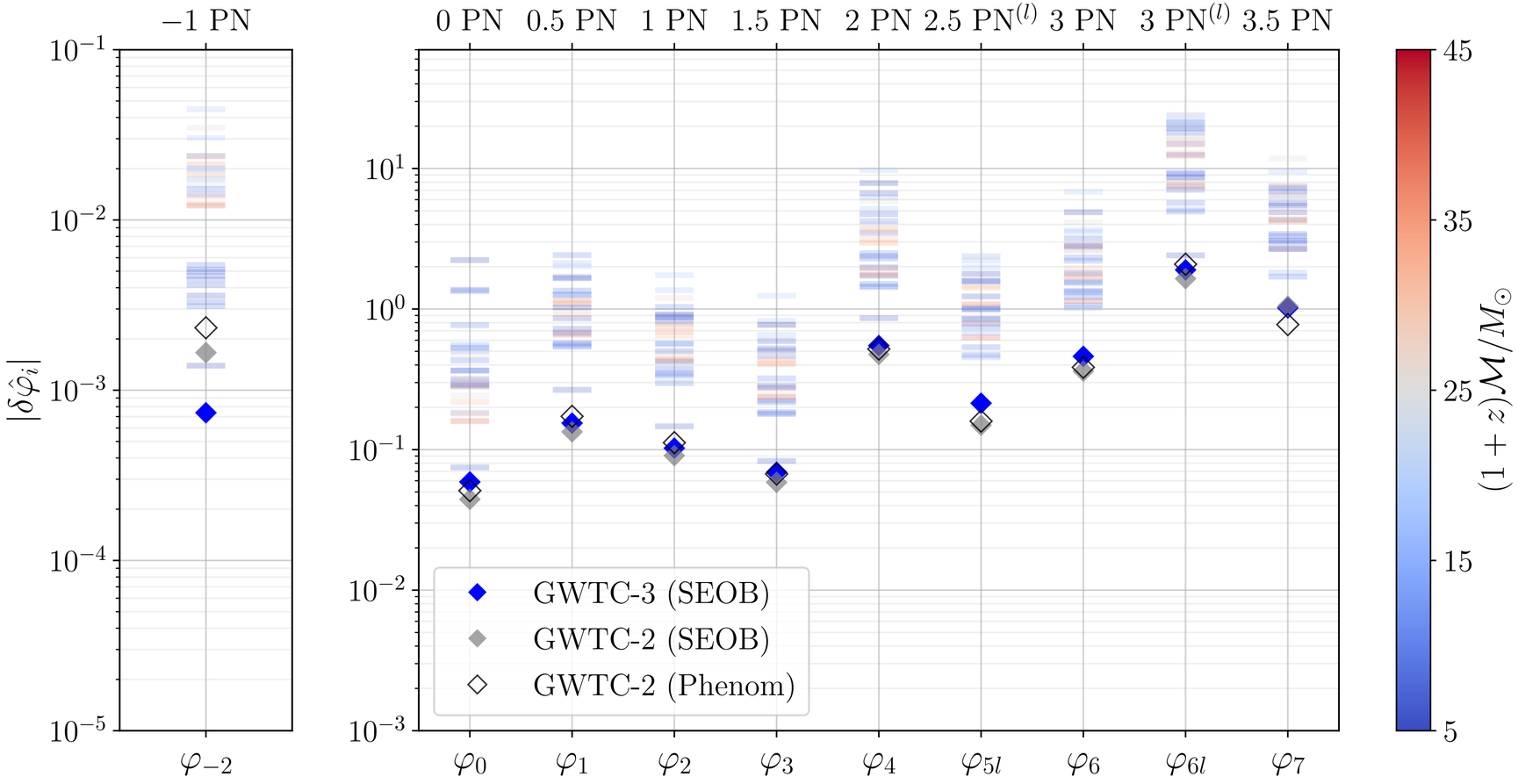




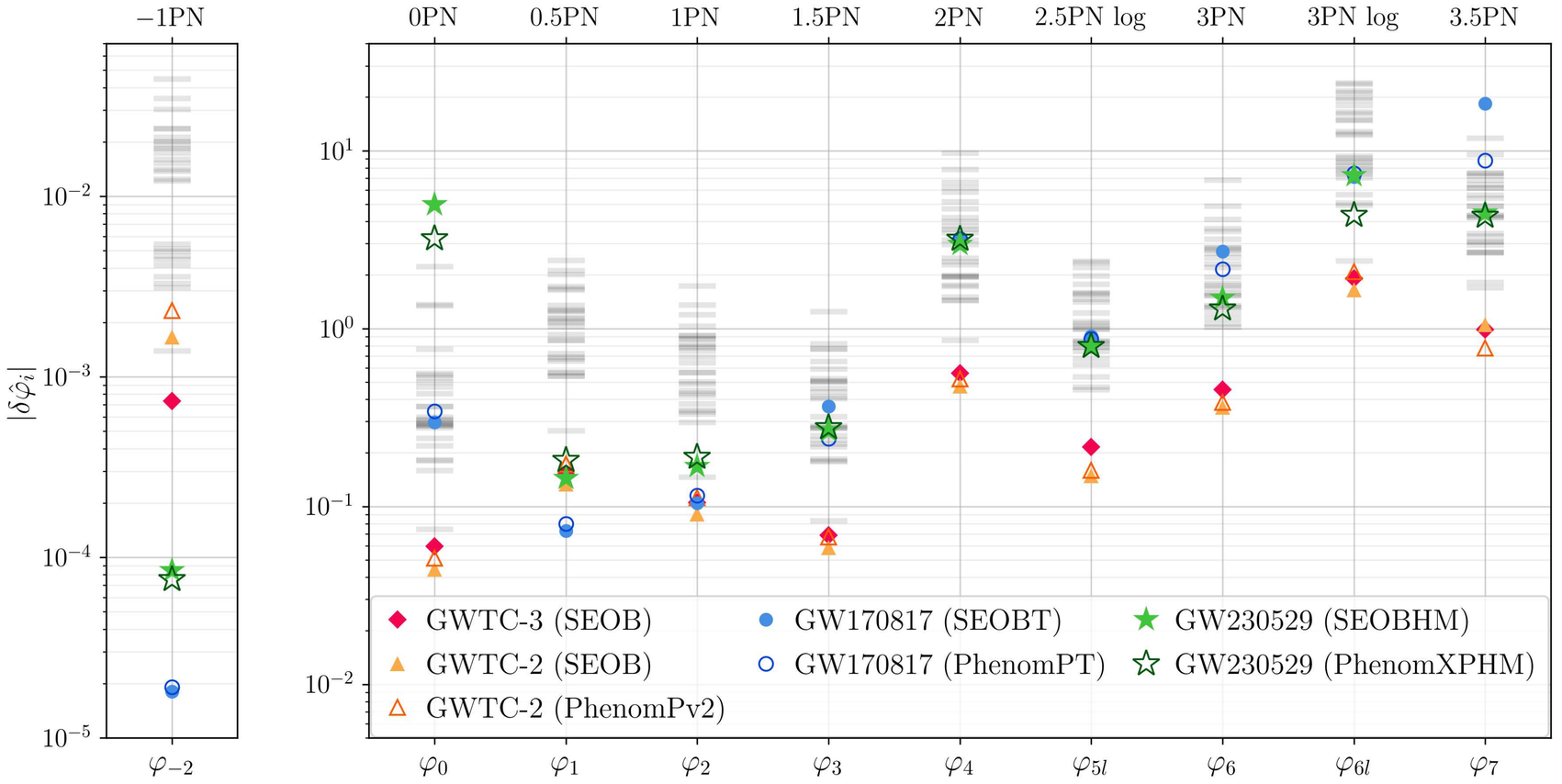
[LVK (2021) arXiv:2112.06861]

Bounds on deviations of PN coefficients

LVK (2021) [arXiv:2112.06861]



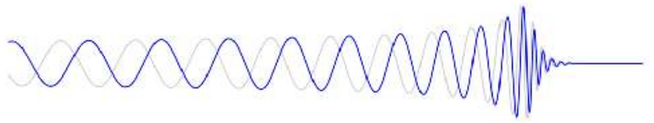
Bounds on deviations of PN coefficients (+GW230529)



[LVK (2024) arXiv:2404.04248]
[Sänger et al. (2024) arXiv:2406.03568]

Tests of GW propagation

GW propagation - Modified Dispersion Relation



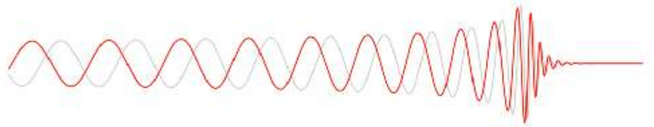
Massive graviton: $E^2 = p^2c^2 + m_g^2c^4$

Generalization: $E^2 = p^2c^2 + \mathbb{A}\, p^\alpha c^\alpha$

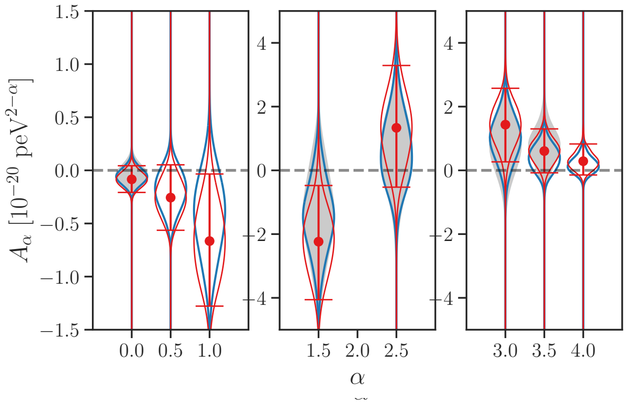
Phase modification:

$$\delta\Psi_\alpha(f) = -\cancel{\beta(\pi\mathcal{M}f)} \quad - \frac{\pi D_\alpha}{(1-\alpha)\lambda_{\mathbb{A}}^{2-\alpha}(1+z)^{1-\alpha}} f^{\alpha-1}$$

[Will 1994]
[Mirshekari+ 2011]



Credits: T.Littenberg, M.Isi, A.Samajdar, A.Ghosh



[LVK (2021) arXiv:2112.06861]

	m_g [10^{-23} eV/c^2]	$\lvert\bar{A}_0\rvert$ < [10^{-45}]	$\lvert\bar{A}_0\rvert$ > [10^{-45}]	Q_{GR} [%]	$\lvert\bar{A}_{0.5}\rvert$ < [10^{-38}]	$\lvert\bar{A}_{0.5}\rvert$ > [10^{-38}]	Q_{GR} [%]	$\lvert\bar{A}_1\rvert$ < [10^{-32}]	$\lvert\bar{A}_1\rvert$ > [10^{-32}]	Q_{GR} [%]	$\lvert\bar{A}_{1.5}\rvert$ < [10^{-26}]	$\lvert\bar{A}_{1.5}\rvert$ > [10^{-26}]	Q_{GR} [%]	$\lvert\bar{A}_{2.5}\rvert$ < [10^{-14}]	$\lvert\bar{A}_{2.5}\rvert$ > [10^{-14}]	Q_{GR} [%]	$\lvert\bar{A}_3\rvert$ < [10^{-8}]	$\lvert\bar{A}_3\rvert$ > [10^{-8}]	Q_{GR} [%]	$\lvert\bar{A}_{3.5}\rvert$ < [10^{-2}]	$\lvert\bar{A}_{3.5}\rvert$ > [10^{-2}]	Q_{GR} [%]	$\lvert\bar{A}_4\rvert$ < [10^{4}]	$\lvert\bar{A}_4\rvert$ > [10^{4}]	Q_{GR} [%]
GWTC-2	1.76	1.75	1.37	66	0.46	0.28	66	1.00	0.52	79	3.35	1.47	83	1.74	2.43	31	1.08	2.17	17	0.76	1.57	12	0.64	0.88	25
GWTC-3	1.27	1.88	0.89	86	0.51	0.19	91	1.16	0.32	96	3.69	0.93	98	1.16	2.95	13	0.66	2.33	2	0.45	1.16	7	0.30	0.74	15

Tests of Gravity with GW170817

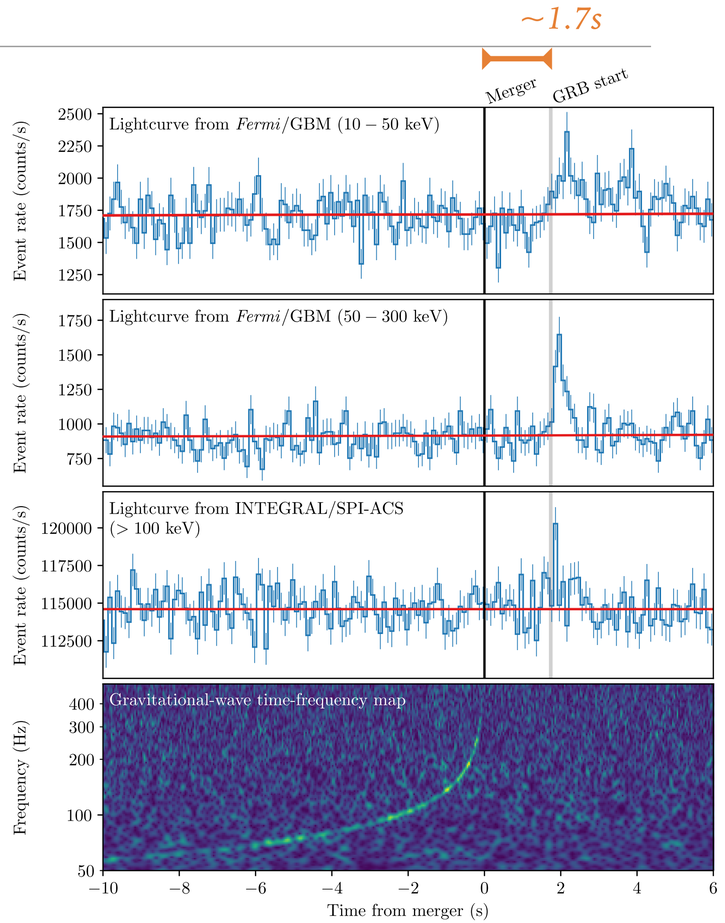
- Coincident GWs and γ-ray detections
- Distance travelled ~40 Mpc
- GW -> GRB time delay (+ reasonable astrophysical priors) => constraints on difference between vEM and vGW:

$$-3 \times 10^{-15} \leq \frac{\Delta v}{v_{\mathrm{EM}}} \leq +7 \times 10^{-16}$$

- Standard Model Extension coefficients
- Test of the equivalence principle (Shapiro time delay):

$$\delta t_{\mathrm{S}} = -\frac{1+\gamma}{c^3} \int_{r_{\mathrm{e}}}^{r_{\mathrm{o}}} U(\boldsymbol{r}(l))\, dl$$

$$-2.6 \times 10^{-7} \leqslant \gamma_{\mathrm{GW}} - \gamma_{\mathrm{EM}} \leqslant 1.2 \times 10^{-6}$$

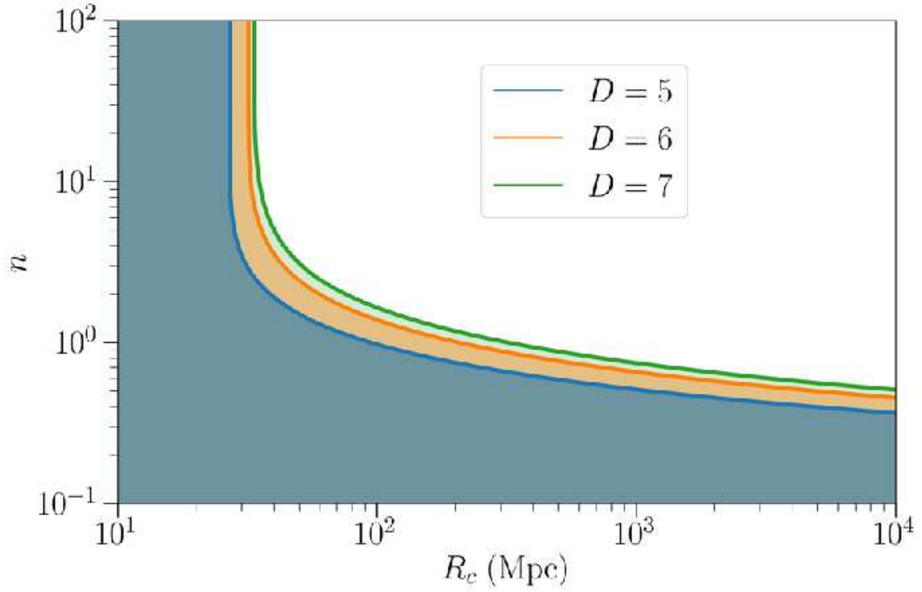


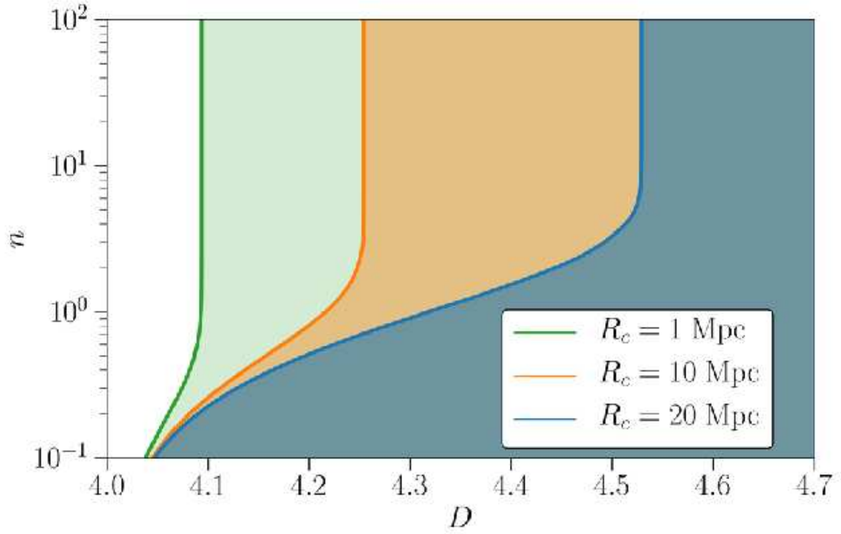
[LVC ApJ Lett. 848, L13 (2017)]

GW170817: TEST OF EXTRA DIMENSIONS

LVC 2018 [arXiv:1811.00364]

- GW detection + EM identification of host galaxy -> independent measurements of distance to source
- In higher-dim gravity, GWs may "leak" into extra dimensions (larger effective distance)
- $$h \propto \frac{1}{d_L^{\rm GW}} = \frac{1}{d_L^{\rm EM}} \left[1 + \left(\frac{d_L^{\rm EM}}{R_c} \right)^n \right]^{-(D-4)/(2n)}$$

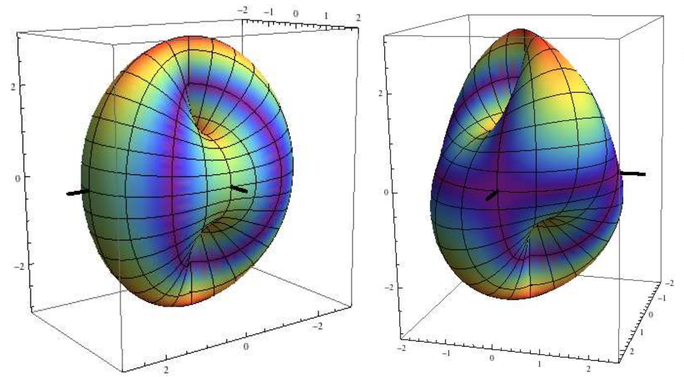


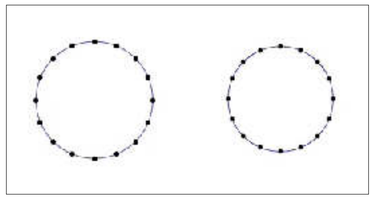


- screening out to "activation radius" R_c
- n: transition steepness
- Astrophysical length scales -> D=4
- Cosmological length scales ($R_c \sim R_H$):
  - slow transition (n~0.1): D=4
  - steep transition: unconstrained

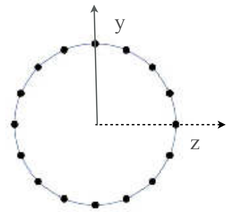
GW Polarizations

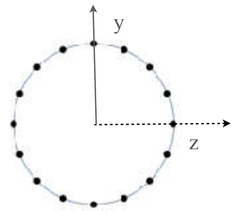
GR

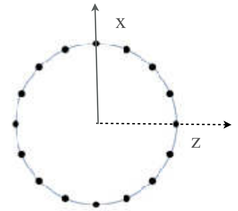


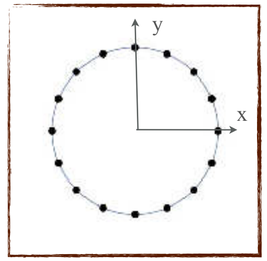


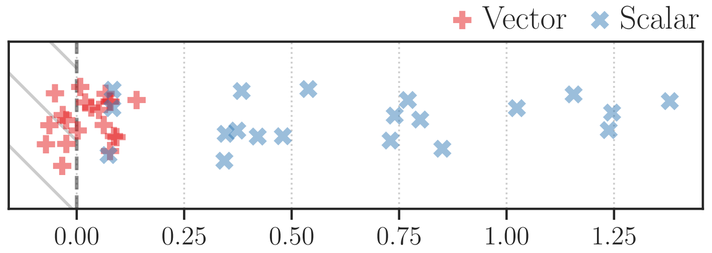
non-GR











$\log_{10} \mathcal{B}^{T}_{V/S}$ (tensor vs non-tensor)

LVC PRD **103**, 122002 (2021)
[arXiv:2010.14529]

Testing the nature of compact objects

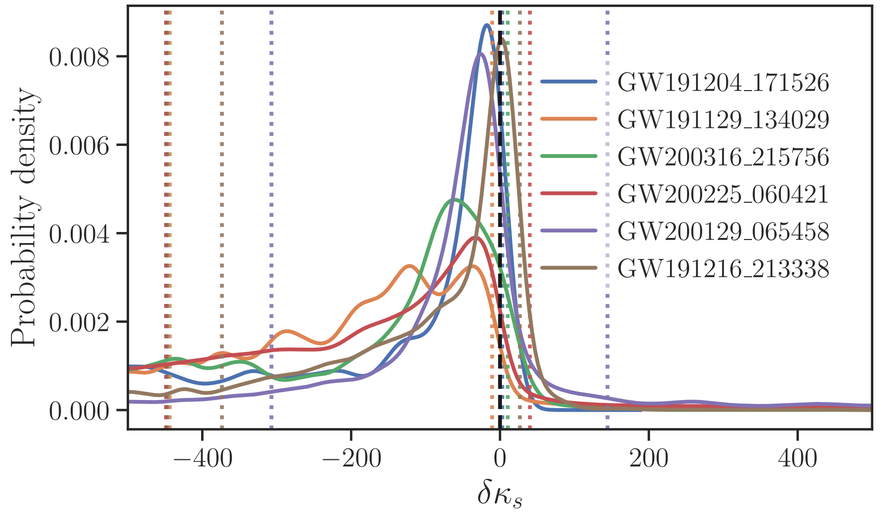
Black Hole Quadrupole

LVC PRD **103**, 122002 (2021)
LVK [arXiv:2112.06861]

- All properties of a Kerr BH are uniquely determined by knowing its mass and spin
- Simple formula for mass and current multipoles
- Spin-induced mass quadrupole:

$$Q = -\kappa \chi^2 M^3 , \quad \kappa_{\rm BH} = 1$$

- Introduces modification in the GW phase that enters @2PN



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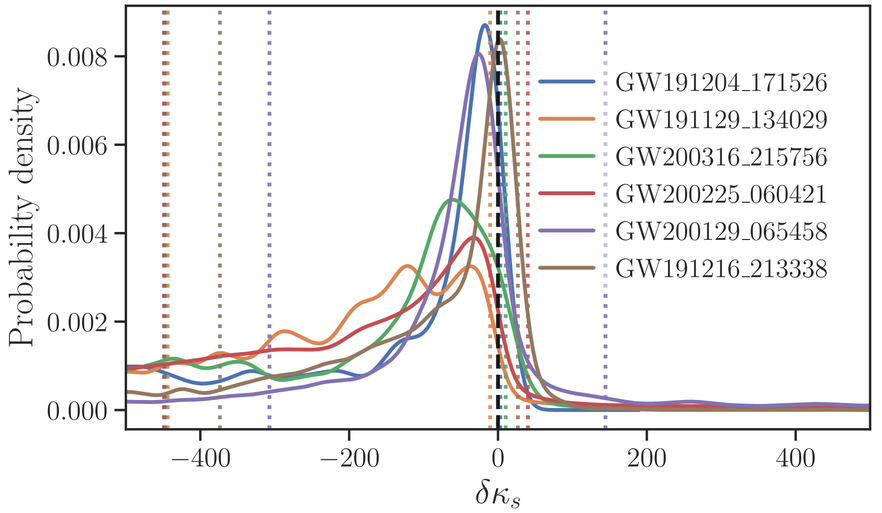
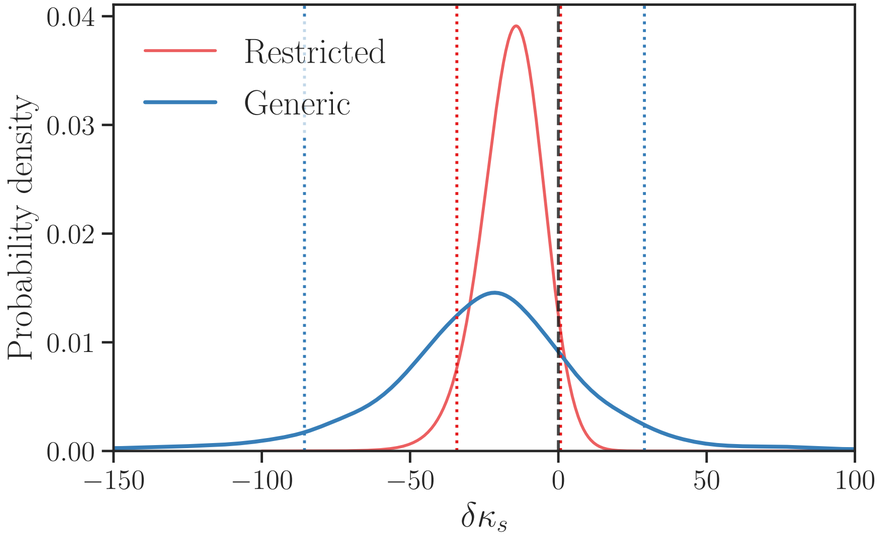
$$Q = -\kappa \chi^2 M^3 , \quad \kappa_{\rm BH} = 1$$

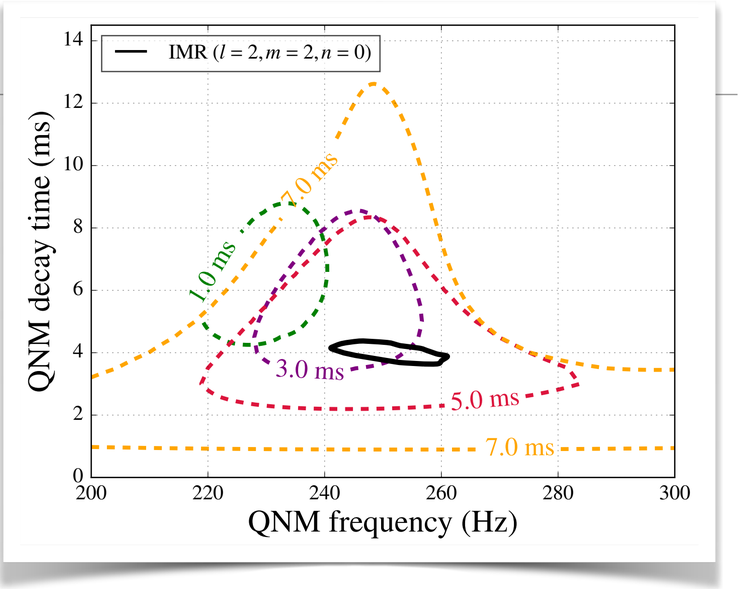
- Introduces modification in the GW phase that enters @2PN
- Non-Kerr compact objects will in general have $\kappa \neq 1$, e.g. neutron-/boson-/grava- stars, etc.
- We measure a combination of κ to be consistent with the Kerr BH value





Ringdown analysis

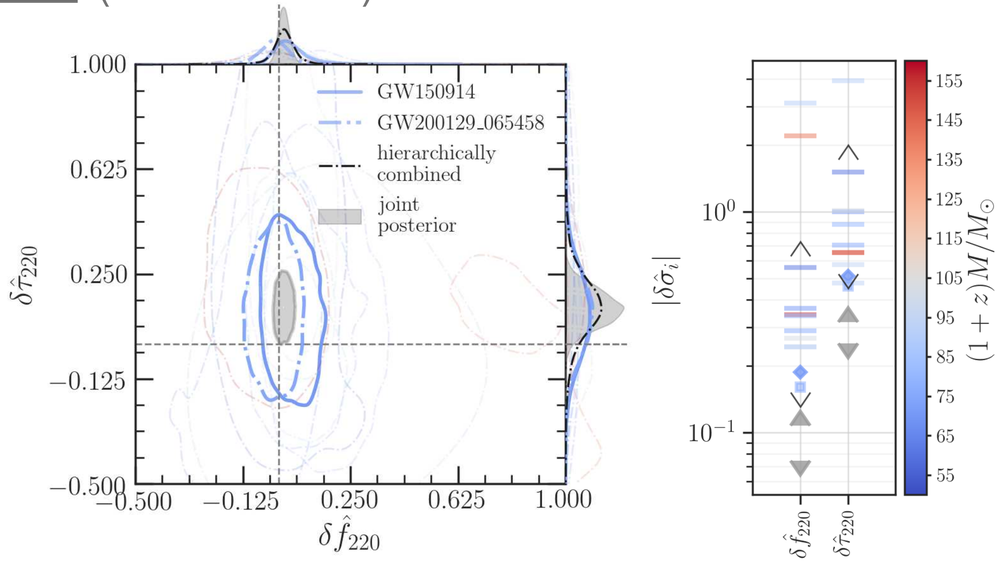
[LVC PRL 116, 221101 (2016)]



- "Is there clear evidence of a quasi-normal ringdown?"
- Final BH radiates damped sinusoids

$$h_+(t) - ih_\times(t) = \sum_{\ell=2}^{+\infty} \sum_{m=-\ell}^{\ell} \sum_{n=0}^{+\infty} \mathcal{A}_{\ell mn} \exp\left[-\frac{t-t_0}{(1+z)\tau_{\ell mn}}\right] \exp\left[-\frac{2\pi i f_{\ell mn}(t-t_0)}{1+z}\right] {}_{-2}S_{\ell mn}(\theta, \phi, \chi_{\rm f}),$$

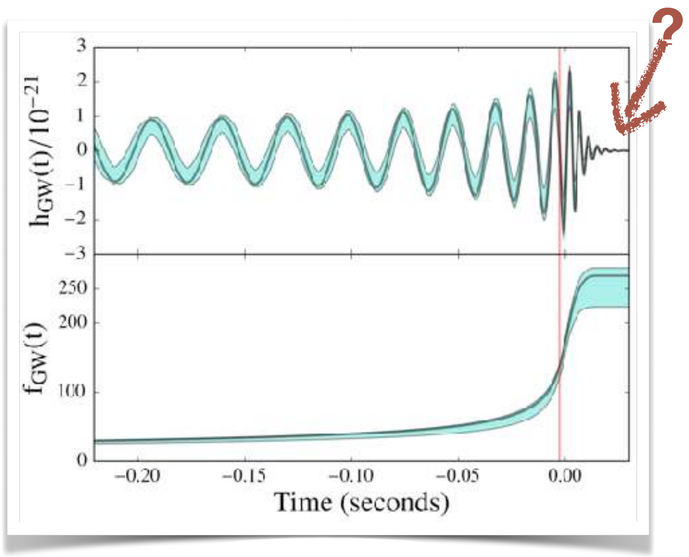
- Estimate f & τ of **dominant** (l=2 m=2) mode

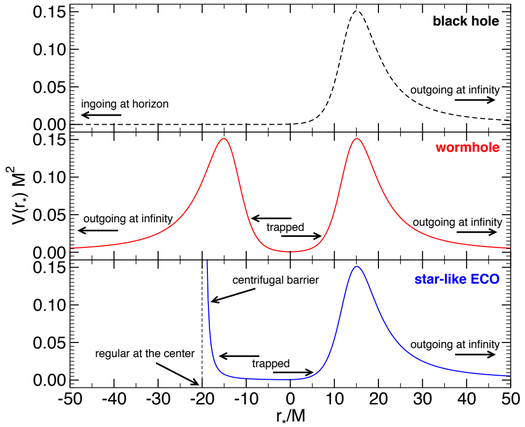


LVK (2021) [arXiv:2112.06861]

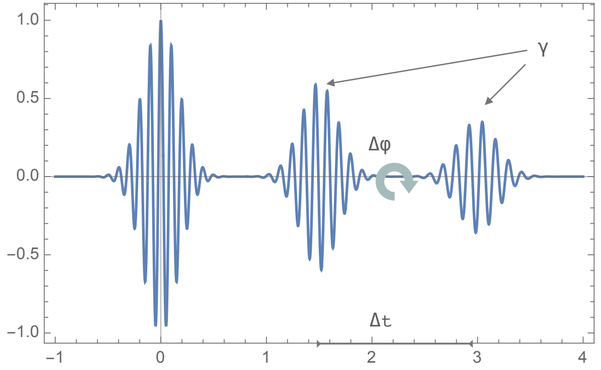
Is there anything beyond ringdown?

Searching for Exotic Compact Objects



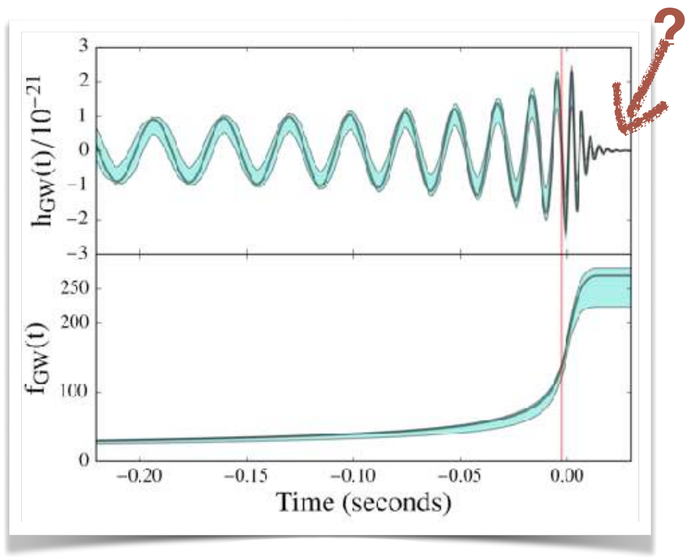


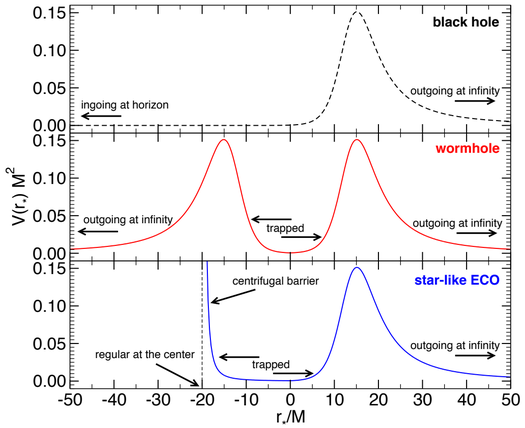
[Cardoso+ 2017]
[Tsang, MA+ arXiv:1804.04877]
[Tsang, MA+ arXiv:1906.11168]
[Lo+ arXiv:1811.07431]



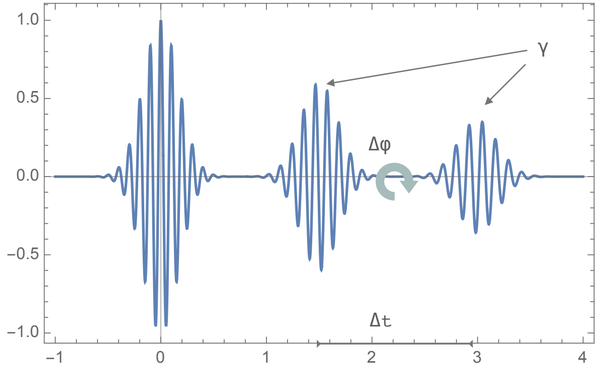
Is there anything beyond ringdown?

Searching for Exotic Compact Objects





[Cardoso+ 2017]
[Tsang, MA+ arXiv:1804.04877]
[Tsang, MA+ arXiv:1906.11168]
[Lo+ arXiv:1811.07431]



LVC PRD **103**, 122002 (2021)
[arXiv:2010.14529]

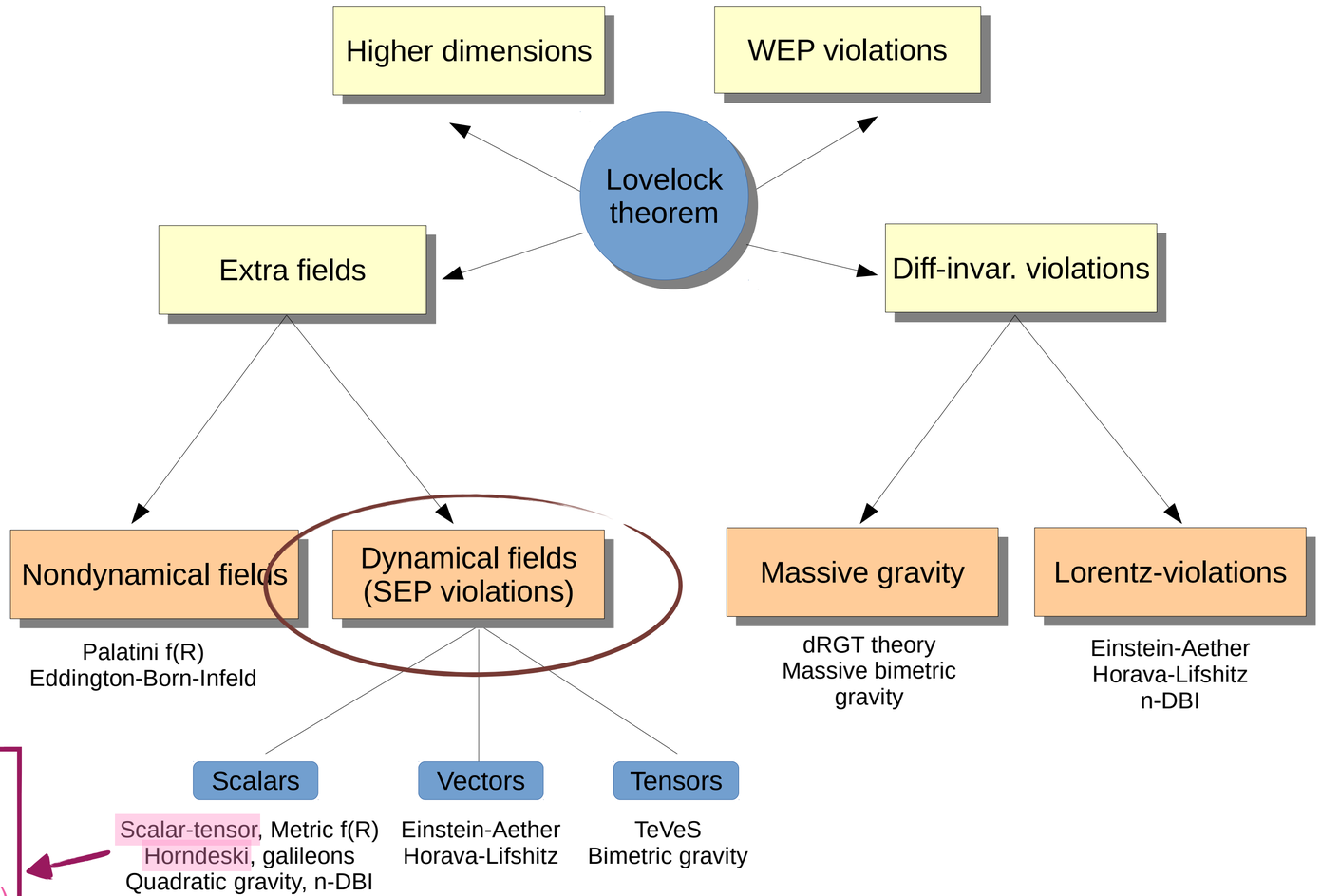
Event	$\log_{10}\mathcal{B}^{\rm IMRE}_{\rm IMR}$	Event	$\log_{10}\mathcal{B}^{\rm IMRE}_{\rm IMR}$
GW150914	−0.57	GW170809	−0.22
GW151226	−0.08	GW170814	−0.49
GW170104	−0.53	GW170818	−0.62
GW170608	−0.44	GW170823	−0.34
GW190408_181802	−0.93	GW190706_222641	−0.10
GW190412	−1.30	GW190707_093326	0.08
GW190421_213856	−0.11	GW190708_232457	−0.87
GW190503_185404	−0.36	GW190720_000836	−0.45
GW190512_180714	−0.56	GW190727_060333	0.01
GW190513_205428	−0.03	GW190728_064510	0.01
GW190517_055101	0.16	GW190828_063405	0.10
GW190519_153544	−0.10	GW190828_065509	−0.01
GW190521	−1.82	GW190910_112807	−0.22
GW190521_074359	−0.72	GW190915_235702	0.17
GW190602_175927	0.13	GW190924_021846	−0.03
GW190630_185205	0.08		

LVK (2021)
[arXiv:2112.06861]

Event	p-value
GW191109_010717	0.35
GW191129_134029	0.35
GW191204_171526	0.37
GW191215_223052	0.23
GW191216_213338	0.88
GW191222_033537	0.89
GW200115_042309	0.44
GW200129_065458	0.33
GW200202_154313	0.43
GW200208_130117	0.24
GW200219_094415	0.18
GW200224_222234	0.59
GW200225_060421	0.69
GW200311_115853	0.42
GW200316_215756	0.27

Towards theory-specific tests

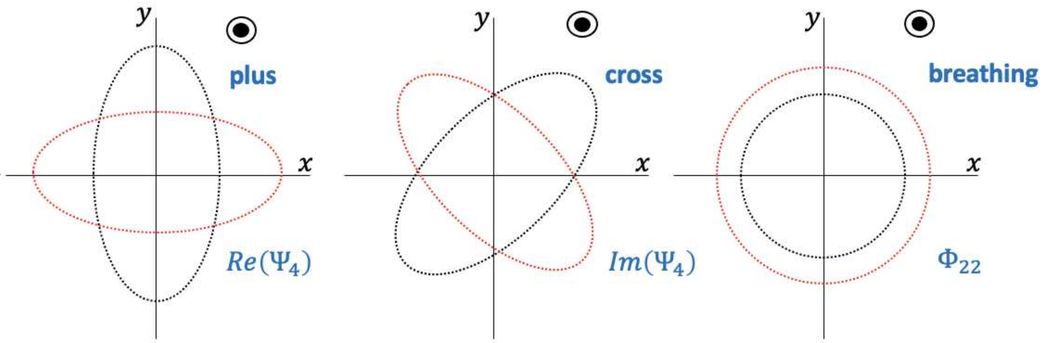
Classical paths away from GR



- Well posed
- NR simulations possible
- Analytical corrections (LO+)

[Berti+ CQG 32, 243001 (2015)]

Scenario I: Ringdown with extra fields



- GR model: superposition of Kerr QNMs

- Einstein-scalar-Gauss-Bonnet is a promising alternative to GR (Horndeski, well posed etc)

$$S = \frac{c^4}{16\pi G}\int d^4x\sqrt{-g}\left[R - \tfrac{1}{2}(\nabla\varphi)^2 + \beta_0\varphi\mathcal{R}_{\rm GB}\right] + S_M\left[\Psi, (1+a_0\varphi)\, g_{\mu\nu}\right]$$

$$G^{(0)}_{\mu\nu} = 0,$$

$$\Box^{(0)}\varphi^{(1)} = -\mathcal{R}^{(0)}_{GB}\beta_0$$

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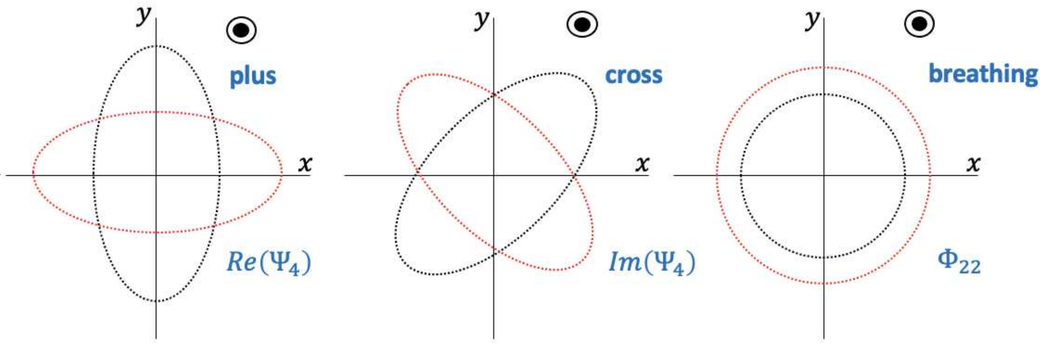

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$$G^{(0)}_{\mu\nu} = 0,$$

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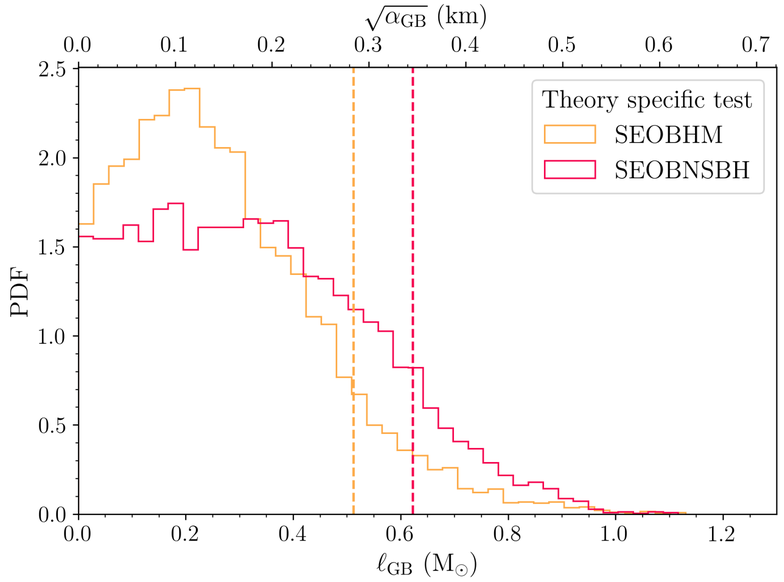
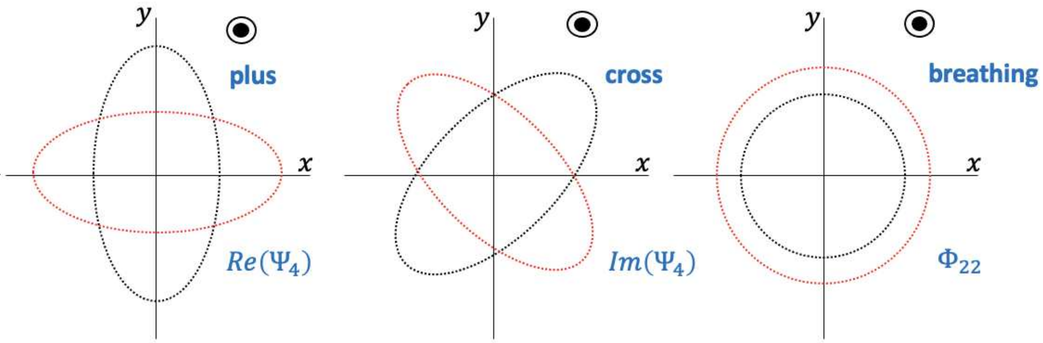
constrained by low-mass CBC inspiral (LO @ -1PN)



[LVK (2024) arXiv:2404.04248]
[Sänger et al. (2024) arXiv:2406.03568]

Scenario I: Ringdown with extra fields



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$$G^{(0)}_{\mu\nu} = 0,$$

$$\Box^{(0)}\varphi^{(1)} = -\mathcal{R}^{(0)}_{GB}\beta_0$$

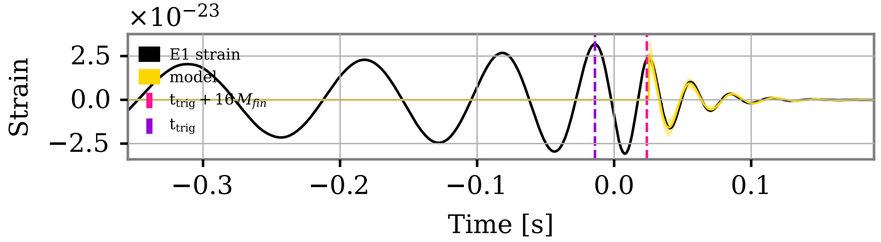
- Decoupling limit -> GR drives the dynamics: Kerr BH background + dynamical scalar

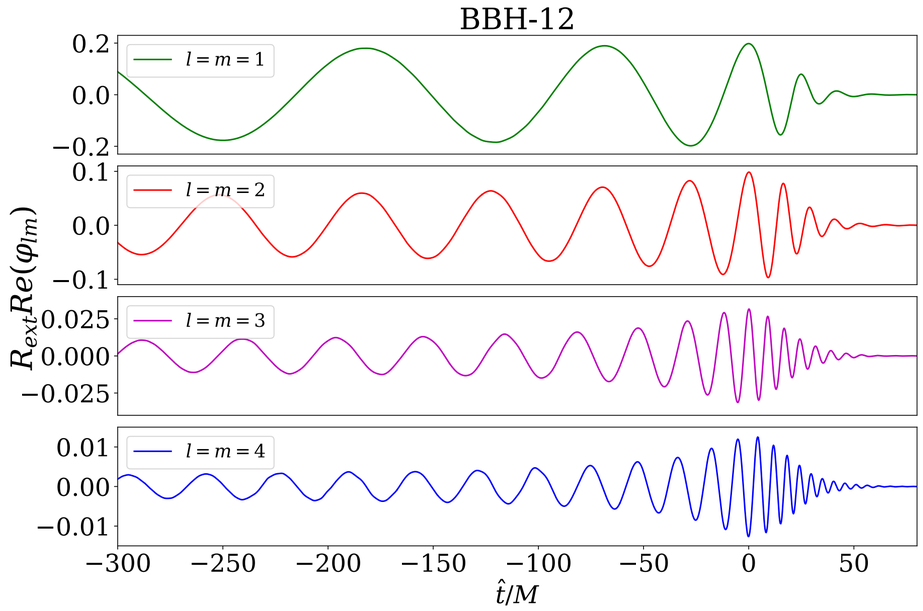
  [Okounkova+ (2019), Witek+ (2019)]

- NR simulations of BBH in EsGB

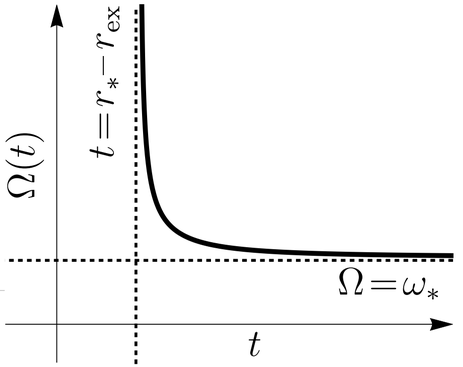
- Remnant BH will also “ring" in **scalar QNMs**

[Tamara Evstafyeva, MA, J.Ripley arXiv:2212.11359]



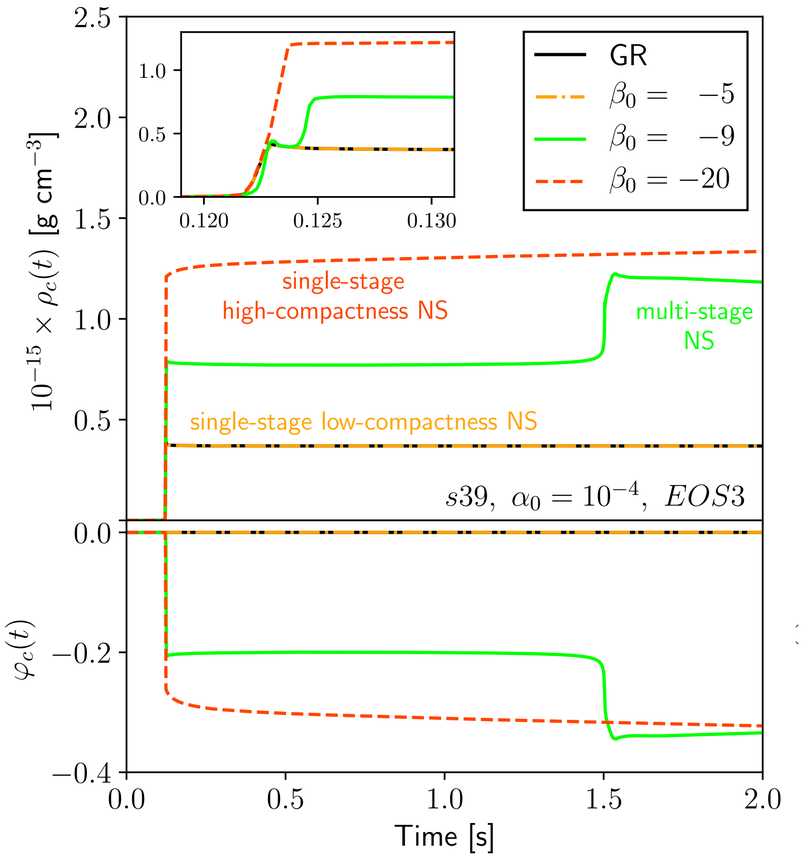


Scenario II: SNe in Massive Scalar-Tensor



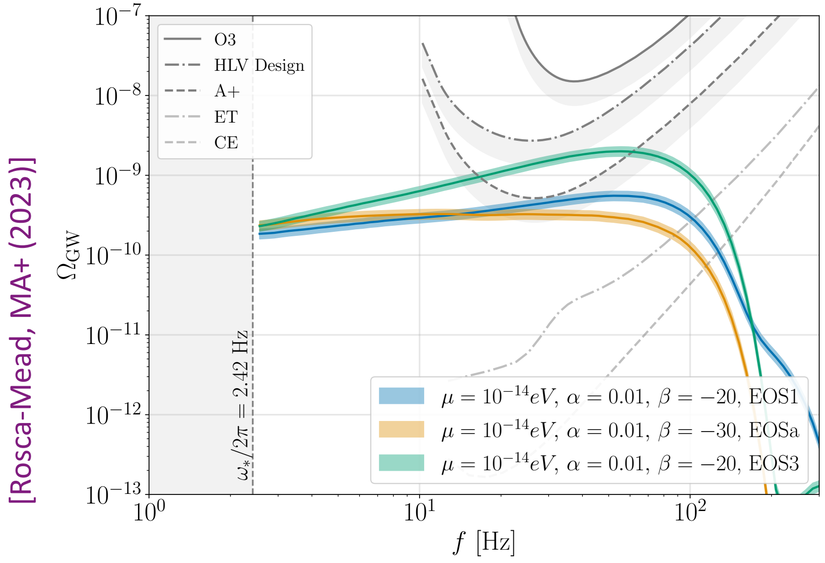
$$S = \int \mathrm{d}x^4\sqrt{-g}\left[\frac{F(\phi)}{16\pi}R - \frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - W(\phi)\right] + S_M\left[\psi_m, g_{\mu\nu}\right]$$

- General Features
  - Dispersion -> **inverse-chirp**!
  - Long-lived, quasi-monochromatic
  - Scalar polarisation
  - Sourced even in sph. symmetry
- Hyper-scalarisation during stellar collapse



Scalar SGWB

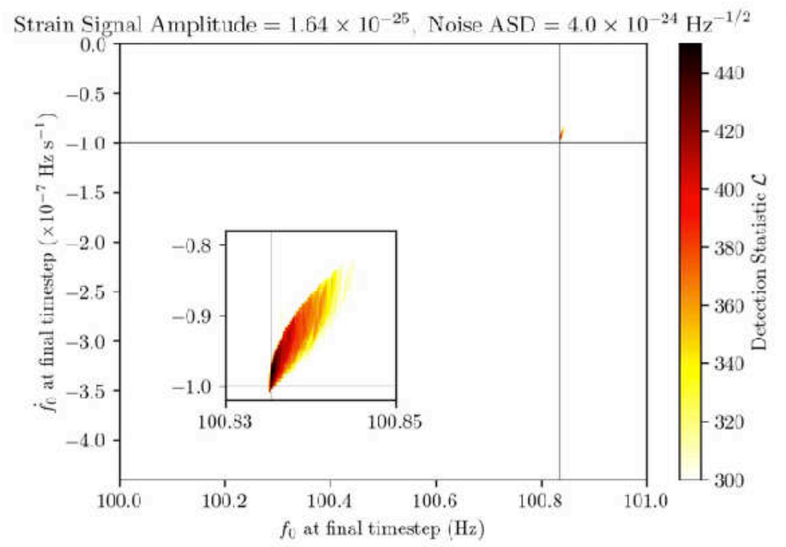
- Scalar SGWB from population of CCSNe
- Low-freq cutoff: $f_* = \mu/\hbar = 2.42 \left(\frac{\mu}{10^{-14}\mathrm{eV}} \right) \mathrm{Hz}$
- Peak @ 60Hz up to O(10^{-9})



[Rosca-Mead, MA+ (2023)]

Continuous Waves

- Target known SNe (Cas A)
- Look for CW with slowly decreasing f
- Search using scalar antenna response



[Kwok & MA, in prep. 2025]

Scanario III: Binary Boson Stars in GR

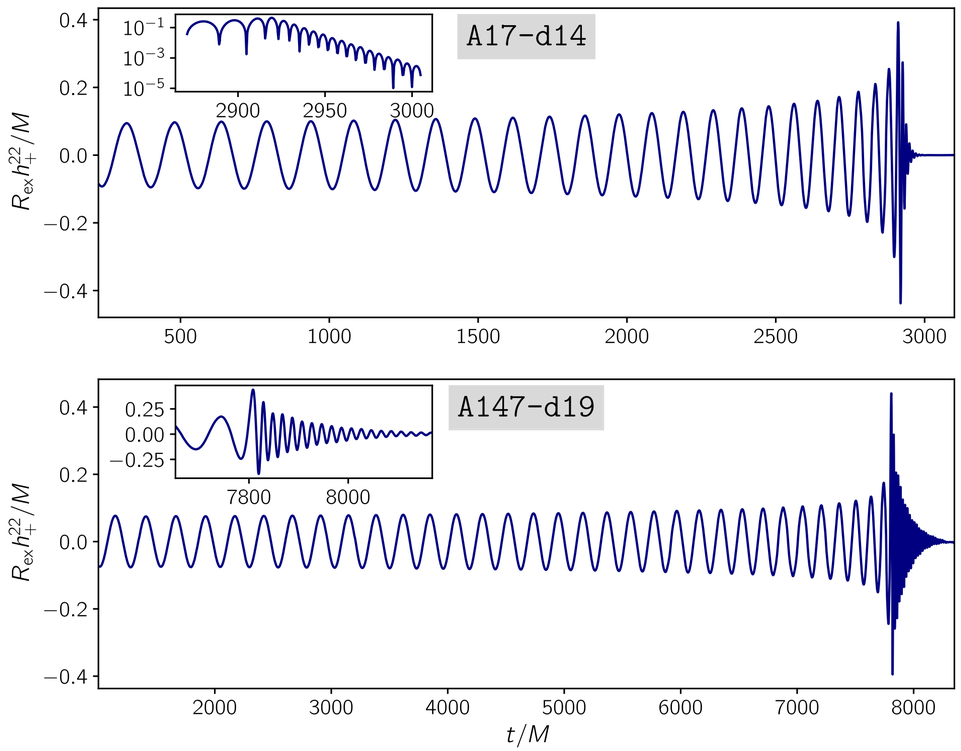
- Action of scalar field minimally coupled to gravity

$$S = \int \frac{\sqrt{-g}}{2} \left\{ \frac{R}{8\pi G} - \left[g^{\mu\nu} \nabla_\mu \bar{\varphi} \nabla_\nu \varphi + V(\varphi) \right] \right\} \mathrm{d}^4 x$$

- Solitonic boson star potential

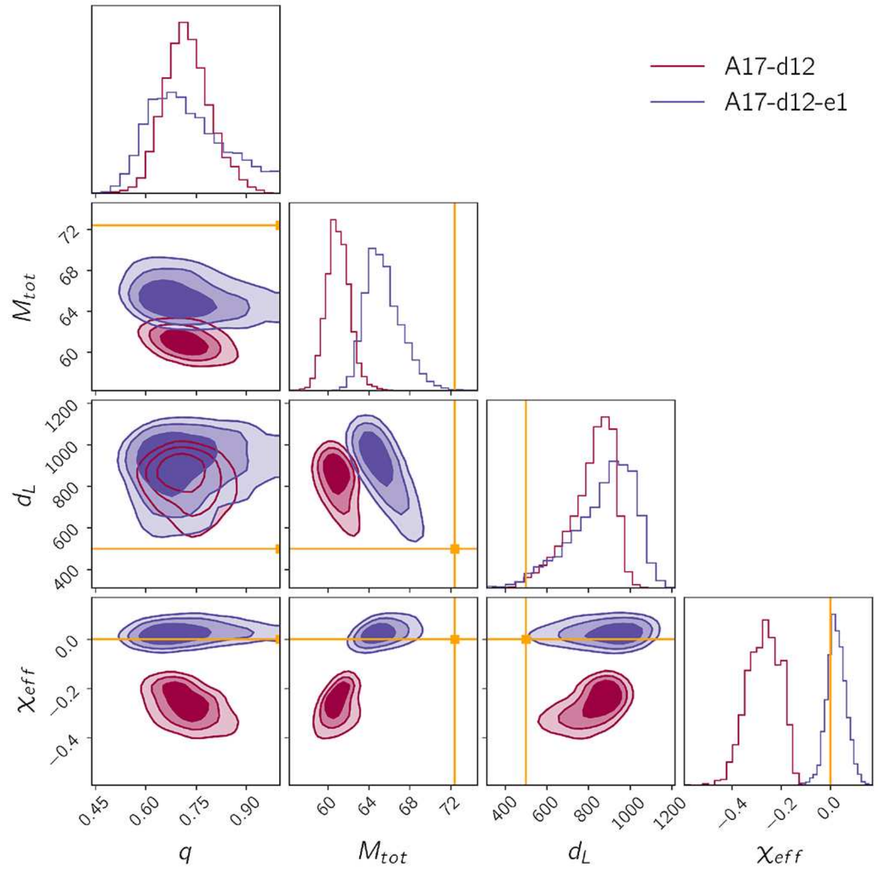
$$V_{\mathrm{sol}} = \mu^2 |\varphi|^2 (1 - 2|\varphi|^2/\sigma_0^2)^2$$

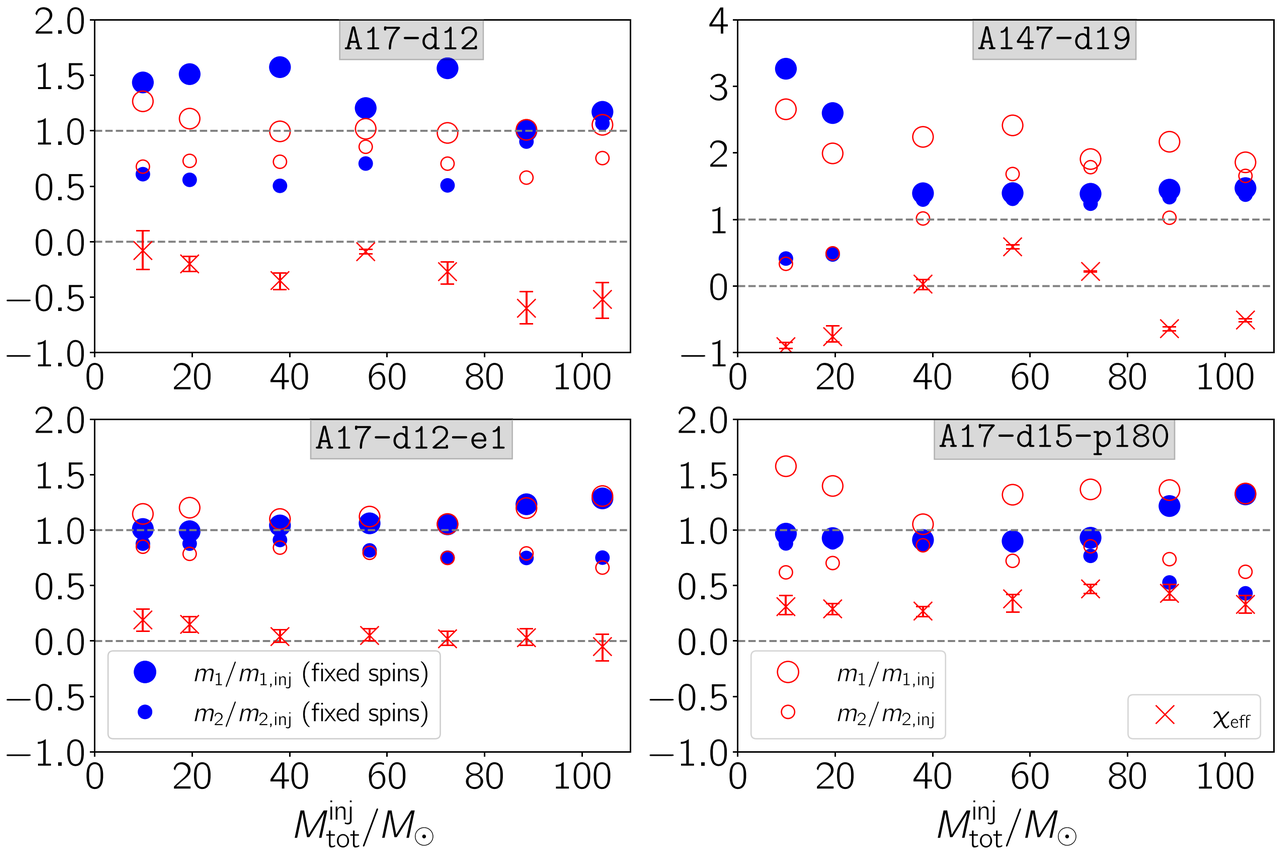
- μ controls the physical scale;
- range of compactnesses close to NS;
- non-zero tides.



Evstafyeva, Sperhake, Romero-Shaw, **MA** (2024) [arXiv:2406.02715]

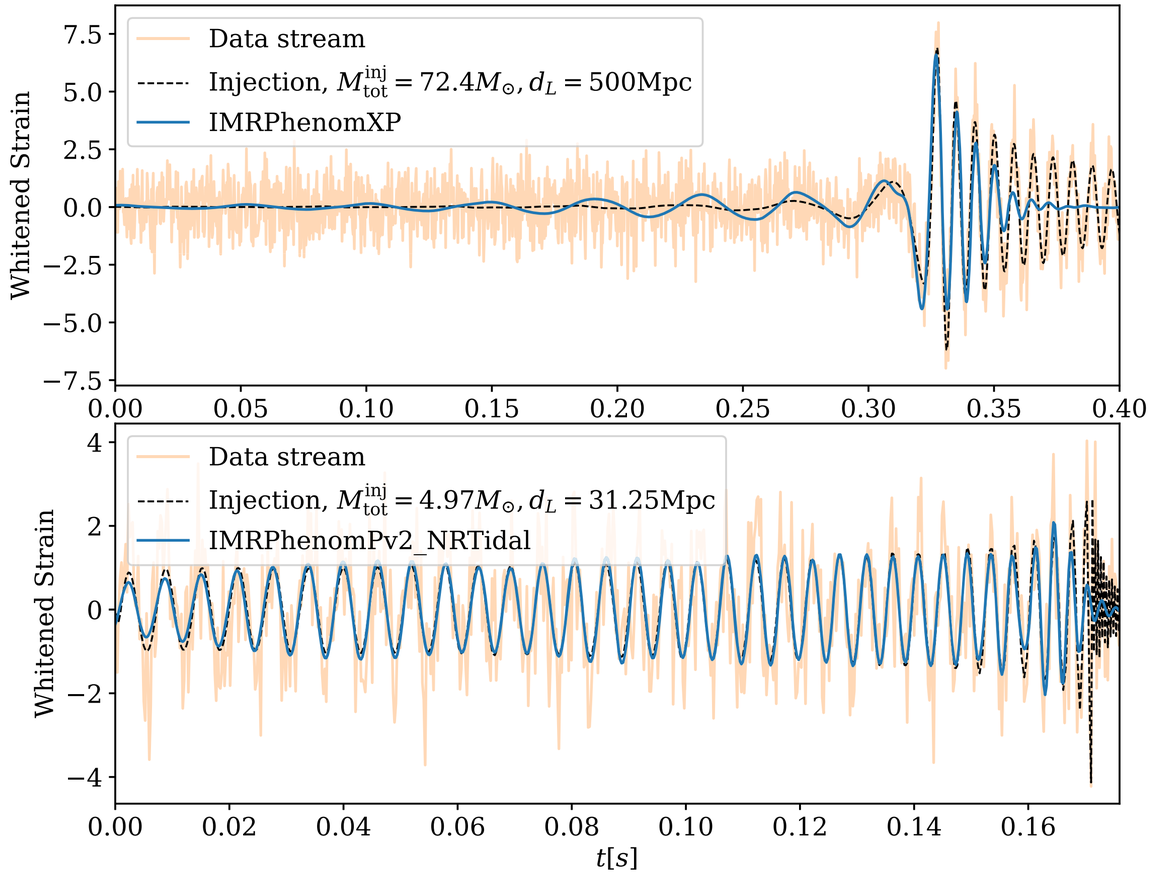
Typical biases in PE [with/without spins]





Evstafyeva, Sperhake, Romero-Shaw, **MA** (2024) [arXiv:2406.02715]

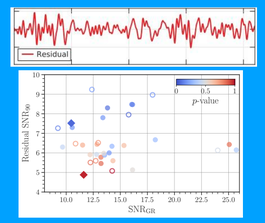
Recovery of a BBS signal



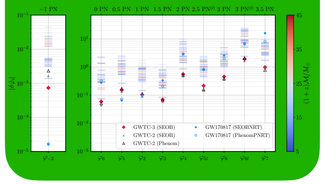
How will tests of GR respond to an accurate IMR signal in this scenario (or other alternatives)?

Where we are

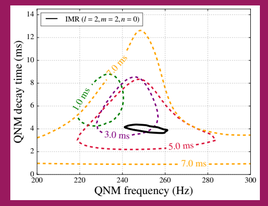
RESIDUALS



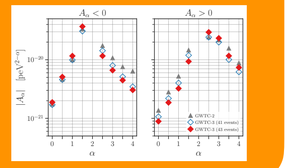
PHASE COEFFICIENTS



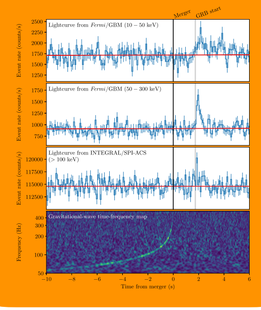
RINGDOWN



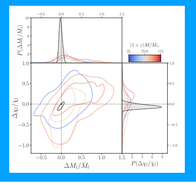
MODIFIED DISPERSION



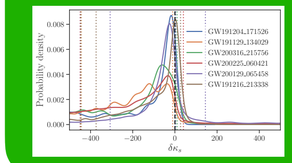
SPEED OF GRAVITY



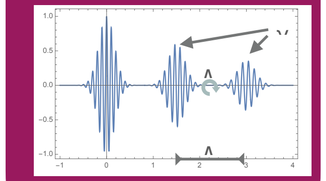
I-MR CONSISTENCY



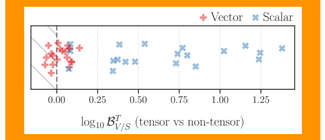
BH SPIN QUADRUPOLE



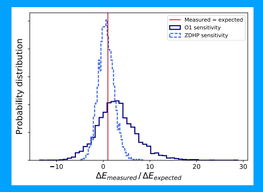
ECHOES



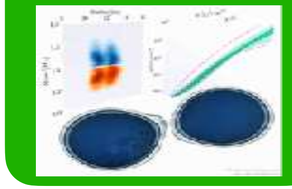
ALTERNATIVE POLARIZATIONS



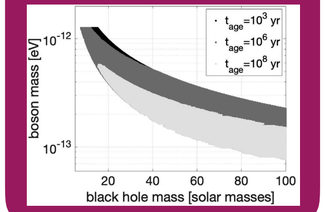
AREA THM



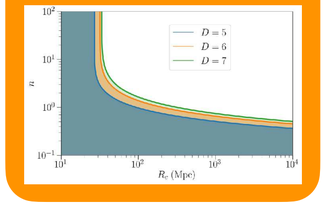
TIDES & RESONANCES

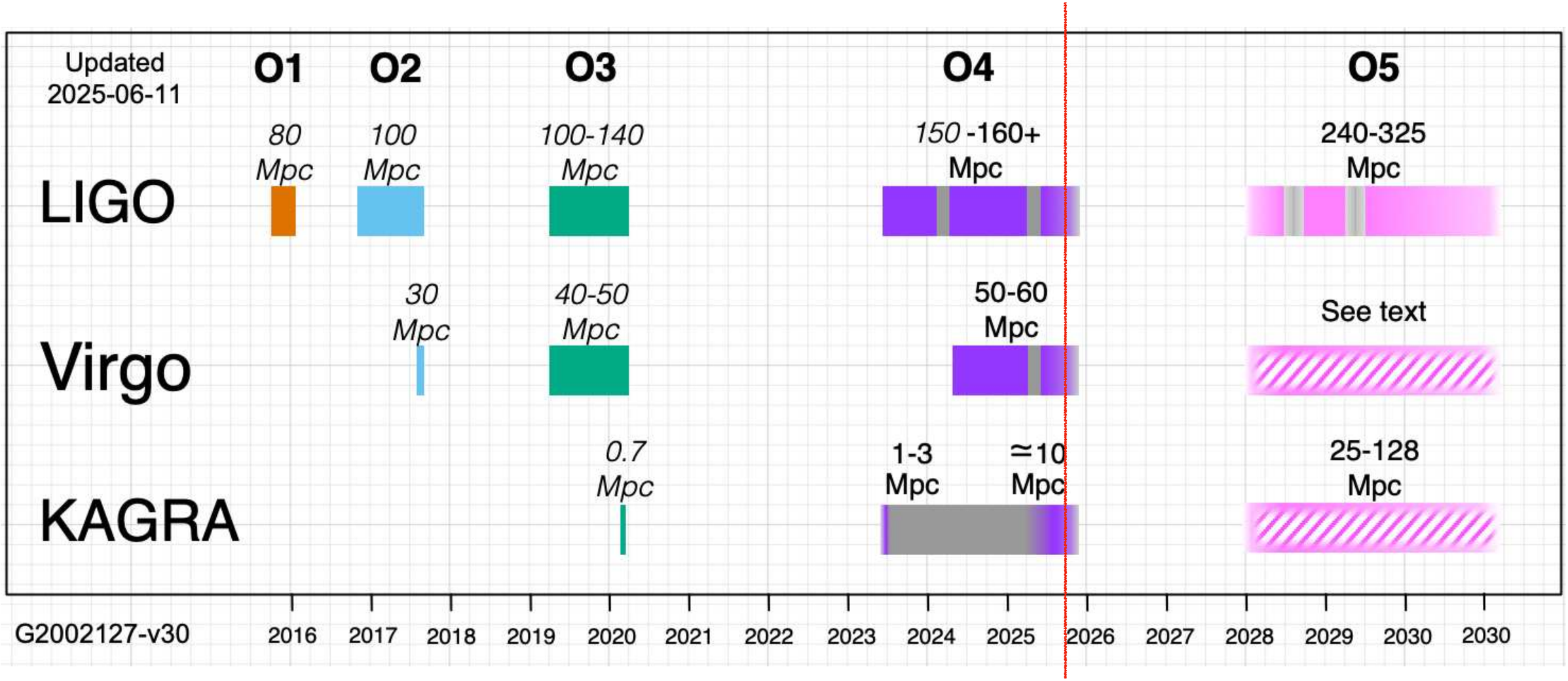


SUPERRADIANCE



EXTRA DIMENSIONS

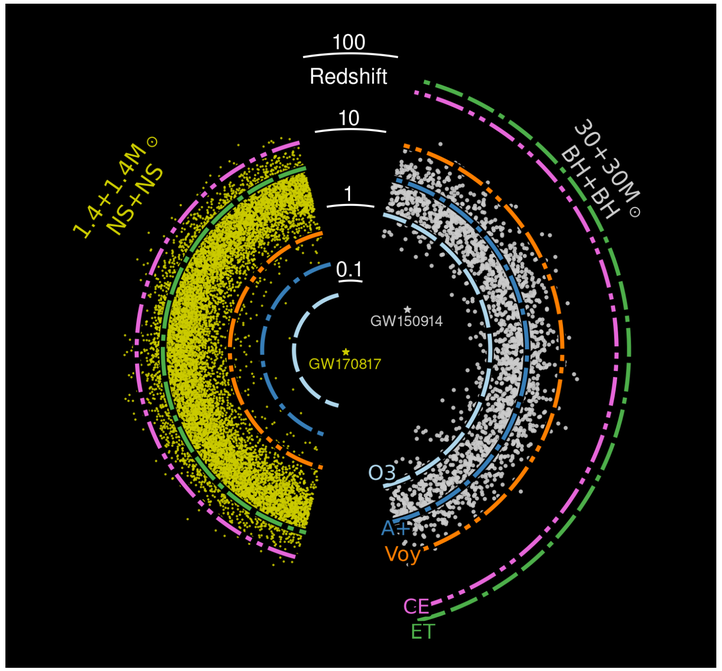


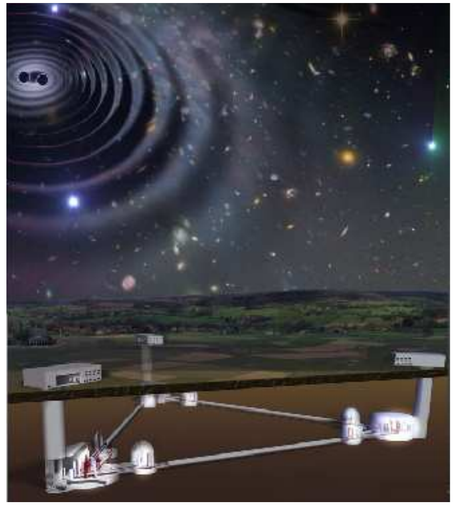


https://observing.docs.ligo.org/plan/

Where we are headed

- Much higher event **rates**
- High-**SNR** measurements
- Both in **inspiral** and **ringdown**/postmerger
- Larger **distances** (propagation, cosmology, astro-pop)
- Expand **frequency range** to lower and higher freqs
- Sensitivity to **non-compact-binary** observations



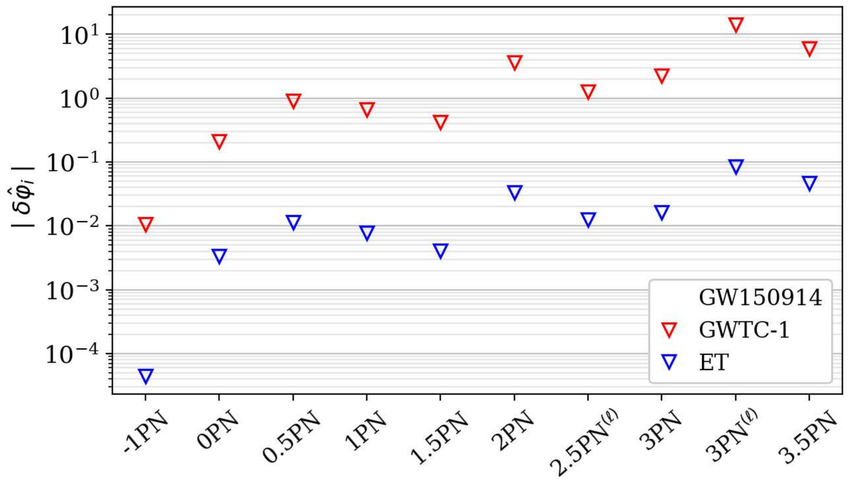


- ET Science Case
- CE Horizon Study

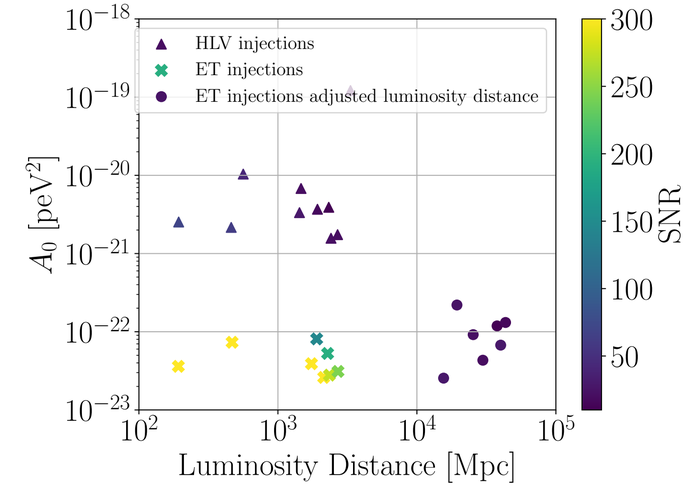
Einstein Telescope projections

The Science of the Einstein Telescope [arXiv:2503.12263]

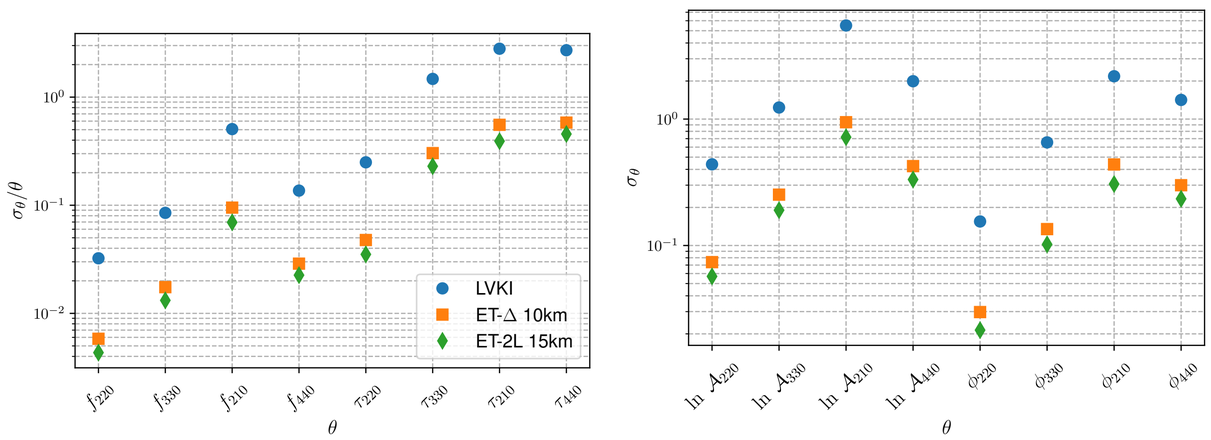
Inspiral dynamics



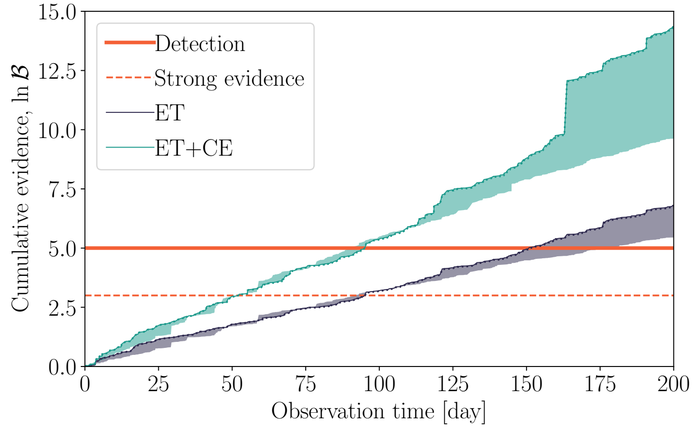
GW Dispersion



Ringdown



Memory

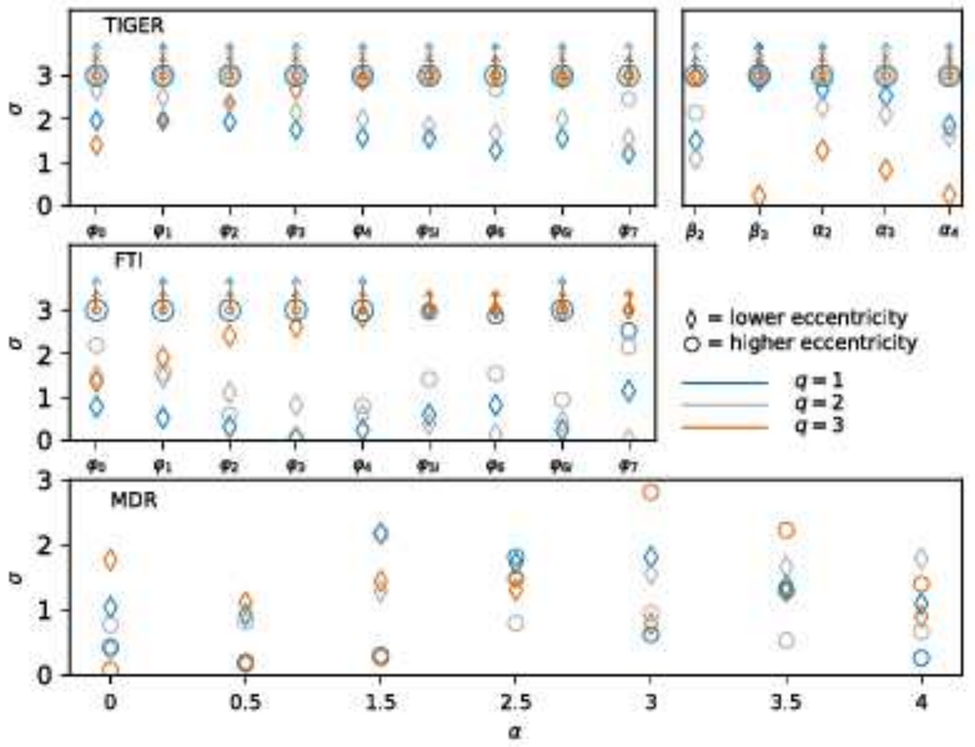


(b) Spin memory detection

The trouble with waveforms

Sources of systematics

- Truncation of known physics (PN, HM, spin-order)
- Missing physics (tides, precession, eccentricity)



[Naraya, Johnson-McDaniel, Gupta arXiv:2306.04068]

Sources of systematics

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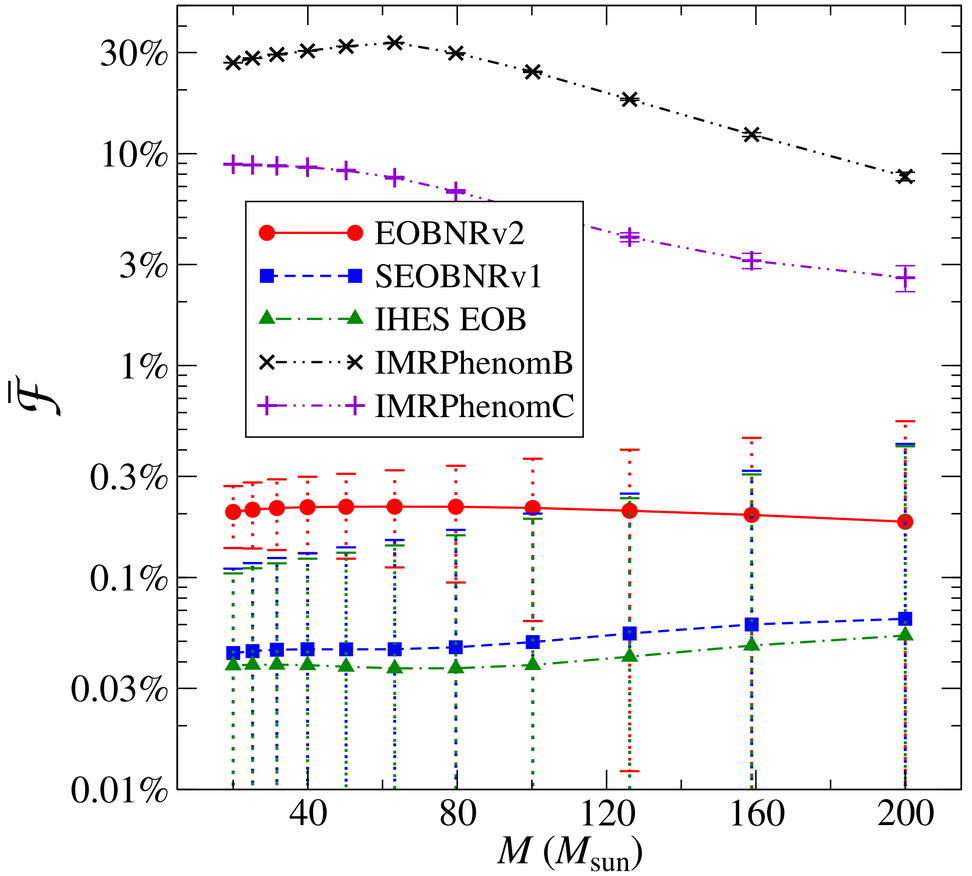
- Mismatches in NR-tuned regime

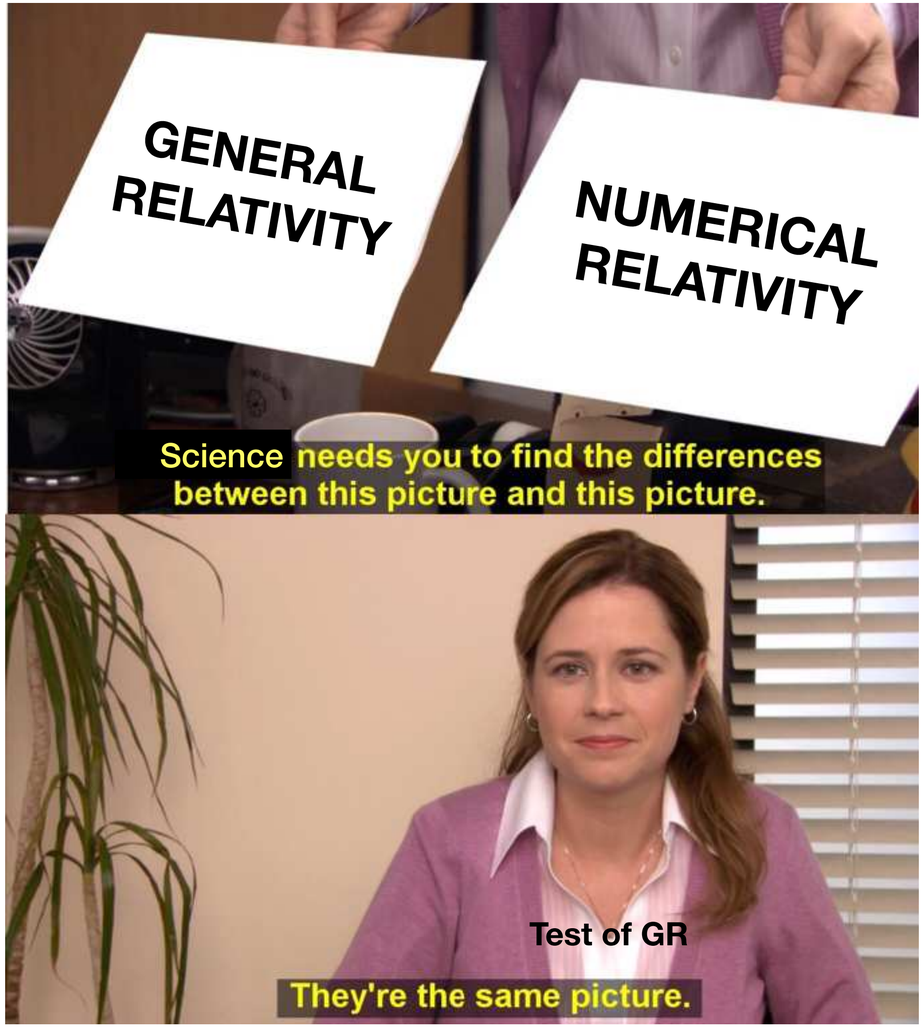
[Hinder+ (2014) CQG 31 025012]



Sources of systematics

- Truncation of known physics (PN, HM, spin-order)
- Missing physics (tides, precession, eccentricity)
- Mismatches in NR-tuned regime
- Numerical Relativity Errors

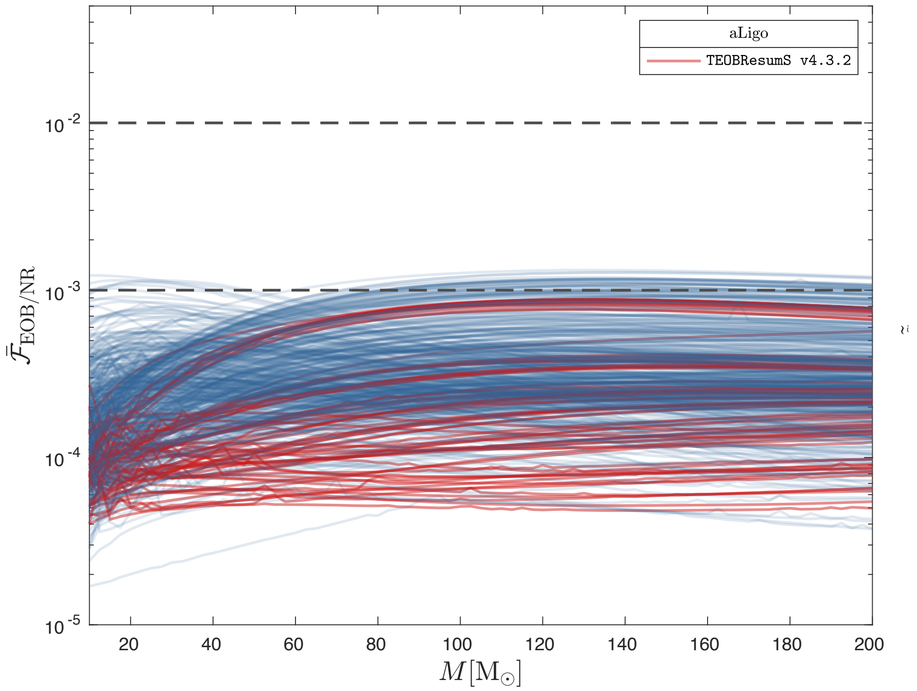
NR has errors?



NR has errors

[Hinder+ (2014) CQG 31 025012]

- Initial data
- Evolution
- Grid resolution
- Waveform extraction

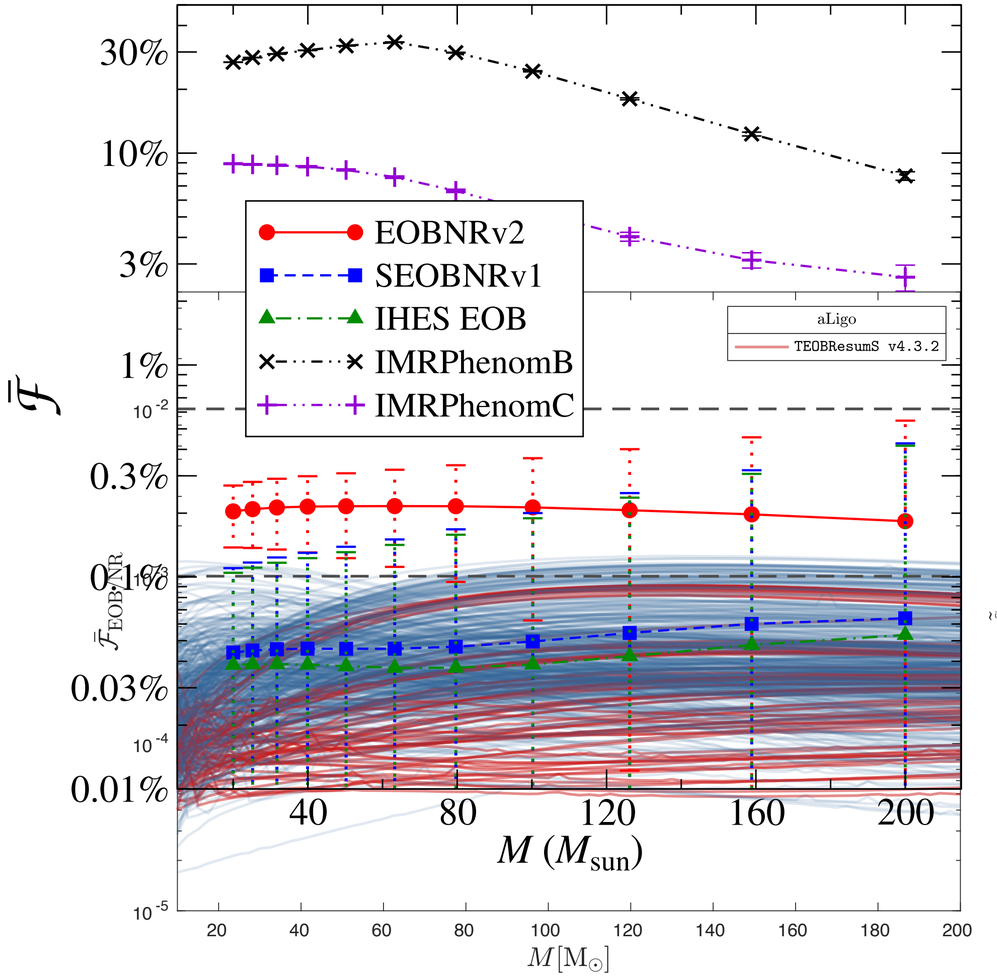


[Nagar+ arXiv:2304.09662]

NR has errors

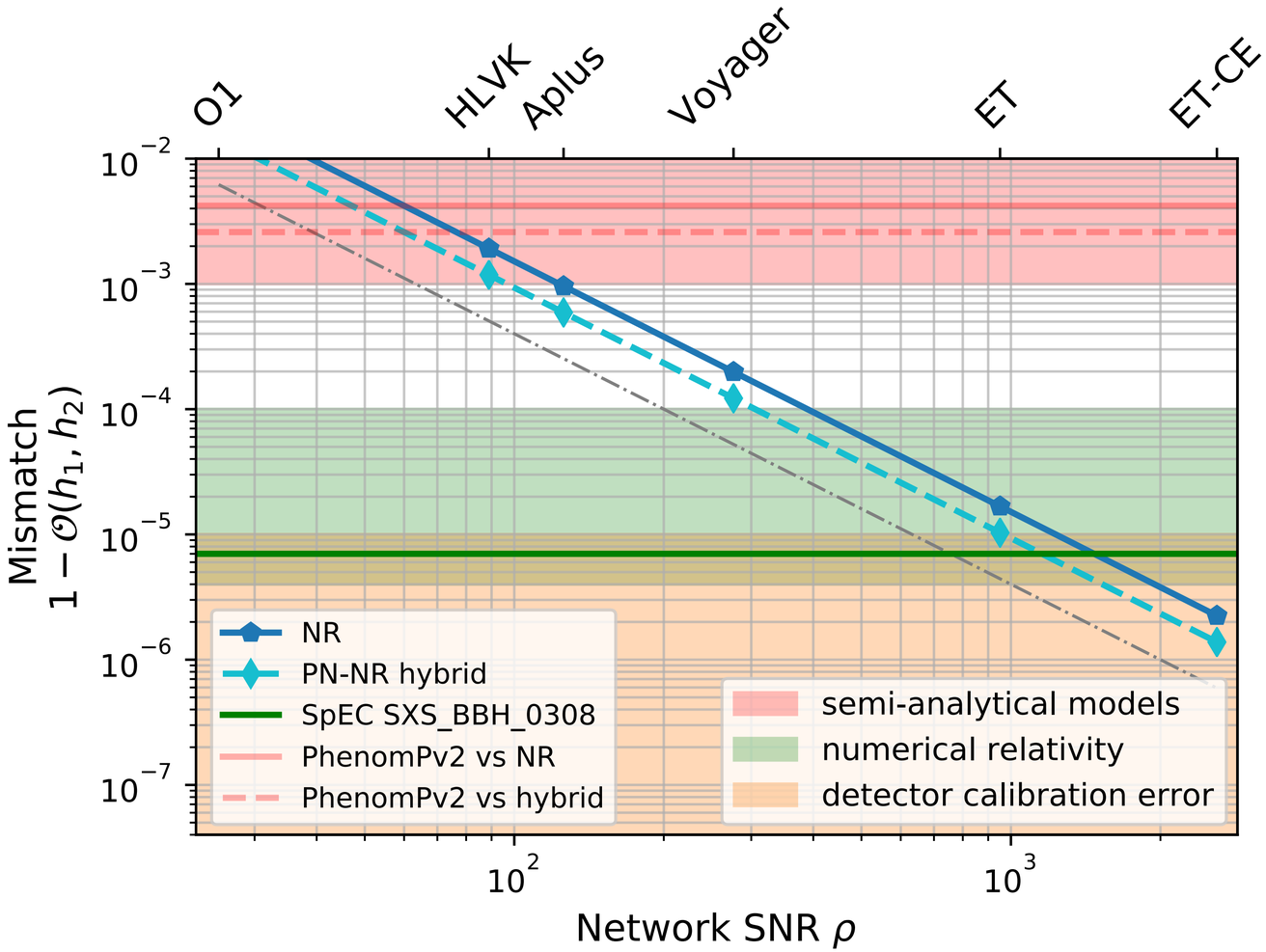
- Initial data
- Evolution
- Grid resolution
- Waveform extraction

[Hinder+ (2014) CQG 31 025012]



[Nagar+ arXiv:2304.09662]

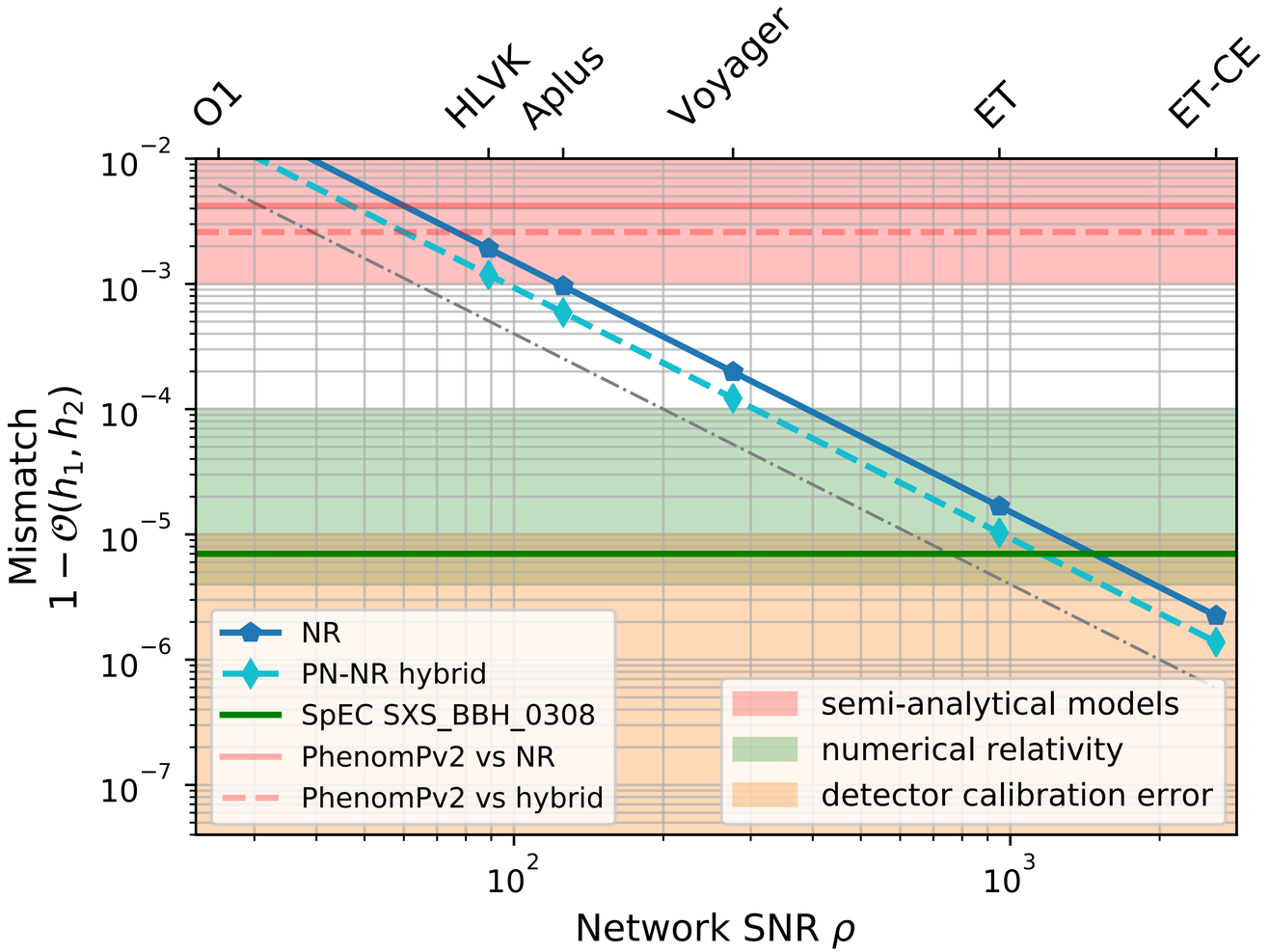
Waveform Accuracy Requirements



$$1-\mathscr{M} < \frac{1}{SNR^2}$$

[M.Pürrer & C.J.Haster arXiv:1912.10055]

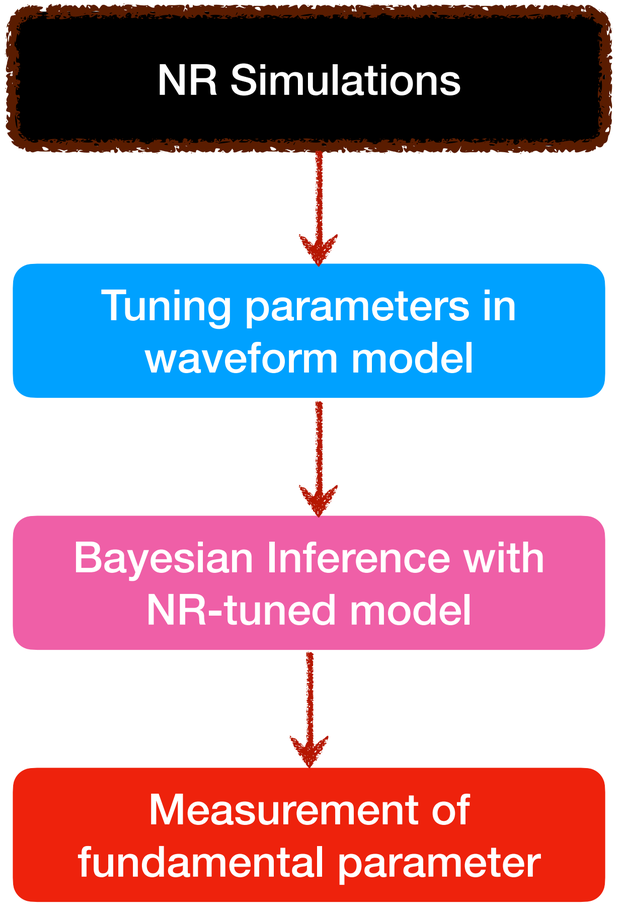
Waveform Accuracy Requirements



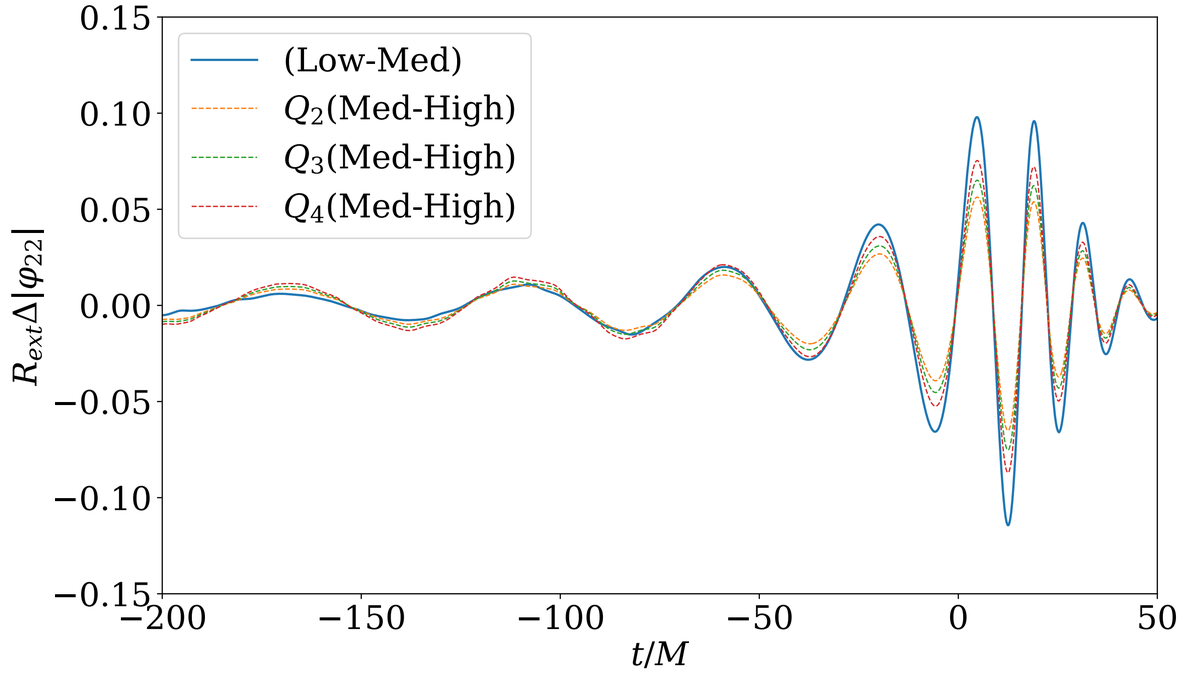
$$1-\mathscr{M} < \frac{1}{N_s\, SNR^2}$$

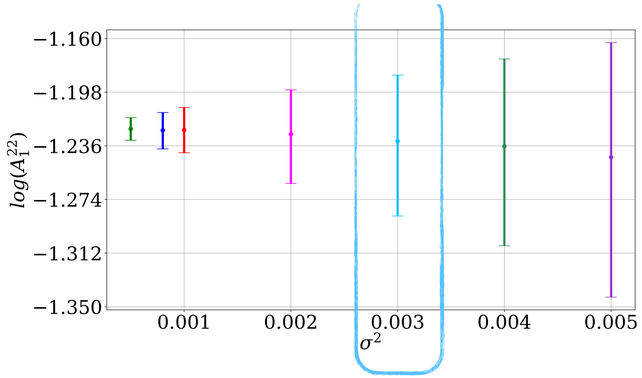
[M.Pürrer & C.J.Haster arXiv:1912.10055]

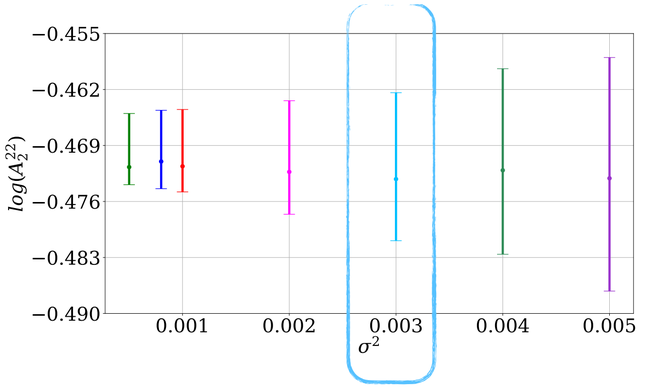
Propagation of Systematics



Measuring QNM Amplitudes

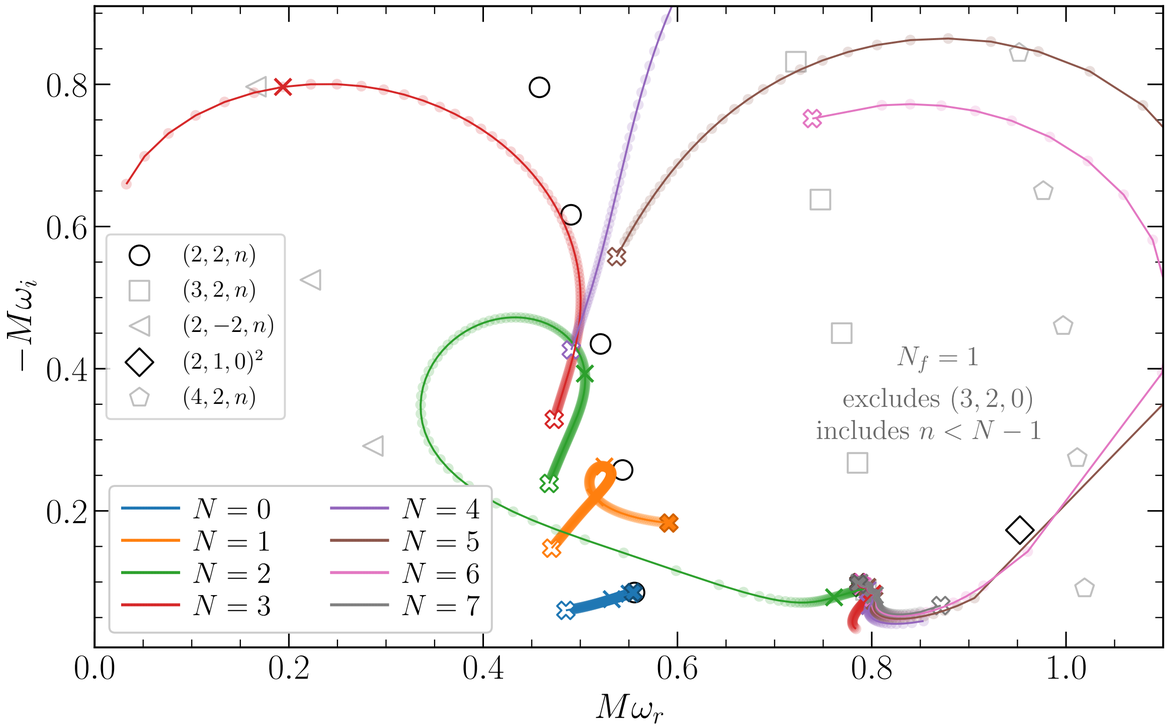






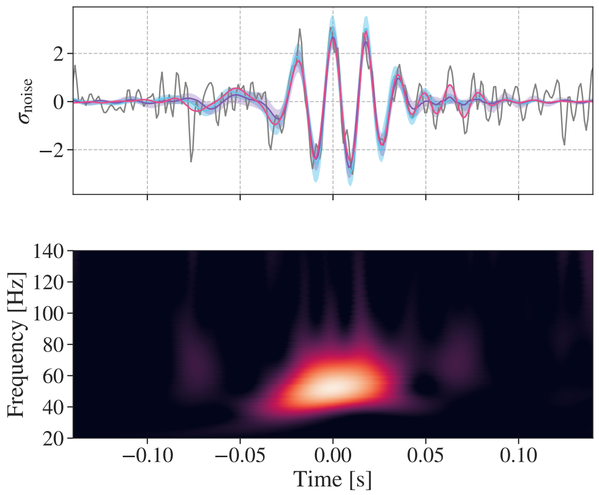
[Tamara Evstafyeva, MA, J.Ripley arXiv:2212.11359]

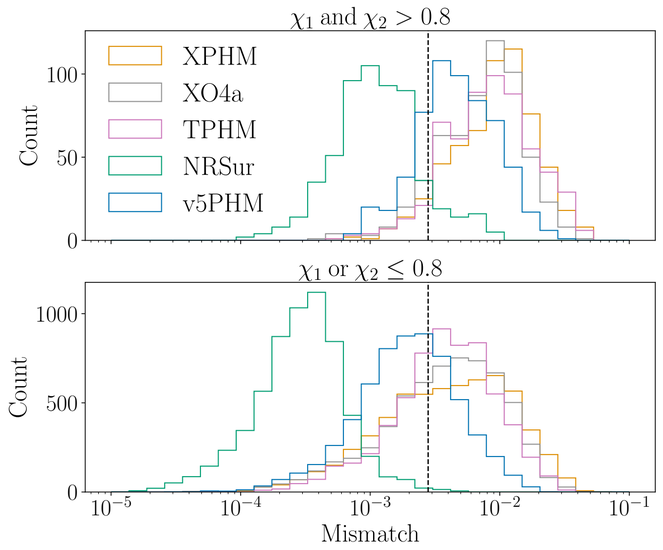
Fitting QNMs (+overtones)

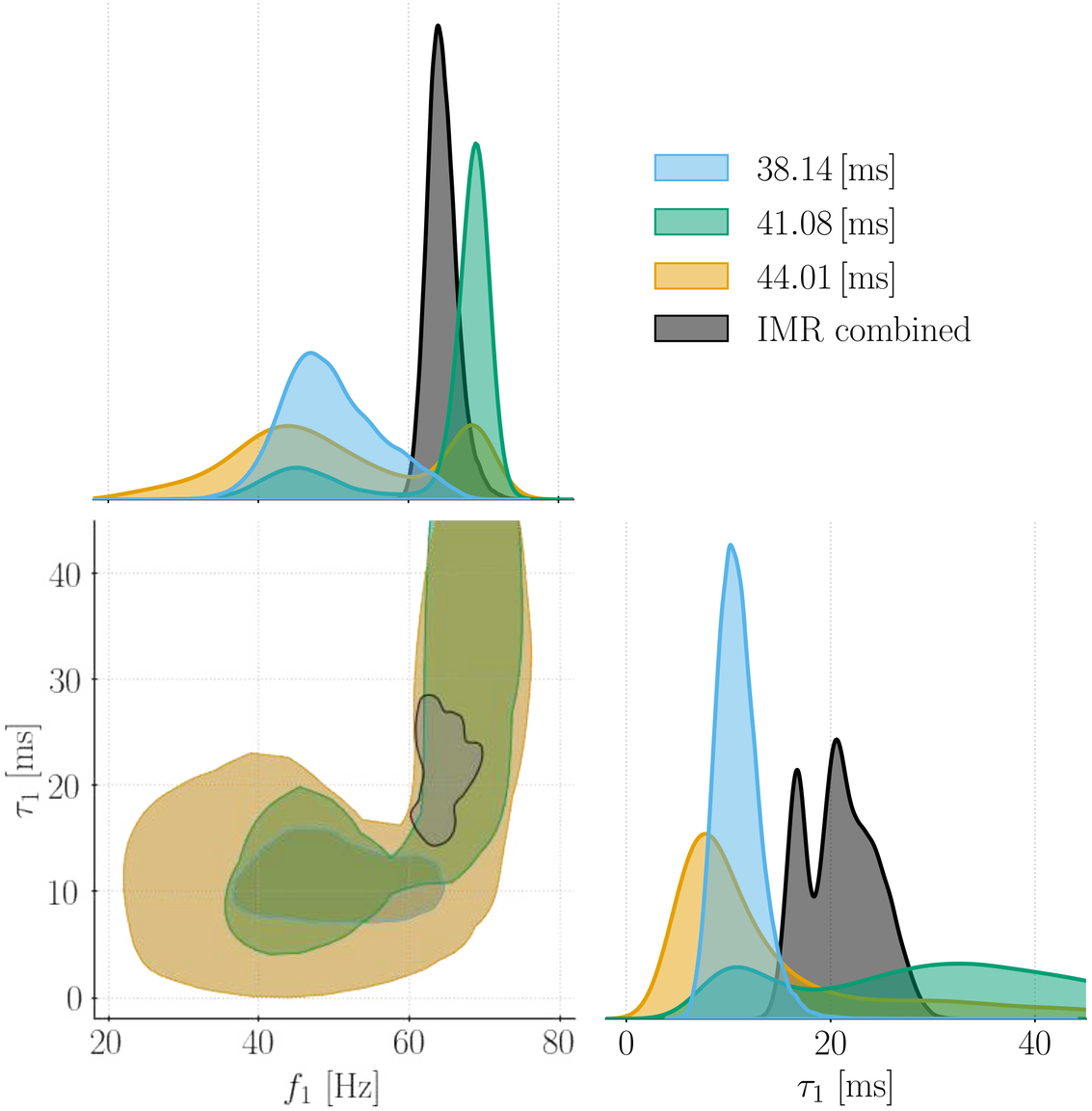


[Baibhav+ arXiv:2302.03050]

GW231123: a BBH with M ~ 190-265 $M_\odot$

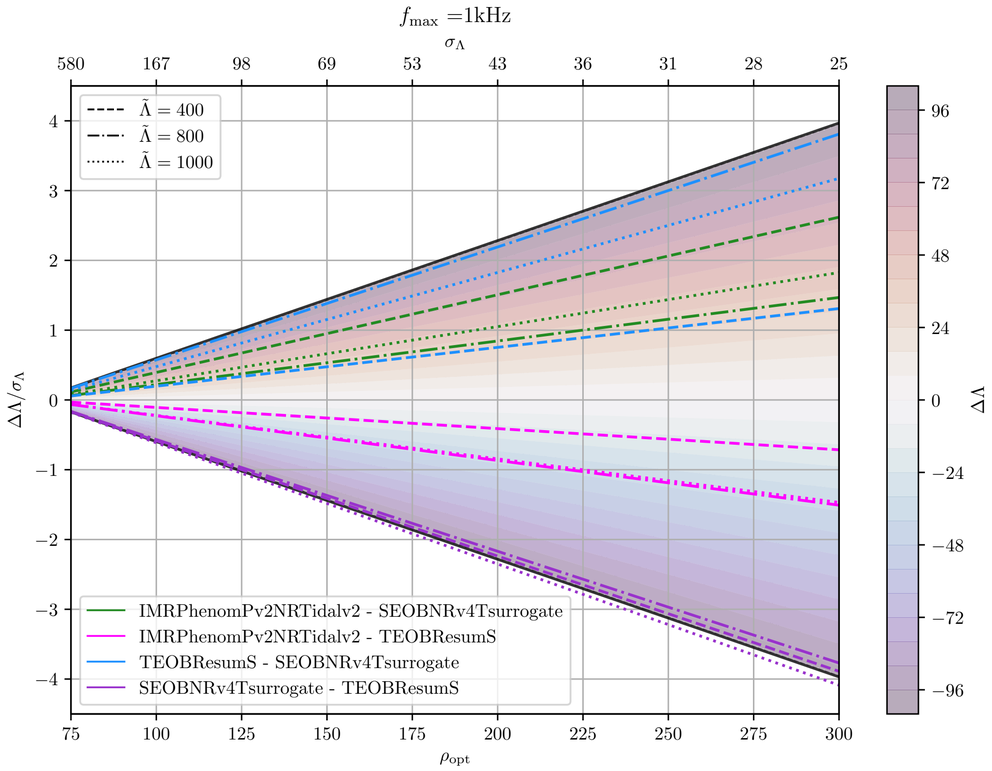






Control the uncertainties

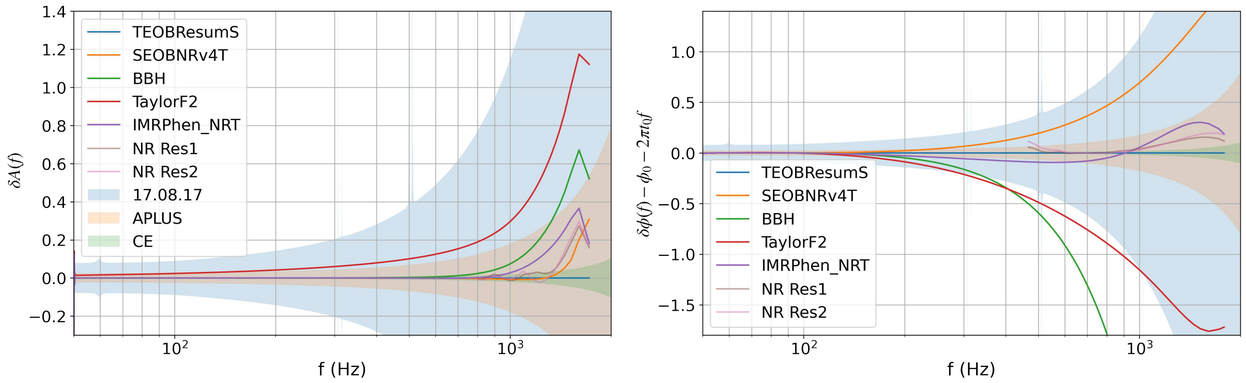
- Similar problem to BNS wfm systematics for tidal parameters



[Gamba+ arXiv:2009.08467]

- Quantifying systematic errors in sampled waveform model

$$\tilde{h}_{\rm true}(f) = \tilde{h}_{\rm model}(f)(1+\delta A(f))\exp(i\delta\phi(f))$$

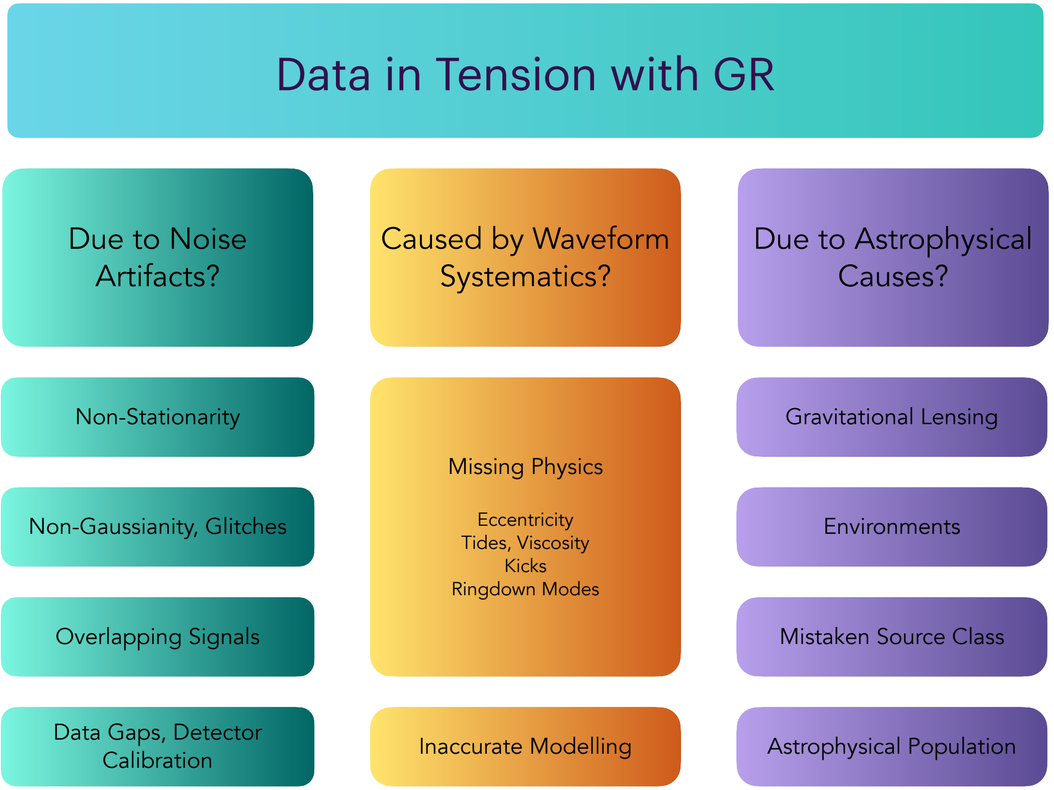


[J. Read arXiv:2301.06630]

[Pompili et al. arXiv:2410.16859]

[Bachhar et al. arXiv:2410.17168]

Possible sources of false positives



Cause	O4	A+	A#	XG
Non-Stationary Noise	✓	✓	✓	✓
Non-Gaussian Noise/Glitches	✓	✓	✓	✓
Overlapping Signals	✗	✗	✗	✓
Data Gaps	✗	✗	✗	✓
Detector Calibration	✗	✗	✗	✓
Eccentricity	✓	✓	✓	✓
Tidal Effects	✗	✓	✓	✓
Kick-induced Effects	✗	✗	✗	✓
Ringdown Modes	✓	✓	✓	✓
Precession and Higher-order Modes	✓	✓	✓	✓
Memory	✗	✗	✓	✓
Sub-optimal Waveform Calibration	✗	✗	✓	✓
Lensing	✗	✗	✗	✓
Environmental Effects	✗	✗	✗	✓
Source Misclassification	✓	✓	✓	✓
Astrophysical Population Assumptions	✓	✓	✓	✓

[Gupta+ arXiv:2405.02197]

Conclusions

- Probing the nature of gravity & compact objects with GWs is a very active field
- We keep pushing the bounds and exploring new phenomenology
- Alternative theories are being explored analytically + numerically; still a lot of work to be done
- Controlling waveform uncertainties all the way to NR level will be crucial for XG network
- Plenty of false-positive possibilities to consider before claiming a violation of GR